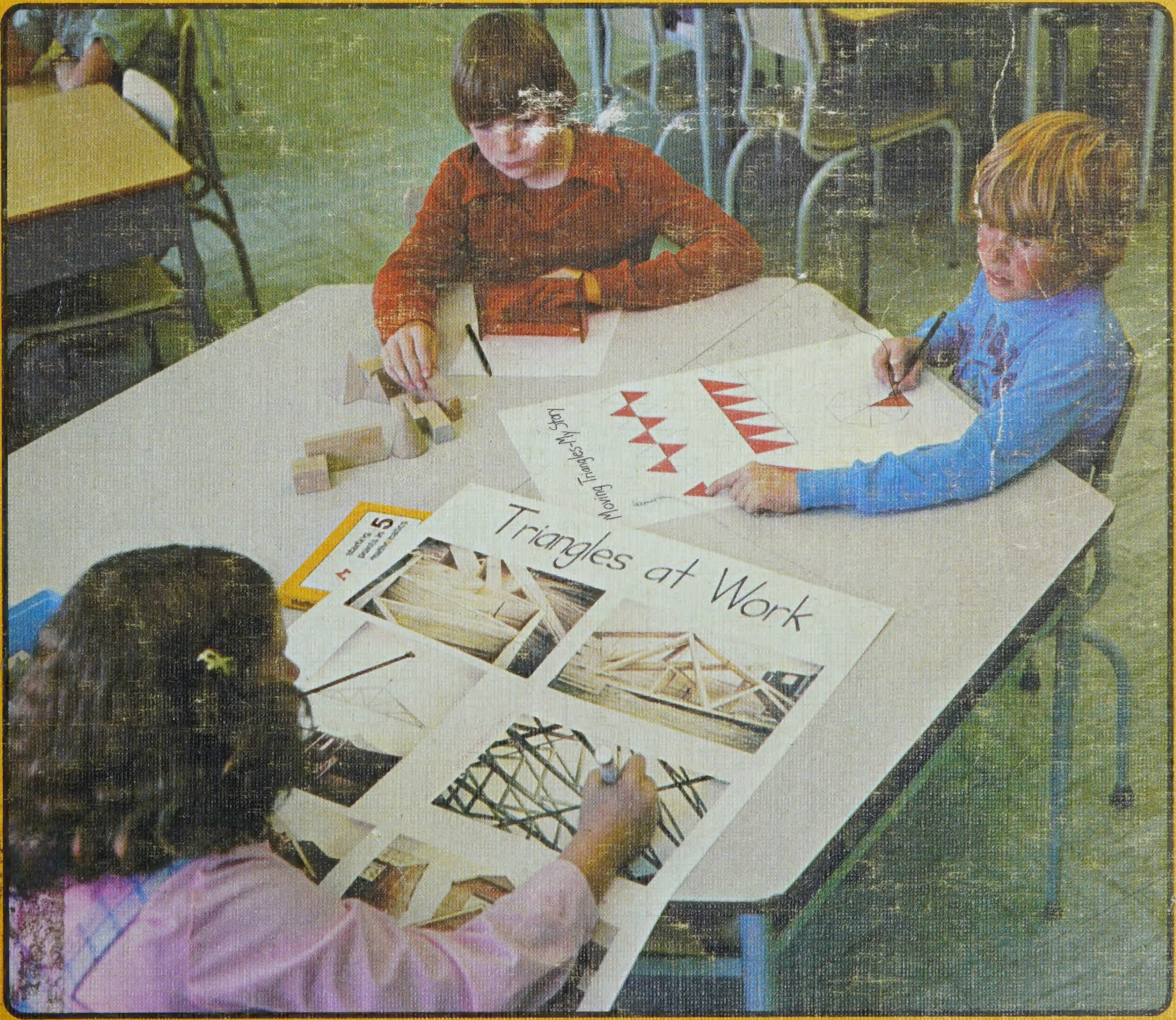
UNIVERSITY OF ALBERTA LIBRARY
0 0001 1546 462

starting points in mathematics 5

teacher's edition



Ex LIBRIS
UNIVERSITATIS
ALBERTÆNSIS

QUAECUMQUE VERA

Digitized by the Internet Archive
in 2021 with funding from
University of Alberta Libraries

https://archive.org/details/startingpointsin05born_2

Teacher's Edition for

starting points in mathematics

BINDERY SPINE SLIP | DATE SENT: | DEPT. EDUC

STYLE	MONO	JNL	INSTRUCTIONS	
CLASS A			STANDARD	X
ALTA BD			HANDSEW	
CANA BD			DO NOT TRIM	
ATLAS BD				
SC CHECK BIND	MONO		OTHER DIRECTIONS	B-4237
REBIND				
CASE				
OTHER				
COLOUR NO.	ANY		WITH IMPRINTING	

(FOLLOW EXACT WORDING AND ARRANGEMENT)

STARTING
POINTS
IN
MATHEMATICS
=
GR.5
TCH.ED.

CHECKBIND

MAKE COVERS USING
COVERS

2800

Ch# 5

UNIVERSITY OF ALBERTA

GINN AND COMPANY
EDUCATIONAL PUBLISHERS

BINDERY SPINE SLIP | DATE SENT: | DEPT. EDUC

	STYLE	MONO	JNL	INSTRUCTIONS	
	CLASS A			STANDARD	X
	ALTA BD			HANDSEW	
	CANA BD			DO NOT TRIM	
	ATLAS BD			OTHER DIRECTIONS	
SC	CHECK BIND	MONO		B-4237	
	REBIND				
	CASE				
	OTHER				
	COLOUR NO.	ANY		WITH IMPRINTING	

(FOLLOW EXACT WORDING AND ARRANGEMENT)

STARTING
POINTS
IN
MATHEMATICS
=
GR. 5
TCH. ED.

CHECKBIND

MAKE COVERS USING
COVERS

2800

Ch#
5

Teacher's Edition for

starting points in mathematics

Mathematics Team
Level 5

Authors:

Trudy Baker
Donald L. Bornhold
Paul Pogue
Stella Tossell

Consultants:

Emery Dosdall
Ralph E. Gardner
Robert Gutcher
Jack A. Hope
Murray McPherson
Edward B. Murrin
Stewart West
Mary Wright

LIBRARY
UNIVERSITY OF ALBERTA

GINN AND COMPANY
EDUCATIONAL PUBLISHERS

For the technical work and illustrations, acknowledgement is made to

Susan Calverley
Irma Ikonen
Sandi Lamanna

LIBRARY
UNIVERSITY OF ALBERTA

© Copyright, Canada, 1982, by Ginn and Company,
A Division of Xerox of Canada Limited

All rights reserved. No part of the material
covered by this copyright may be produced in any
form or by any means of reproduction.

Printed in Canada
ISBN 0-7702-0842-8
BCDEFG . 085432

Contents

In each Teacher's Edition of *Starting Points in Mathematics*, the pages for the student's book are referred to by numeral only, while pages in the teacher's edition are designated by the letter T and a numeral.

Authors and Consultants

Trudy Baker

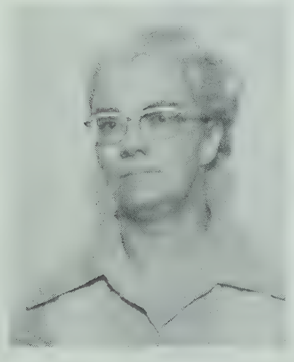
Trudy is currently employed at the Joy in Learning Curriculum Development and Teacher Training Centre in Toronto. She studied at Calvin College in Michigan and at the University of Alberta in Calgary. Her background in mathematics includes teaching in elementary schools in Alberta, British Columbia, and Ontario. She is active in curriculum development for the primary and junior grades and conducts mathematics workshops for teachers. She has worked on the program as an author of material for Grades 3 to 6.

Dr. James E. Beamer

Currently Associate Professor of Mathematics Education at the University of Saskatchewan, Jim has worked closely with the Saskatoon Separate and Public School Boards in developing the mathematics curriculum. He has a Bachelor of Science degree from Parsons College, a Master of Science degree from the University of Notre Dame, and a Doctor of Education degree from the University of Nebraska. He is the author of a number of research papers and publications in North America and has made a major contribution to teacher-in-service work in his home province. He has worked on the program from its beginning, planning the development and evaluating manuscript for the primary phrase.

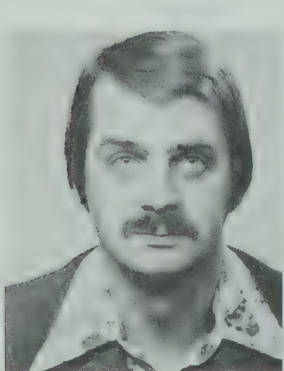
Donald L. Bornhold

Don has a Bachelor of Arts degree from the University of Western Ontario, a Bachelor of Pedagogy degree from the University of Toronto, and courses toward a Ph.D. at Columbia University. His background in mathematics has included teaching elementary and junior-high school in Kitchener and Simcoe; principal in Sherbrooke, Quebec; superintendent in Kirkland Lake, Ontario; inspector and assistant area superintendent of schools for the North York Board of Education; lecturer for the Ontario Department of Education at State College of Victoria, Melbourne, Australia; senior author of another mathematics series. He is the senior author of the program.



Grace Dilley

Grace is currently a Helping Teacher of Mathematics in School District 36, Surrey, British Columbia. She has Bachelor of Education and Master of Arts in Education degrees from the University of British Columbia. Prior to her present position, she was a classroom teacher, and a lecturer at the University of British Columbia. She is active in the British Columbia Association of Mathematics Teachers and is an author of another mathematics series. She has evaluated manuscript for the primary phase of the program.



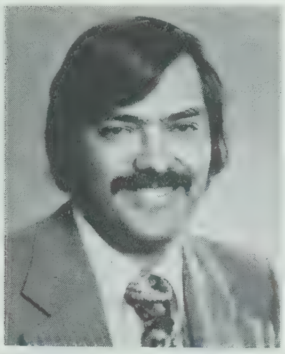
Emery Dosdall

Emery is currently the Director of Program Supervision (K-12) for the Edmonton Public Schools. He has a Bachelor of Education degree and a diploma in Educational Administration from the University of Alberta, and a Master of Education degree from the University of Oregon. Prior to his present appointment, he was Supervisor of Mathematics (K-12), lecturer at the University of Alberta, assistant principal, and consultant. He has worked extensively in elementary mathematics curriculum development for the Edmonton Public Schools and has participated throughout his home province and nationally in the development of mathematics. He has worked on the program from its beginning, evaluating manuscript, consulting at all grade levels, and coordinating and evaluating field-test material prior to publication.

Authors and Consultants

Ralph Gardner

Ralph is currently Principal of Seaview Elementary School in Port Coquitlam, British Columbia, where he is actively teaching mathematics. He has returned to the classroom following a position as Supervisor of Instruction and a position as Mathematics Consultant with the School District of Coquitlam. He has been active in mathematics in a number of capacities — serving on the Provincial Mathematics Committee, developing curriculum in the province, as an author of another mathematics program. He has worked on the program in planning the development, evaluating manuscript, and in field-testing the new approaches.



Robert Gutcher

Bob is currently an Assistant Superintendent with the Metropolitan Toronto Separate School Board, having previously been its Mathematics Coordinator. He has a Bachelor of Arts degree from the University of Western Ontario, a Master of Mathematics degree from the University of Waterloo, and a Master of Education degree from the Ontario Institute for Studies in Education. Prior to his most recent positions, he was head of a mathematics department with the Wellington County R.C.S.S. Board and earlier worked with the Etobicoke and Waterloo County Boards of Education. He has worked on the program in planning the development, evaluating manuscript, consulting at all grade levels, and as the senior author of the Grade 3 material.

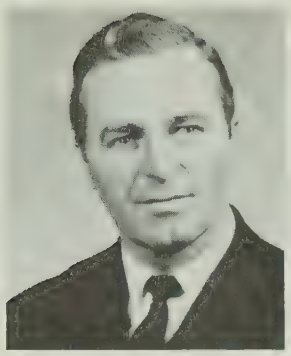
Jack A. Hope

Jack is currently an Assistant Professor with the Department of Curriculum Studies at the University of Saskatchewan in Saskatoon. He is also a part-time consultant with the Saskatoon Separate and Public School Boards. He has Bachelor of Science and Master of Arts in Education degrees from the University of British Columbia and courses toward a Doctor of Education degree. His background in mathematics has included teaching elementary school in Port Alberni, British Columbia, teaching methods courses at the University of British Columbia, and conducting summer school courses at the University of Regina and the University of Victoria. He has worked on the Grades 4 to 6 phase of the program in planning the development and evaluating manuscript.

Jean Lewis

Jean is currently School Supervisor of School District 15, Moncton, New Brunswick. She has Bachelor degrees in Education and Elementary Education from the University of Moncton. She has taught extensively in elementary and junior-high schools and was an elementary school principal prior to her present appointment. She is a Past President of the New Brunswick School Supervisors Organization and represented New Brunswick at N.C.T.M. meetings for a number of years. She continues to be active in curriculum development and is an author of other mathematics publications. She has worked on the program in planning the development and evaluating manuscript for the primary phase.

Authors and Consultants



Dr. Murray McPherson

Murray is currently Professor and Head, Department of Curriculum: Mathematics and Natural Sciences, Faculty of Education at the University of Manitoba. He has Bachelor of Science and Master of Education degrees from the University of Manitoba, and a Ph.D. from Michigan State University. Prior to his present appointment, he was a teacher and head of the department of mathematics at St. John's High School, Winnipeg, and a mathematics teacher at Dauphin Collegiate. He is Past President of the Manitoba Association of Mathematics Teachers and has spoken at a number of N.C.T.M. meetings. He has worked on the program in planning its development and evaluating manuscript.

Edward B. Murrin

Ed is currently teaching mathematics at Antigonish Regional High School. Prior to his present teaching responsibilities, he has been an elementary school principal for sixteen years, a Past President of the Nova Scotia Mathematics Teachers Association, and a lecturer in elementary mathematics at St. Francis Xavier University. He was the Canadian representative for the National Council of Teachers of Mathematics from 1972 to 1975 and has been active in mathematics throughout his home province of Nova Scotia for many years. He has worked on the program in planning its development and evaluating manuscript.

Paul Pogue

Paul is currently teaching mathematics at Barrie North Collegiate Institute in Barrie, Ontario. A graduate of Lakeshore Teachers College, Toronto, he has many years of experience in elementary and high school mathematics classrooms. He has been active in curriculum planning as project director of ''Recommendations for Intermediate Mathematics for the Province of Ontario'', a Ministry of Education publication, and as a member of the writing team for the Intermediate guidelines established by the Ministry of Education. At the present time, he is a director of the Ontario Association of Mathematics Education. He has worked on the program in planning the development, evaluating manuscript, and as an author of material for Grades 4 to 6.

Trudy Stacey

Trudy has a Bachelor of Arts degree from York University. She has been an elementary teacher and is currently a General Consultant for the North York Board of Education and Program Leader in Language Arts and Mathematics. She developed the themes for the primary phase of the program.

Stella Tossell

Stella has a Bachelor of Arts degree from the University of Toronto. Her background in mathematics has included teaching at the secondary level with the Lincoln County and York County Boards of Education. After teaching at the American School in Athens, Greece, she resumed her duties with the York County Board of Education and then joined the North York Board of Education in an advisory position at the junior-high level. More recently she has been a Mathematics Consultant mainly at the elementary level for the North York Board of Education. She has worked on the program in evaluating and writing manuscript, and writing teaching suggestions and activities for the teachers.

Authors and Consultants

Cathie Traynor

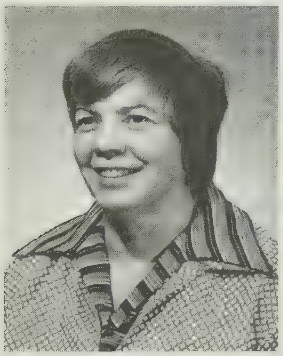
Cathie is a Consultant with the Metropolitan Toronto Separate School Board. A graduate of Lakeshore Teachers College, Toronto, she has continued her education at York University and, through the Ontario and British Ministries of Education, in a number of specialized categories, namely, early childhood education, language development, speech therapy, and special education. She taught in the elementary grades for the Metropolitan Toronto Separate School Board and also the Ontario Ministry curriculum and mathematics courses. She has lectured at York University. She is an author on the program and has coordinated and evaluated the field-test material prior to publication.

Stewart West

Stewart has Bachelor degrees in Arts and Education from the University of New Brunswick and is currently teaching at the Magnetic Hill School in Moncton. He has served in many capacities in the province: as a member of the Provincial Committees in Mathematics and Science; as Conference Chairman for the Association Subject Council Workshop; as President and Workshop Coordinator for Elementary Teachers' Council. He continues to remain active in numerous presentations in his home province, in other centers in Canada, and in the United States. He has worked on the Grade 3 to 6 phase of the program in planning the development, evaluating manuscript, and coordinating and evaluating field-test material prior to publication.

Gary White

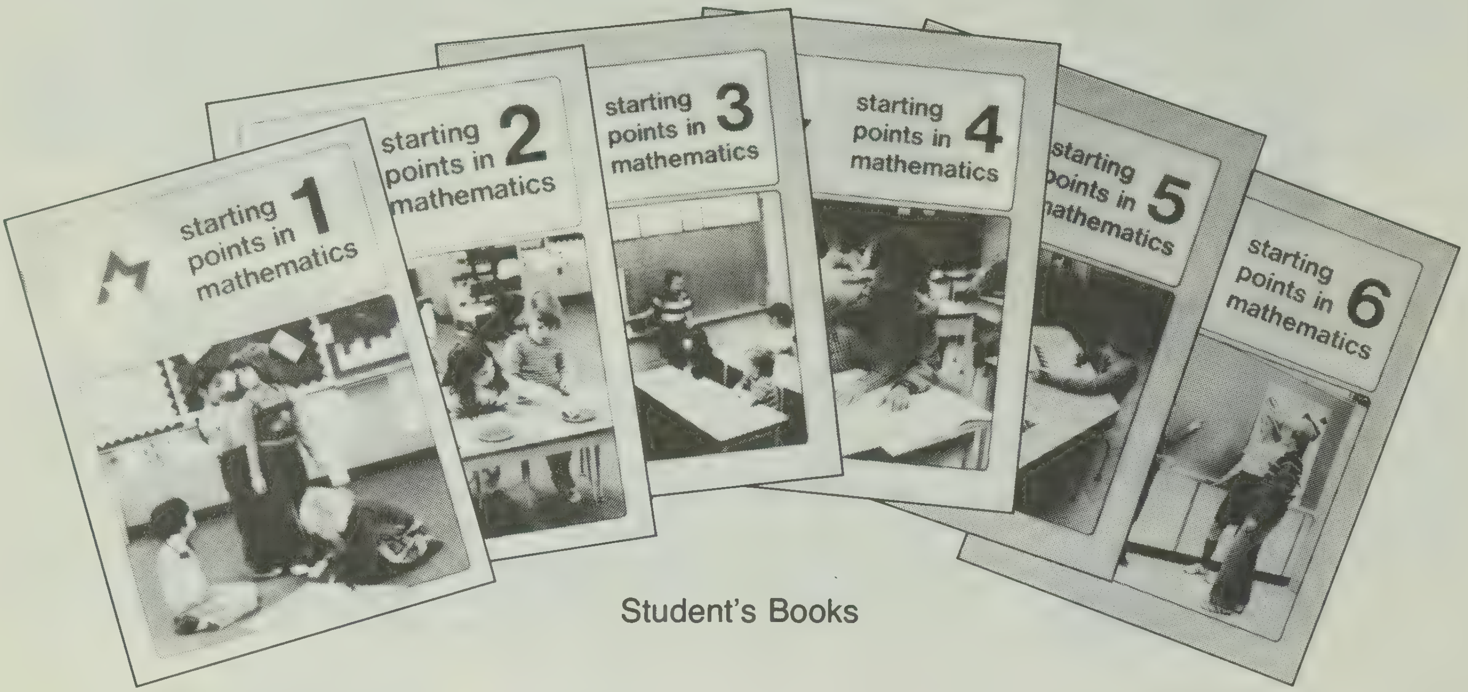
Gary is currently Principal of Robert Meek Public School in Kingston, Ontario. He has a Bachelor of Arts degree from Queen's University and is a graduate of Peterborough Teacher's College. His background in mathematics includes teaching at the primary, junior, and intermediate levels. He is active in curriculum development and writing and presenting suggestions and activities for classroom teachers. He has worked on the program as an author of material for Grade 4.



Mary Wright

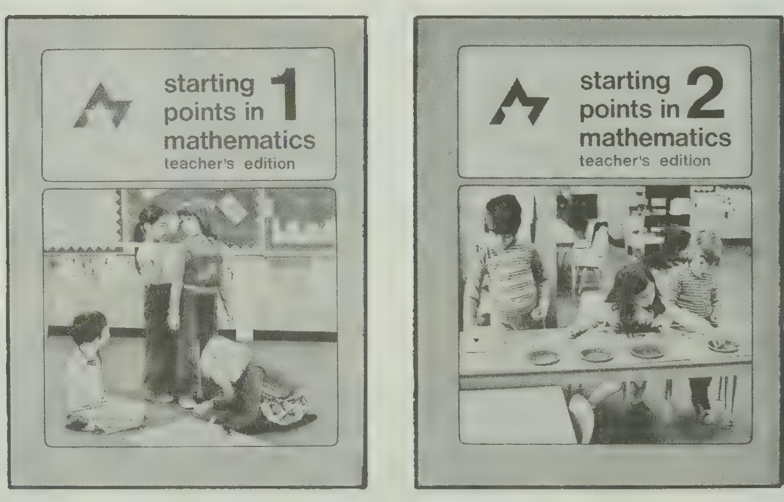
Mary received her education in England and carried on postgraduate work at Acadia University (M.Sc.), and Dalhousie University. She taught high school in England, New Brunswick, Nova Scotia, and Prince Edward Island. She was a lecturer and assistant professor of mathematics at Acadia University for ten years. Currently, she is Mathematics Consultant for the Regional Administrative Office of the Department of Education in Montague, Prince Edward Island. She has been active in mathematics in a number of capacities. She is a recipient of a Canada Council Scholarship for Teachers of Mathematics, and a member of the Canadian Mathematics Congress and of N.C.T.M. She has worked on the Grades 4 to 6 phase of the program in planning the development and evaluating manuscript.

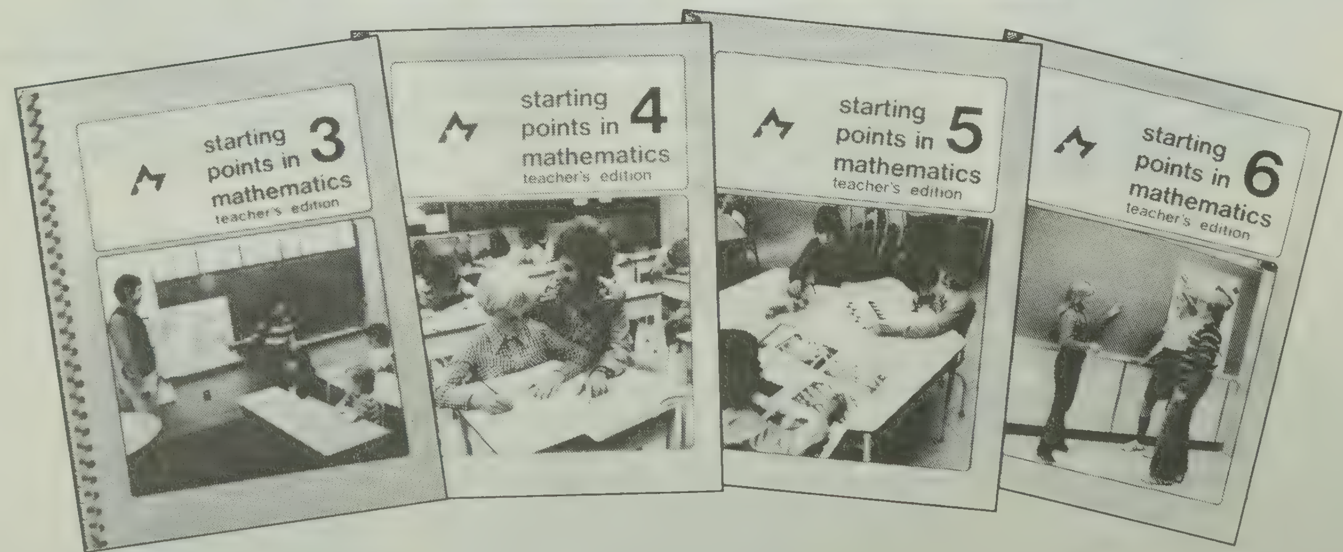
starting points in mathematics



Student's Books

Teacher's Editions





Program Highlights

Content

- Computation strands that maintain a balance between concepts and skills
- A metric Measurement strand using units and symbols in accordance with the National Standards of Canada
- A Decimals and Fractions strand that reflects the more significant role of decimals in a metric world
- A Geometry strand that introduces transformation geometry topics in addition to the more traditional topics
- A Problem Solving strand that identifies specific problem solving skills and strategies
- Lessons on using a calculator to reinforce the understanding of number operations and as an aid for checking results

Development

- Computational concepts and skills built upon the basic facts, the continued manipulation of concrete materials, place value, systematic development of the algorithms, and practice
- Measurement concepts and skills introduced using non-standard units; refined and developed using only approved metric units
- Decimals introduced with the parts-of-a-whole concept and developed by extending the place-value concepts of whole numbers
- Corresponding ideas among the Numeration, Computation, Measurement, and Decimals strands treated as mutually supportive concepts for both development and reinforcement
- Problem Solving strand integrated with the other strands
- Material provided for maintenance of computational skills

For the Student

- A highly visual program placing mathematics ideas and experiences within meaningful settings of real-life objects and situations
- Uniform lesson structure with completed examples to illustrate each objective
- A variety of types of exercises
- Problems that provide reasons for learning mathematics
- *Special Features* showing mathematics in use in real-life situations and providing opportunities to be individually creative with mathematical skills in problem solving and enrichment activities

For the Teacher

- Manageable units for the development of concepts and skills
- Overviews that provide mathematics background and summarize the content of each unit
- Concise statements of lesson objectives
- Suggestions for activities to precede and follow each lesson in the book; suggestions for teaching each lesson in the book
- Uniform lesson structure that is adaptable to a variety of classroom strategies
- Unit themes that support the integration of mathematics with other areas of study, and suggestions on how this integration can be achieved
- Component skills necessary for achieving lesson objectives identified
- Assessment materials included in the book and the teacher's edition

Features of the Student's Book

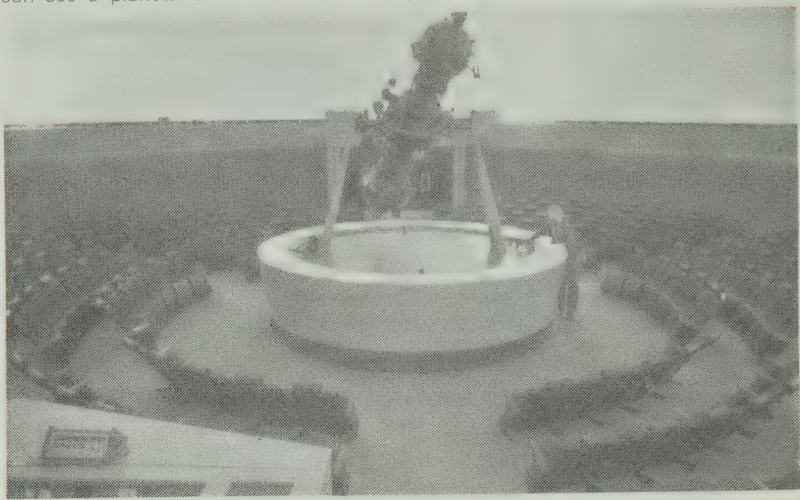
Lesson outcome is stated.

Color and design are used to assist understanding.

A lesson begins with a worked example of what is to be learned.

Multiplying by a One-Digit Number

The planetarium can hold 354 people for each show. On Saturday there are 6 shows. How many people can see a planetarium show on Saturday?



Multiply 6 and 354.

$$\begin{array}{r} {}^{2} \\ 3\,5\,4 \\ 6 \\ \hline 4 \end{array}$$

6 × 4 = 24 or 2 tens 4 ones.

$$\begin{array}{r} {}^{3}\,{}^{2} \\ 3\,5\,4 \\ 6 \\ \hline 2\,4 \end{array}$$

6 × 5 tens = 30 tens
2 more tens make 32 tens or 3 hundreds 2 tens.

$$\begin{array}{r} {}^{3}\,{}^{2} \\ 3\,5\,4 \\ 6 \\ \hline 2\,1\,2\,4 \end{array}$$

6 × 3 hundreds = 18 hundreds.
3 more hundreds make 21 hundreds or 2 thousands 1 hundred.

2124 people can see a planetarium show on Saturday.

46

The development of a concept enables the student to move from the use of concrete materials to the use of abstract number sentences and algorithms.

Special Features

Practice with addition, subtraction, multiplication, and division

KEEPING SHARP

Some interesting ideas for fun and enrichment

try this

Lessons and activities to help in learning the skills needed for solving problems

PROBLEM SOLVING

Working Together

Complete each multiplication.

1. $\begin{array}{r} \blacksquare \\ 47 \\ \underline{5} \\ 23\blacksquare \end{array}$ 2. $\begin{array}{r} 4 \\ 36 \\ \underline{7} \\ \blacksquare\blacksquare 2 \end{array}$ 3. $\begin{array}{r} \blacksquare \\ 63 \\ \underline{4} \\ \blacksquare\blacksquare\blacksquare \end{array}$ 4. $\begin{array}{r} \blacksquare\ \ \ \ \ \ \\ 2830 \\ \underline{3} \\ \blacksquare\blacksquare 90 \end{array}$ 5. $\begin{array}{r} \blacksquare\blacksquare\ \ \\ 134 \\ \underline{6} \\ 8\blacksquare\blacksquare \end{array}$ 6. $\begin{array}{r} \blacksquare\blacksquare\ \ \ \\ 3271 \\ \underline{8} \\ \blacksquare\blacksquare\blacksquare 8 \end{array}$

Multiply.

7. $\begin{array}{r} 75 \\ \underline{4} \end{array}$ 8. $\begin{array}{r} 684 \\ \underline{5} \end{array}$ 9. $\begin{array}{r} 2604 \\ \underline{3} \end{array}$ 10. $\begin{array}{r} 9431 \\ \underline{9} \end{array}$ 11. $\begin{array}{r} 16\,938 \\ \underline{6} \end{array}$ 12. $\begin{array}{r} 40\,536 \\ \underline{8} \end{array}$

Working Together shows the steps of what is to be learned.

Exercises

Multiply.

1. $\begin{array}{r} 59 \\ \underline{5} \end{array}$ 2. $\begin{array}{r} 34 \\ \underline{7} \end{array}$ 3. $\begin{array}{r} 87 \\ \underline{6} \end{array}$

4. $\begin{array}{r} 678 \\ \underline{9} \end{array}$ 5. $\begin{array}{r} 951 \\ \underline{7} \end{array}$ 6. $\begin{array}{r} 606 \\ \underline{6} \end{array}$

7. $\begin{array}{r} 1231 \\ \underline{3} \end{array}$ 8. $\begin{array}{r} 4003 \\ \underline{5} \end{array}$ 9. $\begin{array}{r} 4625 \\ \underline{8} \end{array}$

10. $\begin{array}{r} 20\,968 \\ \underline{4} \end{array}$ 11. $\begin{array}{r} 27\,879 \\ \underline{3} \end{array}$

12. $\begin{array}{r} 72\,342 \\ \underline{7} \end{array}$ 13. $\begin{array}{r} 17\,727 \\ \underline{9} \end{array}$

14. 8 × 941 15. 6 × 9214
16. 9 × 17 17. 2 × 47 216
18. 4 × 634 19. 8 × 5764
20. 5 × 712 21. 4 × 17 819
22. 9 × 5555 23. 7 × 76 008
24. 8 × 880 25. 4 × 9052
26. 3 × 5226 27. 6 × 10 375
28. 2 × 9999 29. 5 × 19 753

Solve.

30. There are 3 planetarium shows Saturday evening. How many people can see these shows?
31. The planetarium has a special show each weekday afternoon. How many people can see this show from Monday to Friday?
32. The school auditorium can hold 1250 people. How many people can see the 3 performances of the school play?
33. The football stadium in Vancouver holds 32 752 people. How many people can go to 8 Canadian Football League games there in a season?
34. The Montreal Forum holds 18 350 people for hockey. How many people can watch 4 Stanley Cup games in the Forum?
35. The baseball stadium in Toronto can hold 46 500 people. How many people can watch a 3-game series in the stadium?

47

Exercises give practice and provide applications of what has been learned.

Word problems whose solutions incorporate the skills taught are included.

Special * exercises provide more practice with problem solving.

Checking Up

End-of-unit lessons provide a check of the understanding of the work of the unit.

Checking Skills

Special reviews provide a check of skills with addition, subtraction, multiplication, and division.

A lesson outline may include some or all of the following:

1 The page references to the student's book

2 The outcome(s) for the lesson

3 Some of the materials that would be desirable for introducing and developing the lesson

4 A reference to a page that may be copied to provide cutouts for the students

5 Mathematical terms used for the first time and other words useful for discussing the development of the topic on the page

6 Identification of concepts and skills that students should be able to perform prior to the lesson

7 Suggested tasks for assessing prerequisite skills

8 Comments about the content of the lesson

9 Activities for developing the lesson concepts, and suggestions for introducing new words and symbols

1 Pages 266-267

2 **LESSON OUTCOME**

Compare fractions with like denominators; compare fractions with unlike denominators by writing equivalent fractions with like denominators

3 **Materials**

models for $\frac{3}{8}$, $\frac{5}{8}$, $\frac{6}{8}$, $\frac{7}{8}$, $\frac{2}{5}$, $\frac{4}{5}$, and $\frac{1}{2}$,
4 prepared from copies of the squares on page T 393

5 **Vocabulary**

like denominators, unlike denominators, common denominators

6 **Prerequisite Skills**

Find the missing term in two equivalent fractions

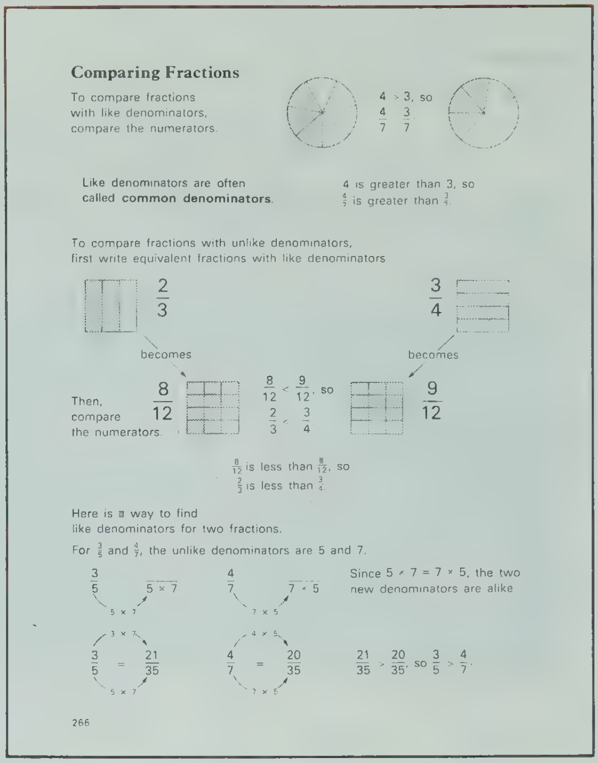
7 **Checking Prerequisite Skills**

Find the missing term.

1. $\frac{2}{3} = \frac{■}{9}$ 6
2. $\frac{3}{4} = \frac{■}{20}$ 15
3. $\frac{7}{25} = \frac{21}{■}$ 75
4. $\frac{2}{7} = \frac{10}{■}$ 35

8 **Background**

The concept of *least common denominator* is introduced at a later level. At this time, a common denominator for two fractions with unlike denominators is found by multiplying their denominators. The result may be the least common denominator, but not necessarily. For example, to compare $\frac{2}{3}$ and $\frac{3}{5}$, their equivalent fractions $\frac{10}{15}$ and $\frac{9}{15}$ are compared; for $\frac{3}{8}$ and $\frac{4}{10}$, $\frac{30}{80}$ and $\frac{32}{80}$ are compared.



LESSON ACTIVITY

9 **Before Using the Pages**

- Display a model of $\frac{3}{8}$ and a model of $\frac{7}{8}$ and have students identify the numbers represented. Ask which number is greater and have students explain their answers.

$\frac{3}{8}$ $\frac{7}{8}$

Repeat the procedure using models of $\frac{5}{8}$ and $\frac{6}{8}$. Ask the students how they can tell which of two fractions is greater without referring to the models. They will likely suggest that they can compare the numerators. Have them compare $\frac{2}{5}$ and $\frac{4}{5}$ and use the models to check their answer. Repeat the procedure for $\frac{1}{2}$ and $\frac{3}{8}$. In this case, although 3 is greater than 1, $\frac{3}{8}$ is less than $\frac{1}{2}$. Ask students to suggest why the procedure of checking the numerators cannot apply in this last example. Lead them to realize that the numerators reveal which fraction is greater only if the denominators are identical. Have students express $\frac{1}{2}$ as $\frac{4}{8}$ and then compare $\frac{4}{8}$ and $\frac{3}{8}$.

Using the Pages

- Lead the students through the three examples on page 266. For $\frac{4}{7}$ and $\frac{3}{7}$, emphasize that the denominators are identical. For $\frac{2}{3}$ and $\frac{3}{4}$, emphasize that the denominators are different and that the fractions in each pair must first be expressed as equivalent fractions so that like denominators are obtained. Draw attention to the diagrams that show $\frac{2}{3}$ as $\frac{8}{12}$, and $\frac{3}{4}$ as $\frac{9}{12}$. Emphasize that $\frac{8}{12}$ represents the same part of the whole as $\frac{2}{3}$; thus, they are different names for the same number. Similarly, $\frac{3}{4}$ and $\frac{9}{12}$ are different names for the same number.

  The third example demonstrates a procedure for obtaining like denominators for two fractions without using

T 290

10

10 Suggestions for using the pages

Overviews

The overview at the beginning of each unit includes a list of the prerequisite skills that are required for successful completion of the unit, the outcomes for the developmental lessons in the unit, mathematical background, comments about the content and how the unit fits with the other units in the program, teaching strategies, materials, and vocabulary.

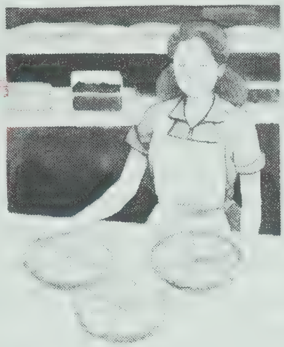
Working Together

For the fractions in each pair, give equivalent fractions with like denominators. Answers may vary

1. $\frac{1}{2}, \frac{4}{9}$ $\frac{9}{18}, \frac{8}{18}$ 2. $\frac{2}{3}, \frac{5}{7}$ $\frac{14}{21}, \frac{15}{21}$ 3. $\frac{3}{4}, \frac{4}{6}$

Use >, <, or = to make true statements.

4. $\frac{1}{2} \bigcirc \frac{4}{9}$ > 5. $\frac{2}{3} \bigcirc \frac{5}{7}$



Exercises

For each of these, show equivalent fractions with like denominators. Then, use >, <, or = to make a true statement.

Example. $\frac{4}{9} = \frac{28}{63}$

$\frac{3}{7} = \frac{27}{63}$

$\frac{28}{63} > \frac{27}{63}$ so $\frac{4}{9} > \frac{3}{7}$

Equivalent fractions may vary

1. $\frac{3}{4} \bigcirc \frac{4}{7}$ 2. $\frac{6}{16} \bigcirc \frac{3}{8}$ $\frac{6}{16}, \frac{6}{16}$

3. $\frac{1}{3} \bigcirc \frac{2}{5}$ 4. $\frac{3}{7} \bigcirc \frac{4}{10}$ $\frac{30}{70}, \frac{28}{70}$

5. $\frac{10}{25} \bigcirc \frac{6}{15}$ $\frac{2}{5}, \frac{2}{5}$ 6. $\frac{2}{9} \bigcirc \frac{1}{5}$ $\frac{10}{45}, \frac{9}{45}$

7. $\frac{7}{12} \bigcirc \frac{2}{3}$ $\frac{7}{12}, \frac{8}{12}$ 8. $\frac{7}{9} \bigcirc \frac{5}{6}$ $\frac{14}{18}, \frac{15}{18}$

List from least to greatest.

•9. $\frac{2}{7}, \frac{3}{7}, \frac{3}{8}$ $\frac{2}{7}, \frac{3}{8}, \frac{3}{7}$ •10. $\frac{4}{12}, \frac{6}{15}, \frac{6}{14}$

List from greatest to least

•11. $\frac{3}{4}, \frac{3}{5}, \frac{4}{6}$ •12. $\frac{3}{7}, \frac{4}{8}, \frac{5}{9}$

Pies are often cut into sixths or eighths.

13. Which is more, $\frac{1}{6}$ of a pie or $\frac{1}{8}$ of a pie? $\frac{1}{6}$

14. Which is more, $\frac{4}{6}$ of a pie or $\frac{5}{8}$ of a pie? $\frac{4}{6}$

15. Which is more, 2 pieces of a pie that is cut into sixths or 3 pieces of a pie that is cut into eighths? the 3 pieces

•16. $\frac{5}{6}$ of the blueberry pie has been eaten. $\frac{7}{8}$ of the apple pie has been eaten. Of which pie is there more left? the blueberry pie

267

diagrams. The new denominator, 35, is the product of the unlike denominators 5 and 7. After discussing the steps shown, develop similar steps on the board for $\frac{2}{3}$ and $\frac{3}{4}$ of the previous example.

$$\frac{2}{3} = \frac{8}{12} \quad (2 \times 4,\ 3 \times 4) \qquad \frac{3}{4} = \frac{9}{12} \quad (3 \times 3,\ 4 \times 3)$$

Working Together: Have students follow the preceding format to obtain equivalent fractions. However, keep in mind that answers may vary because the concept of least common denominator is not dealt with at this time. For example, for Ex. 3, some students may express each fraction with a denominator of 24, others may use 12. Have each method shown on the board.

Exercises: Draw attention to the fact that each of Ex. 9-12 involves three fractions which will have to be expressed with like denominators. You may need to work one of these exercises on the board with the students to provide them with an example.

(11)

Ex. 16 involves the comparison of two fractions. Note that it is not necessary to use subtraction to answer the question "Of which pie is there more left?" By determining that $\frac{5}{6}$ is less than $\frac{7}{8}$, students can reason that less blueberry pie has been eaten and thus more of it is left. However, some students may intuitively think of comparing $\frac{1}{6}$ and $\frac{1}{8}$, rather than $\frac{5}{6}$ and $\frac{7}{8}$.

Assessment

Show equivalent fractions with like denominators. Then use >, <, or = to make a true statement.

1. $\frac{2}{3} \bigcirc \frac{5}{8}$ > $\frac{16}{24}$ $\frac{15}{24}$ 2. $\frac{9}{15} \bigcirc \frac{6}{10}$ = $\frac{54}{90}$ $\frac{54}{90}$ 3. $\frac{3}{4} \bigcirc \frac{4}{5}$ < $\frac{15}{20}$ $\frac{16}{20}$

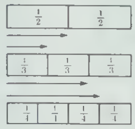
Solve.

4. Which is more, $\frac{5}{6}$ of a pie or $\frac{7}{8}$ of a pie? $\frac{7}{8}$

(12)

RELATED ACTIVITIES (13)

• Have students use the fraction strips described in *Related Activities* on page T285, for a visual comparison of two given fractions. For each comparison, have them write one or two sentences as shown below. (14)



$\frac{1}{2} > \frac{1}{3}$, $\frac{1}{3} < \frac{1}{2}$

$\frac{2}{3} < \frac{3}{4}$, $\frac{3}{4} > \frac{2}{3}$

• For the given set of fractions, have students compare each fraction with $\frac{1}{2}$ and write each in the appropriate column of a chart similar to the following.

$\frac{1}{3}$, $\frac{3}{5}$, $\frac{2}{4}$, $\frac{5}{9}$, $\frac{2}{3}$

$\frac{6}{12}$, $\frac{5}{8}$, $\frac{3}{7}$, $\frac{5}{6}$, $\frac{4}{8}$

$\frac{2}{9}$, $\frac{7}{10}$, $\frac{8}{15}$, $\frac{5}{12}$, $\frac{4}{7}$

Less than $\frac{1}{2}$	Equal to $\frac{1}{2}$	Greater than $\frac{1}{2}$
$\frac{1}{3}$	$\frac{2}{4}$	$\frac{3}{5}$

T 291

13 Suggested activities and games which may be used to reinforce, extend, or enrich a particular topic

14 Illustrations of suggested materials

11 Reduced pages from the student's book with answers indicated

12 Exercises to evaluate the learning outcome

Answers

Answers for the exercises are given on each reduced page from the student's book. Answers that did not fit on the reduced pages or beside them appear on pages T366-T375.

Other Materials

- Games and Teaching Aids
- Pages for Reproduction
- Year-End Evaluation Chart
- Index for Student's Book

Teaching Strategies

Using the Introductory Material

Knowing what the *Starting Points in Mathematics* program can do and what it cannot do is an important place to begin. There are two features to help you. First, the Scope and Sequence chart shows the content by grade level and allows you to locate particular topics in the overall development. Secondly, each unit begins with an overview which summarizes the content of the unit, includes mathematics background for the teacher, and suggests strategies for class organization and teaching.

Presenting the Lessons

The organization of the teaching suggestions for each lesson has built-in strategies to motivate and teach, and for practice and application. The identification of *Prerequisite Skills* and concepts are stated in terms of tasks students should be able to perform prior to the lesson. Teachers will want to assess prerequisite skills in certain instances; for example,

- for topics that have proven traditionally difficult,
- for new mathematics topics in the curriculum,
- with students who have missed school or been transferred,
- at the beginning of the year when the mathematical levels of the students may be uncertain,
- as a starting point to work with students who have not been successful with the previous lesson.

In the *Before Using the Page(s)* section, most lessons are initiated with concrete material to provide a review, an appropriate warm-up, or activities designed to lead students to discover a concept or a skill.

The teaching suggestions in the *Using the Page(s)* section emphasize the key aspects of the teaching example and, at times, suggest a possible sequence. When students are ready to work the exercises independently, they will benefit from a full explanation of the teaching example as a guide for their work. The teaching suggestions follow naturally from the preliminary activities and allow you to develop the lesson outcomes from the students' page(s).

In all teaching lessons, the *Working Together* section provides an examination of the sub-skills and/or the sequence of steps leading to the outcome. This section allows for immediate feedback of the students' understanding of the lesson. In this regard, this section may be completed orally or on the board. Students experiencing difficulty with the exercises can be referred to the appropriate example. *Working Together* provides another opportunity to see how all the steps of the lesson combine and, consequently, provides a means for diagnosing the students' understanding.

Each teaching lesson provides material in the *Assessment* section to evaluate the learning outcome. If the material is not used following the lesson, teachers may consider using it for pretesting or reviewing.

Most of the *Related Activities* can be used for all the students. The suggestions in this section try to provide a balance between reinforcement, enrichment, and review.

Grouping for Instruction

Knowing what to teach is one thing, knowing how to adapt a program for individual differences in ability and capacity for achievement is the ongoing role of all teachers.

It is possible to work with lower achievers and higher achievers by using the same material but by altering the teaching procedure. Lower achievers, as in other subjects, need a slower pace to provide for maximum use of concrete materials and pictorial representations as well as varied activities to ensure understanding.

With higher achievers you will often wish to move at a faster pace. This does not mean a more rapid movement through the lessons, but rather a change in approach. All students need the benefits derived from the use of concrete materials for both present and future understanding, but higher achievers tend to move more readily from the concrete to the abstract levels of mathematical thinking. They grasp concepts and skills quickly and will benefit from exploration and challenges that will allow them to use and broaden their newly acquired abilities in different settings.

Grouping for instruction is dependent on a number of factors, including teacher preference, teaching strategy, social and academic needs of students, abilities and skills of the students, the need to vary instruction, the organization of the classroom. Some possible ways for grouping are given below.

The Whole Class

Instruction of the whole class is appropriate for the introduction of new topics or class projects.

Skill Groups

For this grouping the teacher selects students having similar needs for the teaching of a specific skill. When the skill is mastered, the group is dissolved.

Interest Groups

For this grouping a student chooses to be a member of the group based on interest in the activity being offered. For example, while one concept is being explored by the whole class, a student may have the choice of working at the board or with activity cards. Interest groups may be formed for the study of a theme or for a group project. This kind of grouping promotes sharing among students and offers opportunities for students to display leadership.

Random Groups

This type of grouping may be as arbitrary as the grouping of all students wearing something red or as open as to include groups of friends. It is especially suited to situations involving games, experiments, and making models.

Often students may be part of a group, but they may work independently within the group. It is here that the teacher may observe and plan for individual needs.

Providing for Individual Differences

Use the daily performance and test results to assess the needs and abilities of the students.

For lower achievers, plan to make more extensive use of the *Prerequisite Skills* assessment before starting a lesson, the *Working Together* section, and the appropriate selection of activities from the *Related Activities*. The *Keeping Sharp* features, *Checking Skills* pages, and *Skill Practice* exercises provide continued practice for the basic operations.

Higher achievers will benefit from the flexibility and variation of the teaching model and a wider exposure to the special features *Try This* and *Problem Solving*. The *Related Activities* provide suggestions for enrichment topics.

All students will benefit from the formal lesson on problem solving found at the end of each of the fifteen units.

Problem Solving

The problem-solving strand is integrated and interspersed throughout *Starting Points in Mathematics 5*. There are fifteen teaching lessons on specific skills, twenty-three special feature sections presenting problems, often as extensions of material under study, and starred problems that require special attention or integrate what has been learned in the problem-solving lessons into the exercises.

Students will benefit from specific instruction on problem solving, the concrete presentation of concepts and skills in all the lessons where this is possible, and meeting problem-solving requirements in a meaningful context. The material in this strand, however, is by no means exhaustive. Teachers will capture the right moments in their daily contact with the students to provide the insights and skills to develop better problem-solving techniques.

Testing and Evaluation

The *Checking Up* feature at the end of each unit reviews the skills taught in the unit and helps to evaluate the students' progress. It could also be used as a pretest for the unit.

The chart in the teacher's edition for each *Checking Up* feature is designed to help locate strengths and weaknesses. The *Skills* section of the chart lists the skills taught in the unit. The *Exercises* section lists the exercises in *Checking Up* corresponding to each skill identified. The *Related Pages* section lists the pages in the teacher's edition where the skill is taught. You may wish to refer to these pages for reviewing or reteaching.

The comments below the chart discuss special aspects of the exercises. They point out possible difficulties and give suggestions for remedying the difficulties.

The *Checking Skills* pages at the end of Units 4, 7, 9, 11, and 14 evaluate students' ongoing performance in the four basic operations for whole numbers and decimals.

As well as the *Keeping Sharp* features and the *Checking Skills* pages provided throughout the text, the *Skill Practice* exercises on pages 332 to 337 provide another resource for maintaining and evaluating computational skills during the year. Suggestions are given in the *Related Activities* of appropriate lessons for assigning exercises on these pages, but they may be assigned at any time when skill maintenance is required. The *Skill Practice* exercises are also useful for diagnosing difficulties in a skill taught earlier in the year and for providing practice after a skill has been retaught.

If evaluation is to be an ongoing process, it is important to keep complete and accurate records of the achievement of each student. A file containing remarks on progress based on the observation of the teacher and samples of the student's work is recommended. The remarks can be dated and are an excellent reference when reporting to parents. The samples of work can be selected by both the teacher and the student. If the student plays an active part in contributing work which indicates her/his mastery of a concept, she/he also recognizes that learning is important. This is an essential factor in assuring future success.

The comprehensive evaluation chart on pages T401 and T402 is intended for use at the end of the school year, but it may be adapted for other uses. For example, if the indicated program is too ambitious for all the students in a class, the chart may be used as a guide for obtaining a minimum program or an average program for the students. The format of the chart may also be adapted as a report to show parents their children's progress.

You may wish to adopt a code for recording the stages of development in each student's mastery of concepts and skills. Slashes and dots may be used in such a way that a box marked ⧄ means that the student has been introduced to the concept or skill; a box marked ☒ means that the student understands the concept or skill; a box marked ☒ means that the student has mastered the concept or skill. Examples are shown below.

Expresses fractions as decimals	⧄ (Exposure)
Relates millilitres and litres	☒ (Understanding)
Knows the basic division facts	☒ (Mastery)

Using Calculators

Because calculators are so popular today, special lessons on the use of the calculator as an instructional aid have been included in *Starting Points in Mathematics 5*.

Some educators and parents oppose the use of calculators in the classroom. They fear a decline in computational skills, a dependence on calculators for even the simplest computations, and a premature introduction of numbers beyond the cognitive levels of the students.

Proper use of calculators does not allow these concerns to materialize. Basic skills and facts are developed first, estimation and mental calculations are encouraged, and the more frequent contacts with decimals on digital displays strengthen students' understanding of place value and their skills in rounding numbers. It has been found that students who use calculators to supplement their own computations generally develop better skills than those who do not.

The fundamental objective of all mathematics programs is the development of skills and the ability to solve problems. Problem-solving programs are strengthened when calculators are used. They save time in performing the necessary computations and, hence, more problems can be solved. By removing concerns about computation, more attention can be directed to the selection of pertinent data and to the implied relationships between numbers.

In summary, calculators can strengthen a mathematics program, but they should be used only after basic skills have been developed. They may be used

- to check basic understandings and skills;
- to support basic skills;
- to develop further insight into basic operations;
- to develop estimating skills;
- to solve real-life problems;
- to save computation time in solving problems;
- to verify mental calculations and procedures;
- to provide insights into higher mathematical concepts;
- to search for interesting number patterns.

A Thematic Approach to Mathematics

Teachers who attempt to provide an integrated curriculum frequently encounter difficulty interweaving the mathematics curriculum with other subjects and in creating sufficient real-life situations to make the association a meaningful one. Several units in *Starting Points in Mathematics 5* are theme oriented so that teachers who have never tried integrating mathematics may wish to experiment with one or more themes, for example, the situations and photographs in Unit 2 suggest "Canada" as a theme. Similarly, the theme "Sports" is predominant in Unit 6 and the theme "The Annual Fair" is predominant in Unit 10.

Student's Bibliography

Numbers and Numeration

Adler, Irving. *The Magic House of Numbers*. New York: John Day Co., Inc., 1974. (Fitzhenry and Whiteside Ltd., Don Mills, Ontario.)

______, and Adler, Ruth. *Numbers Old and New*. New York: John Day Co., Inc., 1960. (Fitzhenry and Whiteside Ltd., Don Mills, Ontario.)

______, and Adler, Ruth. *Sets*. New York: John Day Co., Inc., 1967. (Fitzhenry and Whiteside Ltd., Don Mills, Ontario.)

Adler, Peggy. *Second Adler Book of Puzzles and Riddles*. New York: John Day Co., Inc., 1963. (Fitzhenry and Whiteside Ltd., Don Mills, Ontario.)

Adler, Peggy, and Adler, Irving. *Math Puzzles*. New York: Franklin Watts, Inc., 1978. (Grolier Limited, Toronto, Ontario.)

Barr, Donald. *Arithmetic for Billy Goats*. New York: Harcourt Brace Jovanovich, Inc., 1966. (Longman Canada Limited, Don Mills, Ontario.)

Barr, George. *Entertaining with Number Tricks*. New York: McGraw-Hill Book Co., 1971. (McGraw-Hill Ryerson Limited, Scarborough, Ontario.)

Barr, Stephen. *Miscellany of Puzzles: Mathematical and Otherwise*. New York: Thomas Y. Crowell Co., Inc., 1965. (Fitzhenry and Whiteside Ltd., Don Mills, Ontario.)

Bendick, Jeanne. *Names, Sets, and Numbers*. New York: Franklin Watts, Inc., 1971. (Grolier Limited, Toronto, Ontario.)

______, and Levin, Marcia. *Take a Number: New Ideas Plus Imagination Equals More Fun*. New York: McGraw-Hill Book Co., 1961-62. (McGraw-Hill Ryerson Limited, Scarborough, Ontario.)

Bitter, Gary G., and Metos, Thomas H. *Exploring with Pocket Calculators*. New York: Julian Messner, Inc., 1977. (Musson Book Company, Don Mills, Ontario.)

Burns, Marilyn. *The Book of Think: Or How to Solve a Problem Twice Your Size*. Boston, Mass.: Little, Brown and Co., 1976. (Little, Brown and Co. (Canada) Ltd., Toronto, Ontario.)

______. *The Good Time Math Event Book*. Mountain View, Calif.: Creative Publications, 1977. (SETSCO Educational Ltd., Coquitlam, B.C.)

______. *The I Hate Mathematics! Book*. Boston, Mass.: Little, Brown and Co., 1975. (Little, Brown and Co. (Canada) Ltd., Toronto, Ontario.)

Charosh, Mannis. *Mathematical Games for One or Two*. New York: Thomas Y. Crowell Co., Inc., 1972. (Fitzhenry and Whiteside Ltd., Don Mills, Ontario.)

______. *Number Ideas Through Pictures*. New York: Thomas Y. Crowell Co., Inc., 1974. Fitzhenry and Whiteside Ltd., Don Mills, Ontario.)

Dennis, J. Richard. *Fractions Are Parts of Things*. New York: Thomas Y. Crowell Co., Inc., 1971. (Fitzhenry and Whiteside Ltd., Don Mills, Ontario.)

Dilson, Jesse. *Abacus*. New York: St. Martin's Press, Inc., 1968. (Macmillan Company of Canada Ltd., Toronto, Ontario.)

Fago, Vincent. *Fun with Numbers*. West Haven, Conn.: Pendulum Press, Inc., 1974. (School Book Fairs Ltd., Mississauga, Ontario.)

Feravolo, Rocco V. *Wonders of Mathematics*. New York: Dodd, Mead and Co., 1963. (Dodd, Mead and Company (Canada) Ltd., Toronto, Ontario.)

Froman, Robert. *Less Than Nothing Is Really Something*. New York: Thomas Y. Crowell Co., Inc., 1973. (Fitzhenry and Whiteside Ltd., Don Mills, Ontario.)

Gardiner, Martin. *Arrow Book of Brain Teasers*. New York: Scholastic Book Services, 1967. (Scholastic-TAB Publications Ltd., Richmond Hill, Ontario.)

Gregor, Arthur. *One, Two, Three, Four, Five Verses*. Philadelphia, Pa.: J.B. Lippincott Co., 1956. (McClelland and Stewart Limited, Toronto, Ontario.)

Hartman, Arlene. *Calculator Game Book For Kids of All Ages*. New York: New American Library, 1977. (Macmillan Company of Canada Ltd., Toronto, Ontario.)

Jonas, Arthur. *New Ways in Math*. Englewood Cliffs, N.J.: Prentice-Hall, Inc., 1962. (Prentice-Hall of Canada Ltd., Scarborough, Ontario.)

______. *More New Ways in Math*. Englewood Cliffs, N.J.: Prentice-Hall, Inc., 1964. (Prentice-Hall of Canada Ltd., Scarborough, Ontario.)

Kettelkamp, Larry. *Puzzle Patterns*. New York: William Morrow and Co., Inc., 1963. (Gage Publishing Limited, Agincourt, Ontario.)

Luce, Marnie. *Counting Systems: The Familiar and the Unusual*; *One Is Unique*; *Primes Are Builders*; *Ten: Why Is It Important?*; *Zero Is Something*. Minneapolis, Minn.: Lerner Publications Co., 1969. (J.M. Dent and Sons (Canada) Ltd., Don Mills, Ontario.)

______, and Lerner, A.B. *Infinity: What Is It*. Minneapolis, Minn.: Lerner Publications Co., 1969. (J.M. Dent and Sons (Canada) Ltd., Don Mills, Ontario.)

Martin, Bill, Jr. *Wise Owl Books: Number Patterns Make Sense*; *Millions of People*; *I've Got Your Number, John*. New York: Holt, Rinehart and Winston, Inc., 1971. (Holt, Rinehart and Winston of Canada Ltd., Toronto, Ontario.)

O'Brien, Thomas C. *Odds and Evens*. New York: Thomas Y. Crowell Co., Inc., 1971. (Fitzhenry and Whiteside Ltd., Don Mills, Ontario.)

Olney, Ross, and Olney, Pat. *Pocket Calculator Fun and Games*. New York: Franklin Watts, Inc., 1977. (Grolier Limited, Toronto, Ontario.)

St. John, Glory. *How to Count Like a Martian*. New York: Henry Z. Walck, Inc., 1975. (Nelson, Foster and Scott Ltd., Don Mills, Ontario.)

Shimek, William. *Patterns: What Are They?* Minneapolis, Minn.: Lerner Publications Co., 1969. (J. M. Dent and Sons (Canada) Ltd., Don Mills, Ontario.)

Smith, David E. *Number Stories of Long Ago*. Book Tower, Detroit, Mich.: Gale Research Co., 1973.

Walls, Fred. *First Book of Puzzles and Brain Twisters*. New York: Franklin Watts, Inc., 1970. (Grolier Limited, Toronto, Ontario.)

Weiss, Irvin. *Zero to Zillions: The Arrow Book of Number Magic*. New York: Scholastic Book Services, 1969. (Scholastic-TAB Publications Ltd., Richmond Hill, Ontario.)

Whitney, David C. *The Easy Book of Fractions*. New York: Franklin Watts, Inc., 1970. (Grolier Limited, Toronto, Ontario.)

______. *The Easy Book of Sets*. New York: Franklin Watts, Inc., 1972. (Grolier Limited, Toronto, Ontario.)

Xerox Educational Services. *Math Game and Puzzles*. New York: Grosset and Dunlap, Inc., 1977. (Fitzhenry and Whiteside Ltd., Don Mills, Ontario.)

Measurement

Adler, Irving, and Adler, Ruth. *The Calendar*. New York: John Day Co., Inc., 1967. (Longman Canada Limited, Don Mills, Ontario.)

Adler, Peggy, and Adler, Irving. *Metric Puzzles*. New York: Franklin Watts, Inc., 1977. (Grolier Limited, Toronto, Ontario.)

Bendick, Jeanne. *Measuring*. New York: Franklin Watts, Inc., 1971. (Grolier Limited, Toronto, Ontario.)

Bitter, Gary G., and Metos, Thomas H. *Exploring with Metrics*. New York: Julian Messner, Inc., 1975. (Musson Book Company, Don Mills, Ontario.)

Brindze, Ruth. *The Story of Our Calendar*. New York: Vanguard Press, Inc., 1949. (Gage Publishing Limited, Agincourt, Ontario.)

Burns, Marilyn. *This Book Is About Time*. Boston, Mass.: Little, Brown and Company, 1978. (Little, Brown and Co. (Canada) Ltd., Toronto, Ontario.)

Carona, Philip B. *Things That Measure*. Englewood Cliffs, N.J.: Prentice-Hall, Inc., 1962. (Prentice-Hall of Canada Ltd., Scarborough, Ontario.)

Froman, Robert. *Bigger and Smaller*. New York: Thomas Y. Crowell Co., Inc., 1971. (Fitzhenry and Whiteside Ltd., Don Mills, Ontario.)

Gallant, Roy A. *Man the Measurer*. New York: Doubleday and Co., Inc., 1972. (Doubleday Canada Limited, Toronto, Ontario.)

Hahn, James, and Hahn, Lynn. *The Metric System*. New York: Franklin Watts, Inc., 1975. (Grolier Limited, Toronto, Ontario.)

Hallamore, Elizabeth. *The Metric Book of Amusing Things To Do*. Toronto, Ontario: Greey de Pencier Publications Ltd., 1974.

Kettelkamp, Larry. *Kites*. New York: William Morrow and Co., Inc., 1959. (Gage Publishing Limited, Agincourt, Ontario.)

Lamm, Joyce. *Let's Talk About the Metric System*. Middle Village, N.Y.: Jonathan David Publishers, Inc., 1974.

Lieberg, Owen S. *Wonders of Measurement*. New York: Dodd, Mead and Co., 1972. (Dodd, Mead and Company (Canada) Ltd., Toronto, Ontario.)

Luce, Marnie. *Measurement: How Much? How Many? How Far?* Minneapolis, Minn.: Lerner Publications Co., 1969. (J. M. Dent and Sons (Canada) Ltd., Don Mills, Ontario.)

Maher, John E. *Ideas About Measuring and Accounting*. New York: Franklin Watts, Inc., 1974. (Grolier Limited, Toronto, Ontario.)

Pine, Tillie S., and Levine, Joseph. *Measurements and How We Use Them*. New York: McGraw-Hill Book Co., 1974. (McGraw-Hill Ryerson Limited, Scarborough, Ontario.)

Podendorf, Illa. *One Hundred One Science Experiments*. New York: Grosset and Dunlap, Inc., 1974. (Fitzhenry and Whiteside Ltd., Don Mills, Ontario.)

Rice, Stanley. *Tell Time*. New York: Harcourt Brace Jovanovich, Inc., 1963. (Longman Canada Limited, Don Mills, Ontario.)

Russell, Solveig Paulson. *Size, Distance, Weight: A First Look at Measuring*. New York: Henry Z. Walck, 1968. (Nelson, Foster and Scott Ltd., Don Mills, Ontario.)

Schlein, Miriam. *Metric: The Modern Way to Measure*. New York: Harcourt Brace Jovanovich, Inc., 1977. (Longman Canada Limited, Don Mills, Ontario.)

______. *Metric: The New Measurement*. New York: Harcourt Brace Jovanovich, Inc., 1975. (Longman Canada Limited, Don Mills, Ontario.)

Shimek, William J. *The Celsius Thermometer*; *The Gram*; *The Liter*; *The Meter*; *The Metric System*. Minneapolis, Minn.: Lerner Publications Co., 1975. (J. M. Dent and Sons (Canada) Ltd., Don Mills, Ontario.)

Srivastava, Jane Jonas. *Area*. New York: Thomas Y. Crowell Co., Inc., 1974. (Fitzhenry and Whiteside Ltd., Don Mills, Ontario.)

______. *Averages*. New York: Thomas Y. Crowell Co., Inc., 1975. (Fitzhenry and Whiteside Ltd., Don Mills, Ontario.)

______. *Statistics*. New York: Thomas Y. Crowell Co., Inc., 1973. (Fitzhenry and Whiteside Ltd., Don Mills, Ontario.)

______. *Weighing and Balancing*. New York: Thomas Y. Crowell Co., Inc., 1972. (Fitzhenry and Whiteside Ltd., Don Mills, Ontario.)

Tallarico, Tony. *The Metric Activity Book*. New York: Grosset and Dunlap, Inc., 1977. (Grosset and Dunlap, Inc., Scarborough, Ontario.)

Webber, Irma E. *Bits That Grow Big*. Reading, Mass.: Addison-Wesley Publishing Co., Inc., 1949. (Addison-Wesley (Canada) Ltd., Don Mills, Ontario.)

Zim, Herbert S., and Skelly, James R. *Metric Measure*. New York: William Morrow and Co., Inc., 1974. (Gage Publishing Limited, Agincourt, Ontario.)

Geometry

Adler, Irving, and Adler, Ruth. *Directions and Angles*. New York: John Day Co., Inc., 1969. (Fitzhenry and Whiteside Ltd., Don Mills, Ontario.)

Bendick, Jeanne. *Shapes*. New York: Franklin Watts, Inc., 1968. (Grolier Limited, Toronto, Ontario.)

______, and Levin, Marcia. *Take Shapes, Lines and Letters: New Horizons in Mathematics*. New York: McGraw-Hill Book Co., 1962. (McGraw-Hill Ryerson Limited, Scarborough, Ontario.)

Charosh, Mannis. *Straight Lines, Parallel Lines, Perpendicular Lines*. New York: Thomas Y. Crowell Co., Inc., 1972. (Fitzhenry and Whiteside Ltd., Don Mills, Ontario.)

Diggins, Julia E. *String, Straightedge, and Shadow: The Story of Geometry*. New York: Viking Press, Inc., 1965. (Penguin Books Canada Limited, Markham, Ontario.)

Downer, Marion. *Discovering Design*. New York: Lothrop, Lee and Shepard Co., 1947. (Gage Publishing Limited, Agincourt, Ontario.)

Ellison, Elise C. *Fun with Lines and Curves*. New York: Lothrop, Lee and Shepard Co., 1972. (Gage Publishing Limited, Agincourt, Ontario.)

Fago, Vincent: *Fun with Figures*. New York: Pantheon Books, Inc., 1974. (Random House of Canada Limited, Mississauga, Ontario.)

Fowler, H. Waller, Jr. *Kites: A Practical Guide to Kite Making and Flying*. New York: Ronald Press Co., 1953. (John Wiley and Sons Canada Ltd., Rexdale, Ontario.)

Frederique and Papy (Frederique Papy and Georges Papy). *Graph Games*. New York: Thomas Y. Crowell Co., Inc., 1971. (Fitzhenry and Whiteside Ltd., Don Mills, Ontario.)

Luce, Marnie. *Points, Lines, and Planes*; *Polygons: Points in a Plane*; *Polyhedrons: Intersecting Planes*. Minneapolis, Minn.: Lerner Publications Co., 1969. (J. M. Dent and Sons (Canada) Ltd., Don Mills, Ontario.)

Murray, William D., and Rigney, Francis J. *Paper Folding for Beginners*. New York: Dover Publications, Inc., 1960. (General Publishing Co. Limited, Don Mills, Ontario.)

Payne, G.C. *Fun with Shapes and Patterns*. New York: International Publications Service, 1975.

Phillips, Jo. *Right Angles: Paper Folding Geometry*. New York: Thomas Y. Crowell Co., Inc., 1972. (Fitzhenry and Whiteside Ltd., Don Mills, Ontario.)

Sitomer, Mindel, and Sitomer, Harry. *Circles*. New York: Thomas Y. Crowell Co., Inc., 1973. (Fitzhenry and Whiteside Ltd., Don Mills, Ontario.)

______. *Lines, Segments, Polygons*. New York: Thomas Y. Crowell Co., Inc., 1972. (Fitzhenry and Whiteside Ltd., Don Mills, Ontario.)

______. *What Is Symmetry?* New York: Thomas Y. Crowell Co., Inc., 1970. (Fitzhenry and Whiteside Ltd., Don Mills, Ontario.)

Slade, Richard. *Geometrical Patterns*. Levittown, N.Y.: Transatlantic Arts, Inc., 1970.

______. *Patterns in Space*. Salem, New Hampshire: Faber and Faber, Inc., 1969. (Oxford University Press, Don Mills, Ontario.)

Struble, Mitch. *Stretching a Point*. Philadelphia, Pa.: Westminster Press, 1971. (McGraw-Hill Ryerson Limited, Scarborough, Ontario.)

Teacher's Bibliography

Ballew, Hunter. *Teaching Children Mathematics*. Columbus, Ohio: Charles E. Merrill Publishing Company, 1973. (Charles E. Merrill Publishing, Weston, Ontario.)

Baratta-Lorton, Mary. *Mathematics Their Way*. Reading, Mass.: Addison-Wesley Publishing Co., Inc., 1976. (Addison-Wesley (Canada) Ltd., Don Mills, Ontario.)

Biggs, Edith, and MacLean, James R. *Freedom to Learn: An Active Learning Approach to Mathematics*. Don Mills, Ontario: Addison-Wesley (Canada) Ltd., 1969.

Boyer, Carl B. *A History of Mathematics*. New York: John Wiley & Sons, Inc., 1968. (John Wiley and Sons Canada Ltd., Rexdale, Ontario.)

Brumfiel, C.F., and Krause, E. *Elementary Mathematics for Teachers*. Reading, Mass.: Addison-Wesley Publishing Co., Inc., 1969. (Addison-Wesley (Canada) Ltd., Don Mills, Ontario.)

Copeland, Richard W. *How Children Learn Mathematics*. New York: Macmillan Publishing Co., Inc., 1979. (Collier Macmillan Canada, Ltd., Cambridge, Ontario.)

______. *Mathematics and the Elementary Teacher*. Philadelphia, Pa.: W. B. Saunders Company, 1976. (W. B. Saunders Co. Canada Ltd., Toronto, Ontario.)

D'Augustine, Charles H. *Multiple Methods of Teaching Mathematics in the Elementary School*. New York: Harper & Row, Publishers, 1973. (Fitzhenry and Whiteside Ltd., Don Mills, Ontario.)

Devine, Donald F., and Kaufman, Jerome E. *Mathematics for Elementary Education*. New York: John Wiley & Sons, Inc., 1974. (John Wiley and Sons Canada Ltd., Rexdale, Ontario.)

Dienes, Zoltan P. *The Six Stages in the Process of Learning Mathematics*. Atlantic Highlands, N.J.: Humanities Press, Inc., 1974.

Dixon, Lyle J. *Mathematics for Elementary Teachers*. Columbus, Ohio: Charles E. Merrill Publishing Company, 1974. (Charles E. Merrill Publishing, Weston, Ontario.)

Dubisch, Roy. *Basic Concepts of Mathematics for Elementary Teachers*. Reading, Mass.: Addison-Wesley Publishing Co. Inc., 1977. (Addison-Wesley (Canada) Ltd., Don Mills, Ontario.)

Dwight, Leslie. *Modern Mathematics for the Elementary Teacher*. New York: Holt, Rinehart and Winston, Inc., 1966. (Holt, Rinehart and Winston of Canada Ltd., Toronto, Ontario.)

Elliott, H. Andrew, MacLean, James R., and Jorden, Janet Mary. *Geometry in the Classroom: New Concepts and Methods*. Toronto, Ontario: Holt, Rinehart and Winston of Canada Ltd., 1968.

Fehr, Howard F., and Phillips, Jo M. *Teaching Modern Mathematics in the Elementary School*. Reading, Mass.: Addison-Wesley Publishing Co., Inc., 1972. (Addison-Wesley (Canada) Ltd., Don Mills, Ontario.)

Forbes, Jack E., and Eicholz, Robert E. *Mathematics for Elementary Teachers*. Reading, Mass.: Addison-Wesley Publishing Co., Inc., 1971. (Addison-Wesley (Canada) Ltd., Don Mills, Ontario.)

Glennon, Vincent J., and Callahan, Leroy, G. *Elementary School Mathematics: A Guide to Current Research*. Reston, Virginia: N.C.T.M., 1975.

Green, George F. *Elementary School Mathematics: Activities and Materials*. Lexington, Mass.: D. C. Heath & Company, 1974. (D. C. Heath Canada Ltd., Toronto, Ontario.)

Grossnickle, F.E., and Brueckner, L.J. *Discovering Meanings in Elementary School Mathematics*. New York: Holt, Rinehart and Winston, Inc., 1973. (Holt, Rinehart and Winston of Canada Ltd., Toronto, Ontario.)

Jensen, Rosalie. *Exploring Mathematical Concepts and Skills in the Elementary School*. Columbus, Ohio: Charles E. Merrill Publishing Company, 1973. (Charles E. Merrill Publishing, Weston, Ontario.)

Kramer, Klass. *Teaching Elementary School Mathematics*. Boston, Mass.: Allyn & Bacon, Inc., 1978. (Allyn and Bacon Canada Ltd., Toronto, Ontario.)

Marks, J.L. *Teaching Elementary School Mathematics for Understanding*. New York: McGraw-Hill Book Company, 1975. (McGraw-Hill Ryerson Limited, Scarborough, Ontario.)

May, Lola J. *Teaching Mathematics in the Elementary School*. New York: The Free Press, 1974. (Collier Macmillan Canada, Ltd., Cambridge, Ontario.)

National Council of Teachers of Mathematics. *Topics in Mathematics for Elementary School Teachers (Twenty-Ninth Yearbook)*. Reston, Virginia: N.C.T.M., 1964.

______. *More Topics in Mathematics for Elementary School Teachers (Thirtieth Yearbook)*. Reston, Virginia: N.C.T.M., 1969.

Polya, Gyorgy. *Mathematical Discovery on Understanding, Learning, and Teaching Problem Solving*. New York: John Wiley & Sons, Inc., 1962. (John Wiley and Sons Canada Ltd., Rexdale, Ontario.)

Reisman, Fredricka K. *Diagnostic Teaching of Elementary School Mathematics*. Chicago: Rand McNally & Co., 1977. (Gage Publishing Limited, Agincourt, Ontario.)

Riedesel, C. Alan. *Guiding Discovery to Elementary School Mathematics*. New York: Appleton-Century-Crofts, 1973. (Prentice-Hall of Canada Ltd., Scarborough, Ontario.)

______, and Burns, Paul C. *Handbook for Exploratory and Systematic Teaching of Elementary School Mathematics*. New York: Harper & Row Publishers, Inc., 1977. (Fitzhenry and Whiteside Ltd., Don Mills, Ontario.)

______, and Callahan, Leroy G. *Elementary School Mathematics for Teachers*. New York: Harper & Row Publishers, Inc., 1977. (Fitzhenry and Whiteside Ltd., Don Mills, Ontario.)

Rising, Gerald R., and Harkin, Joseph B. *The Third "R". Mathematics Teaching for Grades K-8*. Belmont, Calif.: Wadsworth Publishing Co., Inc., 1978.

Schminke, C. *Teaching the Child Mathematics*. New York: Holt, Rinehart and Winston, Inc., 1973. (Holt, Rinehart and Winston of Canada Ltd., Toronto, Ontario.)

Schultz, James E. *Mathematics for Elementary School Teachers*. Columbus, Ohio: Charles E. Merrill Publishing Company, 1977. (Charles E. Merrill Publishing, Weston, Ontario.)

Smith, Seaton E., Jr. *Bulletin Board Ideas for Elementary and Middle School Mathematics*. Reston, Virginia: N.C.T.M., 1977.

______, and Backman, Carl A. (editors) *Teacher-Made Aids for Elementary School Mathematics*: Readings from the *Arithmetic Teacher*. Reston, Virginia: N.C.T.M., 1974.

Turner, Ethel M. *Teaching Aids for Elementary Mathematics: A Source Book for Elementary Teachers*. New York: Holt, Rinehart and Winston, Inc., 1966. (Holt, Rinehart and Winston of Canada Ltd., Toronto, Ontario.)

Underhill, Robert G. *Teaching Elementary School Mathematics*. Columbus, Ohio: Charles E. Merrill Publishing Company, 1977. (Charles E. Merrill Publishing, Weston, Ontario.)

Weiss, Sol. *Elementary Mathematics: Teaching Suggestions and Strategies*. Boston: Prindle, Weber & Schmidt, Inc., 1978.

Westcott, Alvin M., and Smith, James A. *Creative Teaching of Mathematics in the Elementary School*. Boston: Allyn & Bacon, Inc., 1978. (Allyn and Bacon Canada Ltd., Toronto, Ontario.)

Williams, Elizabeth and Shuard, Hilary. *Elementary Mathematics Today: A Teacher Resource*. Reading, Mass.: Addison-Wesley Publishing Co., Inc., 1972. (Addison-Wesley (Canada) Ltd., Don Mills, Ontario.)

Scope and Sequence

	Grade 4	Grade 5	Grade 6
NUMBER AND NUMERATION	Read, write numerals to 999 999 Place value for six-digit numbers Compare, order numbers to 999 999 Ordinal number concepts to 999 Round in tens' place, in hundreds' place, in thousands' place Roman numerals for 1 to 100	Read, write numerals to 999 999 999 Place value for nine-digit numbers Compare, order numbers 999 999 Round in ones' place, in tens' place, or in hundreds' place of either of the first two periods Roman numerals for 1 to 2000	Read, write numerals to 999 999 999 999 Place value for twelve-digit numbers Compare, order numbers to 999 999 999 Round in ones', tens', or hundreds' place of any period to millions Roman numerals for 1 to 3999 Read, write, compare, order integers
ADDITION	Algorithm, four-digit addends Algorithm, three or more addends Add decimals, to hundredths Round addends, estimate sum; compute result and compare sums Add amounts of money Add proper and mixed-form fractions with like denominators, regrouping for whole-number sums	Algorithm, five-digit and six-digit addends Algorithm, three or more addends Add decimals, to thousandths Round addends, estimate sum; compute result and compare sums Add amounts of money Add proper and mixed-form fractions with like denominators	Algorithm, six-digit addends Algorithm, three or more addends Add decimals, to ten-thousandths Round addends, estimate sum; compute result and compare sums Add amounts of money Add proper and mixed-form fractions with unlike denominators Add integers
SUBTRACTION	Algorithm, four-digit minuends, four-digit subtrahends Check by addition Subtract decimals, to hundredths Subtract amounts of money Subtract proper and mixed-form fractions with like denominators, regrouping of whole-number minuends	Algorithm, five-digit and six-digit minuends Subtract decimals, to thousandths Round and subtract to estimate the difference; compute difference and compare with estimate Subtract amounts of money Subtract proper and mixed-form fractions with like denominators	Algorithm, six-digit minuends Subtract decimals, to ten-thousandths Round and subtract to estimate the difference; compute difference and compare with estimate Subtract amounts of money Subtract proper and mixed-form fractions with unlike denominators Subtract integers, using temperatures as a model
MULTIPLICATION	Algorithm, multiplicand to three digits, multiplier to two digits Multiply up to four one-digit numbers Determine missing factor in multiplication fact Multiply one-place decimal by one-digit whole number Round three-digit multiplicand in its greatest place and multiply by one-digit multiplier to estimate product Round decimal multiplicand to nearest whole number and multiply to estimate product Multiply amounts of money Identify multiples of numbers	Algorithm, multiplicand to five digits, multiplier to three digits Identify prime numbers, prime factors, common multiples of a pair of whole numbers Multiply decimal tenths, hundredths, thousandths by whole numbers Multiply decimal tenths by decimal tenths Round factors, estimate product; compute result and compare products Multiply amounts of money Multiply a fraction and a whole number which is a multiple of the denominator Find equivalent fractions, ratios, rates	Algorithm, multiplicand to seven digits, multiplier to three digits Identify prime numbers, composite numbers, prime factors, common multiples of a pair of whole numbers Multiply decimal and whole number, products to ten-thousandths Multiply decimal by decimal, products to ten-thousandths Round factors, estimate product; compute result and compare products Multiply amounts of money Multiply fraction by fraction Multiply fraction and whole number Find equivalent fractions, ratios, rates
DIVISION	Algorithm, dividend to three digits, one-digit divisor Check by multiplying Divide whole-dollar amounts Find average of set of numbers	Algorithm, dividend to six digits, divisor to two digits Check by using multiplication Divide to thousandths by one-digit or two-digit whole numbers Round dividend, divisor to estimate quotient; compute quotient and compare with estimate Divide amounts of money Find equivalent fractions, ratios, rates Find decimal equivalent to a fraction Find average of a set of numbers and compare it with the given numbers	Algorithm, dividend to six digits, divisor to three digits Check by using multiplication Divide decimal by whole number and by decimal, extra zeros in dividend, divisor to three digits Round dividend, divisor to estimate quotient; compute quotient and compare with estimate Round decimal quotients Divide amounts of money Divide fraction by fraction Divide fraction by whole number Divide whole number by fraction Find equivalent fractions, ratios, rates Find decimal equivalent to a fraction Find average of a set of numbers and compare it with the given numbers

Scope and Sequence

	Grade 4	Grade 5	Grade 6
DECIMALS AND FRACTIONS	Proper and mixed-form halves, thirds, fourths, fifths, tenths, hundredths for part-of-whole and part-of-set models Equivalent decimal tenths, hundredths with part-of-whole model Place value for decimals to hundredths; regrouping Equivalent decimals and proper and mixed-form fractions (100, 10, 4, 2 as denominators) Compare, order decimals to hundredths Add, subtract decimals, to hundredths Multiply one-place decimal by one-digit whole number Round decimal to nearest whole number, estimate sum or product Compare and order fractions using decimal equivalents and models Add, subtract proper and mixed-form fractions with like denominators, regrouping to/from whole numbers	Proper and mixed-form fractions for part-of-whole, part-of-set models Decimal thousandths Equivalent decimal tenths, hundredths, and thousandths Place value for decimals to thousandths; regrouping Compare, order decimals to thousandths Round decimal to nearest whole number, to nearest tenth Add, subtract decimals, to thousandths Multiply decimal tenths, hundredths, thousandths by whole number Multiply decimal tenths by decimal tenths Round decimal factor(s) to whole number(s) and multiply to estimate product Divide decimal by whole number, "terminating" quotients Find equivalent fractions Find the missing term in a pair of equivalent fractions Compare fractions (one denominator not necessarily a multiple of the other) Convert between decimals and fractions (halves, fourths, fifths, eighths, tenths) Convert between improper fractions and mixed-form fractions Add, subtract proper and mixed-form fractions with like denominators Multiply a fraction and a whole number which is a multiple of the denominator	Proper and mixed-form fractions for part-of-whole, part-of-set models Decimal ten-thousandths Find equivalent decimals Place value for decimals to ten-thousandths; regrouping Compare, order decimals Round decimals Add, subtract decimals, to ten-thousandths Estimate sums and differences Multiply decimal and whole number, products to ten-thousandths Multiply decimal by decimal, products to ten-thousandths Divide decimal by whole number and by decimal, extra zeros in dividend, divisor to three digits Round decimal quotients Estimate products and quotients Find equivalent fractions Find the missing term in a pair of equivalent fractions Find like denominators Compare, order fractions Convert between decimals and fractions Convert between improper fractions and mixed-form fractions Add, subtract proper and mixed-form fractions with unlike denominators Multiply fractions by fractions Multiply fractions and whole numbers Identify, find reciprocals Divide fractions by fractions, fractions and whole numbers
PROBLEM SOLVING	Use models to obtain solutions Choose the operation needed Identify relevant, irrelevant, missing information Write an equation for a word problem Multiple-step solutions; addition, subtraction, multiplication, comparison Estimate answers Recognize answers as reasonable Guess and test Multiple solutions Read scales Organize data Logical thinking	Identify relevant, irrelevant, missing information Recognize that different situations affect answers Find the number of possibilities Collect, organize, and display data Give the most reasonable answer Use models to obtain solutions Write and solve an equation for a word problem Multiple-step solutions Find and continue patterns Estimate answers Recognize incorrect results Logical thinking Guess and test Multiple solutions Use a calculator	Consider ways of estimating Identify relevant, irrelevant, missing information Find needed information Recognize that different situations affect answers Give reasonable measurements Find the number of possibilities Multiple-step solutions Write and solve an equation for a word problem Logical thinking Restate word problems Consider the chances Use models to obtain solutions Solve problems without pencil and paper Consider possible solutions Use a calculator
MEASUREMENT	Estimate and measure; choose the preferred linear unit Convert between kilometres and metres; metres and centimetres Measure and add to find perimeter Count, estimate, and calculate area in square centimetres Calculate area in square centimetres, square decimetres, or square metres Count, estimate the number of centimetre cubes; compare objects to centimetre cube, decimetre cube, metre cube	Estimate, measure and record lengths in appropriate units Convert among units of length Find the perimeter of a polygon, of a square, of a rectangle Measure circumference of circular objects Multiply to find area of a rectangle Create rectangular shapes having given perimeter or area Count, calculate to find volume in cubic centimetres Investigate metric prefixes Estimate capacity in millilitres, litres	Estimate, measure and record lengths in appropriate units Convert among units of length Find the perimeter of a polygon, of a square, of a rectangle Measure circumference, radius, diameter of circular objects Multiply to find area of a rectangle, a parallelogram, and a triangle Multiply to find the volume of a rectangular prism in cubic units Understand metric prefixes Estimate capacity in millilitres, litres

	Grade 4	Grade 5	Grade 6
MEASUREMENT (continued)	Compare small amounts of time, of length, of mass, of capacity Classify capacities in comparison with 1 mL, 500 mL, 1000 mL Classify masses in comparison with 1 g, 500 g, 1000 g Convert among units of time Tell and record time to the nearest minute using 12-hour and 24-hour clock; add, subtract time using 24-hour clock	Estimate mass in grams, kilograms Convert between millilitres and litres, grams and kilograms Find capacity in litres, millilitres, by finding volume in cubic centimetres Relate capacity, mass, and volume units for water Numeric dating; time to the second; add, subtract time, no regrouping	Estimate mass in grams, kilograms Convert between millilitres and litres, grams and kilograms Relate capacity, mass, and volume units for water Numeric dating; time to the second; convert among time zones in Canada; standard time, daylight saving time Read and record temperatures, including negative-number readings for temperatures below 0°C Find differences between temperatures
GEOMETRY	Identify, name, and draw lines, line segments and their end points Identify, name, and draw angles; identify and draw right angles Compare angles to right angles Identify, name, and draw triangles; identify and name sides and angles of triangles Identify and draw polygons; identify and name sides and angles of polygons Identify and draw circles; identify and name parts of circle Recognize patterns for, and properties of, solids Use tracing paper to test for congruency in general and under a slide, flip, or turn on a grid Identify and check line of symmetry as a flip line; identify multiple lines of symmetry (on a grid)	Identify, name, and draw lines, rays, and line segments Identify and draw parallel, intersecting, or perpendicular lines, rays, and line segments Identify, name, measure, and draw angles; identify congruent angles by tracing Classify angles as acute, right, or obtuse Classify plane shapes according to number of sides, relationships between sides and lines of symmetry; equilateral, isosceles, and scalene triangles; square, rhombus, rectangle, parallelogram, trapezoid Classify pyramids and prisms according to their faces Recognize cylinders, spheres, cones Identify and sketch lines of symmetry (not on grid) Use tracing paper to test for or help to draw slide, flip, or turn image (on or not on grid) Turn triangles to form regular polygons Fit polygons together to make a pattern without spaces Copy picture from one grid onto another grid (including distortions)	Identify, name, and draw lines, rays, and line segments Identify and draw parallel, intersecting, or perpendicular lines, rays, and line segments Identify, name, measure, and draw angles Classify angles as acute, right, obtuse, or straight Classify plane shapes according to number of sides; equilateral, isosceles, and scalene triangles; square, rhombus, rectangle, parallelogram, trapezoid, regular polygons Identify parts of a circle; relate radius, diameter, and circumference Classify pyramids and prisms Recognize cylinders, spheres, cones Identify and sketch lines of symmetry Identify congruent shapes Identify and draw similar shapes Interpret and make scale drawings Use tracing paper to test for or help to draw slide, flip, or turn image (on or not on grid) Draw slide image using a rule Draw flip image by counting Test for rotational symmetry Make tiling patterns Identify slide, flip, and turn images in tiling patterns Copy picture from one grid onto another grid (including distortions)
GRAPHING	Interpret pictographs and bar graphs Gather and organize information in pictographs and bar graphs Associate ordered pairs of numbers (including 0 as a coordinate) and points on a grid	Interpret pictographs and bar graphs Gather and organize information in pictographs and bar graphs Relate graphing of ordered pairs of numbers to graphing of ordered pairs of data Interpret and draw line graphs	Interpret, draw pictographs, bar graphs, line graphs, and broken-line graphs Gather and organize information for graphs Relate graphing of ordered pairs of numbers to graphing of ordered pairs of data Interpret circle graphs
RATIO AND PERCENT		Write ratios using colon and fraction notation Multiply, divide to find equivalent ratios, rates Find the missing term in a pair of equivalent ratios, rates Use the symbol % for percent Convert among decimals, fractions, percents, and ratios	Write ratios using colon and fraction notation Find equivalent ratios, rates Find the missing term in a pair of equivalent ratios, rates Find unit rates, unit prices Use the symbol % for percent Convert among decimals, fractions, percents, and ratios Find a percent of a number Calculate interest, discount

Contents of Student's Book

The Mathematics Centre

A mathematics centre, like centres for other subject areas, is a place for the storage of certain specific materials and an area for the students to become involved in activities. With careful planning and involvement of the students, the mathematics centre can become a stimulating environment. The students will enjoy bringing materials from home to supplement and add variety to those in the centre. The mathematics centre is an ideal place to display the students' work. If a thematic approach is used for teaching mathematics, the centre can be adapted as a setting for each new theme.

When the students have finished their regular assignments, they may engage in extra activities and projects in the mathematics centre. Activity cards, puzzles, games, and homemade as well as commercial materials will lead the students to broader understanding as well as provide opportunities for the teacher to observe and question the students, and to evaluate their progress. Consideration of the students' interaction in the mathematics centre will suggest adaptations to make this strategy an important part of the learning experience.

STORAGE OF MATERIALS

Materials should be stored where students can have easy access to them. Open shelves and small tables can be used in a pleasing and practical arrangement for holding containers. Containers for materials should be both sturdy and colorful. Vinyl coverings, spray enamel, wallpaper, and fabric will increase the durability of the containers as well as increase the appeal of the mathematics centre. Some ideas for suitable containers are pails (formed by cutting the top off a large plastic bottle), trays (from corrugated boxes in which canned goods are sold), baskets (in which fruit and vegetables are sold), boxes (sturdy ones such as those from small appliances), and other containers (ice cream containers, large cans, plastic tubs).

MATERIALS

The materials used for teaching mathematics need not be expensive commercial materials; simple everyday objects can be used effectively as learning and teaching aids.

A list of the materials suggested for each unit is given in the unit overview. When a unit is almost completed, look ahead to the next unit and begin to collect and have the students help to collect the materials.

The following materials will be helpful for developing the various concepts and skills.

Number

- counters such as buttons, beans, pebbles, bingo chips
- objects for grouping, such as pipe cleaners, drinking straws, stirrers, beans in plastic bags, Unifix cubes
- models for ones, tens, hundreds, and thousands
- flash cards for basic addition, subtraction, multiplication, and division facts
- dominoes and domino cards, playing cards, dice
- shapes marked to show halves, thirds, fourths, fifths, eighths, and tenths of a whole
- set holders such as plastic tubs, egg cartons, plastic hoops, Styrofoam trays, paper or foil plates
- models for tenths and hundredths

Geometry

- a collection of three-dimensional shapes, such as balls, boxes, cores from rolls of paper, funnels
- wooden, plastic, or cardboard plane shapes (triangles, rectangles, squares, pentagons, hexagons, octagons, circles)
- commercial wooden or plastic plane shapes (triangles, rectangles, squares, pentagons, hexagons, octagons, circles)
- geoboards, rubber bands, geopaper from page T 396
- parquetry blocks, gummed shapes for forming patterns
- pictures of symmetrical objects and shapes
- cutouts of plane shapes for showing slides, flips, and turns
- felt, plastic, or cardboard tangram pieces
- materials for constructing models of three-dimensional shapes (straws, pipe cleaners, toothpicks, plasticine, clay)
- square tiles (ceramic tiles, floor tiles)
- samples of fabric, wallpaper, and gift wrapping paper

Measurement

- real money, play money
- non-standard units for measuring length (straws, paper clips, ribbons), capacity (jars, paper cups, milk cartons), mass (washers, plasticine balls)
- unmarked metre sticks, metre sticks and tapes marked in millimetres, centimetres, and decimetres
- rulers or straight edges
- centimetre cubes, decimetre cubes
- containers for comparing capacity (jars, bottles, cans, boxes)
- materials for filling containers (sand, water, rice)
- one-litre containers, other containers marked in litres (juice cans, pails) and in millilitres (soft-drink cans, canned goods)
- objects for comparing masses (pebbles, stones, books)
- one-kilogram masses, objects with masses marked in kilograms (boxes of detergent, bags of sugar) and in grams (boxes of cereal, pasta, or crackers)
- balance scales, kitchen scales, step-on scales
- thermometers

There are many other teaching aids that will be useful in a mathematics program, for example, D-Stix, Multibase Arithmetic Blocks. They may be acquired over a number of years, but the following aids are basic for the program.

Display Board: This may be a flannel board, a magnetic board, or a bulletin board for use with cutouts of objects, numerals, and symbols in demonstrating new concepts.

Attribute Blocks: These are sets of wooden or plastic blocks that show likenesses and differences in color, shape, size, and thickness. One set usually includes 48 pieces made up of four shapes (circle, rectangle, square, triangle), three colors (red, blue, yellow), two sizes, and two thicknesses. (Some sets also include the hexagon.)

If commercial attribute blocks are not available, you may wish to make your own blocks by using the patterns on pages T 382 to T 385. The blocks may be made from plywood of two thicknesses, one of which is about three times as thick as the other.

Number Line: A number line on the chalkboard or display board may be permanently displayed at a level where students can see it and also reach it, if necessary; for example, with number strips in showing extensions of basic addition facts. Each student should have an individual number line.

Other teaching aids are described on pages T 376 and T 377.

Timing Schedule

The following information will guide you in planning your schedule for working through *Starting Points in Mathematics 5* in one year. Depending on the abilities of the students in your class, you will find sufficient material in Book 5 for a minimum program, an average program, and an enriched program.

For this book, most lessons are developed over two pages and these are referred to as "double page" lessons in the table below. There are also "single page" lessons, for example, most *Problem Solving* lessons and *Checking Up* lessons that appear at the end of each unit.

There are approximately 175 days in the school year. The number of lessons in Book 5 is 195. This number excludes the *Checking Skills* exercises provided after the *Checking Up* pages for Units 4, 7, 9, 11, and 14 and the *Skill Practice* exercises on pages 332 to 337. No suggestions have been made for these and other maintenance lessons because the need for these can be determined only by you for the students in your particular class.

The number of days required to complete a unit will depend on the level of the students in your class. For example, students who have completed the first four books of *Starting Points in Mathematics* may be able to complete Unit 1 in 8 days or fewer than 8 days. If much of the work is new for students, you may plan to spend as many as 10 days to consolidate most of the concepts of Unit 1 at this time. Other alternatives would be to omit certain lessons and teach them at a later time. For example, the work on nine-place numerals on pages 6 and 7 may be left until the lessons in Unit 3 which involve a few products having seven digits. The work of Roman numerals can provide a welcome diversion during Unit 5 (division), Unit 6 (decimals), or Unit 13 (fractions).

Since the work of Unit 2 (addition and subtraction) will likely be review for most students, you may plan to begin the program with this unit. At the same time the work of Unit 1 may be introduced gradually. In this way students will have ample opportunity to practice skills with which they are already familiar and still be provided with the challenge of learning new concepts.

Teachers wishing to provide a minimum program may plan to omit part or parts of certain units and, depending on the students' progress, return to some of these later in the year. Also, if the content of Unit 15 (ratio) is beyond the scope of the curriculum guidelines or if there is insufficient time to complete the unit, it may be omitted because the topic is redeveloped in *Starting Points in Mathematics 6*. For skill-developmental lessons, it may not be necessary to assign all the exercises, but only sufficient exercises to ensure that the skills have been mastered. For example, you might assign the odd-numbered exercises and if these are reasonably well completed, there is probably no need for further practice. If, on the other hand, there seem to be difficulties, reteaching and review should be provided before assigning the even-numbered exercises. In planning any schedule, it should also be kept in mind that certain topics such as measurement (Units 7 and 11) and geometry (Units 9 and 14) require more time than others as they involve more activity with concrete objects and manipulative materials.

	Number of Lessons				
Unit	Double Page	Single Page	Lessons in Unit	Number of Days	My Schedule
1	6	2	8	6-10	
2	9	8	17	13-17	
3	9	4	13	11-14	
4	6	2	8	8-10	
5	8	4	12	11-13	
6	11	4	15	12-16	
7	7	7	14	12-15	
8	7	8	15	13-16	
9	9	5	14	12-15	
10	8	6	14	12-15	
11	8	4	12	10-13	
12	8	4	12	10-13	
13	12	6	18	14-18	
14	13	0	13	13-15	
15	6	4	10	8-10	
Total	127	68	195	165-210	

Unit 1 Overview

Numeration

In this unit, the use of numbers in everyday life is explored, with particular emphasis on numeration in our base-ten number system. Place value is analyzed in numerals having up to six places (hundred thousands). The study of numerals having up to nine places (hundred millions) is optional. Expanded notation is presented to point out two aspects of numeration: each digit of a numeral represents a value equal to the product of its face value and its place value; the value of a numeral in standard form is the sum of all the values represented by the digits. Students are shown how to compare and order numbers by analyzing numerals from left to right until they find differences in digits. The rounding of numbers is included in this unit as a foundation for developing skills of estimation in later units. Roman numerals for numbers to 2000 are presented and compared to numerals in our own system of numeration. The last lesson in this unit points out the importance of being able to obtain information from a variety of sources.

Prerequisite Skills

- read and write standard numerals for numbers to 999
- interpret place value in numerals to 999
- write the expanded form for numbers to 999
- compare and order numbers to 999
- name numbers in terms of other numbers, sometimes using addition or subtraction

Unit Outcomes

- read and write standard numerals for numbers to 999 999 999
- interpret place value in numerals to 999 999 999
- write the expanded form for numbers to 999 999 999
- compare and order numbers to 999 999
- round numbers to the nearest thousand, ten thousand, or hundred thousand
- read and write Roman numerals for numbers to 2000
- find the information needed to solve a problem

Background

Two concepts of number are used in everyday life. The more common and more significant is the *cardinal* concept, which refers to the quantitative value expressed. For instance, the librarian's report might show that there are sixteen (16) encyclopedias in the library. The cardinal number "sixteen" and the numeral 16 indicate *how many*. The second basic concept is found in the *ordinal* use, by which the number indicates *which one* in a series of consecutive numbers. The numbers in dates are ordinals. For instance, the year 1984 is one of a series of years, following 1983 and preceding 1985. Most ordinal numbers may also be interpreted to indicate a quantity, but this requires a consideration of the particular one, combined with all the preceding ones in the series. There are a few exceptions to this, however, because sometimes the series is not consecutive. For example, 97 identifies a major highway in one province, but there is no highway 96. Therefore, in this case, 97 is not a cardinal number, neither is it a true ordinal number, although it has ordinal value by identifying a particular highway.

In this unit the words *number*, *numeral*, and *digit* are used. There is often confusion about their correct usage. Briefly, a number is an abstract idea which can be represented by a numeral. A numeral may be a single digit, such as 6, or an arrangement of several digits, such as 235, 27 468, and 145 670. In the Hindu-Arabic system there are ten digits, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, which can be used to represent many different values according to their positions in the numerals. In our base-ten numeration system, the value of each place is a power of ten, increasing from right to left. The value of any one digit in a numeral is essentially a combination of three values: *base*, *face*, and *place*. In our system of numeration the base is ten, the place values are powers of ten, and each digit has its own face value. Therefore, in 6345 the 4 has a value of 4×10, or 4 tens (40), the 3 has a value of $3 \times 10 \times 10$, or 3 hundreds (300), and the 6 has a value of $6 \times 10 \times 10 \times 10$, or 6 thousands (6000). The 5 refers to ones, thus it has a value of 5. These values are shown in the expanded form $6000 + 300 + 40 + 5$. This form also shows the addition of values, which is implied but not shown, in the standard form 6345.

The Roman system of numeration uses seven different symbols, I, V, X, L, C, D, and M, in a variety of combinations. In contrast to the base-ten system, it changes symbols for higher powers. Whereas in the Hindu-Arabic system the 5 in 45 means 5 ones and the 5 in 520 means 5 hundreds, the Roman symbol V is used only for 5 ones and a different symbol, D, is used for 5 hundreds. The Roman system uses both addition and subtraction of the values represented and in this connection the positions of symbols are significant. If a symbol of equal or lesser value is on the right of another symbol, their values are added; but if a symbol of lesser value is on the left of another symbol, the true value of the expression is their difference. For instance, CX and XC both show symbols for one ten and one hundred: in the first case, their values are added (110); in the second case, their values are subtracted (90). The symbols I, X, C, and M may be repeated up to three times (showing addition), but may be used on the left of a greater-value symbol only once (showing subtraction).

To interpret a Roman numeral it is necessary to separate it into groups of symbols which represent specific values, starting at the left. The Roman numeral MDCCCXLVII may be grouped as M DCCC XL VII $(1000 + 800 + 40 + 7)$. In the Hindu-Arabic system, if one numeral has more digits than another it is obviously greater; 1243 is greater than 785. But in the Roman system the number of digits does not have any bearing on the size of the number represented, as shown by DCCCLXXVIII (878) and MMD (2500).

In the base-ten numeration system, digits are considered in groups of three starting from the right. Each group is called a *period* with ones, tens, and hundreds recurring for ones, for thousands, and for millions.

millions' period			thousands' period			ones' period		
h	t	o	h	t	o	h	t	o
3	5	0	4	6	9	7	0	0

In the past, commas have been written between the periods, but now it is customary to leave a space between two consecutive periods, except in cases where not more than four digits are involved. Note how this style is used in working with numbers in the operations.

$$\begin{array}{r} 1934 \\ 1862 \\ +\ 518 \\ \hline \end{array} \qquad \begin{array}{r} 9\ 135 \\ 17\ 648 \\ +\ 6\ 278 \\ \hline \end{array} \qquad \begin{array}{r} 13\ 745 \\ -\ 8\ 892 \\ \hline \end{array}$$

Teaching Strategies

Numeration involving numerals with up to nine digits to show millions may be too advanced for some students at the beginning of the school year or may even be beyond the scope of the program in some schools. For either of these situations, it is suggested that pages 6 and 7 be omitted and only the appropriate exercises on pages 9 and 11 be assigned. Omitting this material will have no impact on later development. Work in addition, subtraction, multiplication, and division in Units 2, 3, and 5 is limited to numbers not greater than ten thousands, with a few exceptions. An alternative approach would be to begin the year's program with Unit 2 in which the familiar operations of addition and subtraction are reviewed and extended. Thus, Unit 1 could be used later in the year as a change of pace from work in other units. Before beginning the unit, you may wish to administer the diagnostic test included at the end of this overview to determine whether students understand numeration concepts for numbers to 9999.

In treating the topic of numeration, emphasis should be given to the place values within periods. Reading a numeral with many places requires only an ability to read meaningfully up to three digits at a time and to ascribe to them the name of the period in which they occur. For example, 375 names the number "three hundred seventy-five" in any period, whether it be thousands, as in 375 420, or ones, as in 580 375 (or millions, as in 375 690 800). It is recommended that reading and writing numerals with up to six places be practised thoroughly in this unit, and that place-value charts be used regularly to direct attention to the names of the periods in which the digits are placed. Place-value charts are also valuable in pointing out any places in which zeros are needed, such as in "two hundred forty thousand fifty-six".

h	t	o	h	t	o
thousands			ones		
2	4	0	0	5	6

200 000
40 000
56
240 056

Expanded form of notation in a vertical arrangement may be used to show the need for zeros in the standard form and it also illustrates that the standard form is the sum of the values represented by the digits.

It is often difficult to derive precise concepts of the large numbers one encounters in everyday experiences. For this reason, rounded numbers are frequently used to convey general information in more easily understood terms. News reporters generally round large numbers. Students need practice in rounding numbers in order to be able to relate to them meaningfully. A simple method of rounding a number is to move the index finger, or a card, over the numeral from left to right, covering the place value to which the number is to be rounded. The next digit to the right is the critical one and determines whether the number should be rounded up or down. To round 758 314 to the nearest ten thousand, the digits 7 and 5 are covered as shown.

$\boxed{75}$8 314
760 000

Since the next digit is 8, the 75 should be increased to 76, and 760 000 written as the number rounded to ten thousands. To round to the nearest thousand, the digits from the left, up to and including the thousands, are covered as shown.

$\boxed{758}$314
758 000

Since the next digit is 3, the 758 is not increased and 758 000 is written as the number rounded to thousands.

Sets of 28 Roman numeral cards for the values shown in the charts on page 12 are recommended for converting Hindu-Arabic numerals in standard form to Roman numerals. It is also relatively easy if the standard form is first changed to expanded form. For example, to express 1981 as a Roman numeral, the expanded form 1000 + 900 + 80 + 1 may be replaced by the four cards shown.

M	CM	LXXX	I

Materials

place-value pocket chart and numeral cards as described in *Before Using the Pages* on page T 4 and extended on page T 6

Vocabulary

expanded form	Roman numerals
standard form	square kilometres, km^2
place-value chart	is greater than (>)
digit	is less than (<)
thousands	round up, round down
millions (optional)	round to the nearest

Diagnostic Test

Write the numerals.

1. eight 8 2. thirty-seven 37 3. one hundred forty 140
4. two hundred six 206 5. four thousand fifty-two 4052

Write each in standard form.

6. 10 + 3 13 7. 400 + 20 + 6 426 8. 800 + 5 805
9. 1000 + 30 1030 10. 4000 + 400 4400
11. 2000 + 900 + 70 + 1 2971
12. 5 tens 1 one 51
13. 2 hundreds 6 tens 0 ones 260
14. 3 thousands 4 hundreds 1 ten 1 one 3411

Write each in expanded form.

15. 63 60 + 3 16. 111 100 + 10 + 1 17. 402 400 + 2 18. 4078 4000 + 70 + 8

For each numeral, tell whether the 2 means 2 thousands, 2 hundreds, 2 tens, or 2 ones.

19. 1426 2 tens 20. 1246 2 hundreds 21. 2461 2 thousands 22. 4612 2 ones

Name the greater number in each pair.

23. 6 or 9 9 24. 46 or 49 49 25. 138 or 83 138
26. 217 or 2217 2217 27. 4014 or 4041 4041

List from least to greatest.

28. 23, 127, 37, 103, 32, 13 13, 23, 32, 37, 103, 127

List from greatest to least.

29. 4142, 442, 44, 4124, 452, 4144 4144, 4142, 4124, 452, 442, 44

Round to the nearest ten.

30. 39 40 31. 85 90 32. 103 100 33. 1199 1200

Round to the nearest hundred.

34. 422 400 35. 499 500 36. 1356 1400 37. 2849 2800

Round to the nearest thousand.

38. 4192 4000 39. 3500 4000 40. 6801 7000

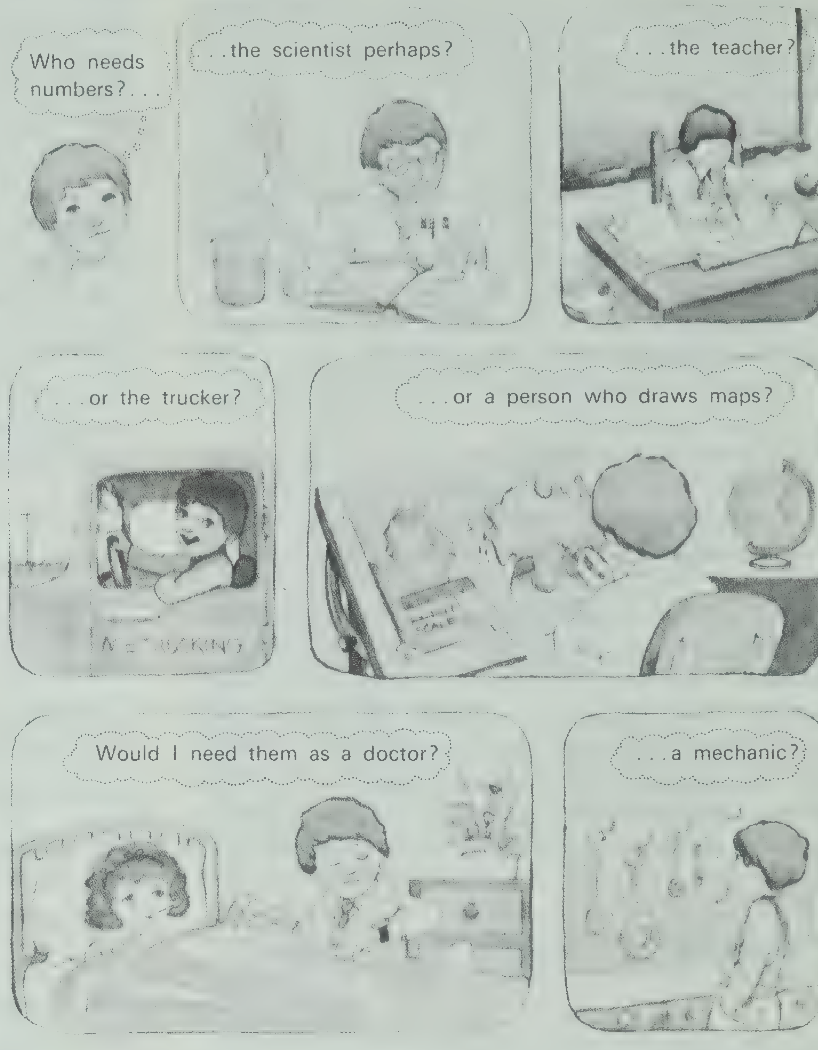
Pages 2-3

OBJECTIVE

Identify the need for and the role of numbers in occupations

1 NUMERATION

Who Needs Numbers?



2

LESSON ACTIVITY

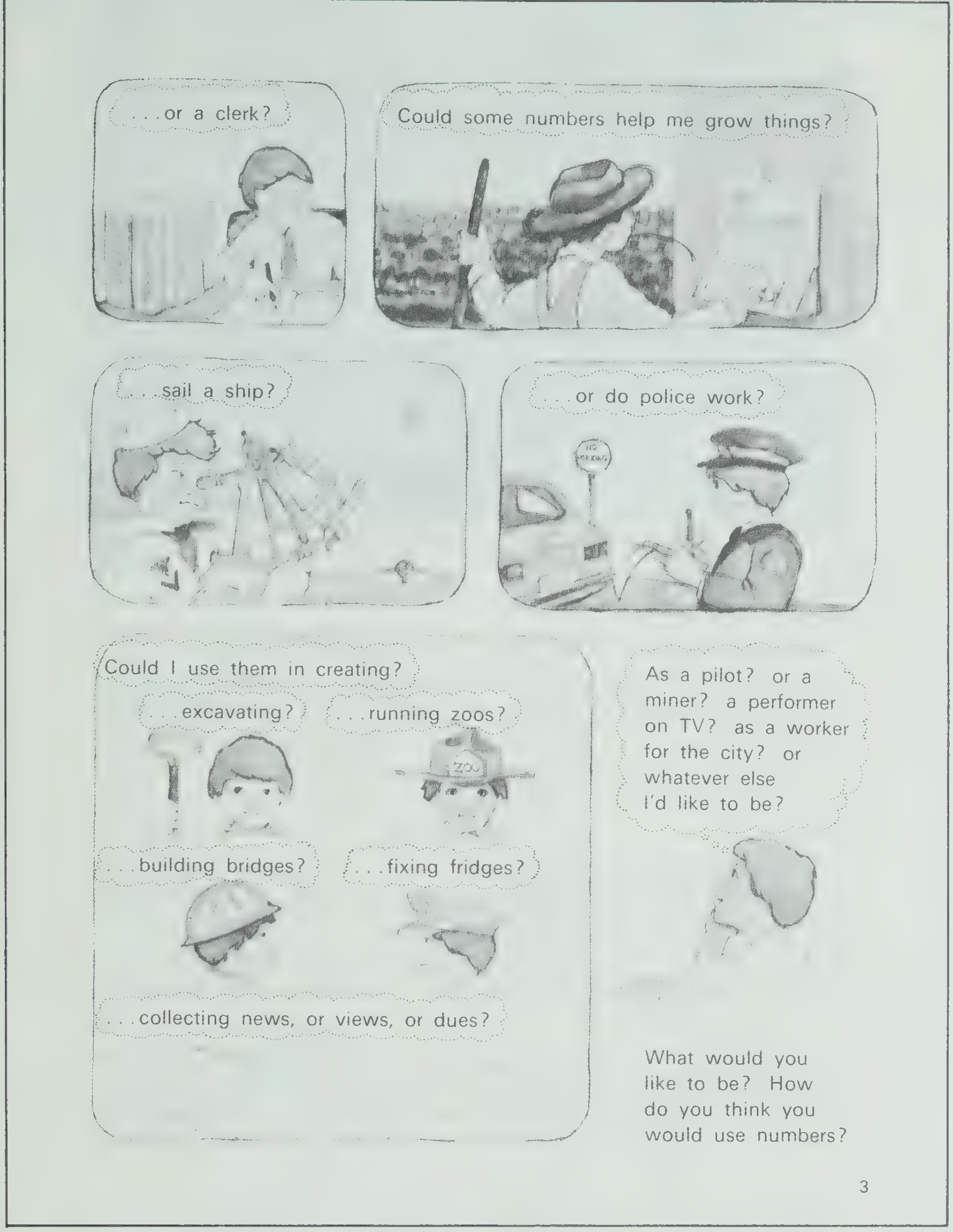
Using the Pages

- The poem on these two pages was designed to stimulate a discussion about the need for numbers and their use in various occupations. To introduce the lesson, you might say, "When someone says the word *mathematics*, many of us probably think of numbers and calculations." Ask the students, "In what ways are numbers important in life outside the mathematics class?" When they have offered several suggestions, prepare them for the concepts in the poem by saying, "Think about what you might like to do when you become an adult and whether numbers would be useful."

  Read the poem or have a student read it aloud. Discuss the meaning of the words "excavating" or "dues", if the students are not familiar with them. Then have them read the poem silently.

  The illustrations are provided to help stimulate thoughts about how numbers are used by persons in different occupations. For a structured discussion, consider each occupation illustrated. Otherwise discuss the occupations of interest to your students and ask for suggestions from them.

- Have the students write a short paragraph about a type of work that interests them or an occupation with which they are familiar. Ask them to suggest several ways in which numbers would be useful in that type of work or occupation.



RELATED ACTIVITIES

- Have the students work together to prepare a mural on the ways in which numbers are used by persons at work. Murals can be made with pictures cut from magazines or pictures drawn and painted by the students.
- Encourage the students to talk with relatives and neighbors about the ways in which numbers are used in their work. Have them write statements about a particular type of occupation. The statements may be read aloud to other students and then displayed. Students who have access to tape recorders can use them to record interviews that can be played by other members of the class.

LESSON OUTCOME

Read and write standard numerals for numbers to 999 999; interpret place value in numerals to 999 999; write the expanded form for numbers to 999 999

Materials

place-value pocket chart and numeral cards as described in *Before Using the Pages*

Vocabulary

expanded form, standard form, place-value chart, digit, square kilometres, km², thousands

Prerequisite Skills

Read and write standard numerals for numbers to 999; interpret place value in numerals to 999; write the expanded form for numbers to 999

Checking Prerequisite Skills

Write the standard numeral.

1. one hundred seventy-two 172

2. 300 + 40 + 5 345

Complete.

3. 309 = _3_ hundreds _0_ tens _9_ ones

Write the expanded form.

4. 382 **5.** 610 **6.** 105

4. 300 + 80 + 2
5. 600 + 10
6. 100 + 5

Numbers to 999 999

Workers in Canada's 28 national parks take care of 129 686 km² (square kilometres) of park property.

In a numeral with four, five, or six digits, the digits in these three places show **thousands**.

hundreds	tens	ones	hundreds	tens	ones
thousands					
1	2	9	6	8	6

The national parks cover *129 thousand 686* km².

In **expanded form**,

129 686 = 100 000 + 20 000 + 9 000 + 600 + 80 + 6

129 686 is the **standard form** for the numeral.

Working Together

Use the place-value chart shown above to help you answer these questions.

Example: The 2 in 129 686 means 2 ten thousands.

1. What does the 9 mean in 129 686?
9 thousands

2. What does each digit mean in 593 064?
5 hundred thousands
9 ten thousands 3 thousands
0 hundreds 6 tens 4 ones

3. What does the 129 mean in 129 686?
one hundred
twenty-nine thousands

Complete.

Leave a space after the thousands.

4.	84 thousand 92	84 092
5.	260 thousand	? 260 000
6.	493 ? thousand 768	493 768
7.	324 ? thousand ?89	324 089

Write each in expanded form.

8. 735 000
700 000 + 30 000 + 5 000

9. 90 420
90 000 + 400 + 20

10. 618 937
600 000 + 10 000 + 8 000 + 900 + 30 + 7

Write each in standard form.

11. 63 thousand 451 63 451

12. five hundred six thousand nine hundred 506 900

13. 400 000 + 70 000 + 300 + 90 + 5 470 395

14. 2 hundred thousands 8 ten thousands 2 hundreds 5 tens 7 ones 280 257

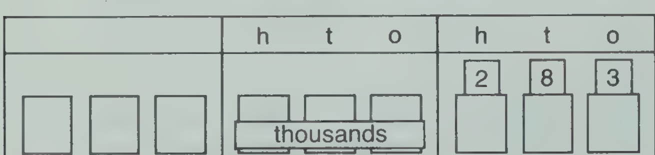
4

LESSON ACTIVITY

Before Using the Pages

- Prepare a pocket chart for nine-place numerals using a sheet of Bristol board and nine library-book pockets. Label the place values to hundred thousands, leaving the millions' period to be labeled during the lesson for pages 6 and 7. Prepare several numeral cards for 0 and at least three for each of the digits 1 to 9.

  Display the chart and have students identify the ones' place, the tens' place, and the hundreds' place. Place a card for 3 in the ones' place and ask what number is shown.



Place a card for 8 in the tens' place and ask what number is shown (83). Show 2 hundreds and ask for the number (283). Remind the students that 283 is a three-place numeral and that such numerals are read without using the word "and", that is, "two hundred eighty-three".

Remove the cards and ask a student to show the cards for the greatest number that can be represented by a three-place numeral. Ask what is needed to show numbers greater than 999 and, in this way, lead into a discussion of the place values in the thousands' period. Direct the students' attention to the order of names in each group (period) from right to left, namely, ones, tens, hundreds, and then (one) thousands, ten thousands, hundred thousands. Have students show and read four-place numerals, then five-place numerals, and then six-place numerals.

After the students have shown that they understand the individual place values, have them examine a few numerals in expanded form, showing 6025, for example, as 6000 + 20 + 5. Review the importance of 0 in the hundreds' place of 6025.

Ask what is the greatest number that can be represented by a six-place numeral. Then have the students turn to page 4 and note the title of the lesson.

Exercises

Complete.

1.	801 thousand 253	? 801 253
2.	74? thousand 6?	74 006
3.	650 thousand	? 650 000
4.	? 438 ? thousand	438 000
5.	137? thousand 430?	137 430

What does each 7 mean?

6. 327 908 7 thousands
7. 723 684 7 hundred thousands
8. 19 704 7 hundreds
9. 175 516 7 ten thousands

Write each in expanded form.

10. 235 000 200 000 + 30 000 + 5 000
11. 98 036 90 000 + 8 000 + 30 + 6
12. 560 804 500 000 + 60 000 + 800 + 4
13. 109 040 100 000 + 9 000 + 40

Write each in standard form.

14. two hundred ten thousand 210 000
15. seventy-three thousand fifty 73 050
16. five hundred forty-seven thousand four hundred one 547 401
17. 900 000 + 20 000 + 500 920 500
18. 40 000 + 1 000 + 80 + 6 41 086
19. 3 hundred thousands 2 thousands 5 hundreds 9 tens 302 590
20. 6 ten thousands 4 hundreds 60 400
21. 2 hundred thousands 8 tens 4 ones 200 084

Write each sentence using a numeral in standard form.

22. The largest national park, Wood Buffalo, covers forty-four thousand eight hundred seven square kilometres.
The largest national park, Wood Buffalo, covers 44 807 km².

23. There are 105 thousand 811 km² in the 5 largest national parks and 23 thousand 875 km² in the other 23.
There are 105 811 km² in the 5 largest national parks and 23 875 km² in the other 23.

5

RELATED ACTIVITIES

- Cards similar to those below are useful for showing a numeral in standard form when the cards overlap and in expanded form when the cards spread apart.

1	6	3	4	2	5

100 000	+	60 000	+	3 000

+	400	+	20	+	5

Have the students work alone or in small groups, showing given numbers to 999 999 in standard and expanded form. They should also practice reading the numerals aloud.

- Have each student make a simple digital device for six-digit numerals and use it as described on page T 377.
- Some students may need to work with models to review the following relationships.

10 ones = 1 ten
10 tens = 1 hundred
10 hundreds = 1 thousand
10 thousands = 1 ten thousand

Copies of page T 392 may be cut to prepare models of ones, tens, and hundreds. To show one thousand, staple together ten models for hundreds.

- Have students locate some of Canada's national parks on a map. For each park, have them write numerals to describe the area, the number of visitors, and so on.

Using the Pages

- Ask the students if they have visited any of Canada's national parks. Have a student read the introductory statement on page 4. Ask how many digits there are in 129 686. Discuss the use of a space between the 9 and the 6, which helps to display two groups of three digits, which in turn helps when reading the numeral.

Review the meaning of the term *standard* (simplest) *form*. Emphasize that the *expanded form* shows the total value of each digit in the numeral. Associate each digit in the place-value chart with the corresponding part of the expanded form. Use the place-value pocket chart to have the students practice reading large numbers and writing the numerals in expanded form.

Working Together: Ex. 1-3 emphasize the place values for six-place numerals. Encourage the students to think of the place values in two groups from right to left: ones, tens, and hundreds, and then thousands, ten thousands, and hundred thousands.

Ex. 4-7 help students to associate a standard numeral with its word name. Ex. 4 and 7 may require more attention because of the 0 in the hundreds' place. The place-value pocket chart may be helpful.

For Ex. 8, mention that since the value represented by a 0 in any place is 0, that place is not shown in the expanded form.

Exercises: Have the students prepare place-value charts, if necessary, to help them answer Ex. 14-21.

Assessment

What does each 4 mean?

1. 230 461 4 hundreds **2.** 147 302 4 ten thousands

Write each in standard form.

3. three hundred four thousand eight hundred sixty-two 304 862
4. 60 000 + 9 000 + 80 + 7 69 087
5. 7 ten thousands 6 thousands 3 hundreds 5 ones 76 305

Write each in expanded form.

6. 987 654 **7.** 302 175 **8.** 602 001

6. 900 000 + 80 000 + 7 000 + 600 + 50 + 4
7. 300 000 + 2 000 + 100 + 70 + 5
8. 600 000 + 2 000 + 1

LESSON OUTCOME

Read and write standard numerals for numbers to 999 999 999; interpret place value in numerals to 999 999 999; write the expanded form for numbers to 999 999 999

Materials

place-value pocket chart and numeral cards as described in *Before Using the Pages* on page T4

Vocabulary

millions

Prerequisite Skills

Read and write standard numerals for numbers to 999 999; interpret place value in numerals to 999 999; write the expanded form for numbers to 999 999

Checking Prerequisite Skills

Write the standard form.

1. 90 000 + 8 000 + 400 + 60 + 6 98 466
2. 130 thousand 42 130 042
3. 5 hundred thousands 4 ten thousands 9 hundreds 4 tens 2 ones 540 942
4. one hundred four thousand 104 000

What does each 4 mean?

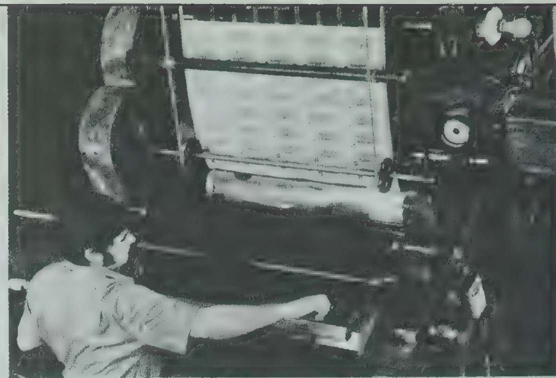
5. 403 485 4 hundred thousands 4 hundreds

Write the expanded form.

6. 62 097 7. 150 702

6. 60 000 + 2 000 + 90 + 7
7. 100 000 + 50 000 + 700 + 2

Numbers to 999 999 999



At the end of 1977 there were 74 788 800 five-dollar bills in use.

In a numeral with seven, eight, or nine digits, the digits in these three places show **millions**.

hundreds	tens	ones	hundreds	tens	ones	hundreds	tens	ones
millions			thousands					
	7	4	7	8	8	8	0	0

At the end of 1977 there were 74 million 788 thousand 800 five-dollar bills in use.

If you know three-digit numerals and the words "thousands" and "millions", you can read any numeral with up to nine digits.

Working Together

Use a place-value chart like the one shown above to help you answer these questions.

Example: The 3 in 436 028 500 means 3 ten millions.

1. What does the 8 mean in 58 064 731? 8 millions
2. What does each digit mean in 207 146 853? 2 hundred millions 0 ten millions 7 millions 1 hundred thousand 4 ten thousands 6 thousands 8 hundreds 5 tens 3 ones
3. What does the 125 mean in 125 280 371? one hundred twenty-five millions

Complete.

4.	384 million 90 thousand 524	384 090 524
5.	9 ? million 403 ? thousand 861	9 403 861
6.	32 million 529 thousand 600	? 32 529 600
7.	627 ? million 306 ? thousand 5 ?	627 306 005

Write each in expanded form.

8. 4 200 000
9. 28 063 400
10. 576 381 429

Write each in standard form.

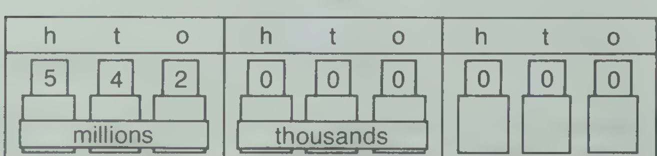
11. eight hundred million 800 000 000
12. seventy million six thousand 70 006 000
13. 10 000 000 + 500 000 + 7 000 + 40 + 8 10 507 048
14. 3 hundred millions 8 ten millions 7 ten thousands 1 thousand 9 tens 380 071 090

8. 4 000 000 + 200 000
9. 20 000 000 + 8 000 000 + 60 000 + 3 000 + 400
10. 500 000 000 + 70 000 000 + 6 000 000 + 300 000 + 80 000 + 1 000 + 400 + 20 + 9

LESSON ACTIVITY

Before Using the Pages

- Display the place-value pocket chart prepared for the previous lesson. Review the names of the place values from right to left, to six places (hundred thousands). Have one student place numeral cards in pockets to show 542. Have another student move the cards to show 542 thousands. Have a student move the cards for 5, 4, and 2 to the three places to the left of the thousands and fill the six pockets to the right with cards showing 0. Ask the students if they know the name of this new period (millions). Have the students turn to the chart on page 6 so that they can suggest the headings for you to complete the pocket chart.



Using the Pages

- The example provided emphasizes that any numeral with up to nine digits can be read if it is known how to read a numeral with up to three digits: the millions are read first, then the thousands, and then the rest of the numeral is read in the usual way. Point out that the spaces separating groups of numerals help to read the entire numeral. As an example, help students read the title of the lesson at the top of page 6 by asking the following questions.

"Are there any millions?" "How many?" (999 millions)
"Are there any thousands?" "How many?" (999 thousands)
"Are there any ones?" "How many?" (999)
"What is the number?" (999 million 999 thousand 999)

Have students read the numeral in the place-value chart. Then ask questions such as the following.

"What digit is in the hundred thousands' place?" (7)
"What digit is in the millions' place?" (7)

Review the names of the place values in sequence from right to left. Then, in preparation for showing numbers in

11. 3 000 000 + 80 000 + 7 000 + 400 + 6
12. 100 000 000 + 20 000 000 + 3 000 000 + 400 000 + 50 000 + 6 000 + 700 + 80 + 9

Exercises

Complete

1.	14	million	519	thousand	628		14 519 628
2.	683	million	59	thousand	327		683 059 327
3.	23	million	5	thousand	30		23 005 030
4.	200	million	49	thousand	8		200 049 008
5.	789	million	723	thousand			789 723 000
6.	120	million	400	thousand	85		120 400 085
7.	5	million			100		5 000 100

What does each 4 mean?

8. 346 763 009 4 ten millions
9. 400 137 998 4 hundred millions
10. 84 000 000 4 millions

Write each in expanded form

11. 3 087 406
12. 123 456 789

Write each in standard form.

13. one hundred ninety-nine million nine hundred fifty-seven thousand eighty 199 957 080
14. 2 ten millions 3 millions 4 thousands 9 hundreds 1 ten 6 ones 23 004 916
15. 700 million 209 thousand 519 700 209 519
16. 70 000 000 + 5 000 000 + 30 000 75 030 000
17. four million six thousand 4 006 000
18. two hundred six million twelve 206 000 012

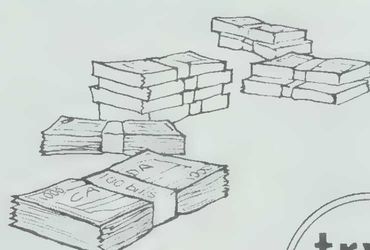
Write the sentence using numerals in standard form.

19. At the end of 1977, there were in use 234 million 887 thousand one-dollar bills, At the end of 1977, there were in use 234 887 000 one-dollar bills,
20. 89 million 602 thousand two-dollar bills, 89 602 000 two-dollar bills,
21. 119 million 636 thousand 400 ten-dollar bills, 119 636 400 ten-dollar bills,
22. 196 million 678 thousand 850 twenty-dollar bills, and 196 678 850 twenty-dollar bills, and
23. 18 million 162 thousand 630 hundred-dollar bills. 18 162 630 hundred-dollar bills.

How much is one million dollars?

$1 000 000 could buy

1. 1 $1 000 000 ranch
2. 10 $100 000 homes
3. 100 $10 000 cars
4. 1000 $1 000 motorcycles
5. 10 000 $100 bicycles
6. 100 000 $10 running shoes
7. 1 000 000 $1 milkshakes
8. What would you do with $1 000 000? Answers will vary.



try this

7

RELATED ACTIVITIES

• The place-value pocket chart described in *Before Using the Pages* may be used with small groups to reinforce concepts of the lesson. For example, instruct different students to show 4 in the ten millions' place, 7 in the millions' place, and so on, and then have them read the numeral.

For variation, have the members of each group "build" their own numbers. Two students may place cards in the millions' places, two may place cards in the thousands' places, and two may place cards in the ones' places. Have all the students interpret the resulting numeral.

• Students will likely enjoy drawing diagrams to show large numbers. For example, over a period of time, students can take turns drawing •'s or X's on a large sheet of paper on the wall, until there are one million marks. Discuss with the students how to keep track of the number of marks throughout the procedure.

• Provide students with copies of the *Guinness Book of World Records*. Have them search for instances in which large numbers are involved and write sentences using these numbers.

• Have students use an abacus to show and interpret numerals with up to nine digits. (See *Peg Abacus* described on page T376.) This could lead them to investigate twelve-place numerals and the billions' period.

expanded form, have students name the place value for each digit in a numeral, starting from the left. Have students help to show the expanded form for 74 788 800 on the board.

74 788 800 = 70 000 000 + 4 000 000 + 700 000
+ 80 000 + 8 000 + 800

Working Together: Ex. 1-3 emphasize the place values for numerals with up to nine digits. Ex. 4-7 deal with the skill of reading numerals. Note, for instance, in Ex. 4, the numeral shows 090 in the thousands' period, but this is read "ninety thousand" as indicated in the chart.

For Ex. 11-14, some students will find it helpful to prepare a place-value chart. The digits may be written in the appropriate columns as the number is interpreted. For example, in Ex. 12, a student might show 7 and 0 first for 70 million, and then 6 for 6 thousand, and finally, show the necessary zeros to complete the numeral.

Exercises: If necessary, have students prepare place-value charts to help in showing the standard form for Ex. 13-18.

Try This: Although the concept of multiplication is inherent in these exercises, the intent is to reinforce the concept of place value to millions. For example, 1 million, 10 hundred thousands, and 100 ten thousands all have the same value. Thus, one million dollars may buy 1 one-million-dollar ranch, 10 hundred-thousand-dollar homes, 100 ten-thousand-dollar cars, and so on.

Assessment

What does each 4 mean?

1. 483 204 514 4 hundred millions 4 thousands 4 ones

Write the standard form.

2. two million four hundred twenty thousand one hundred six 2 420 106
3. 50 000 000 + 3 000 000 + 4 000 + 500 + 9 53 004 509
4. 6 hundred millions 8 ten millions 4 millions 684 000 000

Write the expanded form.

5. 92 040 360
90 000 000 + 2 000 000 + 40 000 + 300 + 60

Pages 8-9

LESSON OUTCOME

Compare and order numbers to 999 999

Materials

place-value pocket chart and numeral cards as described in *Before Using the Pages* on page T4 and extended on page T6

Vocabulary

is greater than (>), is less than (<)

Prerequisite Skills

Compare and order numbers to 999

Checking Prerequisite Skills

Which is greater,

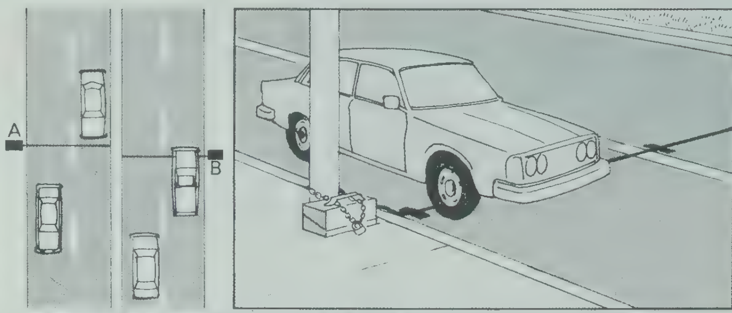
1. 302 or 320? 320
2. 493 or 496? 496
3. 201 or 102? 201

List from least to greatest.

4. 234, 243, 146, 25, 143, 184
   25, 143, 146, 184, 234, 243

Comparing and Ordering Numbers

In one week, machine A counted 131 210 vehicles. Machine B counted 129 984 vehicles. Which machine counted more vehicles?



131 210 } both show 1 hundred thousand.
129 984

131 210 shows 3 ten thousands.
129 984 shows 2 ten thousands.
3 is greater than 2, so
131 210 is greater than 129 984.
Machine A counted more vehicles.

3 > 2, so
131 210 > 129 984.

Working Together

Follow the path for each pair of numbers.

1. 739 461 / 739 416
2. 43 496 / 43 915
3. 384 967 / 385 967

Start at the left. → Compare the first digits | next digits → If they are not equal | equal → Which is greater?

Which is greater,

4. 743 384 or 734 395?
5. 508 733 or 508 377?
6. 84 997 or 89 412?

List from least to greatest.

7. 197 403, 19 574, 195 743, 197 430

19 574
195 743
197 403
197 430

8

LESSON ACTIVITY

Before Using the Pages

- Briefly review the work of the preceding lesson. Use the place-value pocket chart for six-place (or nine-place) numerals and numeral cards. For example, show the numeral 608 143 and have students read the numeral and tell what digit appears in the tens' place and the ten thousands' place. Also, have them describe the group of digits in the thousands' period. For example, 608 in the numeral 608 143 means 608 thousands. Similarly, in the ones' period, 143 means 143 ones.

  Have pairs of students help to show numerals for numbers with up to six digits. For example, have one student show 26 thousands and another show 70. Then, if necessary, ask whether any zeros are needed to complete the numeral. Discuss why a 0 is needed in the hundreds' place but not in the hundred thousands' place.

- Have each student write any two six-place numerals so that corresponding places of the two numerals are aligned vertically.

  213 407
  211 988

  Have one student show her/his example on the board. Ask which number is greater and have students explain how they determined this. Discuss the advantages of one procedure over another.

Using the Pages

- Have the students read the problem at the top of page 8 silently. Ask what information is to be found. Establish that it is necessary to compare two numbers. Discuss that the comparison takes place in a left-to-right order and have students suggest why this is preferable to a right-to-left order. Point out that corresponding digits are highlighted by red to facilitate the comparison at each step.

  You may wish to point out, also, that the space in a numeral separating the ones' and thousands' periods

Exercises

Which is greater,

1. 69 158 or 69 258?
2. 35 609 or 34 609?
3. 17 308 or 162 038?
4. 699 832 or 689 925?
5. 389 502 or 389 503?
6. 247 792 or 247 729?

Use >, <, or = to make true statements.

Example: 321 698 is less than 321 700, so
321 698 < 321 700.

7. 349 527 ○ 449 525 <
8. 18 805 ○ 18 750 >
9. 529 510 ○ 529 509 >
10. 689 412 ○ 689 412 =
11. 573 375 ○ 573 757 <
12. 900 627 ○ 899 235 >

List from least to greatest.

13.

673 058	637 508	637 985
97 805	639 850	637 580

List from greatest to least.

14.

174 215	174 156	71 561
174 216	173 612	174 205

Numbers shown with more than six digits also may be compared.

Example:

7 is greater than 5, so
13 728 651 > 13 582 156.

Use >, <, or =

15. 2 753 900 ○ 2 729 300 >
16. 148 962 705 ○ 184 962 075 <
17. 36 024 571 ○ 34 206 751 >

List from least to greatest.

18.

13 201 596	13 210 569
132 569 713	13 201 956

13. 97 805, 637 508, 637 580, 637 985, 639 850, 673 058

14. 174 216, 174 215, 174 205, 174 156, 173 612, 71 561

18. 13 201 596, 13 201 956, 13 210 569, 132 569 713

try this

even (e)	odd (o)
0, 2, 4, 6, 8	1, 3, 5, 7, 9

The greatest number that can be shown with digits in this pattern oeo eoe is 989 898

What are the greatest numbers that can be shown with digits in these patterns?

1. eoe oeo 898 989
2. ooo eee 999 888
3. eeo oee 889 988
4. oee oee 988 988

What are the least numbers that can be shown with digits in these patterns?

5. oeo eoe 101 010
6. eoe oeo 10 101
7. ooo eee 111 000
8. ooe eoo 110 011

Repeat the above exercises. This time, do not use any digit more than once in each numeral you write.

try this 1. 896 745 2. 975 864 3. 869 742 4. 986 742 5. 103 254 6. 12 345 7. 135 246 8. 130 257

9

RELATED ACTIVITIES

- Have students use the letters e and o to prepare patterns similar to those in *Try This* for six-digit numerals and find the greatest (least) numbers for the patterns.
- If students prepared the digital device suggested in *Related Activities* on page T5, they may now work in groups of two to four. Each student represents a number using her/his digital device. The numbers are then compared and ordered from least to greatest.
- As an activity for a large group, have each of 10 or 12 students write a six-digit numeral using six different digits from 1 to 9. Have the students write their numerals on the board and then rewrite them in order. The same number may occur more than once. To vary the activity, have students use, for example, only the digits 3 and 4 to form a six-digit numeral.
- If students are having difficulty ordering large numbers, it may be helpful to have them work first with three-place numerals, and then with six-place numerals.

suggests that the numerals can be compared not only by individual places, but also by groups (periods). For this example, recognizing that 131 is greater than 129 allows us to conclude that 131 210 is greater than 129 984. This technique is particularly useful for numerals showing more than two periods. For nine-digit numerals, for example, the three digits showing millions are compared first. The digits for thousands are compared next, if necessary, and finally the digits for ones may have to be compared. Emphasize that each of these steps is similar to comparing two three-digit numbers.

Point out the symbol > in the "thought cloud" and have students interpret the sentences. Have a student show the symbol < for "is less than" on the board.

Working Together: Ex. 1-3 establish the left-to-right order of comparison and emphasize the comparison of numbers by digits. Discuss Ex. 1 with the students to help them understand the sequence indicated by the "flow chart". It would be helpful to have a student express the ideas of the "flow chart" in her/his own words for the benefit of others. This will help students with Ex. 4-7. For Ex. 1-6, you may wish to have students use the symbol > to write their answers, as shown in the worked example.

Exercises: Draw attention to the order stated in the instruction for each of Ex. 13-14. Ex. 15-18 extend the ideas of this lesson to numbers having from seven to nine digits.

Try This: These exercises review the concept of even numbers and odd numbers and reinforce the concept of place value. Tell the students that since six letters are shown in each pattern, six-digit numerals are suggested. Ex. 6 can motivate a discussion regarding 0 in a numeral. For example, for Ex. 6, the least number is 010 101, but this usually is written as the five-digit numeral 10 101.

Assessment

Which is greater,

1. 43 182 or 43 482? 43 482
2. 185 253 or 175 532? 185 253

List from least to greatest.

3. 14 320, 147 368, 143 023, 143 022
14 320, 143 022, 143 023, 147 368

LESSON OUTCOME

Round numbers to the nearest thousand, ten thousand, or hundred thousand

Materials

place-value pocket chart and cards described in *Before Using the Pages* on pages T 4 and T 6 (optional)

Vocabulary

round up, round down, round to the nearest

Prerequisite Skills

Interpret place value for six-digit numerals

Checking Prerequisite Skills

What does each 3 mean?

3 thousands / 3 ten thousands

1. 243 135 2. 436 301

3 tens / 3 hundred thousands

Background

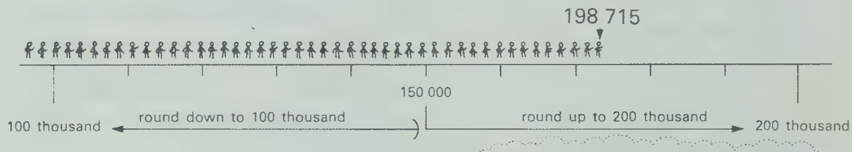
If students have not already experienced the concept of rounding whole numbers, prepare this lesson carefully. Concentrate first on rounding numbers less than 1000 to the nearest ten and then to the nearest hundred. (You may wish to refer to pages T 14 and T 15 of the Teacher's Edition of *Starting Points in Mathematics 4*.) The activity suggested in *Before Using the Pages* is recommended for students who have previously encountered the concept of rounding numbers, but require a review of the basic ideas.

Rounding

In a recent school year, each of five provinces had about 200 000 students in elementary and secondary schools.

Province	Number of Students
Alberta	418 715
British Columbia	511 671
Manitoba	232 470
New Brunswick	155 819
Newfoundland	153 576
Nova Scotia	198 715
Ontario	1 974 702
Prince Edward Island	27 225
Quebec	1 297 690
Saskatchewan	220 979

Nova Scotia had 198 715 students.



198 715 rounded to the nearest hundred thousand is 200 000.

Saskatchewan had 220 979 students.

220 979 rounded to the nearest hundred thousand is 200 000.

Nova Scotia and Saskatchewan were two of the five provinces that had about 200 000 students each.

10

LESSON ACTIVITY

Before Using the Pages

- Draw a number line marked into ten equal parts on the board. Ask questions and direct the students as suggested below, labeling the number line according to their answers. Sample answers are given in parentheses.

"Name a multiple of ten." (30)
"Name the next multiple of ten." (40)
"Name a number between 30 and 40." (36)
"Is the number closer to 30 or to 40?" (40)
"We say that 36 rounded to the nearest ten is 40."
"Round 33 to the nearest ten." (30)
"Is 35 closer to 30 or to 40?"
"By agreement, we round 35 up to 40, rather than down to 30."

Have the students round each number to the nearest ten.

48, 148, 3148 72, 872, 6872

Follow a similar sequence to review rounding numbers to other given places.

- The following activity can prepare the students for focusing on the place values necessary for rounding to a given place. Say, "I am thinking of a number, but I am not going to show you all the digits." Write 31 _ on the board and ask, "Can you round my number to the nearest hundred?" Replace the 1 in the tens' place by a different digit such as 7 and repeat the question. Discuss why they do not need to know the digit that appears in the ones' place. Extend the concept by having the students round 43 _ _, for example, to the nearest thousand.

To lead students toward rounding larger numbers, develop examples similar to the following on the board.

1. Round 362 to the nearest ten.
   Round 362 145 to the nearest ten thousand.
2. Round 362 to the nearest hundred.
   Round 362 145 to the nearest hundred thousand.

Working Together

When rounding to $\boxed{\text{this place}}$, first check the digit in (this place)

hundreds	tens	ones			
thousands			hundreds	tens	ones

1. When rounding to the nearest ten thousand, first check the digit in the ? place.
one thousands
2. When rounding to the nearest thousand, first check the digit in the ? place.
hundreds

If the digit you check is 5, 6, 7, 8, or 9, round up.
If the digit you check is 0, 1, 2, 3, or 4, round down.

Would you round down or up to the nearest hundred thousand?

3. 482 651 up
4. 549 567 down

Would you round down or up to the nearest ten thousand?

5. 27 225 up
6. 511 671 down

Round to the

7. nearest thousand.
324 517
325 000
8. nearest ten thousand.
796 487
800 000
9. nearest hundred thousand.
329 870
300 000

Exercises

Do chart

		thousand	ten thousand	hundred thousand
1.	418 715	419 000 ?	420 000 ?	400 000 ?
2.	232 470	232 000 ?	230 000 ?	200 000 ?
3.	155 819	156 000 ?	160 000 ?	200 000 ?
4.	153 576	154 000 ?	150 000 ?	200 000 ?

The number of students for all ten provinces was 5 191 562. Round this number to the nearest

5. thousand
5 192 000
6. ten thousand
5 190 000
7. hundred thousand
5 200 000
8. million
5 000 000

Round to the nearest

Do chart

		million	ten million	hundred million
9.	254 859 926	255 000 000 ?	250 000 000 ?	300 000 000 ?
10.	839 581 074	840 000 000 ?	840 000 000 ?	800 000 000 ?

11

RELATED ACTIVITIES

- Interesting facts can be taken from such books as a *World Atlas* and the *Guinness Book of World Records*. The numbers given in these facts can be rounded by the students to a particular place named by you or suggested by them.
- Frequently, events reported in newspapers and magazines name numbers that have been rounded to a particular place. Encourage students to find such articles and bring them to school for display and discussion. For example, discuss the possibilities for the exact number which is reported as 375 000.
- Exercises similar to the following may help students who are having difficulty rounding larger numbers.

Round to the nearest ten.
1. 413 410
2. 178 180

Round to the nearest ten thousand.
3. 413 thousand
410 thousand
4. 178 000
180 000

Round to the nearest ten million.
5. 413 million
410 million
6. 178 000 000
180 000 000

Using the Pages

- Draw the students' attention to the chart on page 10. Have them read the numeral that shows the number of students in their province and in neighbouring provinces. Ask which province had the fewest students and which had the most. To review the lesson on pages 8 and 9, you may wish to have students list the provinces in order from least to greatest number of students. For your province, or any province, ask if it is necessary to read the entire numeral to have an idea of the number of students. Establish that noting just those digits to the left of the ones' period gives a clear idea of the number of students in a province, and emphasize this as a practical use of rounded numbers.

  Discuss the number lines shown for rounding the two numbers involved. Pay particular attention to the two "limits" (for example, 100 thousand and 200 thousand for the first number line) and to the "halfway" number (for example, 150 000).

Working Together: The task of rounding a number to a given place is simplified by concentrating on the digit in the place to its immediate right. This concept was hinted at in the second activity in *Before Using the Pages* and is dealt with here in Ex. 1 and 2. Relate the digit 5 in any place value to the "halfway" point between two multiples of 10 (100, 1000, . . .) on the number line. This can help students to understand the statements shown for rounding up and rounding down. Pay particular attention to Ex. 8 for which rounding 796 487 to the nearest ten thousand results in a change of digits for the thousands', ten thousands', and hundred thousands' places: 796 487 → 800 000.

Exercises: It is preferable to have the students complete the chart column by column rather than row by row, referring each time to the original number. That way, they will be considering the same place value in each numeral at one time. Ex. 8-10 extend the ideas of this lesson to numbers requiring seven to nine digits.

Assessment

Round to the

1. nearest thousand.
20 493 20 000
2. nearest hundred thousand.
395 261 400 000
3. nearest thousand.
219 849 220 000
4. nearest ten thousand.
175 063 180 000

Pages 12-13

LESSON OUTCOME

Read and write Roman numerals for numbers to 2000

Vocabulary

Roman numerals

Prerequisite Skills

Name numbers in terms of other numbers, sometimes using addition or subtraction

Checking Prerequisite Skills

Complete.

1. 9 = 10 − 1

2. 14 = 15 − 1

3. 70 = 50 + 20

4. 110 = 100 + 10

5. 400 = 500 − 100

Roman Numerals

Many motion pictures use Roman numerals to show the year in which they were completed.

There are seven basic Roman numerals.

I	V	X	L	C	D	M
1	5	10	50	100	500	1000

The cartoon was completed in

M	CM	LX	VII
1000	900	60	7

1967

Working Together

Choose no more than one numeral from each chart to make Roman numerals.

Numeral for one thousand	Numerals for multiples of 100	Numerals for multiples of 10	Numerals for 1 to 9
1000 = M	100 = C	10 = X	1 = I
	200 = CC	20 = XX	2 = II
	300 = CCC	30 = XXX	3 = III
	400 = CD	40 = XL	4 = IV
	500 = D	50 = L	5 = V
	600 = DC	60 = LX	6 = VI
	700 = DCC	70 = LXX	7 = VII
	800 = DCCC	80 = LXXX	8 = VIII
	900 = CM	90 = XC	9 = IX

Example:

For 1492, use M, CD, XC, and II to get MCDXCII.

1. 1859 MDCCCLIX
2. 1507 MDVII
3. 370 CCCLXX

Ring the thousands, the hundreds, the tens, and the ones. Write the standard form for each.

Examples: M CM LXXX VI = 1986 M XC IV = 1094

4. M DCCC XXX IX 1839
5. M CD V 1405
6. DC XL 640

There is no Roman numeral for zero, so nothing is used to show zero hundreds.

12

LESSON ACTIVITY

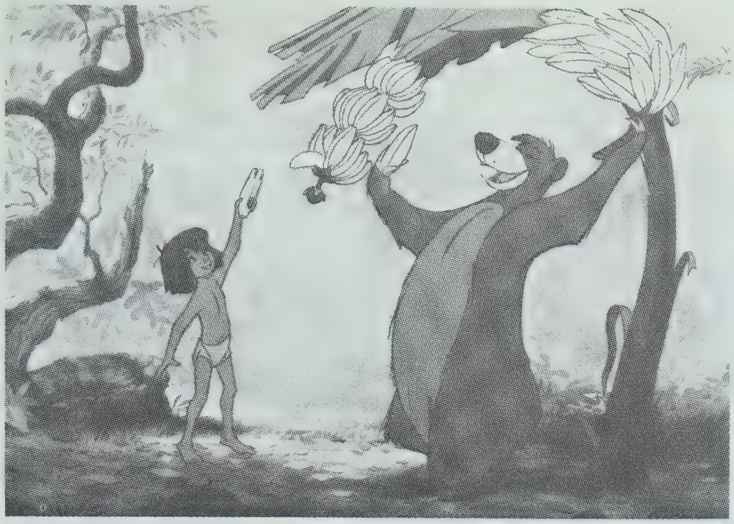
Before Using the Pages

- Determine whether students have encountered Roman numerals before and have them recall what they know about them. They may suggest that Roman numerals were used long ago and that the system uses capital letters to represent numbers. Have a few students write Roman numerals on the board, and ask other students which numbers are represented. Ask the students whether Roman numerals are still in use today.

Using the Pages

- The worked example provides one instance in which Roman numerals are used today. Draw the students' attention to the photograph on page 13 and, if possible, have them identify the movie in which this scene appears (*Jungle Book*). Point out the Roman numeral shown beneath the photograph and tell the students that it names the year in which the cartoon was completed. Then discuss the example at the top of page 12 to show how the numeral is interpreted. Point out that the numerals shown in the "thought clouds" suggest the expanded notation for 1967. Discuss the way of interpreting 7 as 2 more than 5, 60 as 10 more than 50, and 900 as 100 less than 1000. Have students give the significance of placing the letter X to the right of L (LX) as opposed to the left of L (XL).

Working Together: Before assigning the exercises, take a few moments to discuss the numerals shown in each of the four charts. For instance, compare the Roman numerals for 4 and 6, for 40 and 60, and for 400 and 600. Have students who are unfamiliar with Roman numerals represent numbers to 99 by selecting a numeral from one chart, or by combining two numerals, one from each of the two appropriate charts. Then proceed with the exercises provided. Ex. 1-3 deal with writing Roman numerals and Ex. 4-6 deal with interpreting Roman numerals. The latter skill is usually more demanding since it involves associating the letters in correct groupings to obtain thousands, hundreds, tens, and ones. Ringing groups of numerals or



© MCMLXVII Walt Disney Productions

Exercises

Write the Roman numerals.

1. 1741 MDCCXLI
2. 676 DCLXXVI
3. 54 LIV
4. 1490 MCDXC
5. 1089 MLXXXIX
6. 1915 MCMXV
7. 230 CCXXX
8. 509 DIX
9. 1802 MDCCCII
10. 943 CMXLIII

Write the standard form for each.

11. MCCXXXIV 1234
12. DCCXXVI 726
13. LIX 59
14. MCDXL 1440
15. MLXIII 1063
16. CMXCIX 999
17. MCCCVII 1307
18. MIV 1004
19. MX 1010
20. MDCCCLXXXVIII 1888

Copy each sentence. Replace numerals in standard form with Roman numerals. Replace Roman numerals with the standard form.

21. The movie was completed in 1979.
22. The television show was made in MCMLXIV.
    The television show was made in 1964.
23. The book was printed in either 1899 or 1900.
24. The old coin was dated MCDXLVIII.
    The old coin was dated 1448
25. Leonardo da Vinci painted *Mona Lisa* about 1503.
26. The cornerstone on the building showed MCMLX.
    The cornerstone on the building showed 1960
27. The newspaper was first published in 1884.
28. Rome was ruled by an emperor until CDLXXVI.
    Rome was ruled by an emperor until 476.

21 The movie was completed in MCMLXXIX
23 The book was printed in either MDCCCXCIX or MCM.
25 Leonardo da Vinci painted Mona Lisa about MDIII.
27 The newspaper was first published in MDCCCLXXXIV

referring to the charts provided may make it easier for the students to complete the exercises. Use other similar exercises as required.

Exercises: Students will quickly realize the advantages of the base-ten numeration system and the disadvantages of the Roman system. Some may need to refer to the charts shown on page 12. Others may be encouraged to refer to the seven basic numerals shown at the top of page 12. The latter method indirectly promotes memorization of the basic numerals.

Assessment

Write the Roman numerals.

1. 86 LXXXVI **2.** 302 CCCII **3.** 690 DCXC

Write the standard form for each.

4. XIX 19 **5.** LVI 56 **6.** MCCXXIV 1224

RELATED ACTIVITIES

- Have students suggest ways that help them remember what number is represented by each of the seven symbols M, D, C, L, X, V, and I. For example, the word FIVE contains the letter V; the letter X suggests two fives (X) and thus, ten; 100 CENTS have the same value as 1 dollar. Perhaps L (50) can suggest "half" the letter C. The numerals can be written on a chart and displayed for several days.
- Have students search for examples of Roman numerals in use today and share their findings. For example, Roman numerals appear on clock faces and on the introductory pages of some books.
- Some students may want to research other numeration systems or even to develop a simple system of their own.
- Have students write the following number sequence using Roman numerals.

  111, 222, 333, . . ., 999
- Facts similar to those in Ex. 27 and 28 may be researched by students and written as exercises for other students to complete.
- Roman numerals on a number line can help students understand, for example, that 400 is 100 less than 500 and that 600 is 100 more than 500.

C CC CCC CD D DC DCC

Page 14

OBJECTIVE

Find the information needed to solve a problem

Background

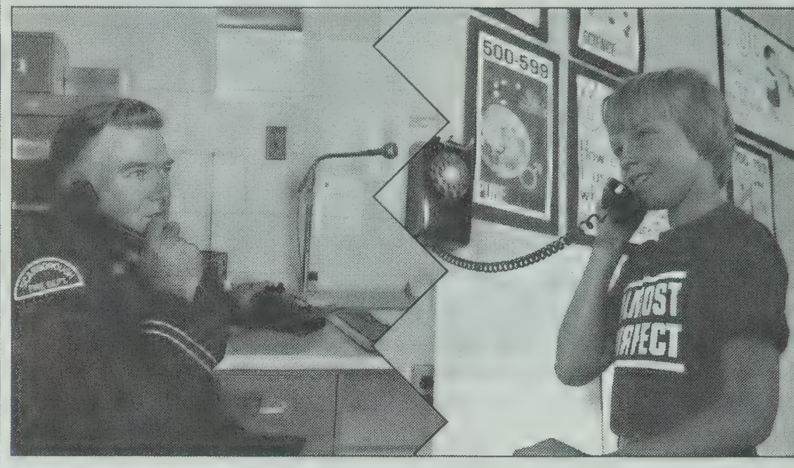
Page 14 is the first of the special problem-solving pages in this book. See page xv for comments on the approach to problem solving in *Starting Points in Mathematics*.

RELATED ACTIVITIES

- Some students may like to write an example of a problem that they encountered outside of school and had to solve by finding the necessary information. They should also explain how they obtained the information. The examples may be displayed and shared with the other students.

Finding the Information Needed

Stuart wanted to know the number of true alarms and the number of false alarms received by the Fire Department.



He called the Fire Department to find out.

How would you get this information?

1. the number of kilometres of highways in Canada
2. the leading batter for the Montreal Expos
3. the distance you plan to travel on a vacation trip
4. the distance you traveled on a vacation trip
5. the number of families named Smith in your town
6. the price of a bicycle that you could afford
7. the current Top Ten records
8. the current best-selling book
9. the number of leaves on a tree
10. how high you can fly a kite
11. the year that Captain Cook landed on the Pacific Coast
12. the number of cars that pass your school in each direction
13. how much money you will need for the next month

PROBLEM SOLVING

Answers will vary.

14

LESSON ACTIVITY

Using the Page

- Of the many strategies that help us solve problems, one that occurs frequently is the need to obtain information from another source. For example, we may ask a particular person, research the information in a book, or conduct a survey. The exercises in this lesson encourage students to consider where or from whom the necessary information might be obtained. A discussion of how to obtain such information might begin this lesson and lead into the example shown at the top of page 14. Note that the students are not required to find the necessary information but only to indicate the source or means for obtaining it. There usually is, of course, more than one way of obtaining the information needed.
- You may wish to have each student choose five or six exercises which interest her/him. However, all the students would benefit from a discussion of the answers suggested for each exercise.

  If a few exercises present particular difficulty, return to them after several days to enable students to consider them at greater length.

Checking Up

Think of a place-value chart to help you tell what the 7 means in each numeral.

1. 407 038 — 7 thousands **2.** 44 702 — 7 hundreds **3.** 708 529 — 7 hundred thousands **4.** 572 681 920 — 7 ten millions

Complete

5. 38 000 = 38 thousand

6. 301 592 = 301 thousand 592

7. 69 005 882 = 69 million 5 thousand 882

Write each in expanded form.

8. 30 200 — 30 000 + 200 **9.** 806 010 — 800 000 + 6000 + 10 **10.** 14 095 370 — 10 000 000 + 4 000 + 90 000 + 5000 + 300 + 70

Write each in standard form.

11. 38 thousand 395 38 395 **12.** 324 thousand 58 324 058

13. four hundred nine thousand 409 000 **14.** sixty-five thousand two hundred 65 200

15. 2 hundred thousands 5 ten thousands 2 thousands 890 907 035

16. 300 000 + 80 000 + 200 + 6 380 206 **17.** 5 000 000 + 70 000 + 3 000 + 20 5 073 020

18. eight hundred ninety million nine hundred seven thousand thirty-five 252 000

19. 6 hundred millions 7 millions 8 ten thousands 3 thousands 5 hundreds 607 083 500

20. LXXXIX 89 **21.** CDLXXIII 473 **22.** MCMXL 1940 **23.** MCXCIV 1194

Use >, <, or = to make true statements.

24. 268 739 > 268 439 **25.** 684 597 = 684 597

26. 21 085 < 20 805 **27.** 64 356 895 > 64 365 598

List from least to greatest.

28. 24 753, 224 537, 224 375, 242 357 — 24 753, 224 375, 224 537, 242 357

List from greatest to least.

29. 807 085, 807 028, 807 580, 89 250 — 807 580, 807 085, 807 028, 89 250

Round to the

30. nearest thousand. 519 800 520 000

31. nearest ten thousand. 376 358 380 000

32. nearest hundred thousand. 349 625 300 000

33. nearest million. 12 903 257 13 000 000

34. nearest ten million. 25 084 311 30 000 000

35. nearest hundred million. 107 399 841 100 000 000

15

Page 15

OBJECTIVE

Demonstrate an understanding of the concepts and skills presented in this unit

RELATED ACTIVITIES

• Many students benefit from reading numerals aloud. Have them read the numerals in Ex. 1-4, 8-10, and 24-35. Others may benefit by showing these numerals on an abacus.

• The digital device suggested in *Related Activities* on page T 5 can be helpful for students having difficulty with rounding numbers. For example, to round 203 157 to the nearest hundred, they can manipulate the loops for hundreds, tens, and ones. The loops are turned forward to show 200, thereby rounding the number up to 203 200. In contrast, to round 203 147 to the nearest hundred, the loops for tens and ones (not hundreds) are turned back to show 00, thereby rounding the number down to 203 100. Also, the devices can help to emphasize that to round a number to a given place, it is necessary to consider only the digit in the place to the immediate right of the given place.

Skills	Exercises	Related Pages
Interpret place value in numerals	1-3	T 4-T 5
	4	T 6-T 7
Read standard numerals	5, 6	T 4-T 5
	7	T 6-T 7
Write numerals in expanded form	8, 9	T 4-T 5
	10	T 6-T 7
Write numerals in standard form	11-16	T 4-T 5
	17-19	T 6-T 7
	20-23	T 12-T 13
Compare numbers	24-27	T 8-T 9
Order numbers	28-29	T 8-T 9
Round numbers	30-35	T 10-T 11

Comments

The chart showing the skill for each exercise on this page is helpful in locating an area of difficulty. Note that Ex. 4, 7, 10, 17-19, 27, 33-35 involve numbers having more than six digits.

Some students may have difficulty placing zeros in the numerals for Ex. 12-15. Working with an abacus, a place-value chart, or the place-value pocket chart described in *Before Using the Pages* on pages T 4 and T 6 may be helpful.

Determine whether errors in Ex. 24-27 are related to actual comparison of numbers or to poor recall of the symbols < and >.

Some students may have poor recall of the seven basic Roman numerals shown on page 12. You may decide to provide them with the basic numerals and test only their ability to interpret the Roman numerals in Ex. 20-23.

Unit 2 Overview

Addition and Subtraction

This unit begins with a review of the basic addition facts and of addition of whole numbers having up to four digits. The process is extended to numbers with five and six digits and to addition of three or four numbers. Basic subtraction facts and subtraction with and without regrouping are reviewed and the process is extended to numbers with five and six digits. Special attention is given to subtraction with zeros in the minuends. Rounding numbers to thousands and to ten thousands is applied in estimating sums and differences. Addition is presented as a means of checking the accuracy of work in subtraction. A lesson is presented on the use of calculators, with the emphasis on determining which operation to use and which keys to press. The problem-solving skill in the last lesson of this unit is related to real life in suggesting that a number of different factors affect the solutions to problems.

Prerequisite Skills

- complete basic addition and subtraction facts
- interpret place value for numerals with up to six digits
- demonstrate competence in lining up numbers in vertical form for addition and subtraction
- regroup for numbers to 9999
- compare two numbers

Unit Outcomes

- complete the basic addition facts
- add two numbers with no regrouping, addends with up to six digits
- add numbers with one, two, or three regroupings, addends with three or four digits
- add two or more numbers with regrouping, sums with up to six digits
- round two or more addends and add to estimate the sum, then compare the estimate of the sum with the exact sum
- write the family of addition and subtraction facts for a given set of numbers
- subtract numbers with no regrouping, minuends with up to five digits
- use addition to check subtraction
- subtract numbers with one, two, or three regroupings, minuends with four or five digits
- subtract with regrouping for zero in one or more places in the minuend, minuends with up to five digits
- solve word problems using addition or subtraction
- round the minuend and the subtrahend and subtract to estimate the difference, then compare the estimate of the difference with the exact difference
- relate additive and subtractive situations to the use of the $\boxed{+}$ and $\boxed{-}$ keys on a calculator
- identify different situations that affect answers to problems

Background

The most basic operation with numbers is addition and the other three, subtraction, multiplication, and division, can all be derived from it directly or inversely. The total number in two or more groups can be found by counting, but a more advanced and speedier method is addition. Speed and accuracy in addition are dependent upon competence in the basic facts.

The basic addition facts are those which have two one-digit addends, such as in the facts $6 + 0 = 6$, $5 + 1 = 6$, $7 + 8 = 15$, and $9 + 9 = 18$. Using the ten digits 0 to 9 in both positions for addends produces a set of 100 addition facts. Of these, 10 are "doubles", such as $2 + 2 = 4$, and the other 90 facts may be reduced to 45 basic relationships through the *commutative property of addition*. Because of this property, the order of two addends may be changed without affecting their sum. So, for every two facts in which the addends are different, only one basic relationship needs to be known, as in $6 + 5 = 11$ and $5 + 6 = 11$. This set of 55 basic facts may be reduced still further to a set of 45 if one rule for zero as an addend is applied; namely, the sum of any number and zero is that number. Mastery of these 45 basic addition facts and the rule for zero is essential.

The inverse relationship between addition and subtraction, by which one operation "undoes" the other, permits the same set of basic facts to be used in both operations. For example, the basic subtraction facts $11 - 4 = 7$ and $11 - 7 = 4$ are derived from the basic addition fact $7 + 4 = 11$ and its commuted fact $4 + 7 = 11$. Sets of four facts for unequal addends, and two facts for equal addends, are called "families" of facts. Efficiency and accuracy are realized if families of basic facts are learned and mastered.

The four basic operations with numbers are *binary*; that is, only two numbers can be combined at any one time and with one unique result. In addition the result is called the *sum*, and in subtraction it is called the *difference*. If three or more numbers are involved in one or more operations, two combine to form a new number which is then combined with another number, and so on, until all the numbers have been used in the operation(s). In addition, three or more addends may be considered in any order with no effect on their sum. This *associative property of addition* is particularly useful in checking the accuracy of column addition by adding in opposite directions.

The commutative and associative properties do not apply in subtraction. Because of the inverse relationship between subtraction and addition, addition may be used to check accuracy in subtraction, as shown.

$$\begin{array}{r} 7603 \\ -\ 1978 \\ \hline 5625 \end{array} \qquad \begin{array}{r} 5625 \\ +\ 1978 \\ \hline 7603 \end{array}$$

The inverse relationship is used in one of the *Try This* features in the unit. An easy way to find the sum of 328 and 497, for example, is to add 3 to 497 to give 500 and then to subtract 3 from the sum of 328 and 500. Briefly, if extra is added, the same amount must be subtracted from the sum. Similarly, for $412 - 99$, subtract 100 and add 1 to the difference. Again, if extra is subtracted, the same amount must be added to the difference.

Regrouping is essentially the same skill in addition and in subtraction. The inverse relationship is again apparent, for in addition the regrouping proceeds from a lesser to a greater place value, and in subtraction, from a greater to a lesser place value. In subtraction the regrouping always involves a change from one unit of a value to ten of a lesser value, but in addition the regrouping may involve two, three, or more greater values. In the example shown, the sum of the ones is 32 (regrouped as 3

tens 2 ones), the sum of the tens is 25 (regrouped as 2 hundreds 5 tens), and the sum of the hundreds is 13 (regrouped as 1 thousand 3 hundreds). Note that partial sums are encountered and these become unseen addends. Extensions of basic addition facts are frequently required. Both of these features contribute to difficulties in addition and deserve special attention. For example, adding downward for the ones gives $9 + 8 = 17$, $17 + 6 = 23$, and then $23 + 9 = 32$. The unseen addends are 17 and 23; $17 + 6 = 23$ and $23 + 9 = 32$ are extensions of the basic facts $7 + 6 = 13$ and $3 + 9 = 12$, respectively. It is important that students have facility in thinking of unseen addends and in using extensions in column addition.

$$\begin{array}{r} {\scriptstyle 23} \\ 479 \\ 258 \\ 346 \\ +\ 269 \\ \hline 1352 \end{array}$$

Subtraction from minuends with zeros often presents considerable difficulty. Difficulties can be minimized by reducing the number of steps. Specific teaching techniques are offered in the *Teaching Strategies* and in the suggestions for teaching pages 32 and 33. It should be noted that a numeral can be interpreted in a variety of ways. For example, a three-place numeral such as 245 shows hundreds, tens, and ones, and may be interpreted as 2 hundreds 4 tens 5 ones, as 24 tens 5 ones, or as 245 ones. Similarly, a number such as 30 000 may be interpreted in the following ways.

$\boxed{3}0\ 000$	3 ten thousands
$\boxed{30}\ 000$	30 thousands
$\boxed{30\ 0}00$	300 hundreds
$\boxed{30\ 00}0$	3 000 tens
$\boxed{30\ 000}$	30 000 ones

In the subtraction of 17 695 from 30 000 it is immediately apparent that regrouping must take place. To subtract ones, a ten must be regrouped as 10 ones. If the minuend 30 000 is interpreted as 3000 tens, the regrouping can occur in one step, changing it to 2999 tens 10 ones (A). With older teaching methods regrouping usually proceeded one step at a time in a much more complex way (B). Flexible interpretation of numbers can lead to efficiency, especially in subtraction with zeros in the minuend.

A
$$\begin{array}{rrrrrr} & 2 & 9 & 9 & 9 & 10 \\ & \cancel{3} & \cancel{0} & \cancel{0} & \cancel{0} & \cancel{0} \\ - & 1 & 7 & 6 & 9 & 5 \\ \hline & 1 & 2 & 3 & 0 & 5 \end{array}$$

B
$$\begin{array}{rrrrrr} & & 9 & 9 & 9 & \\ & 2 & \cancel{10} & \cancel{10} & \cancel{10} & 10 \\ & \cancel{3} & \cancel{0} & \cancel{0} & \cancel{0} & \cancel{0} \\ - & 1 & 7 & 6 & 9 & 5 \\ \hline & 1 & 2 & 3 & 0 & 5 \end{array}$$

The first of seven lessons on the use of a calculator is presented in this unit. Exercises which appear on each such page may be completed without using a calculator, but the benefits of using a calculator should not be overlooked. (See the comments about the use of calculators on page xv.)

Solving problems requires, besides computational skills, an ability to identify whether the relationships are additive, subtractive, multiplicative, or divisive. Although a calculator can compute rapidly, it cannot choose the operations. Additive situations are relatively easy to spot because the operation is performed only for the purpose of finding the total number when two or more groups are combined. On the other hand, subtraction may be used for several purposes: to find how many are left, to find how many are gone, to find which group is larger (smaller), and to find how many more are needed. Of the first two, which may be referred to as "separations", the "how many left" is the more common, although the removal of a subset is involved in both. The third type of subtractive situation compares two groups or two numbers. In this type there is no separation; two sets are matched to see which has more (fewer) members and two numbers are treated by subtraction to find their difference. The fourth type of subtractive situation is actually an incomplete additive situation for which the inverse operation of subtraction is used to find the unknown addend. Students need many experiences with all of these subtractive relationships in order to use subtraction effectively in solving such problems.

Teaching Strategies

Before commencing Unit 2 it is suggested that a survey be taken of students' abilities in addition and subtraction, using numbers with up to four digits, with zeros in the addends and the minuends, and with some regrouping in the two operations. Analysis of the results will indicate areas of strength and weakness which should be considered in planning the lessons. Students with similar needs should be taught as one group with materials and methods selected accordingly. Restructuring of the groups should occur from time to time as individual students indicate changes in their needs.

Place-value charts should be used in many of the lessons to bring meaning to the operations with larger numbers, especially when the regrouping of sums or minuends is required. Models of ones, tens, and hundreds are also useful aids for teaching and learning, but can become rather cumbersome for five-place and six-place numerals. Sets of colored cards with place values written on them, similar to money in the game of Monopoly, may be used effectively. Whenever it is necessary, a card of one denomination may be exchanged for 10 cards of a lesser value, and vice versa.

Regroupings of sums and minuends are shown on the students' pages to ensure meaningful development of skills. Students should continue to show regroupings in their written work as long as they need to, but as they gain confidence they should be encouraged to do as much work as possible by merely thinking of the regroupings.

If there is a limited number of calculators, present the lesson on pages 38 and 39 relatively early in the unit to enable each student to use a calculator. If students work in pairs for some of the *Related Activities* suggested for pages T33, T36, and T37, calculators may be used to check the results.

Materials

stopwatches or watches with second hands (optional)
four-place abacus
models for thousands, hundreds, tens, and ones
map of Canada (optional)
a copy of page T391 for each student (optional)
a blank sheet of paper for each student
calculators (optional)

Vocabulary

addition, add	basic subtraction fact
addend, sum	kilometres, km
basic addition fact	metres, m
vertical form	names of the ten provinces of Canada and their abbreviations
regroup	palindrome
estimate	cross-number puzzle
family	calculator
related	digit display
subtraction, subtract	keys
difference	

LESSON OUTCOME

Complete the basic addition facts

Materials

a copy of the ten-by-ten grid on page T 391 for each student (optional), stopwatches or watches with second hands (optional)

Vocabulary

addition, add, addend, sum, basic addition fact

RELATED ACTIVITIES

- Provide students with number wheels and tables prepared from copies of page T 390, to practice basic addition facts.

+	7
8	15
2	
	12
1	

- Copies of the addition table on page T 380 may be used by students who have poor recall of basic addition facts. If students work in pairs, one student can name pairs of addends and the other can name the sum. If the sum is correct, the squares for that sum are colored.

2 ADDITION AND SUBTRACTION

Basic Addition Facts

There had been 8 members of the Dionne family until May 28, 1934 when 5 baby girls were born.

$8 + 5 = 13$

$$\begin{array}{rl} 8 & \text{addend} \\ \underline{5} & \text{addend} \\ 13 & \text{sum} \end{array}$$

Then there were 13 in the Dionne family.

Exercises

Write the six sums for each row.

Have someone time you for some rows. Which row can you do the fastest?

1. →	5 + 2 7	4 + 3 7	3 + 5 8	2 + 7 9	1 + 4 5	2 + 2 4
2. →	3 + 9 12	3 + 4 7	8 + 7 15	0 + 9 9	3 + 8 11	7 + 7 14
3. →	7 + 1 8	9 + 1 10	4 + 1 5	8 + 1 9	6 + 1 7	1 + 1 2
4. →	6 + 8 14	1 + 9 10	7 + 5 12	3 + 6 9	9 + 7 16	6 + 6 12
5. →	5 + 9 14	5 + 6 11	5 + 7 12	5 + 4 9	5 + 8 13	5 + 5 10
6. →	7 + 8 15	6 + 5 11	9 + 4 13	7 + 2 9	6 + 9 15	8 + 8 16
7. →	4 + 0 4	0 + 7 7	3 + 0 3	9 + 0 9	0 + 6 6	0 + 0 0
8. →	9 + 8 17	7 + 0 7	4 + 8 12	6 + 3 9	8 + 6 14	4 + 4 8
9. →	0 + 3 3	1 + 3 4	1 + 5 6	4 + 5 9	4 + 9 13	9 + 9 18
10. →	6 + 7 13	3 + 1 4	8 + 5 13	1 + 8 9	8 + 4 12	3 + 3 6
	↓ 11.	↓ 12.	↓ 13.	↓ 14.	↓ 15.	↓ 16.

Try the additions again. This time, list the sums for each column.

Which could you list faster, the sums for a row or for a column?

16

LESSON ACTIVITY

Using the Page

- Begin with a discussion of the photograph. Ask how many babies there are and, if you wish, introduce the word *quintuplets*. Have the students read the introductory statement silently and then ask for the date of birth of the five baby girls. Ask how many members there were in the Dionne family after the quintuplets were born. Ask what numbers were used and what operation was performed to obtain the answer. Point out the two forms, horizontal and vertical, for recording the addition, and review the terms *addend* and *sum*.

Exercises: Have the students study the exercises. Ask, "How many addends are there in each exercise?" "What number is the greatest addend?" Establish that an addition fact that involves just two addends, each of which is not greater than nine, names a *basic addition fact*. Ask whether 9 + 3 + 4 = 16 and 12 + 3 = 15 name basic addition facts and have students give reasons for their answers.

- Since the students are to write the sums for each row and for each column, each addition is performed twice. There is no need to copy the addends. (You may wish to have the students write their answers on copies of page T 391.) Note that there are patterns for some of the rows and columns, and these can help students to complete the facts quickly. For example, the commutative (order) property of addition is illustrated in Ex. 12 and it would be beneficial to have students express this property in their own words, after the exercises are completed. Ex. 7 illustrates that zero as an addend does not affect the sum. Discuss the patterns for other rows and columns.

Assessment

Add.

1. 4 + 6 10 **2.** 3 + 9 12 **3.** 8 + 5 13 **4.** 7 + 7 14

5. 5 + 9 14 **6.** 6 + 7 13 **7.** 9 + 9 18 **8.** 4 + 5 9

9. $\begin{array}{r} 3 \\ \underline{7} \\ 10 \end{array}$ **10.** $\begin{array}{r} 2 \\ \underline{9} \\ 11 \end{array}$ **11.** $\begin{array}{r} 8 \\ \underline{0} \\ 8 \end{array}$ **12.** $\begin{array}{r} 9 \\ \underline{6} \\ 15 \end{array}$

Addition with No Regrouping

113 036 people were living in Halifax in 1976. Across the harbor, 64 452 people were living in Dartmouth. How many people were living in both cities?

Add 113 036 and 64 452.

	hundred thousands	ten thousands	thousands	hundreds	tens	ones
Show the addends in vertical form	1	1	3	0	3	6
		6	4	4	5	2

Then use basic addition facts place by place to find the sum.

Add ones. Add tens. Add hundreds.	1	1	3	0	3	6
		6	4	4	5	2
				4	8	8

Add thousands. Add ten thousands. Add hundred thousands.	1	1	3	0	3	6
		6	4	4	5	2
	1	7	7	4	8	8

177 488 people were living in Halifax and Dartmouth.

Exercises

Add.

1. 27 + 42 = 69
2. 234 + 525 = 759
3. 6234 + 3762 = 9996
4. 15 911 + 24 057 = 39 968
5. 144 421 + 14 363 = 158 784
6. 38 + 51 89
7. 707 + 181 888
8. 5235 + 2064 7299
9. 26 381 + 1 608 27 989
10. 27 531 + 30 131 57 662
11. 2 152 + 11 402 13 554

17

LESSON OUTCOME

Add two numbers with no regrouping, addends with up to six digits

Materials

abacus (optional)

Vocabulary

vertical form

Prerequisite Skills

Complete basic addition facts, sums to 9; interpret place value for numerals with up to six digits

Checking Prerequisite Skills

Add.

1. 6 + 3 = 9 **2.** 4 + 2 = 6 **3.** 0 + 9 = 9 **4.** 3 + 5 = 8

What does each 4 mean?

5. 24 365 4 thousands **6.** 423 809 4 hundred thousands

RELATED ACTIVITIES

- For further practice, you may wish to have students complete Ex. 1-5 on page 332.
- Some students may require practice in reading numerals with up to six digits. Work with them in small groups, having them read the numerals for Ex. 1-11 aloud.
- Have students demonstrate several of the exercises in this lesson on an abacus.

LESSON ACTIVITY

Using the Page

- Have the students read the word problem at the top of the page silently. Refer them to the photograph and discuss the fact that the two cities are situated on opposite sides of a harbor. Some students may be able to name another pair of cities that are similarly situated, for example, Vancouver and North Vancouver. Ask what operation is required to find how many people were living in both cities. Have students read the addends aloud and state the number of digits in each. Discuss the implications of not lining up the digits place by place in vertical form. Review that the addition is completed in a right-to-left order and have the students note that basic addition facts are applied in each place. Remind the students that a space is left between the hundreds' and the thousands' places. Have a student read the concluding statement.

  Have one group of students check the addition by changing the order of the addends. Another group may demonstrate the addition on an abacus. If possible, have students explain the meaning of "no regrouping" at the top of the page.

Exercises: Note that if each addend in an exercise has not more than four digits, a space between the hundreds' place and the thousands' place is not required (Ex. 3 and 8, for example). Have the students check their additions in one or both of the ways suggested in *Using the Page*.

Assessment

Add.

1. 4315 + 2063 = 6378 **2.** 372 154 + 13 825 = 385 979

3. 1 417 + 21 332 22 749 **4.** 50 635 + 312 334 362 969

Pages 18-19

LESSON OUTCOME

Add numbers with one, two, or three regroupings, addends with three or four digits; solve related problems

Materials

a four-place abacus or models for thousands, hundreds, tens, and ones

Vocabulary

regroup, kilometres, km

Prerequisite Skills

Complete the basic addition facts; demonstrate competence in lining up addends in vertical form for addition; interpret place value for numerals with up to four digits; regroup for numbers to 9999

Checking Prerequisite Skills

Add.

1. 9 + 4 = 13 **2.** 7 + 9 = 16 **3.** 5 + 6 11 **4.** 6 + 8 14

Show the addends in vertical form with their places lined up.

5. 145 + 2311 145 / 2311 **6.** 2217 + 5420 2217 / 5420

What does each digit mean in

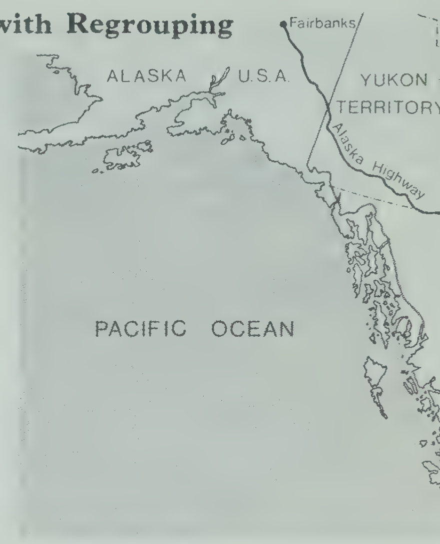
7. 1605? 1 thousand 6 hundreds 0 tens 5 ones

Complete.

8. 17 tens = __1__ hundred __7__ tens

9. 1 thousand 13 hundreds = __2__ thousands __3__ hundreds

Adding Four-Digit Numbers with Regrouping



1965 km (kilometres) of the Alaska Highway are in Canada. 486 km are in the United States. How long is the Alaska Highway?

Add 1965 and 486.

Show the addends in vertical form.

$$\begin{array}{r} 1965 \\ 486 \\ \hline \end{array}$$

Add ones. Regroup 11 ones as 1 ten, 1 one.

$$\begin{array}{r} \scriptstyle 1\; \\ 1965 \\ 486 \\ \hline 1 \end{array}$$

Add tens. Regroup 15 tens as 1 hundred, 5 tens.

$$\begin{array}{r} \scriptstyle 1\;1\;\; \\ 1965 \\ 486 \\ \hline 51 \end{array}$$

Add hundreds. Regroup 14 hundreds as 1 thousand, 4 hundreds.

$$\begin{array}{r} \scriptstyle 1\;1\;1\;\;\; \\ 1965 \\ 486 \\ \hline 451 \end{array}$$

Add thousands.

$$\begin{array}{r} \scriptstyle 1\;1\;1\;\;\; \\ 1965 \\ 486 \\ \hline 2451 \end{array}$$

The Alaska Highway is 2451 km long.

Working Together

Add by following the steps.

1.
$$\begin{array}{r} \scriptstyle 1\;1\;\; \\ 4536 \\ 3725 \\ \hline 8261 \end{array}$$

Add ones and regroup.
Add tens.
Add hundreds and regroup.
Add thousands.

Add.

2. 7039 + 2937 = 9976 **3.** 3508 + 4655 = 8163

4. 3987 + 2835 6822

5. 2296 + 5763 8059

6. 379 + 2389 2768

18

LESSON ACTIVITY

Before Using the Pages

- Write the exercise 1436 + 2140 in vertical form on the board. Have students use an abacus (or models of thousands, hundreds, tens, and ones) to demonstrate the addition. Review that in addition the ones are added first, then the tens, hundreds, and thousands are added in turn. Have a student read the sum shown on the abacus.

  Write the exercise 1436 + 2847 in vertical form on the board. Have the students observe that one addend is the same as in the preceding exercise, but the other addend is different. Have them demonstrate the addition on an abacus, explaining each step of the procedure. Discuss the need to regroup in some places and not in others. Have students explain how they recognize whether regrouping will be required. Summarize by noting that 10 ones equal 1 ten, 10 tens equal 1 hundred, and 10 hundreds equal 1 thousand. You may wish to write the addition in a place-value chart on the board.

thousands	hundreds	tens	ones
1		1	
1	4	3	6
+ 2	8	4	7
4	2	8	3

Using the Pages

- Begin with a discussion of the map shown at the top of pages 18 and 19. Have the students trace the red paths representing the boundaries between Canada and the United States. Have them point to Fairbanks and Dawson Creek and describe the location of each city. Have them name and trace the highway that joins these two cities. Ask whether the greater part of the highway runs through Canada or through the United States. Have them describe the location of Lloydminster. (It is on the boundary between Alberta and Saskatchewan.)
- The worked example reviews the steps in the *algorithm* (arrangement of numerals in vertical form) for addition with regrouping. Each step is highlighted in red. Have a

Exercises

Add.

1. 3467 + 5928 = 9395
2. 4322 + 1494 = 5816
3. 6048 + 1971 = 8019
4. 5227 + 384 = 5611
5. 6294 + 3536 = 9830
6. 5930 + 3491 = 9421
7. 7868 + 673 = 8541
8. 3968 + 3987 = 7955
9. 5397 + 2857
10. 3097 + 989
11. 621 + 6624
12. 1644 + 2948
13. 4937 + 273
14. 3587 + 1441
15. 4016 + 2666
16. 503 + 3826
17. 1175 + 8169
18. 5815 + 1875

9. 8254 10. 4086 11. 7245 12. 4592 13. 5210 14. 5028 15. 6682 16. 4329 17. 9344 18. 7690

Solve.

19. Canada's border with Alaska is 2478 km long. Canada's border with the rest of the United States is 6416 km long. How long is Canada's border with the United States? 8894 km

20. 5715 people lived in Lloydminster, Alberta in 1976. 4432 people lived in Lloydminster, Saskatchewan. How many people lived in Lloydminster? 10 147

21. 398 people lived in Flin Flon, Saskatchewan in 1976. 8033 people lived in Flin Flon, Manitoba. How many people lived in Flin Flon? 8431

19

RELATED ACTIVITIES

- Students may use models or an abacus to demonstrate some of the addition exercises on page 19.
- Provide students with exercises that contain errors. (Examples may be compiled from the students' work.) Have them identify and correct the errors as shown.

$$\begin{array}{r} {}^{1}\ \ \ {}^{1}\ \ \ \\ 3\ 6\ 4\ 2 \\ +\ 1\ 4\ 1\ 7 \\ \hline 5\ 0\ 6\ 9 \end{array} \qquad \begin{array}{r} {}^{1}\ \ \ \ \ \ \ \ \ \ \\ 3\ 6\ 4\ 2 \\ +\ 1\ 4\ 1\ 7 \\ \hline 5\ 0\ 5\ 9 \end{array}$$

- For further practice with basic addition facts, provide students with copies of page T391. Have them write the numbers 0 to 9 in any order as addends in the top row and the left column. Have them complete the table by writing the sums.

+	7	3	6	9	8	2	4	0	5	1
6										
0										
3										
9										
1										
5										
8										
2										
4										
7										

student read the word problem at the top of page 18. Discuss the fact that the solution involves addition, in order to find the number of kilometres in the Alaska Highway. Question the students as you lead them through the steps of the solution. Ask, for example, "How does writing the addends in vertical form help in the addition?" "With which place does the addition begin?" "What is the sum of the ones?" "Why is a small 1 shown above the 6 in the tens' place?" Emphasize the right-to-left order of adding and regrouping. It may not be necessary for some students to write the numerals to indicate the regrouping. Discuss this with them, and then have a student read the concluding statement at the bottom of the page.

Working Together: Ex. 1 emphasizes the right-to-left order of adding and regrouping, where necessary. The four brief statements provided for Ex. 1 can be adapted to describe the additions for the remaining exercises. You may find it beneficial to have students provide similar statements in an oral discussion of these exercises.

Exercises: Remind the students to write the addends in vertical form for Ex. 9-18 and a concluding statement for Ex. 19-21.

Assessment

Add.

1. 4826 + 538 = 5364
2. 1234 + 5678 6912
3. 3647 + 353 4000

Solve.

4. The distance from Thunder Bay to Regina is 1277 km. The distance from Regina to Vancouver is 1809 km. How far is it from Thunder Bay to Vancouver? 3086 km

Pages 20-21

LESSON OUTCOME

Add two or more numbers with regrouping, sums with up to six digits

Materials

abacus (optional), map of Canada (optional)

Vocabulary

names of the ten provinces of Canada and their abbreviations

Prerequisite Skills

Add more than two one-digit numbers; add two numbers with regrouping, sums to 9999

Checking Prerequisite Skills

Add.

1. 4, 4, 7 — 15

2. 3, 8, 9 — 20

3. 1, 5, 6, 8 — 20

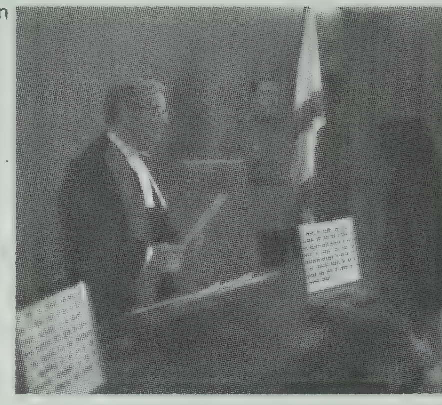
4. 2, 7, 9, 9 — 27

5. 1485 + 796 — 2281

6. 3389 + 1615 — 5004

Addition with Regrouping

More than 137 000 people became Canadian citizens in 1976. Of these, 38 061 had been citizens of the United Kingdom and 58 458 had been citizens of other countries in Europe. How many people from Europe became Canadian citizens?



Add 38 061 and 58 458.

Add ones, tens, and hundreds.

$$\begin{array}{r} \scriptstyle 1 \\ 38\,061 \\ 58\,458 \\ \hline 519 \end{array}$$

Remember to regroup when needed.

Then, add thousands and ten thousands.

$$\begin{array}{r} \scriptstyle 1 \\ 38\,061 \\ 58\,458 \\ \hline 96\,519 \end{array}$$

In 1976, 96 519 people from Europe became Canadian citizens.

In that same year, 40 988 people from other countries of the world became Canadian citizens. How many people became Canadian citizens?

You can add 38 061, 58 458, and 40 988, or . . .

. . . you can add 40 988 to the number who came from Europe.

$$\begin{array}{r} \scriptstyle 1\,1\,\,2\,1 \\ 38\,061 \\ 58\,458 \\ 40\,988 \\ \hline 137\,507 \end{array} \qquad \begin{array}{r} \scriptstyle 1\,\,1\,1 \\ 96\,519 \\ 40\,988 \\ \hline 137\,507 \end{array}$$

137 507 people became Canadian citizens in 1976.

20

LESSON ACTIVITY

Before Using the Pages

- Prepare a place-value chart for six-digit numerals on the board. Name a number, for example, seventy-five thousand two hundred eighty-six. Have a student write the standard numeral on the board. Have another student write the digits in the place-value chart. Repeat the procedure for another number, for example, three hundred twenty-seven thousand six hundred thirty-five.

	hundreds (thousands)	tens (thousands)	ones (thousands)	hundreds	tens	ones
75 286		7	5	2	8	6
327 635	+ 3	2	7	6	3	5

Guide the students in finding the sum of the two numbers, adding place by place from right to left. Have students suggest the regrouping that is required in the addition process. For example, the sum of the thousands is 12 thousands and this is regrouped as 1 ten thousand 2 thousands. You may wish to demonstrate the addition on an abacus.

Using the Pages

- Read the first sentence of the word problem at the top of page 20 and explain the meaning of the word *citizens*. Students or parents of students in your class may have been born in another country and now live in Canada. Ask students who are able to relate to the photograph shown to describe the procedure for taking the oath of citizenship.

Read the rest of the word problem and discuss why the solution can be found by addition. Have students explain the steps of adding and regrouping, and then read the concluding statement about people from Europe who became Canadian citizens. Continue with the next stage of the worked example and discuss the two approaches shown for finding the sum. For the exercise with three addends, have students perform the addition aloud to explain each digit of the sum.

Working Together

Add by following the steps.

1.
```
   1 1 1 1
    34 769
    25 483
    60 252
```
Add ones and regroup.
Add tens and regroup.
Add hundreds and regroup.
Add thousands and regroup.
Add ten thousands.

2.
```
  2 1 1
   1934
   1862
    518
   4314
```
Add ones and regroup.
Add tens and regroup.
Add hundreds and regroup.
Add thousands.

Add.

3. 70 935 + 25 437 = 96 372

4. 21 743 + 43 962 + 64 531 = 130 236

5. 83 457 + 12 568 96 025

6. 9 135 + 17 648 + 6 278 33 061

Exercises

Add.

1. 12 445 + 15 684 = 28 129
2. 66 357 + 53 926 = 120 283
3. 25 256 + 8 359 = 33 615
4. 74 181 + 57 834 = 132 015
5. 19 064 + 2 379 + 27 273 = 48 716
6. 7 352 + 24 378 + 989 = 32 719
7. 6 628 + 26 447 33 075
8. 39 218 + 5 494 + 24 039 68 751
9. 37 947 + 93 627 131 574
10. 97 156 + 8 492 + 178 + 6 076 111 902
11. 40 892 + 20 897 + 59 024 120 813
12. 8 393 + 15 418 + 37 564 61 375
13. 5398 + 1760 + 5187 12 345
14. 19 738 + 14 829 34 567
15. 443 + 2 863 + 53 483 56 789

This chart shows the number of immigrants in each province who became citizens in 1976.

Alta.	8 260	N.S.	1 143
B.C.	22 290	Ont.	78 724
Man.	3 953	P.E.I.	91
N.B.	493	Que.	20 638
Nfld.	260	Sask.	1 371

How many immigrants became citizens in

16. either New Brunswick or Quebec? 21 131

*17. an island province? 351

*18. the Prairie Provinces? 13 584

*19. the three Maritime Provinces? 1727

*20. one of the provinces west of Ontario? 35 874

21

RELATED ACTIVITIES

• For further practice, you may wish to have students complete selected exercises from Ex. 6-26 on page 332.

• In addition of more than two numbers, it is helpful to find number pairs having a sum of 10. For example, 6 + 7 + 9 + 3 + 4 + 1 + 5 can be computed mentally by adding in the following order.

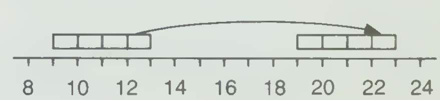
(6 + 4) + (7 + 3) + (9 + 1) + 5

Prepare several similar exercises and encourage the students to add by finding combinations for 10. This approach can then be applied in any column for the addition of more than two numbers.

• Practice with extensions of basic addition facts can improve performance in column addition of more than two numbers. Students need to see that knowing the basic fact 9 + 4 = 13, for instance, can help in finding the following sums.

19	29	39	49
+ 4	+ 4	+ 4	+ 4
23	33	43	53

This relationship of a basic fact and its extensions is probably best observed on the number line. For example, by holding a "four" strip on the number line at 9, 19, 29, . . ., it can be seen that the "four" strip ends at 13, 23, 33, . . . for the sums 9 + 4, 19 + 4, 29 + 4, ...



Working Together: Ex. 1 and 2 emphasize the right-to-left order of adding and regrouping for two addends and three addends, respectively. Because the addends in Ex. 2 and 6 do not have the same number of digits, it is advisable to review the importance of arranging the addends in vertical form with their places lined up. Review that for exercises with more than two addends, a sum found by adding in one direction can be checked by adding in the opposite direction.

Exercises: Before the students begin Ex. 16-20, have them name the provinces indicated by the abbreviations in the chart. Ex. 17-20 are starred because their solutions depend on additional information which the students may need to obtain elsewhere. Discuss where the information can be found and, if necessary, have a map of Canada available for reference. The map on pages 18 and 19 can be helpful for Ex. 18 and 20.

Assessment

Add.

1. 34 169 + 8 943 = 43 112

2. 1452 + 683 + 9325 11 460

3. 274 + 4 870 + 63 194 68 338

OBJECTIVE

Demonstrate competence in addition with and without regrouping; solve related word problems

Vocabulary

palindrome

Practice

Add

1. 2418 + 844 = 3262
2. 3674 + 1353 = 5027
3. 15 236 + 6 376 = 21 612
4. 29 135 + 45 899 = 75 034
5. 98 736 + 22 647 = 121 383
6. 149 + 376 + 192 = 717
7. 5126 + 465 + 5056 = 10 647
8. 7 624 + 18 908 + 758 = 27 290
9. 14 698 + 29 786 + 8 737 = 53 221
10. 17 534 + 26 583 + 17 584 = 61 701

11. 3894 + 519 4413
12. 796 + 2375 3171
13. 5 527 + 97 274 102 801
14. 18 964 + 57 947 76 911
15. 17 971 + 9 479 27 450
16. 3586 + 8498 12 084
17. 854 + 8309 + 986 10 149
18. 6 756 + 4 992 + 10 482 22 230
19. 20 825 + 59 129 + 46 230 126 184
20. 7 196 + 16 073 + 2 282 + 15 464 41 015

Amounts of money are added just like whole numbers.

Examples:

$16 548 + 19 497 = $36 045	$265.93 + 85.46 = $351.39	$306.73 + 242.97 + 918.34 = $1468.04

Add.

21. $8423 + 8683 = $17 106
22. $235.87 + 58.38 = $294.25
23. $ 5 769 + 11 974 + 4 463 = $22 206
24. $221.61 + 63.42 + 90.83 + 102.24 = $478.10

25. $82 942 + $65 329 $148 271
26. $45.67 + $456.78 $502.45
27. $3 698 + $25 858 + $4 476 $34 032
28. $106.65 + $43.75 + $172.95 $323.35

Use >, <, or = to make true statements.

29. 4261 + 4973 ● 7766 + 1666 <
30. $4 498 + $77 167 ● $72 998 + $8 658 >
31. 31 729 + 11 359 ● 12 751 + 27 648 >
32. 843 + 974 + 596 ● 1828 + 585 =
33. $236.67 + $269.48 ● $506.15 =
34. 29 187 ● 7 379 + 949 + 21 672 <

22

LESSON ACTIVITY

Using the Pages

- Before the students begin these exercises, you may wish to conduct an oral review of some of the concepts presented in the preceding lessons. Have them study the pages for a moment. Then ask questions and direct the students in the following manner.
  1. Name an exercise that has three addends.
  2. For an exercise that has four addends,
     a. name the greatest addend.
     b. give the number of digits in the greatest addend.
     c. which digit is in the thousands' place of the greatest addend?
     d. name the four addends from least to greatest.
  3. For an exercise that involves dollars and cents, name the first addend.
  4. What helps in lining up the dollars and cents in vertical form?
  5. For an exercise that involves only dollars, name the first addend.
  6. In the instruction for Ex. 29-34, tell what the first symbol means.
  7. How would you determine which symbol is the correct one for Ex. 29?

  You may wish to work Ex. 29 on the board with the students, thereby reviewing the steps of adding with regrouping and providing a format for the students to follow for Ex. 30-34.

Ex. 29   4261 + 4973 = 9234   7766 + 1666 = 9432

4261 + 4973 < 7766 + 1666

Remind the students to write concluding statements for Ex. 35-39.

Jill's father works full time.
Her mother works part time.

35. Last year Jill's father earned $9348. Her mother earned $4875. How much did they earn together? $14 223
36. Jill's mother had to pay $188.50 and her father had to pay $1351.70 for income taxes. What was their total income tax? $1540.20
37. With income taxes paid, Jill's father earned $7996.30 and her mother earned $4686.50. How much was that in all? $12 682.80
38. Jill's mother paid a babysitter $248.75 and a nursery school $367.50 during the year to help care for Jill's little brother. How much did this cost in all? $616.25
39. Jill's mother saved $475.87 and her father saved $665.58 for the year. How much did they save in all? $1141.45

A *palindrome* reads the same from left to right or from right to left.

DID	TOOT	RADAR
121	7337	54945

Give another example of a palindrome. Answers will vary.

1. with three letters. MOM
2. with four letters. NOON
3. with four digits. 8228
4. with five digits. 61316

786 is not a palindrome, but follow these steps using addition.

$$\begin{array}{r} 786 \\ \underline{687} \\ 1473 \end{array} \qquad \begin{array}{r} 1473 \\ \underline{3741} \\ 5214 \end{array} \qquad \begin{array}{r} 5214 \\ \underline{4125} \\ 9339 \end{array}$$

a palindrome!

Answers will vary.
Choose any number.
An example is given.

5. Reverse the order of its digits and add. Is the sum a palindrome? no

$$\begin{array}{r} 284 \\ \underline{482} \\ 766 \end{array}$$

6. If not, repeat the above step one or more times to see if you can get a palindrome.

$$\begin{array}{r} 766 \\ \underline{667} \\ 1433 \end{array} \rightarrow \begin{array}{r} 1433 \\ \underline{3341} \\ 4774 \end{array}$$

try this

Try This: The investigation of numbers that are palindromes provides practice in addition with regrouping. You may wish to have each student show one example on a separate sheet of paper and display the examples. The example provided for 786 requires three additions. Challenge the students to find an example that requires more than three additions. Challenge some students to find how many additions are required for 276.

Some examples of words that are palindromes are *wow*, *mom*, *dad*, *bib*, *Bob*, *gag*, *sees*, *deed*, *noon*, and *Anna*. Two sentences that are palindromes are "Poor Dan is in a droop" and "Madam, I'm Adam".

RELATED ACTIVITIES

- Further practice with addition of one-digit numbers can be given in the form of a number square as shown. Have the students add horizontally, vertically, and diagonally. Copies of page T 397 may be used to prepare the number squares.

					32
	6	4	9	7	
	8	3	8	3	
	2	9	7	6	
	8	9	4	5	

Note that some students may compute the sum 32, for example, for the indicated square, and then write 32 in the diagonally opposite square without computing the sum again. Such students are demonstrating an understanding of the associative property of addition.

- For further practice in adding amounts of money, cut pictures from catalogs and newspapers of items such as furniture and appliances with the prices shown. Paste the pictures on a sheet of Bristol board. Have students pretend to buy two to four items and find the total cost.

The pictures can also be used by students for writing word problems for other students to solve.

LESSON OUTCOME

Round two or more addends and add to estimate the sum, then compare the estimate of the sum with the exact sum

Vocabulary
estimate

Prerequisite Skills
Add two or more numbers with regrouping, sums with up to six digits

Checking Prerequisite Skills
Add.

1. 14 925 + 3 087 = 18 012

2. 23 046 + 37 159 = 60 205

3. 5 316 + 20 145 + 18 372 = 43 833

4. 15 869 + 22 578 + 39 604 + 24 000 = 102 051

Estimating the Sum

On many weekends over 100 000 people will go to the Canadian Football League games.

Here are how many people went to the games one weekend.

To estimate the total number who went to a game, round to the nearest ten thousand and add.

Ottawa at Montreal	62 157	or about	60 000
British Columbia at Hamilton	19 133	or about	20 000
Calgary at Winnipeg	23 663	or about	20 000
Toronto at Edmonton	25 388	or about	30 000

About 130 000 people went to the Canadian Football League games one weekend.

For the exact sum, add in the usual way.

```
 21 22
 62 157
 19 133
 23 663
 25 388
130 341
```

The exact number of people who went to the games was 130 341.

Working Together

Round to the nearest ten thousand.

1. 10 726 — 10 000 **2.** 15 478 — 20 000

Round to the nearest thousand.

3. 37 298 — 37 000 **4.** 9504 — 10 000

Round to the nearest ten thousand and add to estimate the sum.

5. 29 345 + 32 989 — 30 000 + 30 000 = 60 000

6. 9 823 + 21 498 + 35 000 — 10 000 + 20 000 + 40 000 = 70 000

Round to the nearest thousand and add to estimate the sum.

7. 8 206 + 11 095 + 14 500 — 8 000 + 11 000 + 15 000 = 34 000

8. 23 528 + 19 700 — 24 000 + 20 000 = 44 000

Complete the chart.

		Estimate	Exact sum
9.	11 294 + 4 370 + 13 806 (11 000 + 4 000 + 14 000)	29 000	29 470
10.	48 500 + 22 468 + 18 849 (50 000 + 20 000 + 20 000)	90 000	89 817

When estimating these sums, round *all* the addends to either thousands or ten thousands.

24

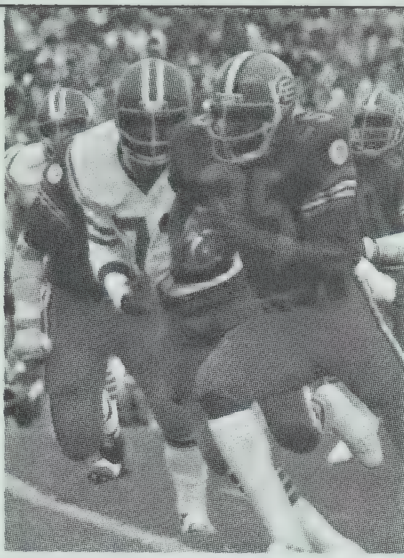
LESSON ACTIVITY

Before Using the Pages

- Spend a few moments reviewing that it is necessary to check only the digit to the right of a given place when rounding a number to that place. If the digit is 5 or greater than 5, the number is rounded up. If the digit is less than 5, the number is rounded down. The addends on pages 24 and 25 have either four or five digits and require rounding to the nearest thousand or ten thousand. Thus, you may focus this review on examples of that kind. For instance, write the numeral 16 358 on the board and have students identify the place value of each digit in the numeral. Then discuss rounding the number to the nearest thousand and to the nearest ten thousand. Use other similar examples.
- Write the exercise 16 358 + 21 746 in vertical form on the board. Have the students suggest how to obtain a quick idea of the sum of these two numbers without any written work. Lead them to suggest that each addend can be rounded to the nearest thousand so that the sum can be thought of as 16 (thousands) + 22 (thousands) = 38 (thousands), to give 38 000. Each addend can also be rounded to the nearest ten thousand. In this case the sum can be thought of as 2 (ten thousands) + 2 (ten thousands) = 4 (ten thousands), to give 40 000 as an estimate.

Using the Pages

- Begin with a brief discussion of the photograph. Some students may be able to identify the teams represented. Point out that attendance at football games is often given in rounded numbers. Introduce the worked example. Have students read the numerals for the exact attendance and explain the rounded number for each. For example, in rounding it to the nearest ten thousand, 62 157 is rounded to 60 000, since the digit in the thousands' place is less than 5. Emphasize that the addition of rounded numbers can usually be carried out fairly quickly to provide an *estimate* (general idea) of the sum. Have the students compare the exact sum and the estimate.



Montreal	69 000
Ottawa	35 342
Toronto	54 040
Hamilton	34 100
Winnipeg	32 950
Saskatchewan	28 000
Calgary	35 500
Edmonton	42 640
British Columbia	32 752

Exercises

Estimates may vary

Round and add
to estimate each sum.
Then find the exact sum.

1. 3467
8920
1674
14 061
(14 000)

2. 6 357
13 698
15 207
35 262
(35 000)

3. 14 689
33 499
29 502
35 000
112 690
(110 000)

4. \$3949 + \$7701 + \$4383 \$16 033

5. 2377 + 6098 + 5500 + 1842 15 817

6. 10 672 + 22 984 33 656

7. \$19 079 + \$7 386 + \$14 768 \$41 233

8. 68 396 + 52 928 + 76 004 197 328

9. 2996 + 3728 + 8276 + 1500 16 500

10. \$13 810 + \$19 475 \$33 285

11. 33 716 + 18 309 + 15 691 67 716

12. 9 748 + 10 874 + 19 487 40 109

Estimates
4. \$16 000 7. \$41 000 10. \$33 000
5. 16 000 8. 200 000 11. 70 000
6. 34 000 9. 17 000 12. 40 000

The chart shows the number of people that Canadian Football League stadiums can hold.

13. Estimate, then find the exact number that these nine stadiums can hold. (360 000) 364 324

At most, four league games can be played on one weekend.

14. Which four stadiums will hold the greatest number of people? Estimate, then find the exact number that these four stadiums can hold. Montreal Toronto Edmonton Calgary (200 000) 201 180

15. Which four stadiums will hold the least number of people? Estimate, then find the exact number that these four stadiums can hold. (120 000) 127 802
Saskatchewan
British Columbia
Winnipeg
Hamilton

25

RELATED ACTIVITIES

• Have students round each of the following numbers to the nearest ten thousand.

	Number	Nearest ten thousand
A	34 680	30 000
B	9 472	10 000
C	24 500	20 000
D	45 126	50 000
E	17 524	20 000
F	52 060	50 000
G	10 423	10 000
H	20 683	20 000
I	39 934	40 000
J	24 495	20 000

Have students use the rounded numbers to complete exercises similar to the following.

Estimate the sum
1. of A and B. 40 000
2. of E and H. 40 000
3. of D, G, and J. 80 000
4. of B, B, and A. 50 000

Which two of the numbers have a sum that is about
5. 70 000?
6. 90 000? D and I, F and I

Which three of the numbers have a sum that is about
7. 50 000?

5. A and I, C and D, C and F, D and E, D and H, D and J, E and F, F and H, F and J
7. A, B, G; B, C, E; B, C, H; B, C, J; B, E, H; B, E, J; B, H, J; C, E, G; C, G, H; C, G, J; E, G, H; E, G, J; G, H, J

Working Together: Ex. 1-4 provide practice with the skill of rounding numbers to the nearest thousand and to the nearest ten thousand. For Ex. 5-8, the students are directed to round the numbers to a particular place for an estimate of the sum. However, for Ex. 9 and 10, they must decide which of two places is preferable for rounding the three addends of each exercise. It is important to discuss what influences this choice. For instance, one of the addends in Ex. 9 has only four digits and thus rounding to the nearest thousand is probably preferable. Because each of the addends in Ex. 10 has five digits, an estimate of the sum is probably easier if the addends are rounded to the nearest ten thousand. (The mental calculation would be (5 + 2 + 2) ten thousands, or 90 000.)

Exercises: It is desirable that eventually students will be able to do a quick mental calculation to estimate a sum. At this time, however, they are required to show two additions for each exercise, one for the estimate of the sum and one for the exact sum. This will enable you to assess their ability to round numbers, to note whether all the addends of an exercise are rounded to the same place, and to discuss the choice of the place for rounding the addends. Ex. 12, particularly, is of interest as the estimates of the sums are the same to the nearest thousand and to the nearest ten thousand, although the rounded numbers differ for two of the addends.

		nearest thousand	nearest ten thousand
Ex. 12	9 748	10 000	10 000
	10 874	11 000	10 000
	19 487	19 000	20 000
		40 000	40 000

Assessment

Round and add to estimate each sum.
Then find the exact sum. Estimates may vary.

1. 10 316
19 875
31 293
61 484 (60 000)

2. 8 347
29 786
13 898
52 031 (52 000)

3. 7 324 + 1 500 + 28 630 + 2 000 39 454 (40 000)

OBJECTIVE

Demonstrate competence in adding and in estimating the sum of two, three, or four numbers

RELATED ACTIVITIES

- Write a selection of numerals with four or five digits on cards. Have the students work in pairs. Each student draws three cards, rounds the numbers, and estimates their sum. One student says, "I think my sum is greater (less) than yours". Then they compare their estimates and verify the situation by finding the exact sums. If the student guessed correctly, he/she scores one point. For the next round, the second student makes the prediction.
- For further practice in rounding numbers and estimating sums, choose exercises from previous lessons on pages 17-22.

Practice

First, estimate the sum without doing any work on paper.

Then, add and compare with your estimate.

Example:

23 749
18 175
41 924
40 000

If the two numbers are quite different, look for a mistake in your work.

	1.	2.	3.	4.
	24 975	38 918	14 500	23 184
	19 498	29 782	29 100	8 762
	37 180	68 700	30 954	10 900
	81 653		18 266	42 846
			92 820	
Estimate to ten thousands. →	80 000	70 000	90 000	40 000

	5.	6.	7.	8.
	5765	13 745	23 702	26 488
	9328	8 892	12 450	3 912
	7923	22 637	33 849	17 500
	23 016		70 001	9 896
				57 796
Estimate to thousands. →	23 000	23 000	70 000	58 000

Estimate the number of ten thousands in each row and column.

9.

14 972	27 013	17 450 →	▒0 000	6
19 288	21 750	9 107 →	▒0 000	5
23 666	15 200	44 879 →	▒0 000	8
▒0 000	▒0 000	▒0 000 →	▒0 000	19
5	7	7		

Estimate the number of thousands in each row and column.

10.

4 098	11 624	8 450 →	▒000	24
15 725	8 902	2 607 →	▒000	28
6 748	9 813	15 450 →	▒000	32
▒000	▒000	▒000 →	▒000	84
27	31	26		

Use *each* of the ten digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 just one time in each of these:

1. Write two numbers that have the greatest sum.
2. Find two other numbers that have the same sum.
3. Write two numbers that have the least sum.
4. Find two other numbers that have the same sum.

PROBLEM SOLVING

Problem Solving   Answers will vary.

1.	2.	3.	4.
97 531	87 520	23 456	20 486
86 420	96 431	10 789	13 759
183 951	183 951	34 245	34 245

26

LESSON ACTIVITY

Using the Page

- This lesson provides practice in estimating sums and also encourages students to compute an estimate mentally rather than in writing. Discuss the example at the top of page 26. Have students explain the process of arriving at 40 000 as the estimate of the sum. Emphasize the importance of comparing the estimate of the sum with the exact sum as a means of checking the exact sum.

  For these exercises, emphasize that estimates should be written before the exact sums are computed. Note that Ex. 9 and 10 involve estimates only, and not exact sums. The sum in the lower right rectangle of each chart serves as a check in much the same way as in a simpler addition square, as shown below.

+ →		
4	3	7
6	8	14
10	11	21

Problem Solving: Ensure that the students understand that the ten digits are to be used in each exercise. Allow them more than one day to explore different addends. You may wish to provide some assistance by asking these questions.

"How many digits must be used?"

"How many digits would there be for each addend?"

"For five-place numerals, what is the greatest place value?"

"For the greatest sum, which place should have the greatest value? the next greatest value?"

Questions such as these can help students "discover" a solution such as A for Ex. 1. Note that solution A is based on the premise that each addend must have five digits. However, some students may suggest that the two addends can have a different number of digits and suggest solution B for Ex. 1. Thus, the solution can depend on the situation involved (see page 40).

A		B	
	97 531		987 654 321
	86 420		0
	183 951		987 654 321

Addition and Subtraction Families

There is a group of four statements that show how the numbers 5, 7, and 12 are related.

This group is called a *family*.

5 + 7 = 12   7 + 5 = 12   12 − 7 = 5   12 − 5 = 7

Working Together

Complete each family.

1. 9 + 6 = 15   6 + 9 = 15
   15 − 6 = 9   15 − 9 = 6
2. 3 + 5 = 8   5 + 3 = 8
   8 − 5 = 3   8 − 3 = 5
3. 8 + 7 = 15   7 + 8 = 15
   15 − 7 = 8   15 − 8 = 7
4. 4 + 6 = 10   6 + 4 = 10
   10 − 6 = 4   10 − 4 = 6

Give the complete family.

5. 7 + 2 = 9
   2 + 7 = 9
   9 − 2 = 7
   9 − 7 = 2
6. 13 − 6 = 7
   13 − 7 = 6
   7 + 6 = 13
   6 + 7 = 13
7. 5 + 5 = 10
   10 − 5 = 5
8. 4 + 8
   4 + 8 = 12
   8 + 4 = 12
   12 − 8 = 4
   12 − 4 = 8
9. 12 − 3
   12 − 3 = 9
   12 − 9 = 3
   9 + 3 = 12
   3 + 9 = 12

Some families have only two members.

Exercises

Write the complete families. Answers are given on page T366

1. 6 + 3 = 9   2. 7 − 4 = 3   3. 7 + 7 = 14   4. 13 − 8 = 5
5. 1 + 4   6. 11 − 9   7. 3 − 0   8. 6 + 0
9. 7 + 4   10. 8 + 4   11. 14 − 6   12. 7 + 3
13. 8 + 8   14. 16 − 7   15. 8 − 8   16. 5 + 6
17. 4 + 9   18. 14 − 9   19. 12 − 6   20. 3 + 8

22. 9 + 9 = 18, 18 − 9 = 9, 9 − 9 = 0, 9 − 0 = 9, 0 + 9 = 9, 9 + 0 = 9

The two numbers 4 and 7 are both part of two families.

Add 4 and 7 and get a member of one family.	4 + 7 = 11
Subtract 4 from 7 and get a member of another family.	7 − 4 = 3

21. 4 + 6 = 10, 6 + 4 = 10, 10 − 6 = 4, 10 − 4 = 6, 6 − 4 = 2, 6 − 2 = 4, 2 + 4 = 6, 4 + 2 = 6

23. 3 + 2 = 5, 2 + 3 = 5, 5 − 2 = 3, 5 − 3 = 2, 3 − 2 = 1, 3 − 1 = 2, 1 + 2 = 3, 2 + 1 = 3

24. 2 + 8 = 10, 8 + 2 = 10, 10 − 8 = 2, 10 − 2 = 8, 8 − 2 = 6, 8 − 6 = 2, 6 + 2 = 8, 2 + 6 = 8

Write *two families* for each pair of numbers.

21. 4 and 6   22. 9 and 9   23. 3 and 2   24. 2 and 8
25. 9 and 5   26. 2 and 7   27. 8 and 9   28. 3 and 3

25. 9 + 5 = 14, 5 + 9 = 14, 14 − 5 = 9, 14 − 9 = 5, 9 − 5 = 4, 9 − 4 = 5, 4 + 5 = 9, 5 + 4 = 9

26. 2 + 7 = 9, 7 + 2 = 9, 9 − 7 = 2, 9 − 2 = 7, 7 − 2 = 5, 7 − 5 = 2, 5 + 2 = 7, 2 + 5 = 7

27. 8 + 9 = 17, 9 + 8 = 17, 17 − 9 = 8, 17 − 8 = 9, 9 − 8 = 1, 9 − 1 = 8, 1 + 8 = 9, 8 + 1 = 9

28. 3 + 3 = 6, 6 − 3 = 3, 3 − 0 = 3, 3 − 3 = 0, 0 + 3 = 3, 3 + 0 = 3

27

Page 27

LESSON OUTCOME

Write the family of addition and subtraction facts for a given set of numbers

Vocabulary

family, related

Prerequisite Skills

Complete the basic addition and subtraction facts

Checking Prerequisite Skills

Add.

1. 4 + 7 = 11	**2.** 8 + 5 = 13	**3.** 9 + 3 = 12	**4.** 7 + 8 = 15

Subtract.

5. 10 − 5 = 5	**6.** 12 − 7 = 5	**7.** 17 − 8 = 9	**8.** 14 − 9 = 5

RELATED ACTIVITIES

- The example on page 27 illustrates how the numbers 4 and 7 generate two families. Challenge the students to name two numbers that can generate only one family. This will occur when one of the two numbers is zero.
- Students may practice writing families of facts using a set of numeral cards, two each for the numbers 0 to 9. The cards are placed in a container. Each student draws two cards and writes as many related addition and subtraction facts as possible.

LESSON ACTIVITY

Using the Page

- The title at the top of page 27 can motivate a discussion of the word *families*. When people are related, they belong to a family. People can belong to more than one family, as any inspection of family trees reveals. Similarly, numbers can be related in families through the operations, in this case, addition and subtraction.
- Discuss the worked example. There are two addition sentences and two subtraction sentences, and each sentence involves the three numbers 5, 7, and 12. Point out that the addition sentences show the same sum because the addends are the same and the order of adding two numbers does not affect the sum. Have the students note that the greatest number, 12, begins each subtraction sentence. Summarize that the numbers 5, 7, and 12 are *related* and the group of four sentences is called a *family*.

Working Together: Ex. 1-4 require the students to complete the basic facts that make up a family. Ex. 5-7 present one complete fact from which students are to deduce the remaining fact(s). Ex. 7 provides an opportunity to discuss that some families consist of just two facts. Have students suggest the reason for this. For Ex. 8 and 9, the third of three related numbers must be determined before the family can be written.

Exercises: Encourage the students to record their sentences in families as suggested in the format shown for Ex. 1 of *Working Together*. Have the students read the example provided and determine for themselves how two numbers can generate two families.

Assessment

Write the complete families.

1. 9 + 4 = 13   **2.** 16 − 7

Write two families for this pair of numbers.

3. 6 and 4

1. 9 + 4 = 13, 4 + 9 = 13, 13 − 4 = 9, 13 − 9 = 4
2. 16 − 7 = 9, 16 − 9 = 7, 9 + 7 = 16, 7 + 9 = 16
3. 6 + 4 = 10, 4 + 6 = 10, 10 − 4 = 6, 10 − 6 = 4, 6 − 4 = 2, 6 − 2 = 4, 2 + 4 = 6, 4 + 2 = 6

Page 28

LESSON OUTCOME

Subtract numbers with no regrouping, minuends with up to five digits; use addition to check subtraction

Materials

abacus (optional)

Vocabulary

subtraction, subtract, difference, basic subtraction fact

Prerequisite Skills

Complete basic subtraction facts, minuends to 9; interpret place value for numerals with up to four digits; compare two numbers

Checking Prerequisite Skills

Subtract.

1. 4 − 3 = 1	**2.** 6 − 0 = 6	**3.** 9 − 5 = 4	**4.** 7 − 7 = 0

What does each digit mean?

5. 2675 2 thousands 6 hundreds 7 tens 5 ones

Tell which is greater,

6. 406 or 604. 604 **7.** 6743 or 6734. 6743

RELATED ACTIVITIES

- For further practice, you may wish to have students complete Ex. 27-31 on page 332.
- Students may use an abacus to demonstrate exercises selected from those on page 28.

Subtraction with No Regrouping

The travel company sold 2675 Canadian tours for the summer.
It sold 1154 Canadian tours for the rest of the year.
How many more Canadian tours did it sell for the summer?

Subtract 1154 from 2675.

Show 2675 − 1154 in vertical form. Then use basic subtraction facts place by place to find the *difference*.

Subtract ones.	Subtract tens.	Subtract hundreds.	Subtract thousands.
2 6 7 5	2 6 7 5	2 6 7 5	2 6 7 5
1 1 5 4	1 1 5 4	1 1 5 4	1 1 5 4
1	2 1	5 2 1	1 5 2 1

The travel company sold 1521 more Canadian tours for the summer than for the rest of the year.

Exercises

Subtract.

1. 9802 − 2501 = 7301	2. 6379 − 5106 = 1273	3. 13 296 − 2 274 = 11 022	4. 4774 − 4351 = 423	5. 2995 − 543 = 2452

6. 6155 − 3014 3141 7. 8862 − 3251 5611 8. 1387 − 1046 341
9. 4595 − 192 4403 10. 7776 − 1420 6356 11. 9566 − 5461 4105

Find the difference of

12. 438 and 216. 222 13. 783 and 5995. 5212 14. 1864 and 731. 1133
15. 7697 and 7215. 482 16. 6437 and 7739. 1302 17. 435 and 8695. 8260

You can use addition to check subtraction.

Example: Subtract. Add.

Subtract.	Add.
3415	1676
1739	1739
1676	3415

If the sum does not match the first number in the subtraction, there is a mistake in your work.

18. Check your results in the odd-numbered exercises above.

28

LESSON ACTIVITY

Before Using the Page

- Write the following information on the board.

  4 trains 7 boats 2 planes

  Review the basic concepts of addition and subtraction by asking the following questions.

  "How many boats and planes are there?"

  "How many more boats than planes are there?"

  "How many fewer trains than boats are there?"

  "If three boats leave the shore, how many boats are left?"

  After each of the preceding questions, have students state a number sentence and the operation used.

Using the Page

- Discuss the word problem and how the solution is found by subtraction, as implied by the words "how many more". Have the students follow the steps highlighted in red. Point out that a *basic subtraction fact* is applied in each place.

  You may wish to have students demonstrate the subtraction on an abacus. Introduce the word *difference* and draw attention to its use in the instruction preceding Ex. 12.

Exercises: Before the students begin, discuss the example showing how to use addition to check subtraction. Have students write an addition exercise on the board to check the subtraction example at the top of page 28. Ensure that the students understand the meaning of the phrase "odd-numbered exercises" in Ex. 18.

Assessment

Subtract.

1. 4936 − 1504 = 3432 **2.** 8462 − 351 8111

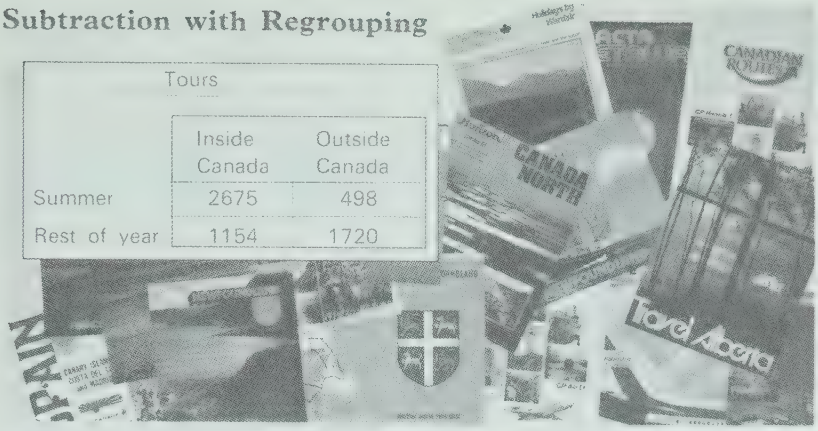
Find the difference. Add to check.

3. 632 and 1835 1203

4. 5498 and 3283 2215

Subtraction with Regrouping

Tours	Inside Canada	Outside Canada
Summer	2675	498
Rest of year	1154	1720



How many more summer tours for inside Canada were sold than summer tours for outside Canada?

Subtract 498 from 2675.

Show 2675 − 498 in vertical form and try to subtract ones.

$$\begin{array}{r} 2\,6\,7\,5 \\ 4\,9\,8 \\ \hline \end{array}$$

Cannot subtract 8 ones from 5 ones

To subtract ones, regroup 7 tens, 5 ones as 6 tens 15 ones and subtract

$$\begin{array}{r} 2\,6\,\overset{6}{\cancel{7}}\,\overset{15}{\cancel{5}} \\ 4\,9\,8 \\ \hline 7 \end{array}$$

To subtract tens, regroup 6 hundreds, 6 tens as 5 hundreds, 16 tens and subtract

$$\begin{array}{r} 2\,\overset{5}{\cancel{6}}\,\overset{16}{\cancel{7}}\,\overset{15}{\cancel{5}} \\ 4\,9\,8 \\ \hline 7\,7 \end{array}$$

Then, subtract hundreds and subtract thousands

$$\begin{array}{r} 2\,\overset{5}{\cancel{6}}\,\overset{16}{\cancel{7}}\,\overset{15}{\cancel{5}} \\ 4\,9\,8 \\ \hline 2\,1\,7\,7 \end{array}$$

The company sold 2177 more summer tours for inside Canada than for outside Canada.

Add to check:

$$\begin{array}{r} 2177 \\ 498 \\ \hline 2675 \end{array}$$

TRAVEL to the next page for more work with subtraction.

Pages 29-30

LESSON OUTCOME

Subtract numbers with one, two, or three regroupings, minuends with four or five digits

Materials

models for thousands, hundreds, tens, and ones, or an abacus

Prerequisite Skills

Complete the basic subtraction facts; interpret place value for numerals with up to five digits; demonstrate competence in lining up numbers in vertical form for subtraction

Checking Prerequisite Skills

Subtract.

1. $\begin{array}{r} 13 \\ 5 \\ \hline 8 \end{array}$ **2.** $\begin{array}{r} 14 \\ 6 \\ \hline 8 \end{array}$

3. $\begin{array}{r} 11 \\ 8 \\ \hline 3 \end{array}$ **4.** $\begin{array}{r} 17 \\ 9 \\ \hline 8 \end{array}$

What does each digit mean?

5. 10 438

6. 43 259

5. 1 ten thousand
0 thousands
4 hundreds
3 tens
8 ones

6. 4 ten thousands
3 thousands
2 hundreds
5 tens
9 ones

Show the numbers in vertical form and subtract.

7. 1324 − 213 $\begin{array}{r} 1324 \\ 213 \\ \hline 1111 \end{array}$

8. 6955 − 3443 $\begin{array}{r} 6955 \\ 3443 \\ \hline 3512 \end{array}$

LESSON ACTIVITY

Before Using the Pages

- Students generally experience more difficulty with regrouping in subtraction than in addition. Working with concrete models helps them to visualize the regrouping that occurs at the abstract level of subtracting numbers.

  Write 45 − 23 in vertical form on the board and have students demonstrate the subtraction, using an abacus or models of tens and ones. Write 45 − 29 in vertical form on the board and repeat the procedure. Ask how this exercise differs from the previous one. Establish that there are not enough ones to permit subtraction of 9 ones. Ask how this difficulty is overcome and have students explain the regrouping necessary. Emphasize that 4 tens 5 ones and 3 tens 15 ones are different names for the same number, 45, and that 3 tens 15 ones is preferable at times as just illustrated. You may wish to show these subtraction exercises and other similar exercises in place-value charts on the board.

tens	ones
3	15
~~4~~	~~5~~
− 2	9
1	6

hundreds	tens	ones
2	14	
~~3~~	~~4~~	7
− 1	6	5
1	8	2

Include exercises that require more than one regrouping, as in 324 − 168, but note that regrouping with 0 in the minuend is delayed until the following lesson.

Using the Pages

- You may wish to introduce the worked example by conducting a brief survey of the students in your class. Ask how many spent their summer vacation in Canada and how many spent their vacation outside Canada. Have the students note which number is greater and how much greater. Then have them interpret the numbers in the chart at the top of page 29. Discuss that the words "how many more" indicate that subtraction will be used to answer the question below the photograph.

RELATED ACTIVITIES

- For further practice, you may wish to have students complete Ex. 32-39 on page 332.
- Have students demonstrate a few of the exercises on page 30, using an abacus, or, where practical, models of thousands, hundreds, tens, and ones.
- Some students may find it helpful to write subtraction exercises using a place-value chart. A grid (copies of page T 391 or T 397) is helpful in spacing digits of the numerals to allow ample space for showing the regrouping. The following example shows Ex. 11 from page 30.

	th	h	t	o	
		10			
	8	~~0~~	14		
	~~9~~	~~1~~	~~4~~	9	
−	6	6	6	7	
	2	4	8	2	

Working Together

Regroup to show 10 more tens. Then complete the exercises.

1. 3524
   462
   3062

2. 2746
   1298
   1448

Regroup to show 10 more hundreds. Then complete the exercises.

3. 8247
   3655
   4592

4. 15 520
   1 875
   13 645

Subtract by following the steps.

5. 55 157
   37 324
   17 833

Subtract.
Regroup and subtract.
Regroup and subtract.
Subtract.

Subtract.

6. 6243
   3537
   2706

7. 7235
   3846
   3389

8. 8151 − 6394 1757

9. 32 109 − 5 123 26 986

Exercises

Subtract.

1. 3829
   2378
   1451

2. 27 339
   4 964
   22 375

3. 9441
   1542
   7899

4. 16 426
   4 869
   11 557

5. 7117
   5929
   1188

6. 64 197
   9 876
   54 321

7. 27 060
   17 236
   9824

8. 66 666
   8 888
   57 778

9. 8790 − 5272 3518
10. 1691 − 749 942
11. 9149 − 6667 2482
12. 13 098 − 1 729 11 369
13. 6342 − 1398 4944
14. 9685 − 3889 5796
15. 41 597 − 14 751 26 846

Remember, you can add to check your work.

Tours

	Inside Canada	Outside Canada
Summer	2675	498
Rest of year	1154	1720

Solve.

16. For the rest of the year, were more tours sold for outside or inside Canada? How many more?
    outside Canada, 566
17. How many fewer tours outside Canada were sold for the summer than for the rest of the year? 1222
*18. In all, were more tours sold for outside or for inside Canada? How many more?
    inside Canada, 1611
*19. In all, were more tours sold for the summer or for the rest of the year? How many more?
    summer, 299

30

Guide the students through the steps of the example which are highlighted in red. Have them explain why regrouping is needed to subtract the ones and the tens but not in the other places. An effective procedure is to read the steps that are listed as students use an abacus or models to demonstrate the subtraction. After discussing the example, have the students complete the same subtraction on their own and check their work with the worked example. Have them note that addition is used to check the subtraction exercise at the bottom of page 29. *Working Together* and *Exercises* for this lesson are given on page 30.

Working Together: Ex. 1-4 provide partially completed subtraction exercises. Emphasize that subtraction is carried out from right to left and the students should begin with the ones to understand the steps that have already been completed. Have students explain how they can tell that regrouping will be needed in the two places specified for Ex. 5.

Exercises: Remind the students to allow plenty of space in their written work to show the numerals for regrouping. Suggest that some students may be able to remember the numerals and not need to write them. Although it is not indicated that a written addition exercise be shown as a check, emphasize the importance of adding mentally to detect errors. Using addition to check subtraction is particularly important for exercises that require several regroupings, as in Ex. 8.

Ex. 18 and 19 are starred because their solutions require more than one step.

Assessment

Subtract.

1. 3484
   1656
   1828
2. 5213 − 682 4531
3. 24 426 − 15 639 8787

Practice

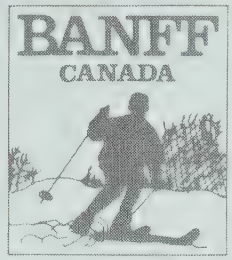
Subtract to decode the message.

A	C	D
7774 − 1796 = 5978	9610 − 2012 = 7598	9547 − 3589 = 5958

E	L	N
8366 − 788 = 7578	1353 − 395 = 958	6421 − 623 = 5798

R	T	V
5432 − 4454 = 978	12 808 − 4 850 = 7958	8255 − 2677 = 5578

7958 T	7598 C
978 R	5978 A
5978 A	5798 N
5578 V	5978 A
7578 E	5958 D
958 L	5978 A



Subtract.

1. 4931 − 2907 = 2024
2. 5385 − 3776 = 1609
3. 18 204 − 5 841 = 12 363
4. 8352 − 3867 = 4485
5. 77 275 − 29 637 = 47 638
6. 5120 − 669 4451
7. 35 825 − 3 658 32 167
8. 3140 − 2798 342
9. 14 689 − 8 995 5694
10. 8350 − 4973 3377
11. 21 744 − 19 858 1886

Some additions and subtractions are easier than they look.

For 375 + 298, think 375 + 300 = 675, then subtract 2. 673

For 820 − 595, think 820 − 600 = 220, then add 5. 225

Try to do these without using paper or pencil.

1. 287 + 594 881
2. 374 − 99 275
3. 625 + 297 922
4. 705 − 493 212
5. 139 + 692 831
6. 268 − 196 72
7. 377 + 485 862
8. 612 − 387 225
9. 468 + 375 843
10. 460 − 175 285
11. 1492 + 2950 4442
12. 4635 − 1980 2655

try this

31

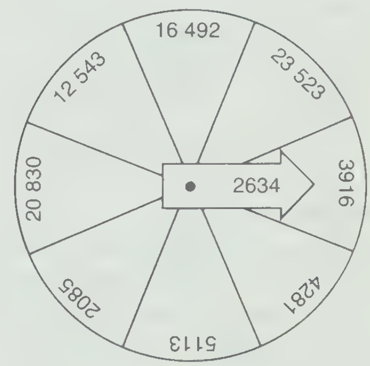
Page 31

OBJECTIVE

Demonstrate competence in subtraction with regrouping

RELATED ACTIVITIES

- A spinner similar to the one shown below can provide subtraction exercises. The number on the arrow is subtracted from the number on the dial. Have students work in pairs, one student completing a subtraction exercise and the other checking the subtraction using addition. After four exercises, have the students exchange roles.



LESSON ACTIVITY

Using the Page

- Most students have likely encountered codes in which numbers represent letters. If the concept is new to some students, illustrate the procedure by representing the letters of the alphabet, A, B, C, and so on, by the numbers 1, 2, 3, and so on. Have students write the numerals that identify their first name, for instance, 19 20 5 12 12 1 for Stella. Have them decode the name of the animal represented by 4 9 14 15 19 1 21 18.
- You may wish to work Ex. 1 with the students to review the steps of subtraction with regrouping.

Try This: The procedures suggested here are useful for mental computations. For the addition of two numbers, one is usually rounded up to the nearest hundred and then added to the other number. Then, to compensate for the rounding up, a subtraction is necessary. Similarly, the subtrahend in a subtraction exercise is usually rounded up, subtracted from the minuend, and compensation is made by addition. Let the students discover the procedure without your assistance, if possible. They may enjoy working in pairs.

Pages 32-33

LESSON OUTCOME

Subtract with regrouping for zero in one or more places in the minuend, minuends with up to five digits

Materials

models for thousands, hundreds, tens, and ones

Vocabulary

metres, m

Prerequisite Skills

Subtract with regrouping, minuends with up to five digits

Checking Prerequisite Skills

Subtract.

1. 4238 − 1674 = 2564 **2.** 31 674 − 8 692 = 22 982

3. 35 126 − 16 148 18 978

Background

The approach to subtraction with zeros in the minuend utilizes place value to facilitate the regrouping. (See the overview on pages T 16 and T 17.)

Subtraction, Regrouping with Zeros

Mount Logan, the highest peak in Canada, is 6050 m (metres) above sea level.

How much higher is Mount Logan than Mount Robson?

Subtract 3954 from 6050.

Show 6050 − 3954 in vertical form and regroup to subtract ones.

		4	10
6	0	~~5~~	~~0~~
3	9	5	4
			6

To subtract tens, think of 6 thousands as 60 hundreds and regroup.

		14	
5	9	~~4~~	10
~~6~~	~~0~~	~~5~~	~~0~~
3	9	5	4
			6

Then, subtract tens.

		14	
5	9	~~4~~	10
~~6~~	~~0~~	~~5~~	~~0~~
3	9	5	4
		9	6

Subtract hundreds. Subtract thousands.

		14	
5	9	~~4~~	10
~~6~~	~~0~~	~~5~~	~~0~~
3	9	5	4
2	0	9	6

Mount Logan is 2096 m higher than Mount Robson.

Here are some other examples that show regrouping with zeros.

3 9 9 12	1 9 9 9 10	12 / 2 9 ~~2~~ 9 10
~~4002~~	~~20 000~~	~~30 300~~
1675	15 276	2 587
2327	4724	27 713

32

LESSON ACTIVITY

Before Using the Pages

- Display 1 thousand and have students identify the number represented. Display 1 hundred, have students identify the number, and ask how many hundreds are the same as 1 thousand. Demonstrate this with models. In a similar manner, show that 100 tens and 1000 ones are the same as 1 thousand, illustrating the difficulty of demonstrating the last example, in particular. You may wish to prepare 1000 ones and 100 tens in two plastic bags in advance of this lesson and display them to emphasize the concept. Summarize the concept on the board in a list as shown.

one thousand	$\boxed{1}$ 0 0 0	1 thousand
	$\boxed{1\ 0}$ 0 0	10 hundreds
	$\boxed{1\ 0\ 0}$ 0	100 tens
	$\boxed{1\ 0\ 0\ 0}$	1000 ones

  Develop similar examples for 2004 and 6039, for instance, without models where their use is impractical.

- Review that 1000 can be thought of as 100 tens. Ask if the number represented changes if 1 ten is regrouped as 10 ones. Ask how many tens and ones there would be if this is done. Write the following on the board, saying, as you regroup in each example, "One hundred tens, zero ones is equal to ninety-nine tens, ten ones".

	tens	ones	
1000	~~100~~	~~0~~	9 9 10
	99	10	~~1 0 0 0~~

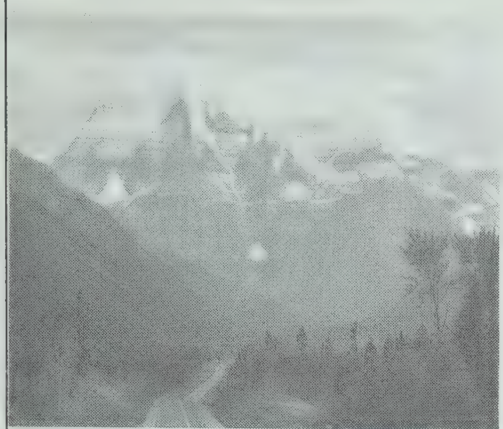
  Use a similar procedure to develop other examples as shown.

	hundreds	tens	ones	
6039	~~60~~	~~3~~	9	5 9 13
	59	13		~~6 0 3~~ 9

Using the Pages

- Have the students read the statements above the photographs on pages 32 and 33. Ask them for the name and height of each mountain pictured. Ask which mountain is higher, and develop that subtraction would be used to answer the question "How much higher?"

Mount Robson, the highest peak in the Canadian Rockies, is 3954 m above sea level.



Working Together

Complete.

1. $\overset{3\,9\,9\,9\,10}{\cancel{40000}}$
2. $\overset{4\,9\,9\,10}{\cancel{5000}}$ 14
3. $\overset{2\,9\,12}{\cancel{30}26}$
4. $\overset{19\;\;12}{2\cancel{05}\cancel{0}2}$ 4 9

Complete the regrouping and the subtraction.

5. 599 10
   ~~6000~~
   2584
   3416
6. 99 13
   ~~10038~~
   5290
   4748

Subtract.

7. 4000
   1427
   2573
8. 30 000
   12 756
   17 244
9. 7020 − 4827 2193
10. 50 075 − 23 294 26 781

Exercises

Subtract.

1. 6000
   3267
   2733
2. 30 000
   13 614
   16 386
3. 8002
   4955
   3047
4. 20 500
   17 815
   2685
5. 44 000 − 6 081 37 919
6. 7000 − 2472 4528
7. 30 402 − 25 894 4508
8. 20 055 − 3 878 16 177

Remember, you can add to check your work.

How much higher is Mount Logan than

9. Cirque Mountain (1573 m) in Newfoundland? 4477 m
10. Mount Royal (233 m) in Quebec? 5817 m
11. Barbeau Peak (2603 m) on Ellesmere Island in the Northwest Territories? 3447 m
12. Mount Fairweather (4663 m) on the British Columbia border with Alaska? 1387 m

Use *each* of the ten digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 just one time in each of these:

1. Write the two numbers that have the greatest difference. 98 765 − 10 234 = 88 531
2. Write the two numbers that have the least difference. 97 531 − 86 420 = 11 111

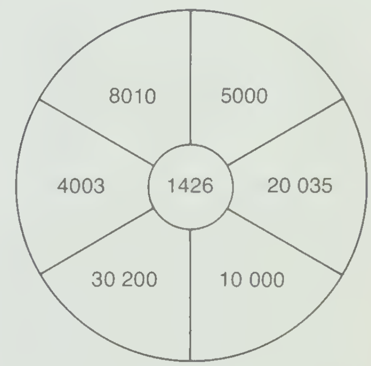
PROBLEM SOLVING

RELATED ACTIVITIES

- For further practice, you may wish to have students complete a selection of exercises from Ex. 40-72 on page 332.
- Repeated use of the same minuend can help improve the skill in subtraction with zeros in the minuend. Such exercises can be generated from number wheels similar to number wheel A, for which the minuend is shown at the center. Number wheel B provides practice in subtracting the same subtrahend from different minuends.

A

B



Lead the students through the steps of the solution. Ask why regrouping is needed to subtract the ones and the tens. Emphasize that since there are 0 hundreds to regroup as tens, it is convenient to think of 6 thousands as 60 hundreds, and regroup it as 59 hundreds and 10 more tens. The fact that one stroke is used to "cross out" 60 helps to emphasize the procedure of regrouping in terms of hundreds. Discuss the steps shown for completing the subtraction exercise and have students explain how each digit in the difference is obtained.

Discuss the kind of regrouping suggested for the minuend of each example at the bottom of page 32. For example, 20 000 is thought of as 2000 tens 0 ones and is regrouped as 1999 tens 10 ones. Discuss each example on the board, or have the students try the subtractions independently and check their solutions with those on the page.

Working Together: Ex. 1-4 offer practice with the kind of regrouping relevant to the subtraction exercises in this lesson. This skill is applied in completing Ex. 5-10.

Exercises: Encourage the students to check their subtraction exercises by using addition. Remind them to write concluding statements for Ex. 9-12.

Problem Solving: These exercises are similar to the ones involving addition on page 26. The solution depends on the number of digits permitted for the minuend and the subtrahend. Without restrictions, the subtraction 987 654 321 − 0 gives the greatest difference. Adapting the questions suggested on page T 28 can lead students to suggest the subtraction 98 765 − 01 234 as having the greatest difference for five-digit minuends and subtrahends. But since the digit 0 is never used in this way, it is preferable to consider 98 765 − 10 234 as having the greatest difference for this situation.

Assessment

Subtract.

1. 5000
   2369
   2631
2. 60 200
   34 871
   25 329
3. 40 305 − 26 719 13 586
4. 20 044 − 3 165 16 879

Pages 34-35

OBJECTIVE

Demonstrate competence in addition and subtraction; solve related word problems

Materials

a copy of page T 391 for each student (optional), a blank sheet of paper for each student

Vocabulary

cross-number puzzle

Practice

Subtract.

1. 7507	2. 33 609	3. 47 439	4. 5341	5. 69 350
4633	9 142	37 766	3554	16 894
2874	24 467	9673	1787	52 456
6. 8000	7. 70 000	8. 40 506	9. 8000	10. 90 300
4915	30 718	3 858	2234	46 788
3085	39 282	36 648	5766	43 512

11. 5255 − 1628 3627 12. 10 634 − 7 945 2689 13. 65 811 − 37 559 28 252
14. 68 010 − 49 027 18 983 15. 2230 − 1761 469 16. 89 352 − 9 779 79 573
17. 41 000 − 16 964 24 036 18. 13 431 − 1 458 11 973 19. 7103 − 5784 1319

Amounts of money are subtracted just like whole numbers.

Examples:

$\overset{7\,16}{\$8\cancel{8}\cancel{6}2}$	$\overset{5\,\overset{11}{\cancel{1}}\,15}{\$7\cancel{6}\cancel{2}.\cancel{5}8}$
3182	624.65
\$5680	\$137.93
$\overset{8\,9\,9\,9\,10}{\$\cancel{90\,000}}$	$\overset{2\,9\,9\;\,9\,10}{\$\cancel{300.00}}$
18 329	197.54
\$71 671	\$102.46

Subtract. Add to check.

20. \$9392 − \$2503 \$6889 21. \$46.32 − \$40.84 \$5.48 22. \$66 784 − \$42 987 \$23 797
23. \$265.21 − \$35.53 \$229.68 24. \$21 325 − \$9 829 \$11 496 25. \$100.00 − \$64.30 \$35.70
26. \$7000 − \$6706 \$294 27. \$501.01 − \$56.67 \$444.34 28. \$60 700 − \$53 292 \$7408

Use >, <, or = to make true statements.

29. 7906 − 1174 > 9100 − 2728	30. 7984 − 3028 < 3028 + 1937
31. \$12 376 = \$32 166 − \$19 790	32. 3292 + 4321 > 9034 − 2321
33. 13 940 − 1 068 = 12 872	34. \$39.62 − \$15.24 > \$23.48
35. 16 923 < 9 733 + 9 890	36. \$500.00 − \$147.52 = \$352.48
37. 2518 + 1438 > 5034 − 1438	38. \$12 028 < \$20 000 − \$7 918
39. 19 134 − 6 789 = 12 345	40. 5784 − 1895 < 1995 + 1895

34

LESSON ACTIVITY

Using the Pages

- Before the students begin, work one or two exercises on the board with them to review the procedure for subtracting when there are zeros in the minuend. One or more of the examples provided for subtracting amounts of money may be discussed orally. Also, students who are not familiar with cross-number puzzles will need some assistance in understanding how to record sums and differences in the puzzle. Review how the puzzle can reveal that an error in computation has been made. For example, the numerals for b down and e across share a common digit. You may wish to provide the students with copies of page T 391 on which to show the cross-number puzzle. Have them color the unused squares and copy the letters in the appropriate blank squares before they write the numerals.

  Note that the minuend and the subtrahend for Ex. 41 have six digits, but the students will likely be able to extend the concept of regrouping to the sixth place with little difficulty. Ex. 42 requires the students to recall which three provinces constitute the Maritime Provinces. For Ex. 43, they will have to select two provinces that are "nearly equal" in size. The choice depends on their interpretation of "nearly equal". As long as the choice is a reasonable one it is acceptable. The two provinces closest in area are Saskatchewan and Manitoba, which differ by 1813 km². It would be beneficial to have different solutions shown on the board and discussed.

  Have the students write their questions for Ex. 44 on a separate sheet of paper. The papers can be exchanged at this time or collected and redistributed on another day.

  These pages provide many exercises for practice and review. You may prefer to assign one section at this time and assign the remaining exercises in groups over a longer period of time. However, be certain to discuss the cross-number puzzle with the students when they have finished it. Although the concept of rotational symmetry has not been introduced formally, the students will likely be able to recognize this feature of the puzzle. It may be

Copy and complete the cross-number puzzle.

Across

c. 12 537 − 3 770
e. 28 069 + 37 387
f. 10 817 − 8 472
g. 6000 − 3899
j. 8371 − 7359
m. 4198 + 1234
n. 90 000 − 24 544
o. 9547 − 1869

a 7		b 5			c 8	7	d 6	7
e 6	5	4	5	6			5	
7		3			f 2	3	4	5
8		g 2	h 1	0	1		5	
	i 6		0		0		6	
	5		j 1	0	1	k 2		l 8
m 5	4	3	2			3		7
	5			n 6	5	4	5	6
o 7	6	7	8			5		7

Down

a. 10 300 − 2 622
b. 865 + 4567
d. 83 810 − 18 354
f. 5043 − 2942
h. 30 008 − 28 996
i. 28 577 + 36 879
k. 4003 − 1658
l. 9661 − 894

This chart shows the number of square kilometres in the provinces and territories.

Newfoundland	404 517	Saskatchewan	651 900
Prince Edward Island	5 657	Alberta	661 185
Nova Scotia	55 490	British Columbia	948 596
New Brunswick	73 437	Yukon	536 324
Quebec	1 540 680	Northwest Territories: Franklin	1 422 559
Ontario	1 068 582	Northwest Territories: Keewatin	590 932
Manitoba	650 087	Northwest Territories: Mackenzie	1 366 193

41. How much larger is British Columbia than Saskatchewan? 296 696 km²

42. How many square kilometres are there in the three Maritime Provinces? 134 584

43. Two provinces are nearly equal in size. How much larger is one than the other?
Saskatchewan and Manitoba
1813 km²

44. Use the chart to make up a question to ask a classmate.

35

RELATED ACTIVITIES

- The pictures of items for sale cut from catalogs and newspapers for the activity on page T25 are useful for generating subtraction exercises. Have students pretend to purchase one item and pay for it by cheque. Have them use subtraction to find how much money they would have left in a bank account if the present balance is $1000, for example.
- You may wish to have students play the game "Shopping Spree" described on page T377. It provides practice with addition and subtraction of amounts of money.

rotated through a quarter turn about the center, and all the digits will be the same as those in the original position of the puzzle. In other words, the difference obtained for c across (8767) is the same as the difference for 1 down, and the digits of the answers for o across and a down are identical to those, but their order is reversed (7678). Have the students find other exercises for which the answers have identical digits but in the reverse order.

Pages 36 - 37

LESSON OUTCOME

Round the minuend and the subtrahend and subtract to estimate the difference, then compare the estimate of the difference with the exact difference

Prerequisite Skills

Subtract with regrouping, minuends with up to five digits

Checking Prerequisite Skills

Subtract.

1. 4130
1645
2485

2. 66 321
38 573
27 748

3. 40 300
9 524
30 776

4. 25 555
12 769
12 786

Estimating the Difference

There are 650 087 km² in Manitoba.

101 592 km² are covered with water.

About how many square kilometres of land are in Manitoba?

Round and subtract.

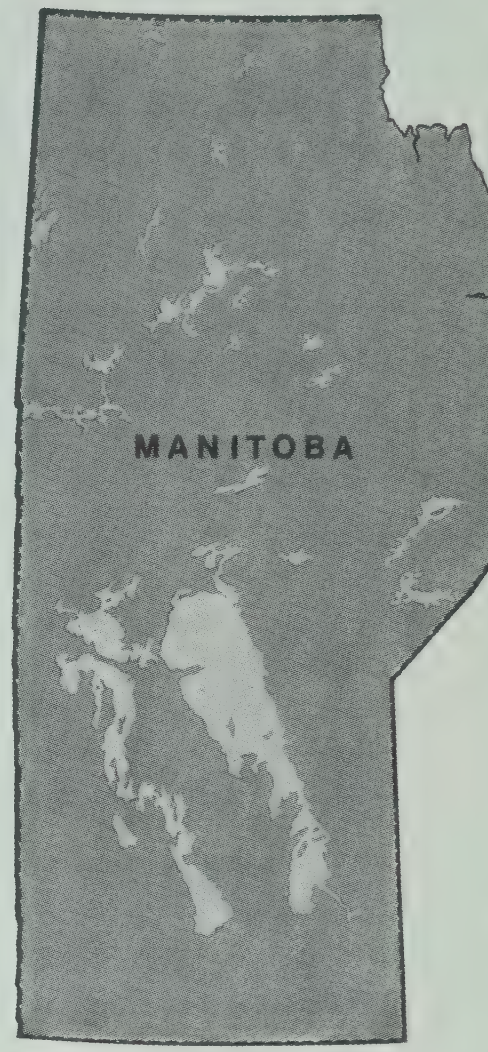
650 087 ⟶ 650 000
101 592 ⟶ 100 000
550 000

There are about 550 000 km² of land in Manitoba.

For the exact amount of land, subtract in the usual way.

4 9 918
6~~50~~ ~~087~~
101 592
548 495

There are 548 495 km² of land in Manitoba.



36

LESSON ACTIVITY

Before Using the Pages

- The development of this lesson for estimating a difference is similar to that given on pages 24 and 25 for estimating a sum. The procedure for reviewing the rounding of numbers described on page T 26 may be repeated at this time. As before, focus the review on numerals with four or five digits and rounding to the nearest thousand or the nearest ten thousand.
- Write the exercise 42 456 − 17 524 in vertical form on the board. Ask how to obtain an estimate of the difference without any written work. Point out that since each numeral has five digits, it is convenient to round each number to the nearest ten thousand. The mental computation then becomes 4 − 2 (ten thousands) is equal to 2 (ten thousands), or 20 000.

Using the Pages

- Have a student read the word problem to introduce the situation. Discuss why subtraction is used to solve the problem. Draw attention to the word ''about'' and develop that it suggests an estimate of the difference rather than an exact difference. Have students explain how each number is rounded. (You may need to remind them of the meaning of the symbol km² which appeared first in Unit 1 on page 4.) Have them compare the estimate of the difference with the exact difference shown at the bottom of page 36 and note that the two numbers are about the same. Have students explain the steps of subtracting and regrouping to obtain the exact difference.

Working Together: Ex. 1-4 deal with the skill of rounding numbers. This skill is applied in Ex. 5-10. For Ex. 9 and 10, discuss whether rounding to the nearest thousand or to the nearest ten thousand is preferable, and have students give reasons for their answers.

Working Together

Round to the nearest ten thousand.

1. 22 900 — 20 000
2. 38 250 — 40 000

Round to the nearest thousand.

3. 4835 — 5000
4. 27 376 — 27 000

Round to the nearest ten thousand. Subtract to estimate the difference.

5. 82 360 − 29 184 — 80 000 − 30 000 = 50 000
6. 58 192 − 11 779 — 60 000 − 10 000 = 50 000

Round to the nearest thousand. Subtract to estimate the difference.

7. 7495 − 3266 — 7000 − 3000 = 4000
8. 47 915 − 25 087 — 48 000 − 25 000 = 23 000

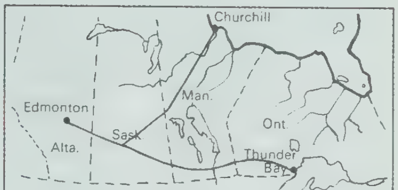
Complete the chart.

		Estimate	Exact difference
9.	12 126 − 4 892	? 7000	? 7234
10.	35 690 − 19 275	20 ? 000	? 16 415

Exercises Estimates may vary.

Round and subtract to estimate each difference. Then find the exact difference.

1. 9163 − 3748 = 5415 (5000)
2. 6272 − 4159 = 2113 (2000)
3. 29 116 − 21 082 = 8034 (10 000)
4. 4833 − 3917 = 916 (1000)
5. 9726 − 2150 = 7576 (8000)
6. 21 970 − 14 038 = 7932 (10 000)
7. 11 784 − 6 118 5666 (6000)
8. 88 056 − 59 894 28 162 (30 000)
9. 67 682 − 9 724 57 958 (58 000)
10. 7854 − 2799 5055 (5000)
11. 23 097 − 14 523 8574 (10 000)
12. 31 235 − 8 477 22 758 (23 000)
13. 27 954 − 25 326 2628 (3000)
14. 39 857 − 38 905 952 (1000)
15. 47 804 − 15 682 32 122 (30 000)
16. 2813 − 2782 31 (30)



From Edmonton it is 1887 km by train to Churchill and then 4668 km by ship to England.

Also, it is 1953 km by train to Thunder Bay and then 6454 km by ship to England.

Which route would be faster for shipping things from Edmonton to England

1. in August? through Churchill
2. in November? through Thunder Bay
3. in February? both routes closed
4. today? depends on the month

PROBLEM SOLVING

37

RELATED ACTIVITIES

• The activity described for addition in *Related Activities* on page T 27 may be adapted for subtraction. Use exercises similar to the following.

1. Estimate the difference of A and B.
2. Which two numbers have a difference of about 40 000?

• The cards suggested for use in the first activity in *Related Activities* on page T 28 may be used for estimating differences.

• For further practice in rounding numbers and estimating differences, choose exercises from previous lessons on pages 28-35.

1. 20 000
2. D and B
   D and G
   F and B
   F and G

Exercises: The students must decide to which place to round the numbers. Usually, for five-digit numerals, the numbers are rounded to the nearest ten thousand. If an exercise involves a four-digit numeral, then each number is rounded to the nearest thousand. Thus, in Ex. 9, for example, 67 682 is rounded to 68 000 and 9 724 is rounded to 10 000 (to the nearest thousand). If rounding to the nearest ten thousand, the numbers would be 70 000 and 10 000.

You may wish to have the students write their exercises using the format shown on page 26. Ensure that they record their estimates before computing the exact differences, as indicated below.

Ex. 3 — 29 116 − 21 082

exact difference →

estimate of the difference → $\boxed{10\ 000}$

Note that the estimate of the difference for Ex. 16 will be 0 if the two numbers are rounded to the nearest thousand or to the nearest hundred.

Problem Solving: These problems suggest that the faster route depends upon the time of year, in other words, on weather conditions. The concept of answers being affected by different situations is dealt with on page 40.

Assessment

Round and subtract to estimate each difference. Then find the exact difference. Estimates may vary.

1. 8413 − 3562 = 4851 (4000)
2. 46 172 − 11 895 = 34 277 (34 000)
3. 41 600 − 9 764 = 31 836 (32 000)

Pages 38-39

OBJECTIVE

Relate additive and subtractive situations to the use of the $\boxed{+}$ and $\boxed{-}$ keys on a calculator

Materials

calculators (optional)

Vocabulary

calculator, digit display, keys

Background

This is the first of seven special lessons devoted to the calculator. Although it is desirable that students have access to calculators, the exercises may be completed without using one. The exercises are designed to support and reinforce the work of numbers and operations by presenting the concepts from a different point of view. Because of the many differences among even the simplest of calculators, some lesson suggestions may have to be modified to fit the calculators you use.

The $\boxed{+}$ and $\boxed{-}$ Keys on a Calculator

People who know how to work with numbers can use a calculator to save them time and effort.

38

LESSON ACTIVITY

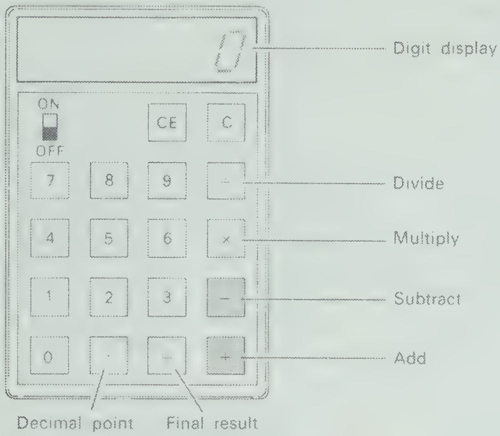
Using the Pages

- The photograph on page 38 can motivate a discussion to introduce this lesson. Point out that there are many different kinds of calculators designed to perform tasks from the very simple to the very complicated. Read the statement above the photograph, and emphasize the need to understand numbers and operations in order to use a calculator. This lesson deals with the skill of knowing which keys to press. This involves interpreting a given situation as one that is additive or subtractive.

  Question the students about the diagram shown at the top of page 39. Have them identify the keys for the digits 0 to 9 and relate these to the section labeled "digit display". Identify the remaining keys that are labeled. Note that the $\boxed{CE}$ (Clear Entry) and $\boxed{C}$ (Clear) keys are not referred to at this time. Explain that the keys indicated in red are the subject of today's lesson. Draw attention to the statement in the "thought cloud" at the top of page 39.

  The students may indicate their answers by writing $\boxed{+}$ or $\boxed{-}$ for each exercise. Ex. 11 is starred because its solution involves more than one step and the use of both the $\boxed{+}$ and $\boxed{-}$ keys. The sequence for carrying out those steps may vary, but this aspect is not dealt with until page 61.

To use a calculator, you need to know which keys to press.



Calculator keys are not always in the same place on different calculators.

Tell which red key you would use to find each of these.

1. 4398 + 1978 $\boxed{+}$
2. 23 846 − 17 248 $\boxed{-}$
3. the difference of 5 375 and 26 849 $\boxed{-}$
4. the sum of 59 848 and 26 973 $\boxed{+}$
5. the cost of the items in your grocery basket $\boxed{+}$
6. how much you will have left in your bank account after you make a withdrawal $\boxed{-}$
7. the number of people who went to the fair in all when you know the number for each day $\boxed{+}$
8. how many more or how many fewer people went to the fair today than yesterday $\boxed{-}$
9. how many more square kilometres there are in Alberta than in Saskatchewan $\boxed{-}$
10. how many square kilometres there are in Canada when you know the number in each province and territory $\boxed{+}$
11. how many fewer people live in the provinces east of Quebec than in Quebec itself when you know the population of each province $\boxed{+}$ and then $\boxed{-}$

Calculator

RELATED ACTIVITIES

• On page 39 it is suggested that calculator keys are not always in the same place on different calculators. This can motivate a discussion of the differences among calculators that students have encountered. Have students cut pictures of calculators from catalogs and magazines. Display them on a large sheet of paper in one area of the room, and have students note their similarities and differences.

• Discuss the abacus as an early form of calculator still in use in parts of the world. Students may be able to bring examples of a Chinese abacus to the classroom.

Page 40

OBJECTIVE

Identify different situations that affect answers to problems

RELATED ACTIVITIES

- Students may enjoy making up problems similar to those on the page for other students to solve. The suggestions may be drawn from their own experiences in decision making. A problem of particular interest to students that suggests a variety of different situations can be written on a large sheet of paper and displayed for several days to give students an opportunity to contribute ideas.

Situations That Affect Answers

An answer sometimes depends upon the situation.
What other answers are possible for Kit?

BICYCLE SALE

This Week Only!

DeLuxe Bicycle $ 99.99 Sale! ~~$115.00~~

Super Bicycle $ 89.99 Sale! ~~$ 99.00~~

Standard Bicycle $ 79.99 Sale! ~~$ 84.00~~

Answers will vary.

Different answers are possible for each of these.
Tell how different situations would affect the answers.

1. What postage stamps can you buy with one dollar?
2. How long does it take to go from Calgary to Vancouver?
3. How many hamburgers should we make for lunch?
4. How long should the table legs be?
5. How big should we make the boat?
6. How much sand should be put into the bag?
7. What shape should we cut out of the cardboard?
8. How big should we make the opening in the birdhouse?
9. How many pages will you read today?

PROBLEM SOLVING

40

LESSON ACTIVITY

Before Using the Page

- Introduce this lesson by suggesting that, sometimes, when you ask a person a question, that person replies, ''It depends.'' Have students relate situations of this nature that they have experienced. For example, a student asks a parent, ''May I go to the movies on Saturday?'' and the parent replies, ''It depends,'' and proceeds to state situations that affect an answer of yes or no.

Using the Page

- For the worked example, have students name as many situations as possible that affect the answer to ''Which bicycle should I get?'' Do not overlook the possibility that although Kit suggests she will earn more money this week, she may not earn enough to buy any of the bicycles on sale. Note that the sign says ''This week only!''

  You may wish to conduct an oral discussion of these exercises. Have the students record ideas in point form for some or all of the exercises to aid them in the discussion. For example, for Ex. 3, the answer can depend on the following.

1. How many people are there for lunch?
2. How many hamburgers will each person want to eat?
3. How large are the hamburgers?
4. How many rolls are there?
5. How much meat is there?
6. How much time do we have to eat?
7. How hungry are we?

Checking Up

Add.

1. 7345 2651 9996	2. 16 942 23 016 39 958	3. 1083 8561 9644	4. 21 541 13 935 35 476
5. 2494 3046 5540	6. 35 261 42 787 78 048	7. 2587 4739 7326	8. 57 642 4 685 62 327
9. $9346 1755 $11 101	10. $19 675 47 785 $67 460	11. $21.83 26.59 $48.42	12. $267.83 75.82 $343.65
13. 2504 635 3071 6210	14. 3 438 12 607 14 869 30 914	15. 14 372 21 143 12 486 48 001	16. 2 987 11 635 13 753 22 717 51 092

17. 4 953 + 14 983 19 936

18. $60 936 + $3 684 $64 620

19. 13 574 + 2 398 + 45 463 61 435

20. $131.56 + $257.43 + $23.31 $412.30

Subtract.

21. 9682 7471 2211	22. 19 658 16 213 3445	23. 8267 6834 1433	24. 34 577 12 097 22 480
25. 6237 3908 2329	26. 44 189 37 452 6737	27. 5133 3765 1368	28. 62 180 45 921 16 259
29. $7911 5242 $2669	30. $44 444 24 865 $19 579	31. $175.48 87.91 $ 87.57	32. $576.60 258.69 $317.91
33. 8000 3832 4168	34. 90 000 65 205 24 795	35. 10 050 5 983 4 067	36. 42 003 10 166 31 837

37. 7353 − 1570 5783

38. 92 209 − 47 862 44 347

39. $30 030 − $24 798 $5232

40. $180.02 − $11.34 $168.68

41. 9765 adults and 14 288 children visited the park in May. What was the total number of visitors to the park in May? 24 053

42. The contest had 35 500 entries and 1750 prizes. How many more entries than prizes were there? 33 750

41

Page 41

OBJECTIVE

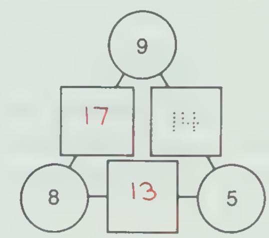
Demonstrate an understanding of the concepts and skills presented in this unit

RELATED ACTIVITIES

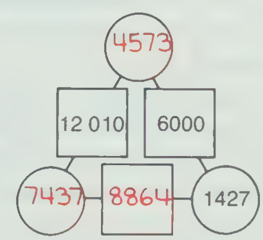
• Select a few numbers from exercises on this page and have students round the numbers to given places.

• On copies of page T 390, write addends in the circles as shown in A. Have students show the sums in the squares. When they are familiar with the procedure, provide more challenging diagrams, as suggested in B.

A



B



• For practice in column addition, have students find the sum for each column of their answers for Ex. 11-16 on page 16.

Skills	Exercises	Related Pages
Add, no regrouping	1, 2	T 19
Add, one regrouping	3	T 20-T 21
	4	T 22-T 23
Add, two regroupings	5	T 20-T 21
	6, 17	T 22-T 23
Add, three regroupings	7	T 20-T 21
	8	T 22-T 23
Add amounts of money	9-12, 18, 20	T 24-T 25
Add more than two numbers	13-16, 19	T 22-T 23
Subtract, no regrouping	21, 22	T 30
Subtract, one regrouping	23, 24	T 31-T 32
Subtract, two regroupings	25, 26, 37	T 31-T 32
Subtract, three regroupings	27, 28, 38	T 31-T 32
Subtract amounts of money	29-32, 39, 40	T 36
Subtract with zeros in the minuend	33-36	T 34-T 35
Solve word problems	41, 42	

Comments

The chart showing the skill required for each exercise facilitates locating an area of difficulty. Exercises involving more than two addends, or addition and subtraction with amounts of money, may involve as many as four regroupings. The skill of lining up numerals in vertical form for addition and subtraction is assessed in Ex. 17-20 and Ex. 37-40. Errors may result from copying numerals incorrectly, regrouping incorrectly (for example, regrouping in addition where not needed), poor recall of basic facts, or poor understanding of the skill. Students should review a skill for which they answer only half the exercises correctly. If fewer than half the exercises are correct, the skill should be retaught.

If all the exercises on page 332 were not assigned and completed earlier, you may wish to have the students complete some or all of the remaining exercises at this time.

Unit 3 Overview

Multiplication

This unit begins with a review of the basic multiplication facts and of multiplying two-digit numbers by one-digit numbers. A transition is made quickly from the longer form of algorithm requiring addition of partial products to the standard form showing only the product. The operation is then extended to multiplying numbers with up to five digits by one-digit multipliers. Multiplication by multiples of 10, 100, and 1000 is presented in preparation for multiplying by two-digit and three-digit multipliers. Medial zeros appear in the multiplicands but not in the multipliers. Estimating products is achieved by rounding both factors. The lesson on the use of the calculator deals with solving one-step and two-step problems using addition, subtraction, and multiplication. The last lesson deals with finding the number of different choices that are possible for a particular situation.

Prerequisite Skills

- multiply a multiple of ten from 10 to 90 by a one-digit number
- regroup among the places for numerals with up to six digits
- read numerals with up to seven digits

Unit Outcomes

- complete the basic multiplication facts
- use the standard algorithm to multiply a two-digit number by a one-digit number
- use the standard form to multiply by a one-digit number, multiplicands with up to five digits
- multiply a number to 999 by a multiple of ten from 10 to 90, a multiple of one hundred from 100 to 900, and a multiple of one thousand from 1000 to 9000
- multiply by a two-digit number, multiplicands with up to four digits
- multiply three two-digit numbers
- multiply by a three-digit number, multiplicands with up to four digits
- round two factors and multiply to estimate the product, then compare the estimate of the product with the exact product
- solve word problems using multiplication
- prepare a keychart to show the order of pressing the keys $\boxed{+}$, $\boxed{-}$, and $\boxed{\times}$ on a calculator to solve a problem
- find the number of possibilities of an event

Background

Multiplication is a binary operation in which two numbers called *factors* are combined to produce a third number called their *product*. Multiplication is related to addition in that the sum of equal addends is the same as the product of the particular addend and the number of such addends.

4	×	6	=	24	6 + 6 + 6 + 6 =	24
factor		factor		product	addends	sum

In discussing multiplication it is sometimes necessary to identify the two factors separately, in which case one is called the *multiplier* and the other is called the *multiplicand*.

factor		factor		product
7	×	48	=	336
multiplier		multiplicand		product

factor	48	multiplicand
factor	× 7	multiplier
product	336	product

It is generally accepted that in a concrete situation the multiplier refers to the number of groups or items and the multiplicand to the size of each group or item. For example, in buying postage stamps, 17 × 5 relates to 17 five-cent stamps and 5 × 17 relates to 5 seventeen-cent stamps; in both cases the cost (product) is the same.

The *commutative property of multiplication* states that the order of two factors may be changed without affecting their product. Therefore, in 7 × 48 = 336 and 48 × 7 = 336 the products are identical. There often are advantages in interchanging the multiplier and the multiplicand.

$$\begin{array}{r} 7 \\ \times\ 48 \\ \hline 56 \\ 280 \\ \hline 336 \end{array} \qquad \begin{array}{r} 48 \\ \times\ 7 \\ \hline 336 \end{array}$$

The *associative property of multiplication* states that the order of multiplying three or more factors does not affect the product. This property is particularly useful in simplifying some calculations, as shown.

$$\begin{aligned} 245 \times 25 \times 8 &= 245 \times (25 \times 8) \\ &= 245 \times 200 \\ &= 49\ 000 \end{aligned}$$

The *distributive property of multiplication over addition* states that if one factor is expressed as the sum of two or more numbers, each of the numbers is multiplied by the other factor and these products are then added. This property is probably used more widely than any other, for without it multiplication with two-digit and three-digit factors would be most cumbersome. Compare the steps and partial products in A and B.

A

$$\begin{aligned} 6 \times 345 &= 6 \times (300 + 40 + 5) \\ &= (6 \times 300) + (6 \times 40) + (6 \times 5) \\ &= 1800 + 240 + 30 \\ &= 2070 \end{aligned}$$

B

$$\begin{array}{rl} 345 & \\ \times\ \ 6 & \\ \hline 30 & (6 \times 5) \\ 240 & (6 \times 40) \\ 1800 & (6 \times 300) \\ \hline 2070 & \end{array}$$

C

$$\begin{array}{r} {\scriptstyle 2\,3}\ \\ 345 \\ \times\ \ 6 \\ \hline 2070 \end{array}$$

The distributive property is also operative in the standard form of the algorithm, as shown in C. Here, as well as in B, products are obtained by considering the place values separately; but the addition (regrouping) is integrated after each partial product. When both the multiplier and the multiplicand have two or more digits, the application of the distributive property is even more significant. In 27 × 365, each value in the multiplicand 365 is multiplied by 7 and by 20 and the partial products are added. Compare the steps shown in D, E, and F.

D

$$\begin{aligned} 27 \times 365 &= (20 + 7) \times (300 + 60 + 5) \\ &= (20 \times 300) + (20 \times 60) + (20 \times 5) \\ &\quad + (7 \times 300) + (7 \times 60) + (7 \times 5) \\ &= 6000 + 1200 + 100 + 2100 + 420 + 35 \\ &= 9855 \end{aligned}$$

E

365	
× 27	
35	(7 × 5)
420	(7 × 60)
2100	(7 × 300)
100	(20 × 5)
1200	(20 × 60)
6000	(20 × 300)
9855	

F

365	
× 27	
2555	(7 × 365)
7300	(20 × 365)
9855	

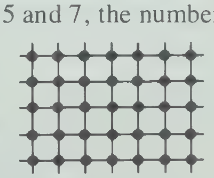
The major difficulty in using the standard form is in the regrouping that occurs from one place to the next. Consider the steps in the example shown. For multiplying ones, $5 \times 7 = 35$, only the 5 is written in the ones' place; the 3 tens of 35 must be remembered while the product 5×9 (tens) is obtained; then the 3 (tens) is added to the 45 (tens), making 48 (tens); again, only the 8 is written in the tens' place, and the 4 (hundreds) must be remembered while the product 5×3 (hundreds) is obtained; then the 4 (hundreds) is added to the 15 (hundreds), making 19 (hundreds). Briefly, the procedure is to multiply first, then add the regrouped number, but many students make errors by reversing these steps.

$$\begin{array}{r} {\scriptstyle 43} \\ 397 \\ \times \quad 5 \\ \hline 1985 \end{array}$$

The four basic operations with numbers are *binary*; that is, only two numbers can be combined at one time to produce a new number. If an expression involves three or more numbers, its value is determined in stages using two numbers at a time. Three elements are required to indicate each step — two numbers and an operation — as shown by $6 + 7$, 4×5, and $28 \div 7$. The lesson on page 61 in this unit introduces the keychart as an aid for showing such steps in solving one-step and two-step problems. Although a keychart for using a calculator may show several steps, the calculator itself is a "binary operator" and each step uses two numbers and an operation. The result of any single step appears on the digital display when a key is pressed for the next operation, and this displayed number is used with the next number — again, two numbers and an operation. For the keychart shown, after pressing keys for the first step (14 × 35), the product appears when the next operation key, $\boxed{+}$, is pressed; then the product 490 is added when 12 is entered, and their sum, 502, appears when the final key, $\boxed{=}$, is pressed.

$$14 \ \boxed{\times}\ 35\ \boxed{+}\ 12 \ = \ \_\_\_$$

In the *Problem Solving* lesson on page 62 another interpretation and use of multiplication is involved in finding the number of possibilities. For example, to multiply 5 and 7, the number of intersections, rather than the repeated addition of one number, provides the solution. Part of a screen illustrates this — 7 vertical wires cross 5 of the horizontal wires at 35 points.



Teaching Strategies

The success of the students in working with this unit is dependent, in part, on complete mastery of all the basic multiplication facts. The first lesson reviews these quickly, but the teacher may need to provide additional surveys and practice for the students to achieve this level of competence.

In the preceding section the difficulties associated with regrouping for the standard algorithm are outlined. Students may need to be reminded frequently that the addition of "carried" numbers occurs after the next multiplication step. Oral and written drills may be helpful in this connection, using examples such as $9 \times 8 + 3$ and $6 \times 8 + 5$. Notice how extensions of basic addition facts are utilized frequently in the process: in the first example, the sum remains in the same decade, and $72 + 3 = 75$ is an extension of $2 + 3 = 5$; in the second example, the extension of the "teen" fact $8 + 5 = 13$ gives a sum in the next higher decade, $48 + 5 = 53$. Thus, multiplication with "carrying" (regrouping) can often present considerable difficulty, since so many things must be considered mentally.

During the early stages many students will probably write the "carried" digits above the multiplicands, but when the multipliers have two or three digits there could be several sets of these. It is important, therefore, that students acquire skill in performing these successive steps without the use of such devices. Practice of the type outlined above is recommended to achieve this.

It should be pointed out that zeros are retained in the ones' place (in the ones' and tens' places) when multiplication is performed using a tens' digit (a hundreds' digit) of the multiplier. Not only do these zeros keep the partial products aligned properly, but they also emphasize the place values of the digits in the multiplier. In presenting the lessons in *Starting Points in Mathematics* it was considered more important to include the zeros, rather than to omit them and to substitute the meaningless and mechanical rule "move over one place".

Differences in abilities among students will probably result in some grasping the new concepts and skills quickly, while others experience difficulty and frustration. It is suggested that the results of each lesson be examined carefully to discover which students require extra assistance and what their specific types of needs are. Rearranging of instructional groups may be needed several times in the unit. Suggestions are offered in the *Related Activities* for each lesson to provide for the divergent needs of both the more capable and the less capable students.

If calculators are not available for the lesson on page 61, the lesson may be adapted by having the students write an open mathematical sentence to structure the problem situation. For example, the open sentence for Ex. 3 in this lesson would be $(18 + 17) \times 16 = \square$; in this instance, parentheses are required to indicate that the addition must occur before the multiplication. The other problems do not require parentheses in the number sentences.

Materials

models for thousands, hundreds, tens, and ones
calendars
copy of the telephone book for your area
telephone book showing long-distance rates
copies of page T 391 for each student (optional)
calculators (optional)

Vocabulary

multiplication	day, d
multiply	hour, h
multiplication sentence	century
factor	overtime
product	minutes, min
multiple	encyclopedia
standard form	dozen
planetarium	keychart

Pages 42-43

LESSON OUTCOME

Complete the basic multiplication facts

Materials

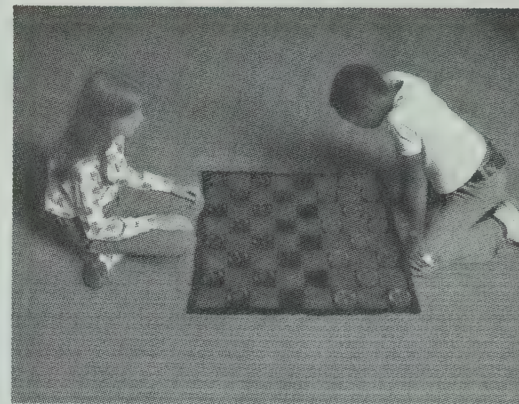
copies of page T 391 for each student (optional)

Vocabulary

multiplication, multiply, multiplication sentence, factor, product, multiple

3 MULTIPLICATION

Basic Facts



How many red checkers are there?

3 rows, 4 in each row

$3 \times 4 = 12$

There are 12 red checkers.

How many checkers are there?

6 rows, 4 in each row

$6 \times 4 = 24$

There are 24 checkers.

How many squares are on the checkerboard?

8 rows, 8 in each row

$\underbrace{8}_{\boxed{\text{factor}}} \times \underbrace{8}_{\boxed{\text{factor}}} = \underbrace{64}_{\boxed{\text{product}}}$

There are 64 squares on the checkerboard.

Working Together

Make a chart and list the multiples.

A **multiple** of a number is the product of that number and any other number.

	0	1	2	3	4	5	6	7	8	9
1. 0 ×	0	0	0	0	0	0	0	0	0	0
2. 1 ×	0	1	2	3	4	5	6	7	8	9
3. 2 ×	0	2	4	6	8	10	12	14	16	18
4. 3 ×	0	3	6	9	12	15	18	21	24	27
5. 4 ×	0	4	8	12	16	20	24	28	32	36
6. 5 ×	0	5	10	15	20	25	30	35	40	45
7. 6 ×	0	6	12	18	24	30	36	42	48	54
8. 7 ×	0	7	14	21	28	35	42	49	56	63
9. 8 ×	0	8	16	24	32	40	48	56	64	72
10. 9 ×	0	9	18	27	36	45	54	63	72	81

42

LESSON ACTIVITY

Using the Pages

- Begin with a discussion of the photograph. Have students name the game shown and briefly describe how it is played. Have them note that the checkers are arranged in rows to begin the game.

  Lead the students in a discussion of the example. Have them recall that $3 \times 4 = 12$ is read "Three times four equals twelve" and that it describes the number for 3 groups (rows) of 4. Have a student suggest the addition sentence that illustrates the concept 3 groups of 4 $(4 + 4 + 4 = 12)$, to review that multiplication is related to repeated addition. Follow a similar discussion for $6 \times 4 = 24$ and $8 \times 8 = 64$, drawing attention to the terms *factor* and *product*. You may wish to introduce the term *multiplication sentence* at this time. Have students identify the factors and products in the sentences $3 \times 4 = 12$ and $6 \times 4 = 24$.

  Explain that each of the three multiplication sentences shows a *basic multiplication fact* since the two factors are less than ten. Emphasize that knowing the basic multiplication facts helps in completing the multiplication of larger numbers accurately and quickly.

  Have students name a few basic facts they recall from earlier work in multiplication.

Working Together: Have a student read the statement describing the meaning of the term *multiple*. Discuss how the multiples of 2, shown in the chart, are obtained. Explain that, for example, from the multiplication sentence $5 \times 2 = 10$, it is seen that 10 is a multiple of 2.

To copy and complete the chart, have the students turn their lined notebook pages sideways and space their work so that the chart fills a whole page. You may prefer to provide each student with a copy of page T 391 for making the chart.

The completed chart presents students with a summary of all the basic multiplication facts.

Exercises

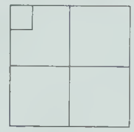
Multiply.

1. 2 × 3 6
2. 4 × 5 20
3. 5 × 1 5
4. 2 × 0 0
5. 5 × 3 15
6. 4 × 2 8
7. 2 × 6 12
8. 1 × 3 3
9. 0 × 4 0
10. 3 × 3 9
11. 2 × 7 14
12. 6 × 3 18
13. 7 × 1 7
14. 6 × 0 0
15. 2 × 9 18
16. 8 × 6 48
17. 4 × 9 36
18. 9 × 3 27
19. 9 × 7 63
20. 6 × 6 36
21. 3 × 8 24
22. 7 × 4 28
23. 5 × 8 40
24. 4 × 6 24
25. 6 × 8 48
26. 3 × 6 18
27. 9 × 5 45
28. 8 × 4 32
29. 7 × 7 49
30. 9 × 4 36

Write the ten products for each column. Which column can you do the fastest?

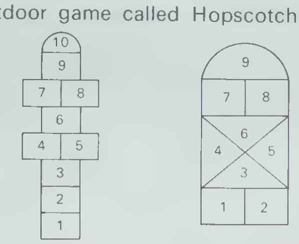
31.	32.	33.
2 × 4 8	0 × 0 0	7 × 8 56
3 × 2 6	1 × 1 1	8 × 7 56
8 × 3 24	2 × 2 4	9 × 6 54
7 × 5 35	3 × 3 9	6 × 9 54
6 × 4 24	4 × 4 16	9 × 8 72
9 × 2 18	5 × 5 25	8 × 9 72
5 × 6 30	6 × 6 36	6 × 7 42
3 × 7 21	7 × 7 49	7 × 6 42
4 × 8 32	8 × 8 64	7 × 9 63
2 × 8 16	9 × 9 81	9 × 7 63

This is the diagram for an outdoor game called Four Square.



1. How many squares can you find in the four-square diagram? 6

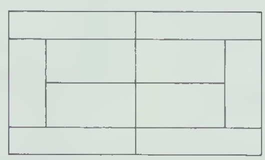
These are diagrams for an outdoor game called Hopscotch.



2. How many rectangles can you find in this diagram? 14

3. How many triangles can you find in this diagram? 8

This is the diagram of a tennis court.



4. How many rectangles can you find in the diagram? 31

5. How many squares can you find in a checkerboard? 204

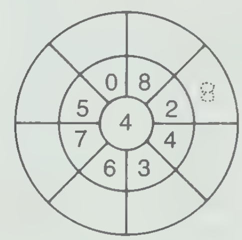
try this

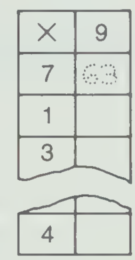
RELATED ACTIVITIES

• Have students refer to the completed chart from *Working Together* to ask and answer questions similar to the following. Note that, at this time, answers are restricted to numbers in the chart.

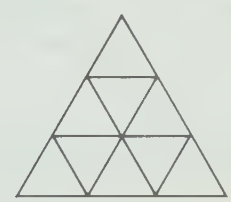
1. I am a multiple of 2.
   The sum of my digits is 5.
   Who am I? 14
2. I am a multiple of 6 and also a multiple of 7.
   Who am I? 42
3. I am a multiple of every number.
   Who am I? 0
4. I am a multiple of 2, of 3, and of 6.
   I am greater than 15.
   Who am I? 18

• Prepare number wheels and tables from copies of page T 390 to provide further practice in multiplication.





• Ask students to see how many triangles they can find in this diagram.



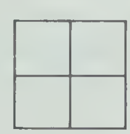
Exercises: Instruct the students to work as quickly as they can. You may prefer to have them write only the products rather than the multiplication sentences. Ask them to ring the exercise numbers in their notebooks of the facts they were unable to recall quickly. Note that Ex. 32 involves equal factors and Ex. 33 presents unequal factors with the factors repeated in the reverse order. Ex. 33 presents an opportunity to discuss the commutative (order) property of multiplication, which states that the order of multiplying two numbers does not affect the product.

Try This: Some students may need to copy the diagrams and trace the line segments using colored pencils to show the different shapes. The emphasis is on how many squares (rectangles, triangles) "can *you* find". Thus, answers may vary. However, the possibilities for each exercise are as follows: 6 for Ex. 1; 14 for Ex. 2; 8 for Ex. 3; 31 for Ex. 4; 204 for Ex. 5. Students may need to return to Ex. 4 and 5 over a period of several days. It will be helpful to number the regions of the diagram for Ex. 4. Probably the best strategy to apply in Ex. 5 is to investigate the number of squares on a copy of page T 391. Consider the squares one by one (A), then two by two (B), and so on, to determine a pattern for reaching the eight-by-eight board as shown in the photograph on page 42. Note that the statement beneath the photograph on page 42 implies 64 small squares.

A

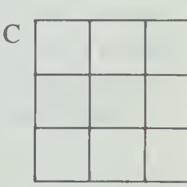
1 square

B



5 squares

C



14 squares

Assessment

Multiply.

1. 4 × 7 28
2. 6 × 9 54
3. 5 × 5 25
4. 0 × 4 0
5. 7 × 1 7
6. 7 × 8 56

Pages 44-45

LESSON OUTCOME

Use the standard algorithm to multiply a two-digit number by a one-digit number; solve related word problems

Materials

models for hundreds, tens, and ones

Vocabulary

standard form

Prerequisite Skills

Complete the basic multiplication facts; multiply a multiple of ten from 10 to 90 by a one-digit number; regroup ones as tens and tens as hundreds

Checking Prerequisite Skills

Complete.

1. 4 × 2 = 8
4 × 2 tens = 8 tens
4 × 20 = 80

2. 6 × 7 = 42 | 6 tens × 7 = 42 tens | 60 × 7 = 420

3. 7 × 9 = 63 | 7 tens × 9 = 63 tens | 70 × 9 = 630

4. 42 ones = 4 tens 2 ones
42 tens = 4 hundreds 2 tens

5. 63 ones = 6 tens 3 ones
63 tens = 6 hundreds 3 tens

Multiplying a Two-Digit Number by a One-Digit Number

3 school buses are used for the field trip. Each bus will carry 57 children. How many children will the buses carry in all?

Multiply 3 and 57.

tens	ones
5	7
×	3
2	1

3 × 7 = 21 or 2 tens 1 one.

tens	ones
2	
5	7
×	3
	1

	5	7
×		3
	2	1
1	5	0

3 × 5 tens = 15 tens and 2 more tens make 17 tens.

	2	
	5	7
×		3
1	7	1

This is the standard form for multiplying.

	5	7
×		3
	2	1
1	5	0
1	7	1

Add.

The standard form is one of the fastest ways I can multiply using a pencil.

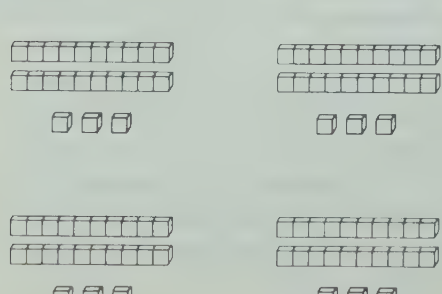
The buses will carry 171 children in all.

44

LESSON ACTIVITY

Before Using the Pages

- Write 4 × 23 = ▒ on the board and ask for the number of groups and the number in each group. Then write the two factors in vertical form. Have each of four students represent 23 using models and display them as shown below. Describe 4 groups of 23 as 4 groups of 2 tens and 3 ones, and point out the 4 groups of 2 tens and the 4 groups of 3 ones in the display.



tens	ones
2	3
×	4
$\boxed{1}$	2
8	0
9	2

Ask for the number represented by 4 groups of 3 ones and have a student write the partial product, 12, in the exercise while another student regroups the models. Draw a square around the 1 in the tens' place to highlight the 1 ten regrouped from 10 ones. Similarly, ask for and show the number represented by 4 groups of 2 tens. Have a student write the final product. Summarize the steps: multiply the ones first; regroup; multiply the tens; and add.

Suggest that the product 92 can be written without having to write the lines that show 12 and 80. In other words, the written work can be reduced from three lines to one line. If possible, have a student demonstrate this at the board, explaining the procedure.

Using the Pages

- The worked example presents the long form and the standard form for multiplying 3 and 57. Have a student read the word problem aloud. Discuss the facts presented in the problem and point out that because 3 groups of 57 are described, multiplication is used in the solution. Have

Working Together

Show each multiplication in standard form.

Example:

63 × 8	becomes	63 × 8 (regroup 2)
24		504
480		
504		

1. 34 × 2: 8, 60, 68 — 34 × 2 = 68
2. 52 × 6: 12, 300, 312 — 52 × 6 = 312 (regroup 1)
3. 49 × 9: 81, 360, 441 — 49 × 9 = 441 (regroup 8)

Multiply. Use the standard form.

4. 42 × 3 = 126
5. 38 × 4 = 152
6. 45 × 7 = 315

Exercises

Multiply. Use the standard form.

1. 21 × 4 = 84
2. 72 × 2 = 144
3. 40 × 6 = 240
4. 62 × 7 = 434
5. 68 × 6 = 408
6. 25 × 5 = 125
7. 83 × 4 = 332
8. 52 × 9 = 468
9. 49 × 8 = 392
10. 87 × 9 = 783
11. 29 × 7 = 203
12. 16 × 8 = 128
13. 2 × 23 46
14. 4 × 72 288
15. 3 × 84 252
16. 5 × 37 185
17. 8 × 77 616
18. 3 × 38 114
19. 7 × 38 266
20. 9 × 79 711
21. 5 × 74 370
22. 6 × 99 594

Solve.

23. The buses will travel 48 km each way on the field trip. How long is the round trip? 96 km
24. What will the total distance be for all 3 buses? 288 km
25. How many children could go on the field trip in 4 buses? 228
26. How many children could go on the field trip in 7 buses? 399
27. 6 teachers or their helpers ride each bus with 57 children. How many people are going on the field trip in all 3 buses? 189
28. If 57 children and 6 adults are driven on the field trip in each of 4 buses, how many people would be on the buses in all? 252

45

RELATED ACTIVITIES

- For further practice, have students complete some or all of Ex. 1-17 on page 334.
- Prepare word problems referring to school buses and trips taken by students in your school or class. Have students help to prepare the problems prior to or immediately following a trip. Ask them to contrast the number of seats available on their buses with the buses described on pages 44 and 45. Assign the problems for extra practice.
- Have students investigate the seating capacity of vehicles other than school buses and use the information to prepare word problems for other students to solve.
- Students having difficulty with the standard form can be helped through the use of felt numerals and a flannel board showing the place values for three-digit numerals. As the multiplication is carried out, felt numerals are placed to show the product and the regrouping. For example, in 4 × 67, the felt numerals 2 and 8 are selected for 4 × 7. The felt numeral 2 is seen first in the ones' column and is then moved above the 6 in the tens' column.

h	t	o
	2	
	6	7
×		4
2	6	2 8

students name the number of digits in each factor and relate this to the title at the top of page 44.

Question the students about the steps shown for each form. For example, ask how many tens and how many ones there are for 57. Ask what is multiplied first, what the partial product is, and how this can be shown in the exercise. Continue the discussion in a similar manner. Emphasize that the standard form is preferable because it involves less written work, but point out that addition of the regrouped tens (2 tens, in this instance) is carried out after multiplication of the 5 tens.

Working Together: Ex. 1-3 provide practice in writing multiplication exercises in standard form. The standard form is then applied in completing Ex. 4-6. Use other similar exercises as required. It would be beneficial to have students explain their work on the board.

Exercises: Remind the students to write Ex. 13-22 in vertical form. For the word problems, the students will need to recall from the worked example that each bus can carry 57 children. The question asked in Ex. 24 refers to the trip described in Ex. 23. If necessary, explain what is meant by "round trip". The solutions to Ex. 27 and 28 involve more than one step. Be certain to note different solutions that students give for these, and provide an opportunity for the solutions to be displayed and discussed.

Assessment

Multiply. Use the standard form.

1. 41 × 3 = 123
2. 26 × 6 = 156
3. 35 × 9 = 315
4. 7 × 80 560
5. 6 × 66 396

Solve.

6. Each bus can carry 49 children. How many children can be carried in 4 buses? 196

Pages 46-47

LESSON OUTCOME

Use the standard form to multiply by a one-digit number, multiplicands with up to five digits; solve related word problems

Vocabulary
planetarium

Prerequisite Skills
Multiply a two-digit number by a one-digit number; regroup among the places for numerals with up to six digits

Checking Prerequisite Skills
Multiply. Use the standard form.

1. $\begin{array}{r} 14 \\ \underline{2} \\ 28 \end{array}$ **2.** $\begin{array}{r} 32 \\ \underline{5} \\ 160 \end{array}$ **3.** $\begin{array}{r} 27 \\ \underline{8} \\ 216 \end{array}$ **4.** $\begin{array}{r} 69 \\ \underline{4} \\ 276 \end{array}$

Complete.

5. 54 tens = _5_ hundreds 4 tens

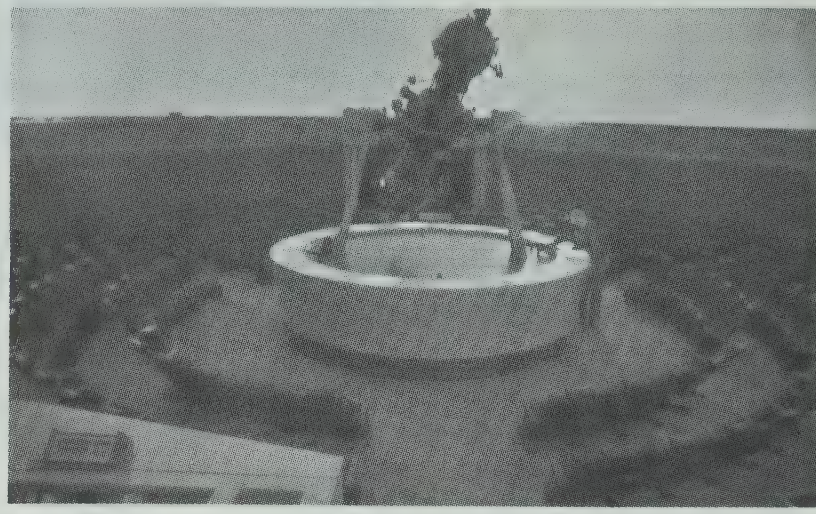
6. 39 hundreds = _3_ thousands 9 hundreds

7. 81 thousands = _8_ ten thousands 1 thousand

8. 47 ten thousands = 4 hundred thousands _7_ ten thousands

Multiplying by a One-Digit Number

The planetarium can hold 354 people for each show. On Saturday there are 6 shows. How many people can see a planetarium show on Saturday?



Multiply 6 and 354.

$$\begin{array}{r} {}^{2} \\ 354 \\ \underline{6} \\ 4 \end{array}$$

6 × 4 = 24 or 2 tens 4 ones.

$$\begin{array}{r} {}^{3}\,{}^{2} \\ 354 \\ \underline{6} \\ 24 \end{array}$$

6 × 5 tens = 30 tens. 2 more tens make 32 tens or 3 hundreds 2 tens.

$$\begin{array}{r} {}^{3}\,{}^{2} \\ 354 \\ \underline{6} \\ 2124 \end{array}$$

6 × 3 hundreds = 18 hundreds. 3 more hundreds make 21 hundreds or 2 thousands 1 hundred.

2124 people can see a planetarium show on Saturday.

46

LESSON ACTIVITY

Before Using the Pages

- You may wish to carry out an oral review of exercises similar to the following.

  6 × 4 tens
  How many tens?
  3 × 9 hundreds
  How many hundreds?
  7 × 8 thousands
  How many thousands?
  8 × 4 ten thousands
  How many ten thousands?

  You may wish to have students express each answer in regrouped form as well, for instance, 24 tens is regrouped as 2 hundreds 4 tens.

- Write the following multiplication exercises on the board.

$$\begin{array}{r} 4 \\ \underline{\times\ 6} \end{array} \quad \begin{array}{r} 54 \\ \underline{\times\ \ 6} \end{array} \quad \begin{array}{r} 354 \\ \underline{\times\ \ \ 6} \end{array} \quad \begin{array}{r} 2354 \\ \underline{\times\ \ \ \ 6} \end{array} \quad \begin{array}{r} 12\ 354 \\ \underline{\times\ \ \ \ \ \ 6} \end{array}$$

  Have students explain how they are alike and how they are different and suggest whether the basic procedure for completing the multiplication would be the same for each exercise.

Using the Pages

- The worked example illustrates that multiplication of a three-digit number is an extension of multiplying a two-digit number. Basic multiplication facts and the concept of place value are applied.

  Introduce the word problem and have a student explain why multiplication can be used to solve the problem. Question the students about the solution. For example, ask what is multiplied first, what the partial product is, how many tens and ones there are, where this is written in the exercise, what is multiplied next, and so on. Have a student read the concluding statement.

- Have the students study the exercises (not the word problems) on page 47. Ask how they are similar. There are two factors

Working Together

Complete each multiplication.

1.	2.	3.	4.	5.	6.
3	4	1	2	22	25
47	36	63	2830	134	3271
5	7	4	3	6	8
23	2		90	8	8
5	25	252	84	04	2616

Multiply.

7.	8.	9.	10.	11.	12.
75	684	2604	9431	16 938	40 536
4	5	3	9	6	8
300	3420	7812	84 879	101 628	324 288

Exercises

Multiply.

1. 59	2. 34	3. 87
5	7	6
295	238	522
4. 678	5. 951	6. 606
9	7	6
6102	6657	3636
7. 1231	8. 4003	9. 4625
3	5	8
3693	20 015	37 000
10. 20 968	11. 27 879	
4	3	
83 872	83 637	
12. 72 342	13. 17 727	
7	9	
506 394	159 543	

14. 8 × 941 7528
15. 6 × 9214 55 284
16. 9 × 17 153
17. 2 × 47 216 94 432
18. 4 × 634 2536
19. 8 × 5764 46 112
20. 5 × 712 3560
21. 4 × 17 819 71 276
22. 9 × 5555 49 995
23. 7 × 76 008 532 056
24. 8 × 880 7040
25. 4 × 9052 36 208
26. 3 × 5226 15 678
27. 6 × 10 375 62 250
28. 2 × 9999 19 998
29. 5 × 19 753 98 765

Solve.

30. There are 3 planetarium shows Saturday evening. How many people can see these shows? 1062
31. The planetarium has a special show each weekday afternoon. How many people can see this show from Monday to Friday? 1770
32. The school auditorium can hold 1250 people. How many people can see the 3 performances of the school play? 3750
33. The football stadium in Vancouver holds 32 752 people. How many people can go to 8 Canadian Football League games there in a season? 262 016
34. The Montreal Forum holds 18 350 people for hockey. How many people can watch 4 Stanley Cup games in the Forum? 73 400
35. The baseball stadium in Toronto can hold 46 500 people. How many people can watch a 3-game series in the stadium? 139 500

47

RELATED ACTIVITIES

- For further practice, you may wish to have students complete Ex. 18-22 on page 334.
- Adapt the fourth activity in *Related Activities* on page T 49 for a larger group. The steps are shown below. Ask five students in turn to complete one of the five steps necessary to calculate 7 × 21 836.

A	B	C
$\boxed{4}$	$\boxed{2}4$	$\boxed{5}24$
21 836	21 836	21 836
× 7	× 7	× 7
$\boxed{2}$	$\boxed{5}2$	$\boxed{8}52$

D	E
$\boxed{1}5\ 24$	15 24
21 836	21 836
× 7	× 7
$\boxed{2}852$	$\boxed{15}2\ 852$

As a silent activity, it requires the continued concentration of each member of the group. For a more challenging activity, have the students show only the digits of the product.

- Some students are handicapped in working with the standard algorithm because of poor recall of basic multiplication facts. These students should be allowed time each day for practice. To diagnose which facts students are having difficulty with, give each student a copy of pages T 381 and T 391. Ask them to write the factors 0 to 9 and the symbol × on their copy of page T 391. Then adapt the procedure suggested for addition facts in *Related Activities* on page T 18.

in each exercise and one of the factors is a number less than ten. Relate this to the title of the lesson at the top of page 46.

Working Together: Ex. 1-3 review the steps of multiplying a two-digit number with regrouping. This concept is applied in Ex. 4 for which the multiplication of the ones and the tens (shown completed) requires no regrouping, but regrouping is necessary for multiplication of the hundreds. Gradually, the concept is extended for exercises that require two and three regroupings (Ex. 5 and 6). Students are required to apply these skills independently in Ex. 7-12.

It would be beneficial to have some students complete their exercises on the board and explain their work. Point out that the procedure of multiplying and regrouping from right to left is applied each time in multiplying by a one-digit number.

Exercises: Note in Ex. 31 that a multiplier of 5 is implied in the wording of the problem.

Assessment

Multiply.

1. 93	2. 275	3. 6130
4	3	5
372	825	30 650

4. 7 × 5642 39 494
5. 9 × 14 069 126 621

Solve.

6. An auditorium can hold 1350 people. How many people can hear the 4 performances of a concert? 5400

Pages 48-49

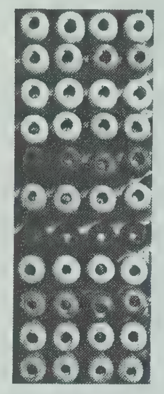
OBJECTIVE

Demonstrate competence in multiplying by a one-digit number; solve related word problems

Practice

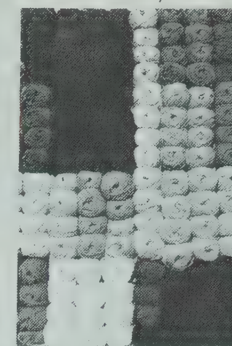
Use multiplication to help you answer the questions.

1. How many balls of crochet cotton are there? 44

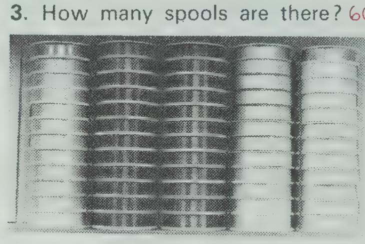


11 rows, 4 in each row

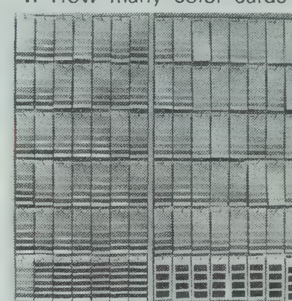
2. How many balls of wool are there? 128



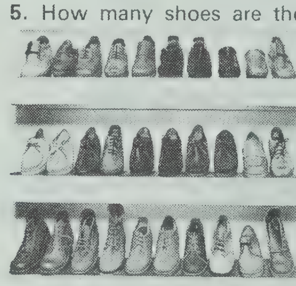
3. How many spools are there? 60



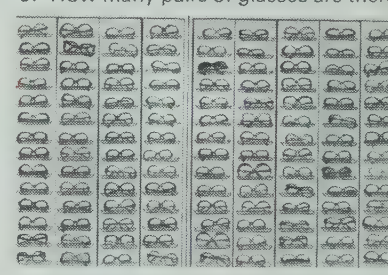
4. How many color cards are there? 90



5. How many shoes are there? 30



6. How many pairs of glasses are there? 126



48

LESSON ACTIVITY

Before Using the Pages

- Remind the students that multiplication can be used to find the number of objects in a rectangular array. For example, show a four-by-seven array of dots on the board and ask how many dots are in the array. Have students indicate whether they found their answer by counting by ones, by addition, or by multiplication. Emphasize that multiplication is the most efficient approach. Tell the students that a rectangular array has seven rows and twelve columns. Ask how many objects there are in the array. Use other similar examples.

Using the Pages

- The photographs on page 48 suggest where arrays are seen in everyday life. The examples at the top of page 49 indicate that amounts of money are multiplied using the same procedure as for multiplying whole numbers. It may be beneficial to discuss the three exercises with the students, having them explain the steps for each multiplication. Draw attention to the symbol $ in each product.

Try This: The students will likely use a ''guess and test'' (trial and error) approach in replacing the gray squares with digits. As indicated, the more digits involved, the more difficult the exercise. Depending on the level of the students in your class, you may wish to direct them to spend more time exploring different solutions for Ex. 1 and 2, for example, and less time trying to find one solution for Ex. 6 or 7. Over a period of several days the students may share their solutions with the rest of the class. Copies of page T381 would be helpful in finding all the possible solutions for Ex. 1. Also, a sequential approach is helpful for Ex. 2 and 3. In other words, have students consider all the possibilities for 2 as the multiplier, 3 as the multiplier, and so on, to 9 as the multiplier. For instance, Ex. 2 may begin in the following way.

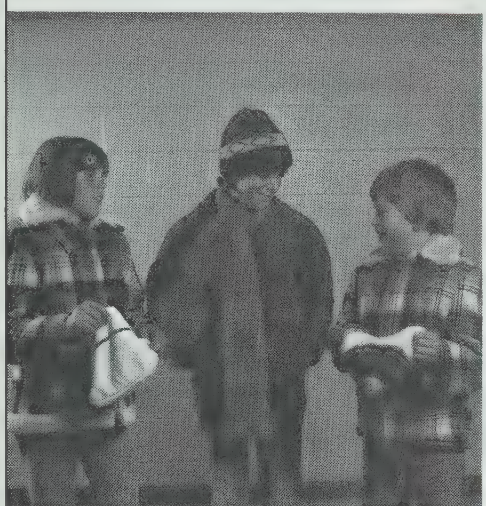
Amounts of money are multiplied just like whole numbers.

Examples:

4 2 1 $3 742 × 6 $22 452	1 $5.40 × 3 $16.20	6 6 3 $19.95 × 7 $139.65

Mrs. Burton bought winter clothes for her 3 sons.

7. Each hat cost $4.99. How much did 3 hats cost? $14.97
8. Each coat cost $23.89. How much did 3 coats cost? $71.67
9. Each pair of boots cost $11.47. How much did 3 pairs of boots cost? $34.41
10. Mrs. Burton bought 2 pairs of socks for each boy. Each pair cost $1.49. How much did the socks cost? $8.94



Multiply.

11. 7 × 273 1911	**12.** 5 × 8478 42 390
13. 4 × 8526 34 104	**14.** 4 × $47.88 $191.52
15. 5 × $295 $1475	**16.** 9 × 30 894 278 046
17. 2 × 58 390 116 780	**18.** 7 × 5185 36 295
19. 3 × 4477 13 431	**20.** 3 × $8592 $25 776
21. 9 × $3.03 $27.27	**22.** 6 × 73 195 439 170
23. 8 × 13 279 106 232	**24.** 8 × 1250 10 000

Replace each ▒ with a digit to show a multiplication. Do not use the same digit more than once in each exercise.

Example: For ▒ ▒ ▒ use 3 2 6

Answers will vary.

1. ▒ × ▒ = ▒▒ (5 × 2 = 10)
2. ▒▒ × ▒ = ▒▒ (13 × 6 = 78)
3. ▒▒ × ▒ = ▒▒▒ (52 × 7 = 364)
4. ▒▒▒ × ▒ = ▒▒▒ (218 × 3 = 654)
5. ▒▒▒ × ▒ = ▒▒▒▒ (453 × 6 = 2718)
6. ▒▒▒▒ × ▒ = ▒▒▒▒ (4835 × 2 = 9670)
7. ▒▒▒▒ × ▒ = ▒▒▒▒▒ (3907 × 4 = 15 628)

The farther you go, the harder they get. How many can you do?

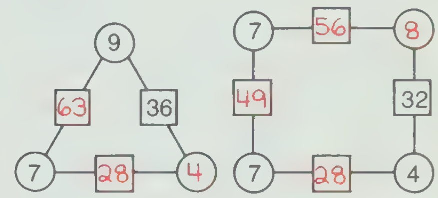
try this

15	17	19	34	35	38
× 2	× 2	× 2	× 2	× 2	× 2
30	34	38	68	70	76

Note, however, that the goal described on page 49 is to find just one exercise to fit each pattern.

RELATED ACTIVITIES

- For further practice, you may wish to have students complete Ex. 23-27 on page 334.
- Have students search for different examples of arrays in everyday life. They may cut pictures from magazines and use these to generate word problems.
- Pictures of items cut from magazines and catalogs may generate multiplication exercises involving amounts of money. For instance, students may find the cost of buying four automobile tires if the price of one tire is $36.67.
- Have students practice basic multiplication facts by completing diagrams similar to the following on copies of page T 390. Factors are shown in the circles; products are shown in the squares.



- The list for the game "Shopping Spree" described on page T 377 may now include such statements as "From now on, every item you purchase is twice the marked price" and "Your shopping limit is now three times the original value".

Pages 50-51

LESSON OUTCOME

Multiply a number to 999 by a multiple of ten from 10 to 90, a multiple of one hundred from 100 to 900, and a multiple of one thousand from 1000 to 9000

Materials

models for thousands, hundreds, tens, and ones (optional)

Prerequisite Skills

Multiply a number to 999 by a one-digit number; read numerals with up to seven digits

Checking Prerequisite Skills

Multiply.

1. $\begin{array}{r} 19 \\ 3 \\ \hline 57 \end{array}$ 2. $\begin{array}{r} 141 \\ 6 \\ \hline 846 \end{array}$

3. $\begin{array}{r} 223 \\ 4 \\ \hline 892 \end{array}$ 4. $\begin{array}{r} 352 \\ 7 \\ \hline 2464 \end{array}$

Complete.

5. 13 047 = 13 thousand 47

6. 5 092 006 = 5 million
92 thousand
6

Multiplying by Multiples of 10, 100, and 1000

The stamp store set up a "Grab Bag Bin". It put 175 different stamps into each bag and filled 300 bags for the bin. How many stamps did it put into the grab bags?

Multiply 300 and 175.

For the product

$$\begin{array}{r} 1\,7\,5 \\ 3\,0\,0 \\ \hline \end{array}$$

3 hundreds 0 tens 0 ones

you need to know how to multiply 0 and 175,

$$\begin{array}{r} 1\,7\,5 \\ 3\,0\,0 \\ \hline 0 \end{array}$$

When 0 is a factor, the product is 0.

$$\begin{array}{r} 1\,7\,5 \\ 3\,0\,0 \\ \hline 0\,0 \end{array}$$

0 × 175 = 0

0 tens × 175 = 0 tens

and how to multiply 3 and 175.

$$\begin{array}{r} {\scriptstyle 2\ 1} \\ 1\,7\,5 \\ 3\,0\,0 \\ \hline 5\,2\,5\,0\,0 \end{array}$$

3 hundreds × 175 = 525 hundreds or 52 thousands 5 hundreds.

The stamp store put 52 500 stamps into the grab bags.

50

LESSON ACTIVITY

Before Using the Pages

- Review that numbers such as 10, 40, and 370 are multiples of ten, and that the standard numerals for multiples of ten show zero in the ones' place. Then have students express multiples of ten as a number of tens. Models are useful in reviewing this concept.

  10 = 1 ten   40 = 4 tens   370 = 37 tens

  Extend this concept to multiples of one hundred and one thousand.

1 hundred	= 100	1 thousand	= 1000
4 hundreds	= 400	4 thousands	= 4000
37 hundreds	= 3700	37 thousands	= 37 000

- Tell the students that thinking of multiples of ten (one hundred, one thousand) as a number of tens (hundreds, thousands) helps to complete multiplication exercises with large numbers. Write the exercises 1 × 23 and 10 × 23 in vertical form on the board and have the students find the products. Ask how the two exercises are alike and how they are different. Demonstrate the similarity by showing 10 × 23 = 230 as 1 ten × 23 = 23 tens. Then extend the concept to multiplying by 100 and by 1000.

23 × 1 = 23	23 × 1 ten = 23 tens	23 × 10 = 230

23 × 1 hundred = 23 hundreds	23 × 100 = 2300

23 × 1 thousand = 23 thousands	23 × 1 000 = 23 000

Use other similar exercises, for example, 1 × 142, 10 × 142, and 1000 × 142. Restricting the multipliers to 10, 100, and 1000 enables students to focus on the zeros in the multiplier and the product.

Using the Pages

- Introduce the word problem and discuss why multiplication can be used to solve the problem. Ask how many digits there are in 300 and have the students identify each. Point out that 300 is a multiple of one hundred. Rewrite the

Working Together

Use the first statement to help you complete the others.

1. 4 × 17 = 68	4 tens × 17 = 68 tens 40 × 17 = 680	4 hundreds × 17 = 68 hundreds 400 × 17 = 6800
2. 6 × 39 = 234	6 tens × 39 = 234 tens 60 × 39 = 2340	6 hundreds × 39 = 234 hundreds 600 × 39 = 23 400

3.

453	453	453	453	453
9	9 hundreds	900	9 thousands	9 000
4077	4077 hundreds	407 700	4077 thousands	4 077 000

Write 0 in the ones place. Then multiply by 7 (tens).

Write 0 in the ones and tens places. Then multiply by 3 (hundreds).

Multiply.

4. 28	**5.** 369	**6.** 128	**7.** 709	**8.** 276	**9.** 784
70	70	300	300	20	500
1960	25 830	38 400	212 700	5520	392 000

Exercises

Multiply.

1. 46	**2.** 141	**3.** 163	**4.** 695	**5.** 108	**6.** 942
30	20	500	60	800	700
1380	2820	81 500	41 700	86 400	659 400

7. 70 × 53 3710 **8.** 200 × 17 3400 **9.** 50 × 47 2350 **10.** 900 × 141 126 900

11. 142 × 60 8520 **12.** 75 × 80 6000 **13.** 36 × 4000 144 000 **14.** 2000 × 358 716 000

Study these multiplication sentences.

3 × 10 = 30	7 × 100 = 700	4 × 1000 = 4 000
28 × 10 = 280	35 × 100 = 3 500	79 × 1000 = 79 000
10 × 65 = 650	100 × 35 = 3 500	1000 × 64 = 64 000
429 × 10 = 4290	631 × 100 = 63 100	152 × 1000 = 152 000

Give a rule that helps you find the product when

For the other factor, move the digits

1. 10 is a factor. **2.** 100 is a factor. **3.** 1000 is a factor.

one place to the left and write one zero. two places to the left and write two zeros. three places to the left and write three zeros.

Make up some multiplication exercises and test your rule.

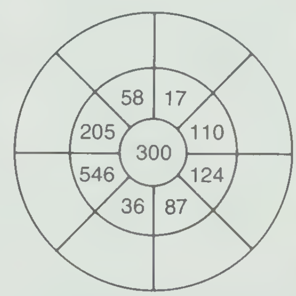
try this

51

RELATED ACTIVITIES

- For further practice, you may wish to have students complete Ex. 28-37 on page 334.
- Demonstrate products obtained by multiplying by 10 on an abacus. For example, have a student show "four" on the abacus, then show "ten times four", "ten times forty", and so on. Point out the corresponding changes such as four rings on the ones' peg replaced by four rings on the tens' peg. Repeat this for numbers involving two digits and three digits. For example, show 397, then show the number that is ten times 397.

  Use a similar procedure to demonstrate multiplying a number by 100 or 1000. Have students describe the results.
- The above procedure can be adapted for use with models of thousands, hundreds, tens, and ones, to offer a different view of the same concept.
- Have students complete number wheels and tables similar to the following on copies of page T 390.



×	48
100	
200	
300	
900	

multiplication on the board as shown and have the students complete it, explaining each step.

175	175
× 3 hundreds	× 300
525 hundreds	52 500

Suggest that the product can also be found by considering each digit of 300 in turn. To do this, it is necessary to recall how zero as a factor affects the product. Lead the students through the worked example of multiplying 175 by 0 ones, 0 tens, and 3 hundreds. Emphasize the right-to-left order of considering the digits of 300. Have a student read the concluding statement.

Working Together: Ex. 1-3 help students to relate multiplication by multiples of 10, 100, and 1000 to multiplication by a one-digit number. Have them note that multiplying by a number of tens results in a zero in the ones' place of the product. Multiplying by a number of hundreds gives a product with 0 tens and 0 ones. Finally, multiplying by a number of thousands gives a product with 0 hundreds, 0 tens, and 0 ones. Since the above results are consistent, Ex. 4-9 suggest that zeros may be written in the appropriate places of the product first, and then multiplication can continue as if by a one-digit number.

Exercises: For Ex. 7-14, you may need to remind the students to write the factors in vertical form so that the multiple of ten (one hundred or one thousand) is the multiplier.

Try This: The skill of multiplying a number by 10, 100, and 1000 is of particular importance in relating metric units of measurement. These exercises encourage students to find a pattern and formulate a rule for multiplying by these numbers. Students who discover patterns on their own and can describe them in their own words are more likely to recall the rules and apply them.

Assessment

Multiply.

1. 52	**2.** 204	**3.** 576
30	800	200
1560	163 200	115 200

4. 173 × 50 8650

5. 12 × 4000 48 000

6. 600 × 397 238 200

Pages 52-53

LESSON OUTCOME

Multiply a two-digit number by a two-digit number

Prerequisite Skills

Multiply a two-digit number by a one-digit number; multiply a two-digit number by a multiple of ten from 10 to 90

Checking Prerequisite Skills

Multiply.

1. $\begin{array}{r} 34 \\ \underline{2} \\ 68 \end{array}$	2. $\begin{array}{r} 76 \\ \underline{5} \\ 380 \end{array}$
3. $\begin{array}{r} 49 \\ \underline{7} \\ 343 \end{array}$	4. $\begin{array}{r} 88 \\ \underline{9} \\ 792 \end{array}$
5. $\begin{array}{r} 27 \\ \underline{80} \\ 2160 \end{array}$	6. $\begin{array}{r} 41 \\ \underline{30} \\ 1230 \end{array}$
7. $\begin{array}{r} 65 \\ \underline{60} \\ 3900 \end{array}$	8. $\begin{array}{r} 22 \\ \underline{40} \\ 880 \end{array}$

Multiplying a Two-Digit Number by a Two-Digit Number

When the puzzle is put together there are 36 rows with 42 pieces in each row. Are there more than 1500 pieces in the puzzle?

Multiply 36 and 42.

For the product

$$\begin{array}{r} 42 \\ \underline{36} \end{array}$$

3 tens 6 ones

you need to know how to multiply 6 and 42.

$$\begin{array}{r} \overset{1}{4}2 \\ \underline{36} \\ 252 \end{array}$$

You do not have to write this numeral if you can remember it.

and how to multiply 3 and 42.

$$\begin{array}{r} 42 \\ \underline{36} \\ 252 \\ 1260 \end{array}$$

3 tens × 42 = 126 tens or 1260.

Then add.

$$\begin{array}{r} 42 \\ \underline{36} \\ 252 \\ \underline{1260} \\ 1512 \end{array}$$

There are more than 1500 pieces in the puzzle.

You can change the order of the factors to check your work.

$$\begin{array}{r} 36 \\ \underline{42} \\ 72 \\ \underline{1440} \\ 1512 \end{array}$$

If this result does not match the first result, there is a mistake in your work.

52

LESSON ACTIVITY

Before Using the Pages

- Write exercises on the board to review multiplication of a two-digit number by multiples of ten from 10 to 90. Some examples are provided. Emphasize that multiplying by a number of tens results in zero in the product.

$$\begin{array}{r} 34 \\ \underline{\times\ 20} \end{array} \quad \begin{array}{r} 76 \\ \underline{\times\ 50} \end{array} \quad \begin{array}{r} 49 \\ \underline{\times\ 70} \end{array} \quad \begin{array}{r} 88 \\ \underline{\times\ 90} \end{array}$$

- Write the following exercises on the board.

$$\begin{array}{r} 34 \\ \underline{\times\ \ 6} \end{array} \quad \begin{array}{r} 34 \\ \underline{\times\ 20} \end{array}$$

Have the students find the product for each exercise. Then have them find the sum of the two products. Explain that in this lesson they will learn to combine the three steps into one multiplication exercise to find a product such as 26 × 34.

Using the Pages

- Briefly discuss the photographs on pages 52 and 53. Have the students note that the cover of the puzzle shown on page 52 suggests there are about 1500 pieces. Ask students whether they enjoy doing jigsaw puzzles and if so, to describe why. Discuss the fact that the assembled puzzle has the shape of a rectangle, and that the pieces of the puzzle fit together in rows with the same number of pieces in each row.

  Introduce the word problem at the top of page 52 and have students recall that the number of objects in a rectangular array can be found by multiplication. The situation presented emphasizes the advantage in knowing how to multiply.

  The worked example demonstrates that multiplication by a two-digit number can be performed by multiplying by the ones as usual, then multiplying by the tens (with care to place 0 ones in the partial product), and then adding the partial products. Guide the students in a discussion of the

Working Together

Multiply by following the steps.

1.
```
                                  27
                                  32
Multiply 2 and 27.          →     54
Write 0 in the ones place.  →    810
Multiply 3 (tens) and 27.
Add.                        →    864
```

2.
```
                                  84
                                  96
Multiply 6 and 84.          →    504
Write 0 in the ones place.  →   7560
Multiply 9 (tens) and 84.
Add.                        →   8064
```

Multiply. Change the order of the factors to check your work.

3. 37 × 26 = 962 4. 46 × 72 = 3312 5. 97 × 85 = 8245

Exercises

Multiply. Change the order of the factors to check your work.

1. 37 × 45 = 1665 2. 41 × 87 = 3567 3. 59 × 63 = 3717

4. 25 × 93 = 2325 5. 84 × 44 = 3696 6. 83 × 78 = 6474

7. 78 × 56 4368 8. 92 × 98 9016

9. 54 × 89 4806 10. 55 × 52 2860

11. 65 × 68 4420 12. 48 × 25 1200

13. 31 × 84 2604 14. 73 × 92 6716

How many pieces are in each of these jigsaw puzzles?

15. 22 rows with 34 pieces in each row 748

16. 28 rows with 28 pieces in each row 784

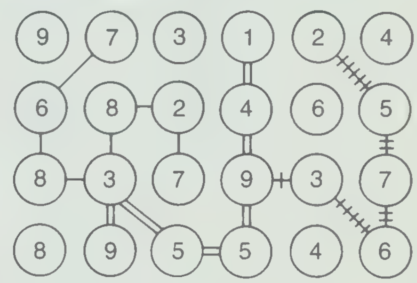
53

RELATED ACTIVITIES

- For further practice, you may wish to have students complete Ex. 38-48 on page 334.
- Students who are having difficulty may find it helpful to perform a multiplication in three parts as described in *Before Using the Pages*.
- Exercises similar to the following help students to focus on the place value of each digit of the multiplier and provide practice in writing corresponding products.

29	43	67	38
× 11	× 22	× 33	× 44
29	86	201	152
290	860	2010	1520
319	946	2211	1672

- Prepare a work sheet showing several one-digit numbers within circular shapes. Form "chains" by joining a few consecutive circles with line segments. Have students select one or more chains and find the product of the numbers in each chain.



The chains can be displayed on an overhead projector rather than on individual work sheets.

steps shown. Afterward, develop the solution on the board in the following way, adding the two partial products as the final step.

42	42	252
× 6	× 3 tens	+ 1260
252	126 tens or 1260	1512

At the bottom of the page, the worked example shows the factors of the original example in the reverse order. Have students describe each step of the multiplication. Point out that by changing the order of the factors and multiplying again the students can check their work.

Working Together: Ex. 1 and 2 identify the steps required to complete the multiplication. Emphasize the reason for writing 0 in the ones' place for the second partial product. Use other similar exercises as required. Have students give statements similar to those in Ex. 1 and 2 in an oral explanation of their work for Ex. 3-5.

Exercises: Students are asked to complete each multiplication twice in Ex. 1-14, by reversing the order of the factors so they can detect errors and make the necessary corrections. Encourage them to space their exercises well to avoid crowded work.

Assessment

Multiply. Change the order of the factors to check your work.

1. 34 × 21 = 714 **2.** 27 × 52 = 1404 **3.** 67 × 33 = 2211 **4.** 78 × 94 = 7332

Pages 54-55

LESSON OUTCOME

Multiply by a two-digit number, multiplicands with up to four digits; multiply three two-digit numbers; solve related word problems

Materials
calendars

Vocabulary
day, d, hour, h, century

Prerequisite Skills
Multiply a number to 9999 by a multiple of ten from 10 to 90; multiply a two-digit number by a two-digit number

Checking Prerequisite Skills
Multiply.

1. 42 × 30 = 1260	2. 154 × 20 = 3080	3. 367 × 40 = 14 680
4. 1235 × 70 = 86 450	5. 4826 × 60 = 289 560	6. 5019 × 50 = 250 950
7. 97 × 41 = 3977	8. 35 × 68 = 2380	9. 76 × 72 = 5472

Multiplying by a Two-Digit Number

About how many breakfasts will Mike eat if he eats one each day for 75 years?

Most years have 365 d (days).

Multiplying 75 and 365 will give *about* how many times Mike will eat breakfast.

For the product

$$\begin{array}{r} 365 \\ \underline{75} \end{array}$$

7 tens 5 ones

you need to know how to multiply 5 and 365,

$$\begin{array}{r} {}^{3\,2} \\ 365 \\ \underline{75} \\ 1825 \end{array}$$

and how to multiply 7 and 365.

$$\begin{array}{r} {}^{4\,3} \\ 365 \\ \underline{75} \\ 1825 \\ 25550 \end{array}$$

7 tens × 365 = 2555 tens or 25 550.

Then add.

$$\begin{array}{r} 365 \\ \underline{75} \\ 1825 \\ \underline{25550} \\ 27375 \end{array}$$

Mike will eat about 27 375 breakfasts in 75 years.

Working Together

Multiply by following the steps.

1. 1548 × 26
   - Multiply 6 and 1548. → 9288
   - Write 0 in the ones place. Multiply 2 (tens) and 1548. → 30 960
   - Add. → 40 248

Multiply.

2. 278 × 32 = 8896
3. 3459 × 27 = 93 393
4. 83 × 296 = 24 568
5. 94 × 6857 = 644 558
6. 26 × 42 × 38 = 41 496

54

LESSON ACTIVITY

Before Using the Pages

- This would be an appropriate time to review the product of three one-digit numbers. Begin with an example such as 2 × 3 × 5 and have students name the product, explaining the procedure they used. Show them that when three factors are multiplied in any order the product remains the same. (This is the associative property of multiplication.) Have students explain why the order (2 × 5) × 3 may be preferable for the preceding example. Use other similar examples as required. Then write the exercise 12 × 27 × 9 on the board and have students suggest how to find the product in more than one way. Develop their suggestions on the board. Some possibilities are given here. Note that method A provides an opportunity to review the steps in multiplying two two-digit numbers. Methods B and C enable you to determine whether the students can extend this procedure on their own for use with three-digit multiplicands.

A
$$\begin{array}{r} 27 \\ \underline{\times\ 12} \\ 54 \\ \underline{270} \\ 324 \end{array} \qquad \begin{array}{r} 324 \\ \underline{\times\ \ 9} \\ 2916 \end{array}$$

B
$$\begin{array}{r} 27 \\ \underline{\times\ 9} \\ 243 \end{array} \qquad \begin{array}{r} 243 \\ \underline{\times\ 12} \\ 486 \\ \underline{2430} \\ 2916 \end{array}$$

C
$$\begin{array}{r} 12 \\ \underline{\times\ 9} \\ 108 \end{array} \qquad \begin{array}{r} 108 \\ \underline{\times\ 27} \\ 756 \\ \underline{2160} \\ 2916 \end{array}$$

Using the Pages

- Introduce the word problem and discuss the use of the word "about". Students may recall that every fourth year is a leap year, which has 366 days. Point out the symbol d for days. Have students name the number of digits in each factor. Guide them in a discussion of the procedure by questioning them about the steps. For example, begin by asking what is multiplied first and have a student explain the multiplication aloud. Point out that the procedure used in multiplying two two-digit numbers is also used when one factor has more than two digits.

Working Together: The procedure shown in the worked example is extended in Ex. 1 in which one factor has four

Exercises

Multiply.

1. 376 × 28 = 10 528	2. 1473 × 95 = 139 935	3. 83 × 36 = 2988	4. 406 × 78 = 31 668	5. 4592 × 43 = 197 456
6. 583 × 64 = 37 312	7. 637 × 26 = 16 562	8. 6205 × 55 = 341 275	9. 5224 × 74 = 386 576	10. 195 × 37 = 7215

11. 56 × 789 44 184
12. 39 × 6437 251 043
13. 27 × 58 1566
14. 47 × 3197 150 259
15. 49 × 863 42 287
16. 86 × 6407 551 002
17. 24 × 17 × 53 21 624
18. 25 × 75 × 16 30 000
19. 27 × 38 × 37 37 962

Solve. Use the table on page 342 if needed.

20. How many hours are there in a year? 8760
21. How many hours are there in a leap year? 8784
22. How many minutes are there in October? 44 640
23. How many seconds are there in a day? 86 400
24. How many months are there in 75 years? 900
25. How many more days are there in a year than in 52 weeks? 1
*26. How many hours are there in each month? 31 d - 744 30 d - 720 28 d - 672
*27. How many days are there in a **century**? 36 525
28. The light bulb was supposed to last for 1000 h (hours). It burned 24 h each day for 45 d. Did it last for 1000 h? yes
29. Mr. Hill said that he watches television about 19 h each week. About how many hours will that be in a year? 988

The office manager has to buy the office supplies.

One package of typing paper is used in the office each day.

The office is open from Monday to Friday except on 10 holidays during the year.

1. How many packages of typing paper should the office manager buy for next year? about 250

PROBLEM SOLVING

(The exact answer will depend on the year. The number of weekdays will vary from 250 to 252, depending on the number of Saturdays and Sundays in a particular year.)

55

RELATED ACTIVITIES

• For further practice, you may wish to have students complete Ex. 49-57 on page 334.

• Students may be interested in calculating the approximate number of breakfasts each member of their family has eaten, assuming they have eaten one breakfast each day of their lives.

Karen (age 6)	Me (age 10)	Bob (age 13)
365 × 6	365 × 10	365 × 13

• Exercises similar to the following may be helpful for students having difficulty.

374 × 2 = 748	374 × 30 = 11 220	374 × 32: 748 + 11 220 = 11 968

• Have students find the product of three numbers such that the product of two of them is a multiple of ten. Ask them to perform the calculations mentally and write the final product.

5 × 7 × 4
6 × 9 × 5
2 × 12 × 5
10 × 18 × 2

digits. The steps for completing the multiplication are provided. It would be beneficial to have students give similar statements in an oral explanation of their work for Ex. 2-5. For Ex. 6, ask the students to find the product in more than one way. Show the different methods on the board.

Exercises: Before the students begin, ask questions relevant to the units of time referred to in Ex. 20-28. For example, ask for the number of days in a week, the number of weeks in a month, the number of hours in a day, and so on. Point out the suggestion given in the instructions that precede Ex. 20. Note that Ex. 26 and 27 are starred: the number of hours in a month depends on the number of days in that month; and the number of leap years in a century affects the number of days in a century. Ask students to determine how many leap years occur in a century (25). If necessary, tell them that the year 1960 was a leap year and remind them that every fourth year is a leap year.

Problem Solving: Remind the students to determine whether "next year" refers to a leap year. If so, an extra package of typing paper may be needed. Provide students with calendars for assistance in solving the problem. Note the different approaches students use to solve the problem and provide an opportunity for these to be shared with the rest of the class.

Assessment

Multiply.

1. 326 × 34 = 11 084
2. 1158 × 49 = 56 742
3. 56 × 4087 228 872
4. 27 × 13 × 88 30 888

Solve.

5. How many months are there in 125 years? 1500

Pages 56-57

LESSON OUTCOME

Multiply by a three-digit number, multiplicands with up to four digits

Materials

copy of the telephone book for your area

Prerequisite Skills

Multiply by a two-digit number, multiplicands with up to four digits; multiply by a multiple of one hundred from 100 to 900

Checking Prerequisite Skills

Multiply.

1. 42 37 1554	2. 217 25 5425
3. 3041 79 240 239	4. 6324 82 518 568
5. 541 700 378 700	6. 672 400 268 800
7. 908 900 817 200	8. 835 600 501 000

Multiplying by a Three-Digit Number

To estimate the number of telephone numbers in the Toronto telephone book, Luisa counted the numbers in one column and multiplied by the number of columns on the page.

109 in one column
4 columns on the page
436 numbers on the page

Then she multiplied 436 by the number of pages that listed telephone numbers.

1971 pages with numbers . . .
Multiply 436 and 1971.

For the product
$$\begin{array}{r} 1971 \\ 436 \\ \hline \end{array}$$

you need to know how to multiply 6 and 1971,

$$\begin{array}{r} 1971 \\ 436 \\ \hline 11826 \end{array}$$

how to multiply 3 (tens) and 1971,

$$\begin{array}{r} 1971 \\ 436 \\ \hline 11826 \\ 59130 \end{array}$$

and how to multiply 4 (hundreds) and 1971.

$$\begin{array}{r} 1971 \\ 436 \\ \hline 11826 \\ 59130 \\ 788400 \end{array}$$

Then add.

$$\begin{array}{r} 1971 \\ 436 \\ \hline 11826 \\ 59130 \\ 788400 \\ \hline 859356 \end{array}$$

The product is 859 356, but Luisa rounded her result and said:

There are about 860 000 telephone numbers in the Toronto telephone book.

56

LESSON ACTIVITY

Before Using the Pages

- Write the following exercises on the board.

$$\begin{array}{r} 634 \\ \times\ 2 \\ \hline 1268 \end{array} \qquad \begin{array}{r} 634 \\ \times\ 2 \text{ tens} \\ \hline 12\,680 \end{array} \qquad \begin{array}{r} 634 \\ \times\ 2 \text{ hundreds} \\ \hline 126\,800 \end{array}$$

  Ask how they are alike and how they are different. Then ask how the products are alike and how they are different. Write the products on the board. Emphasize that multiplying by a number of tens gives a product with 0 ones and multiplying by a number of hundreds results in a product with 0 tens and 0 ones. Emphasize that these zeros are shown first in the product and that the multiplication continues as if by a one-digit number.

- Write the exercise 25 × 217 in vertical form on the board. Have a student explain how to find the product. (Multiply 217 by 5; multiply 217 by 2 tens; add.)

  Change the exercise to show 125 × 217 and have students suggest the steps for finding the product. Ask students to predict the number of digits in the product.

Using the Pages

- The photograph on page 57 can stimulate a discussion of where large numbers are found in everyday life. Telephone books for large metropolitan areas contain thousands of telephone numbers. Have students suggest other examples, such as the number of words in a dictionary.

  Introduce the word problem at the top of page 56. Display the telephone book for your area and point out that each page shows columns of telephone numbers. Contrast the number of columns on each page of your book with the book described in the example. Ask how Luisa found the number of telephone numbers on one page without counting all of them and how she used the number of the pages in the book to find how many telephone numbers were listed. Remind the students how useful it is to know how to multiply.

Working Together

Multiply by following the steps.

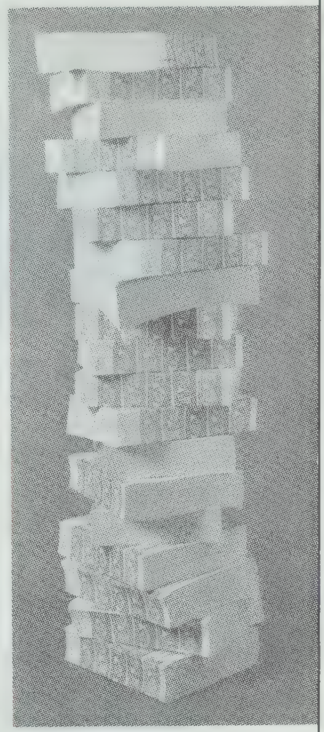
1.

$$\begin{array}{r} 416 \\ \underline{235} \\ 2\,080 \\ 12\,480 \\ 83\,200 \\ \hline 97\,760 \end{array}$$

Multiply 5 and 416. → 2 080
Write 0 in the ones place. Multiply 3 (tens) and 416. → 12 480
Write 0 in the ones and tens places. Multiply 2 (hundreds) and 416. → 83 200
Add. → 97 760

Multiply.

2. 478 × 263 = 125 714
3. 3846 × 352 = 1 353 792
4. 723 × 219 158 337
5. 186 × 24 × 359 1 602 576



Exercises

Multiply.

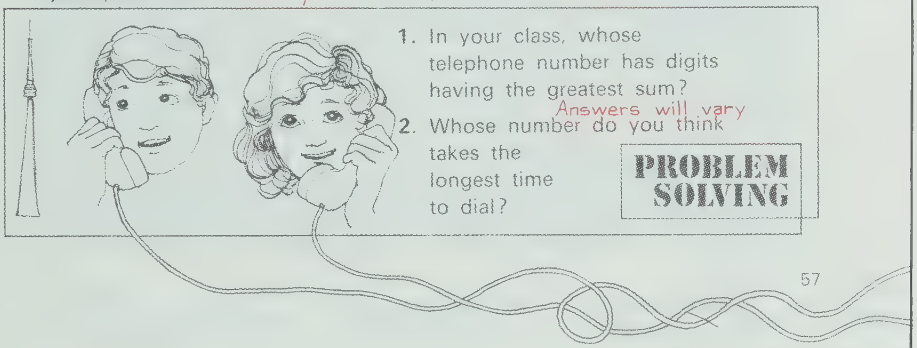
1. 759 × 384 = 291 456
2. 665 × 597 = 397 005
3. 6842 × 196 = 1 341 032
4. 8006 × 348 = 2 786 088
5. 737 × 599 441 463
6. 518 × 4231 2 191 658
7. 667 × 3072 2 049 024
8. 749 × 458 343 042
9. 9 × 87 × 2732 2 139 156
10. 35 × 265 × 19 176 225

Follow Luisa's steps.

11. Estimate how many telephone numbers are in your phone book. Answers will vary.

Estimate

12. How many pages of telephone numbers are in this stack of Toronto phone books? 38 000



1. In your class, whose telephone number has digits having the greatest sum?
2. Whose number do you think takes the longest time to dial? Answers will vary

PROBLEM SOLVING

RELATED ACTIVITIES

- For further practice, you may wish to have students complete Ex. 58-65 on page 334.
- Assign exercises similar to the following to reinforce the concept of place value in multiplication.

$$\begin{array}{r} 312 \\ \times\ 111 \\ \hline 312 \\ 3\,120 \\ 31\,200 \\ \hline 34\,632 \end{array} \qquad \begin{array}{r} 312 \\ \times\ 222 \\ \hline 624 \\ 6\,240 \\ 62\,400 \\ \hline 69\,264 \end{array} \qquad \begin{array}{r} 312 \\ \times\ 333 \\ \hline \end{array} \qquad \begin{array}{r} 312 \\ \times\ 444 \\ \hline \end{array}$$

- Students may be interested in a short method for multiplying when one factor is 11. The digits of the other factors are used to write the product in one line. The steps are shown here for three examples.

No regrouping

$11 \times 234 = \underline{2}\ \underline{5}\ \underline{7}\ \underline{4}$

2 3 $\boxed{4}$
2 $\boxed{3\ 4}$ (3 + 4)
$\boxed{2\ 3}$ 4 (2 + 3)
$\boxed{2}$ 3 4

One regrouping

$11 \times 239 = \underline{2}\ \underline{6}\ \underline{2}\ \underline{9}$

2 3 $\boxed{9}$
2 $\boxed{3\ 9}$ (3 + 9)
$\boxed{2\ 3}$ 9 (2 + 3 + 1)
$\boxed{2}$ 3 9

Two regroupings

$11 \times 279 = \underline{3}\ \underline{0}\ \underline{6}\ \underline{9}$

2 7 $\boxed{9}$
2 $\boxed{7\ 9}$ (7 + 9)
$\boxed{2\ 7}$ 9 (2 + 7 + 1)
$\boxed{2}$ 7 9 (2 + 1)

Have students state the number of digits in 436 and the place value of each digit. Ask students to explain each step of the multiplication aloud, paying particular attention to the zeros in the partial products. Emphasize that there are three multiplication steps before the addition is performed, since there are three non-zero digits in 436. A rounded product is used in the concluding statement and is probably preferable to the exact product. Review the procedure for rounding to the nearest ten thousand.

Working Together: Ex. 1 identifies the steps for completing the multiplication. Have students give similar statements in an oral explanation of their work for Ex. 2-5. Have students find the product for Ex. 5 in more than one way. Note, however, that multiplying 186 and 359 first will result in the need to multiply a five-digit number by a two-digit number.

Exercises: Provide the students with pages from an old telephone book to obtain the information needed to solve Ex. 11. Remind them to space their work well.

Problem Solving: You may wish to prepare the appropriate list of telephone numbers in advance of this lesson and give a copy to each student. You may prefer to have each student write her/his telephone number on the board, in turn, while the others are working at the exercises.

Assessment

Multiply.

1. 243 × 132 = 32 076
2. 2596 × 347 = 900 812
3. 517 × 912 471 504
4. 297 × 3009 893 673
5. 6 × 42 × 315 79 380

Pages 58-59

OBJECTIVE

Demonstrate competence in multiplying by two-digit and three-digit numbers; solve related word problems

Materials

telephone book showing long-distance rates

Vocabulary

overtime, minutes, min

Practice

How many pieces are in this jigsaw puzzle? 627

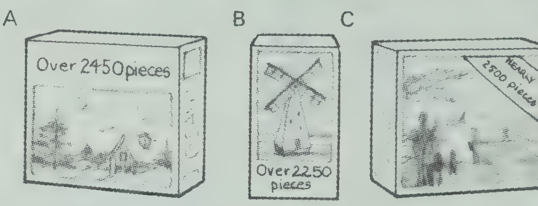
11 rows, 57 pieces in each row

1.

Which box most likely belongs to each puzzle?

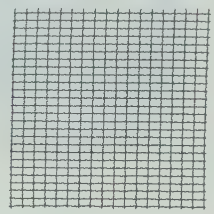
2. 65 rows, 35 pieces in each row B
3. 49 rows, 51 pieces in each row C
4. 43 rows, 57 pieces in each row A

A B C



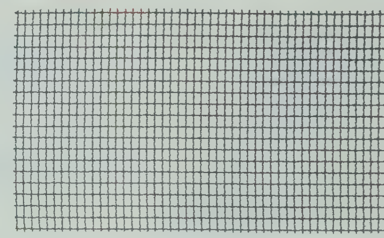
How many holes are in each piece of screen? 931

5.



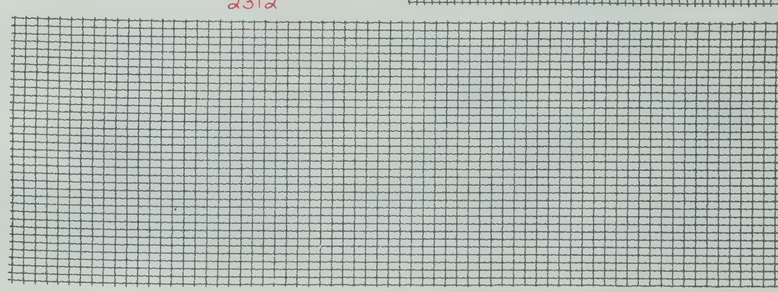
425

6.



2312

7.



LESSON ACTIVITY

Using the Pages

- Students should realize that multiplication can be used to find the total number of pieces, since the puzzle in the photograph has 11 rows with the same number of pieces in each row. Have students suggest whether or not multiplication can be used to solve Ex. 5-7 and explain what must be determined before multiplication can be used. Counting the rows and columns will give the following.

  Ex. 5: 25 rows, 17 holes in each row
  Ex. 6: 19 rows, 49 holes in each row
  Ex. 7: 34 rows, 68 holes in each row

  The examples at the top of page 59 demonstrate that amounts of money are multiplied in the same way as whole numbers. Point out the decimal point and the symbol $ in the products. You may wish to develop one or more of these examples on the board to review the steps in multiplying by a two-digit or a three-digit number. It would be beneficial to have students read the products aloud, particularly those numerals with six or seven digits.

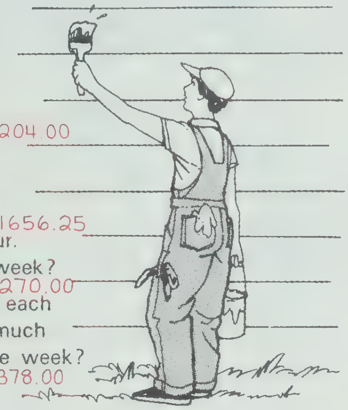
  Ex. 11 is starred because its solution requires more than one step. The information in Ex. 10 is needed for Ex. 11. Have a student explain the meaning of the word *overtime* in Ex. 11.

Problem Solving: These exercises present an everyday application of multiplication. Have students share copies of a telephone book for Ex. 3.

Amounts of money are multiplied just like whole numbers.

Examples:	\$3 285	\$193	\$8.13	\$28.56
	64	438	129	753
	13 140	1 544	73 17	85 68
	197 100	5 790	162 60	1 428 00
	\$210 240	77 200	813 00	19 992 00
		\$84 534	\$1048.77	\$21 505.68

Solve.



8. The painter earned \$8.50 for each hour it took to paint the house. It took 24 h. How much did the painter earn? \$204.00
9. The plumber worked 125 h one month. The plumber earned \$13.25 for each hour. How much did the plumber earn? \$1656.25
10. The factory worker earns \$6.75 each hour. How much would be earned in a 40 h week? \$270.00
*11. The factory worker earns twice as much each hour for any *overtime* after 40 h. How much would be earned for 48 h of work in one week? \$378.00

Multiply.

12. 74 × 76 = 5624
13. 509 × 69 = 35 121
14. \$744 × 582 = \$433 008
15. 9486 × 676 = 6 412 536
16. 1007 × 36 = 36 252
17. \$9.56 × 215 = \$2055.40
18. 79 × 643 50 797
19. 85 × \$3268 \$277 780
20. 42 × \$6.41 \$269.22
21. 625 × 392 245 000
22. 189 × \$6325 \$1 195 425
23. 789 × \$72.86 \$57 486.54

1. The first 3 min (minutes) cost \$2.80. Each extra minute costs 80¢. How much would 15 min cost? \$12.40
2. The first 3 min cost \$7.35. Each extra minute costs \$2.45. How much would 8 min cost? \$19.60

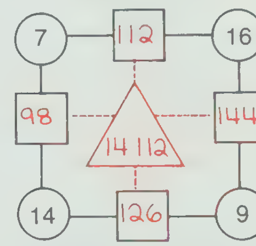
PROBLEM SOLVING

3. Look in your telephone book. Can you find how much a 5 min call to Montreal would cost?

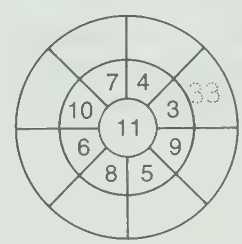
Answers will vary

RELATED ACTIVITIES

- Exercises similar to Ex. 3 of *Problem Solving* may be assigned.
- Use copies of page T 390 to assign practice in multiplication as indicated in the diagram below. As usual, factors are shown in the circles and products are shown in the squares. However, when the diagram is completed, have students multiply the numbers in opposite squares of the diagram. They will discover that, if their work is correct, the products obtained are equal. The product may be written in a triangular shape in the diagram as shown.



- Students will find it beneficial to memorize multiplication facts of 11 and 12 to 11 × 11 and 12 × 12, and of 15 to 6 × 15. These may be practiced in tables and number wheels using copies of page T 390.



×	12
4	
12	
6	
9	

LESSON OUTCOME

Round two factors and multiply to estimate the product, then compare the estimate of the product with the exact product

Vocabulary

encyclopedia, dozen

Prerequisite Skills

Multiply by a three-digit number

Checking Prerequisite Skills

Multiply.

1. 43 × 29 = 1247
2. 652 × 84 = 54 768
3. 408 × 259 = 105 672
4. 3216 × 176 = 566 016

RELATED ACTIVITIES

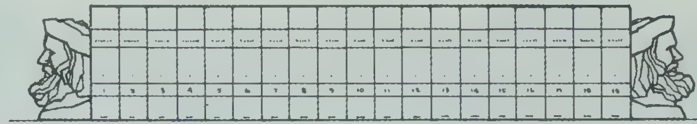
- Some students may require more experience with multiplication exercises similar to the following.

50	600	3000	9000
× 40	× 20	× 40	× 500
20$\boxed{00}$	12 $\boxed{000}$		

- For further practice in estimating a product, choose exercises from the preceding pages of this unit.

Estimating the Product

There are 19 books in the encyclopedia. Each book has 832 pages. About how many pages are there in the encyclopedia?



To estimate the product of 19 and 832,

round 832 ⟶ 800
and round 19 ⟶ 20
then multiply. ⟶ 16 000

For the exact product, multiply in the usual way.

$$\begin{array}{r} 832 \\ \underline{19} \\ 7\,488 \\ \underline{8\,320} \\ 15\,808 \end{array}$$

There are about 16 000 pages in the encyclopedia.

Working Together

Round to the nearest hundred.

1. 482 — 500
2. 209 — 200
3. 650 — 700

Round to the nearest thousand.

4. 5495 — 5000
5. 912 — 1000
6. 3710 — 4000

Study these multiplication examples.

60	700	5000
40	30	600
2400	21 000	3 000 000

7. Give a rule for multiplying when the only digit not 0 in each factor is the one on the left.

Round and multiply to estimate the product.

8. 68 × 82 — 5600
9. 309 × 75 — 24 000
10. 6824 × 198 — 1 400 000

Exercises

Estimates may vary.

Round and multiply to estimate the product. Then find the exact product.

1. 84 × 78 — (6400) 6552
2. 482 × 71 — (35 000) 34 222
3. 674 × 841 — (560 000) 566 834
4. 1257 × 293 — (390 000) 368 301
5. 756 × 542 — (400 000) 409 752
6. 928 × 41 — (36 000) 38 048
7. 77 × 126 — (8000) 9702
8. 948 × 4735 — (4 500 000) 4 488 780
9. 67 × 97 — (7000) 6499
10. 225 × 958 — (200 000) 215 550

Estimate the total number

11. of eggs in 425 cartons, when each carton holds three dozen eggs. 16 000
12. of days in 1980 years. 730 000
13. of pages in all the copies of this book used by your class. Answers will vary.

 7. Write as many zeros as there are in the factors and then multiply the non-zero digits.

LESSON ACTIVITY

Before Using the Page

- Write 23 × 59 in vertical form on the board. Have students choose the best estimate for the product from 120, 1200, and 12 000, without using pencil and paper. Have them explain their choice. Then have them find the exact product.

Using the Page

- The worked example presents a situation in which an estimate of the product is required. Emphasize the word "about" in the question at the top of the page. Have students explain that 832 is rounded to the nearest hundred and 19 is rounded to the nearest ten. Pay careful attention to the product 20 × 800. Have the students note the exact product and compare it with the estimate of the product.

Working Together: Ex. 1-6 review the skill of rounding to the nearest hundred or to the nearest thousand. Review the procedure of inspecting the digit to the right of the given place. Ex. 7 is of particular importance in developing the skill of multiplying efficiently and estimating products. The red on the numerals highlights the basic multiplication fact in each exercise, as well as the relationship between the "end zeros" of a product and the "end zeros" of each factor. Use other similar examples.

Exercises: Note that each factor of an exercise need not be rounded to the same place. To apply a basic multiplication fact in estimating a product, all the digits but one in each factor must be zero. In Ex. 6, for example, 40 × 930 = 37 200 will give a more accurate estimate of the exact product, 38 048, than 40 × 900 = 36 000. However, the latter is easier to compute mentally and is an acceptable estimate.

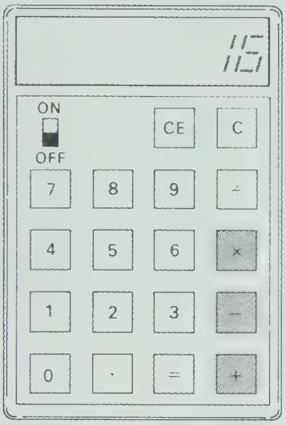
Assessment

Round and multiply to estimate the product. Then find the exact product.

1. 64 × 45 — 2880 (3000)
2. 682 × 39 — 26 598 (28 000)
3. 938 × 211 — 197 918 (180 000)
4. 1219 × 362 — 441 278 (480 000)

Keycharts and the $\boxed{+}$, $\boxed{-}$, and $\boxed{\times}$ Keys

To use a calculator you need to know which keys to press and the order in which to press them.



Sue bought 16 packages of beads. Each package had 144 beads. She used 1575 beads in the belts and bracelets she made. How many beads did she have left?

I have to multiply, then subtract.

... $\boxed{\times}$, then $\boxed{-}$

To show which keys to use to solve this problem, Sue made a **keychart**.

From left to right this keychart shows the keys to press.

16 $\boxed{\times}$ 144 $\boxed{-}$ 1575 $\boxed{=}$ ____

Make a keychart that shows how to solve each of these.

975 $\boxed{+}$ 1368 $\boxed{=}$ 2343

1. 975 people went to the afternoon show. 1368 went to the evening show. How many went to both shows?

175 $\boxed{\times}$ 125 $\boxed{=}$ 21 875

2. The machine planted 175 seeds in each of the 125 rows. How many seeds were planted?

3. 18 girls and 17 boys are in each class. How many girls and boys are in 16 classes?

18 $\boxed{+}$ 17 $\boxed{=}\boxed{\times}$ 16 $\boxed{=}$ 560

4. Each page has 3 lists with 75 names on each list. How many names are listed on 45 pages?

3 $\boxed{\times}$ 75 $\boxed{=}\boxed{\times}$ 45 $\boxed{=}$ 10 125

5. The car sells for \$4150, but it is on sale for \$875 less. Tax on the sale price is \$262. How much would you have to pay for the car?

4150 $\boxed{-}$ 875 $\boxed{+}$ 262 $\boxed{=}$ 3537 (\$3537)

6. How many days are there in the last six months of the year?

31 $\boxed{+}$ 31 $\boxed{+}$ 30 $\boxed{+}$ 31 $\boxed{+}$ 30 $\boxed{+}$ 31 $\boxed{=}$ 184

Can you solve each problem?

Calculator

61

Page 61

OBJECTIVE

Prepare a keychart to show the order of pressing the keys $\boxed{+}$, $\boxed{-}$, and $\boxed{\times}$ on a calculator to solve a problem

Materials

calculators (optional)

Vocabulary

keychart

RELATED ACTIVITIES

- Provide students with exercises similar to those on the page.
- Some students may wish to research the device known as ''Napier's Bones'' which was popular in Europe in the seventeenth century. The device was developed for assisting with multiplication. A short description of the device is given on page T378.

LESSON ACTIVITY

Using the Page

- The lesson on page 39 emphasizes knowing which keys to press on a calculator. This aspect is recalled now, but the concept of knowing the order in which to press the keys is included. Keep in mind that a keychart that shows

  8 $\boxed{+}$ 4 $\boxed{\times}$ 2 $\boxed{=}$ ____

  is not the same as one that shows

  4 $\boxed{\times}$ 2 $\boxed{+}$ 8 $\boxed{=}$ ____.

  For the former, the keychart indicates that the sum of 8 and 4 is to be found first, and the result is to be multiplied by 2, giving the value 24. For the latter, multiplication is performed first, giving the value 16 to the expression.
- Discuss the example provided. Have students explain why multiplication is required first. Draw attention to the squares that are drawn around operation symbols and the symbol = in the keychart and emphasize the left-to-right order. Remind the students that the calculator is a device that carries out functions but does not make decisions. Emphasize that the person using the calculator decides which operations are performed and the order in which they are performed.
- Draw attention to the fact that the symbol \$ does not appear on the calculator, but there is a decimal point key for separating dollars and cents. (Decimal points are not encountered at this time, however.) Since the products involved are similar to those encountered in the preceding lessons, students can complete the exercises without using a calculator.

Page 62

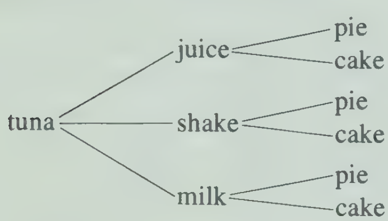
OBJECTIVE

Find the number of possibilities of an event

RELATED ACTIVITIES

- Students may like to test whether the rule involving multiplication applies when there are three categories from which to select a combination lunch. The situation should be relatively simple, as in the example given, and a diagram rather than a chart will be useful.

Sandwich (Choose one)	Drink (Choose one)	Dessert (Choose one)
tuna egg	juice shake milk	pie cake

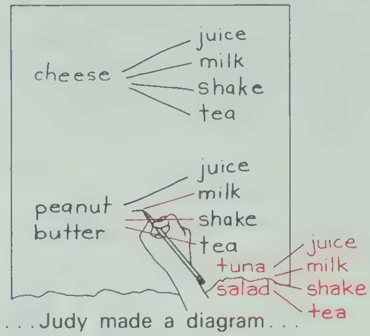


If necessary, a similar diagram may be drawn to show the choices starting with an egg sandwich. Then all the possibilities can be counted.

Finding the Number of Possibilities

To think about the possible choices of a sandwich and a drink . . .

LUNCH ORDER FORM	
SANDWICH (Choose one)	DRINK (Choose one)
cheese peanut butter tuna salad	juice milk shake tea
Each order will include 1 apple and 2 cookies.	



. . . Judy made a diagram . . .

	juice	milk	shake	tea
cheese	cheese juice	cheese milk	cheese shake	cheese tea
peanut butter	peanut butter juice	peanut butter milk	peanut butter shake	peanut butter tea
tuna salad	tuna salad juice	tuna salad milk	tuna salad shake	tuna salad tea

. . . and Roger made a chart.

The chart and the diagram will both show that there are 12 ways to choose a sandwich and a drink.

1. Complete Judy's diagram.
2. Complete Roger's chart.

Draw a diagram or a chart that shows the different ways

3. the club election could turn out when Agnes, Beth, Charles, Doug, and Ellen are running for president and Frank, Gary, and Helen are running for vice-president. How many president and vice-president pairs are possible? 15
4. the faces of two dice could turn up. How many number pairs are possible? 36
5. one of the adjectives: jolly, angry, silly, friendly, foolish, bashful, or gentle, could be paired with one of the nouns: rabbit, elf, boy, or giant. How many word pairs are possible? 28
6. you could choose two different letters of the alphabet so that the second one is a vowel. How many letter pairs are possible? 125 (for a, e, i, o, u)

PROBLEM SOLVING

Can you give a rule that tells how to find the number of pairs without drawing a chart or diagram?

Charts for Ex. 3–6 are shown on page T366.

62

LESSON ACTIVITY

Using the Page

- You might begin with a brief discussion of the difficulty in making up one's mind when deciding what to order from a menu when eating at a restaurant. Part of the difficulty arises from the fact that there are so many different possibilities.

  Draw the students' attention to the lunch order form illustrated at the top of the page. Emphasize that only one choice is permitted from each category. Have students name their preferences. Explain that the problems in this lesson involve finding how many ways there are to make a choice. Suggest that a diagram helps to determine this.

  Have students describe the two methods shown. Some students may sense the relationship of multiplication to the solution at this point, but do not emphasize this until they have attempted several of the exercises.

For Ex. 6, emphasize that if a letter is selected for the first place, it is not available for the second place. Thus, a letter pair such as (a,a) is not acceptable. If it is agreed to include ''y'' as a vowel, then the number of letter pairs is determined by 26 letters, six of which are vowels. One of the six vowels must be chosen as the second letter of the pair. Then, the first letter may be selected from the remaining 25 letters. Thus, the number of letter pairs that are possible is given by 25 × 6. If ''y'' is not considered a vowel, the number of letter pairs is given by 25 × 5.

Checking Up

Multiply.

1. 32 4 128	**2.** 87 7 609	**3.** 319 6 1914	**4.** 684 2 1368	**5.** $461 5 $2305
6. 6905 7 48 335	**7.** 4188 8 33 504	**8.** 78 495 9 706 455	**9.** 35 926 3 107 778	**10.** $84.47 4 $337.88
11. 72 20 1440	**12.** 925 50 46 250	**13.** 3816 90 343 440	**14.** 162 400 64 800	**15.** $734 600 $440 400
16. 32 28 896	**17.** 58 67 3886	**18.** 706 98 69 188	**19.** 195 19 3705	**20.** $5.79 35 $202.65
21. 4675 58 271 150	**22.** 6868 14 96 152	**23.** 238 357 84 966	**24.** 404 717 289 668	**25.** $19.98 66 $1318.68
26. 5932 487 2 888 884	**27.** 4026 639 2 572 614	**28.** $471 394 $185 574	**29.** $5.76 638 $3674.88	**30.** $11.95 285 $3405.75

Round each factor and multiply to estimate the product.

31. 42 68 2800	**32.** 875 33 27 000	**33.** 6904 49 350 000	**34.** 826 417 320 000	**35.** 3802 628 2 400 000

Solve.

36. The potato plants are in 47 rows with 175 plants in each row. How many potato plants are there in all? 8225

37. In one year Mr. Coe sold 124 of one kind of car for $4375 each. What was Mr. Coe's sales total for the car that year? $542 500

38. 275 children signed up for each of the 5 day camps. How many signed up in all? 1375

39. Mrs. Martin earned $4.55 an hour for 36 h of work. How much did she earn in all? $163.80

Multiply.

40. 6 × 62 372	**41.** 4 × 593 2372	**42.** 8 × 1820 14 560
43. 2 × 64 358 128 716	**44.** 97 × 32 3104	**45.** 41 × 723 29 643
46. 15 × 2386 35 790	**47.** 593 × 980 581 140	**48.** 282 × 7935 2 237 670
49. 4 × $590 $2360	**50.** 74 × $67 $4958	**51.** 321 × $5.79 $1858.59

63

Page 63

OBJECTIVE

Demonstrate an understanding of the concepts and skills presented in this unit

RELATED ACTIVITIES

- Multiplication squares similar to the following can provide practice with basic facts and multiplication with two-digit and three-digit numbers. You may wish to cut copies of page T 391 to prepare the number squares.

×		
7	8	56
9	5	45
63	40	2520

×		
9	12	
8	20	

Note that the product in the lower right square serves as a check for the number square.

Another check for a multiplication square involves multiplying the two numbers on each diagonal and then multiplying those two products. The result should be equal to the number in the lower right square. For the first number square above,

$$7 \times 5 = 35 \qquad 9 \times 8 = 72$$

$$\begin{array}{r} 35 \\ \times\ 72 \\ \hline 70 \\ 2450 \\ \hline 2520 \end{array}$$

Skills	Exercises	Related Pages
Multiply by a one-digit number	1, 2, 40	T 48-T 49
	3, 4, 6-9, 41-43	T 50-T 51
	5, 10, 49	T 52-T 53
Multiply by multiples of 10 and 100	11-15	T 54-T 55
Multiply by a two-digit number	16, 17, 44	T 56-T 57
	18, 19, 21, 22, 45, 46	T 58-T 59
	20, 25, 50	T 62-T 63
Multiply by a three-digit number	23, 24, 26, 27, 47, 48	T 60-T 61
	28-30, 51	T 62-T 63
Estimate a product	31-35	T 64
Solve multiplication problems	36-39	

Comments

Use of the multiplication algorithm involves the following skills.

1. Complete basic multiplication facts
2. Show 0 ones in multiplying by the number of tens
3. Show 0 ones and 0 tens in multiplying by the number of hundreds
4. Regroup in multiplying
5. Add partial products

It is important to analyze the results to determine which particular skill, or skills, should have further attention. Appropriate remedial assistance may then be planned and provided to overcome any difficulty.

The steps outlined in *Working Together* and *Related Activities* for many lessons of this unit are useful for reteaching or reviewing necessary skills.

If all the exercises on page 334 were not assigned and completed earlier, you may wish to have the students complete, for example, Ex. 66-81.

Unit 4 Overview

Graphing

This unit begins with the use of tallies for collecting and organizing information of interest to the students. From such data, simple pictographs and bar graphs are developed. In the latter, numbered scales appear on one of the axes, but in the next lesson, *Reading Line Graphs*, scales appear on both axes. Graphs with numbers on both axes lead directly to the lesson on ordered pairs of numbers by which points on a grid may be identified. The work with graphs concludes with a lesson on drawing line graphs. A limited amount of guidance is offered in the lesson on problem solving, which requires the students to utilize the concepts and skills presented in this unit to collect, organize, and display information. At the end of the unit, four sets of exercises are provided to maintain and to assess skills in addition, subtraction, multiplication, and solving related problems.

Prerequisite Skills

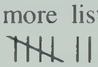
- read a scale
- match whole numbers with points on a number line

Unit Outcomes

- prepare a chart for collecting and recording information
- interpret information shown in a tally sheet and a check list
- interpret and draw a pictograph
- interpret and draw a bar graph
- interpret a line graph
- match points on a grid with ordered pairs of numbers
- draw a line graph
- collect, organize, and display information to solve a problem

Background

Numbers are used frequently for naming and comparing quantities. Numbers themselves, however, are abstract and require the formation of meaningful concepts before any intelligent responses or comparisons can be made. It is usually much easier if the real objects are present, but this is often impractical and sometimes, if the quantities are very large, this could be even more confusing than the abstract numbers. To bridge the gap between real objects and abstract numbers, graphs are useful. They provide a visual means of organizing, classifying, and recording information.

The first step in constructing a graph involves collecting and organizing information. A tally system is often employed to record the number of items in one or more lists. Tallies are usually grouped in fives, for example,  represents 7. The tallies may be counted by fives and ones to determine the actual numbers. The use of fives in the tallying helps in organizing the data and in preparing pictographs and bar graphs, especially where one picture, symbol, or unit on the scale represents five items.

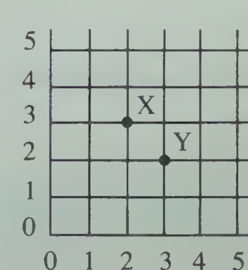
A pictograph uses pictures, or representative symbols, either on a one-to-one basis to real objects or on a one-to-many basis. Each pictograph, therefore, requires an explanatory statement called a *key* to establish and to reveal this relationship. The key for the following pictograph indicates that each symbol represents 20 homes. The unit value of  is 20. Multiples of the unit value of a symbol are easily shown by a number of whole symbols, and other numbers are either rounded to the nearest multiple or are represented by parts of symbols. In the pictograph, the number of homes receiving the evening paper only is represented by eight symbols which may indicate exactly 160 homes or about 160 homes, for example, 162. Similarly, half a symbol for "no papers delivered" may represent from 8 to 12 homes.

Newspaper Delivery for Homes in Our Area	
Morning paper only	
Evening paper only	
Morning and evening papers	
No papers delivered	
Each  stands for 20 homes.	

It is important to keep in mind the reason for displaying information in the form of a graph; usually it is easier to interpret the information from a graph than from a chart that merely lists the numbers. Therefore, for graphs that the students draw, it is important to discuss the results. Questions suggested by the teacher and the students will help to improve the skill of interpreting information shown in a graph. The reading of graphs requires examination of three features: the title of the graph, the names on one of the axes or the meaning of the numerical scale found there, and the meaning of the scale on the other axis.

Drawing graphs is much more difficult than reading them, since a number of decisions always have to be made after the information has been collected: What kind of graph will best represent the data? What will be a suitable title? What should the headings be? What scales or symbols are to be used? Of these, the last is probably the most difficult. On a pictograph, poorly chosen symbols and unit values for them can result in either too many (few) symbols or an inability to adapt the symbols to represent less than unit values. For example, the pictograph above would involve too many symbols if each symbol represented 5 homes. On the other hand, it would be awkward to represent 25 homes if the unit value for  were 75 homes. Similarly, poorly chosen intervals on scales can result either in bar graphs and line graphs that are too large to be shown completely, or in graphs that are too small to show differences in the data. The cliché "learn to do by doing" is very apt in this connection, because students can best acquire the skills required in making graphs by doing just that. Also, the teacher can assist the students by conducting a brief discussion of the exercises before they prepare the graphs.

Locating places on a map is relatively easy when letters along one edge and numerals along the other edge identify regions. On a grid, points where lines intersect can be accurately named by using pairs of numbers, one from each edge.



To avoid confusion, it is universally accepted that the first number of an ordered pair refers to the horizontal distance from the starting point, and the second number refers to the vertical distance from the same starting point, designated by the letter O, representing "origin". Briefly, one counts *over* first and then *up* from zero, hence the term *ordered pair*. The order of the numbers 2 and 3 is significant in locating the points X and Y, since (2,3) names point X and (3,2) names point Y.

Teaching Strategies

This is a relatively short unit, although the varied activities of collecting and organizing information and of preparing graphs to display it are time-consuming. Also, the work requires a considerable amount of discussion to help the students to acquire understanding of the basic elements of graphs and to develop the necessary skills of interpreting and making graphs. Since any discussion is more beneficial if the number of participants is relatively small, it is recommended that the class be divided into groups. Limited quantities of materials and lack of space may also suggest the advisability of small groups. Besides these considerations, small groups are usually more manageable when the students move about in their various activities. Since basic computational skills are not required in much of the work, students with different abilities in these areas can be grouped together. Under these circumstances, it is more important that the members of groups be co-operative with one another, and it is advisable to include a balance of both leaders and followers. By this means, students can learn good social work habits from one another, as well as mathematical skills and concepts.

The guidance of the teacher in the activities and discussions is important in the development of meaningful concepts and skills in graphing. While the teacher is working with one or more of these small groups, others may be assigned exercises to maintain basic skills in computation and problem solving. For this purpose, two *Keeping Sharp* features on pages 65 and 73 and two *Checking Skills* pages at the end of the unit are provided. The latter exercises may be divided into smaller sets for use on several days, and the assignments may either concentrate on one operation at a time, or may include exercises for the three operations. The number of exercises and problems should be enough for about seven assignments.

A number of activities which the students can carry out without the guidance of the teacher are found in the *Related Activities* for each lesson. Even the *Assessment* exercises may be used for this purpose.

Working with graphs is confined to Unit 4 in *Starting Points in Mathematics 5*, but the teacher should use any opportunities during the rest of the school year for students to use their skills of graphing to portray information from other subject areas, as well as day-to-day items of interest to them.

Materials

graph paper, colored paper
overhead projector and a transparency showing a grid (optional)
copies of page T 397 and a straight edge for each student
several sheets of graph paper (copies of page T 397 or T 398) and a straight edge for each student
graph paper (copies of pages T 397 and T 398) or large sheets of paper and construction paper

Vocabulary

ballot
survey form
tally
pictograph
bar graph
line graph
seconds, s
ordered pair
grid
plot a point
horizontal
vertical

Pages 64-65

LESSON OUTCOME

Prepare a chart for collecting and recording information

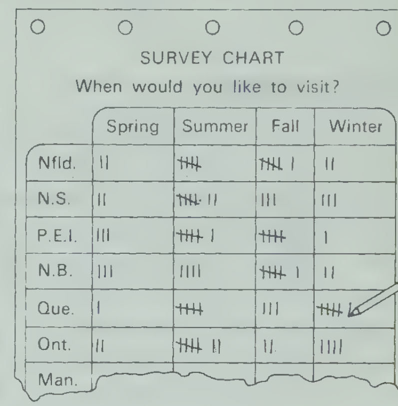
Vocabulary
ballot, survey form

4 GRAPHING

Collecting Information

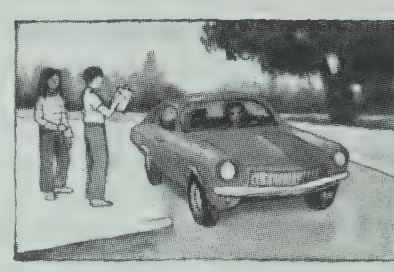
Sometimes, information is given to you.

Sometimes, you must ask for it.



Sometimes, you must look it up,

or perform a task for it.



Working Together

To keep a chart of what you and your classmates eat for breakfast each day of the week,

1. how many rows would you draw on your chart? What name would you give each row?
   One for each kind of food eaten
   The name of one food
2. how many columns would you draw on your chart? 7 What would be their titles?
   The days of the week

64

LESSON ACTIVITY

Using the Pages

- The illustrations on page 64 can motivate a discussion of ways to obtain information. Have a student read the title of the lesson and the introductory statement at the top of page 64. Then discuss each of the illustrations in turn. For example, for the first illustration, discuss the meaning of the word *ballots*. Have students suggest situations for which ballots are used and the kind of information they provide. Discuss the need to interpret the given information in an organized way. (This is dealt with on pages 66 and 67.)

  A second method of obtaining information is to ask for it. Have students suggest what information was needed for the survey chart and how it was obtained. Discuss the ways in which a survey chart helps to organize the information in a logical manner. Ask how the person conducting the survey would know how many rows and how many columns to prepare for the chart before conducting the survey.

  For the third illustration, ask what kinds of information might be found through research in books, and what books might provide the information needed. Finally, have students interpret the fourth illustration and also suggest other similar examples of information that can be obtained through performing a certain task. Some examples are identifying the kinds of trees along the sides of streets and the makes and kinds of bicycles in the bicycle racks at school.

Working Together: These exercises help students to consider the different aspects of the information they need. The survey chart in the second illustration on page 64 can help them visualize a similar chart for Ex. 1 and 2. Since the survey continues for a period of one week, there would likely be seven columns in the chart, one column for each day of the week. Then, a decision is required regarding the number of rows, which is based on the kinds of food eaten for breakfast. These might be considered in categories such as juice, fruit, hot cereal, cold cereal, eggs, toast or bread,

Exercises

Collecting information can be made easier if you are prepared. Try some of these.

Answers will vary

1. Draw a ballot you could use in an election for club president, vice-president, and secretary.
2. Draw a survey form you could use to find out the three favorite songs, in order, of your classmates.
3. Draw a chart you could use to list the time it takes you to go from one place to another for each of five places in your town.
4. Draw a chart you could use for a week to keep track of the television shows you watch and the time you spend watching them.
5. Draw a survey form you could use to find out which television shows people watch.
6. Draw a chart you could use to list the height, mass, and age of each person in your class.
7. Draw a chart you could use to compare grocery prices in three grocery stores.
8. Draw a chart you could use to compare prices in three take-out pizza restaurants.
9. Draw a diagram you could use for counting cars passing a street corner near your school.
10. Draw a chart you could use to keep track of how you use your money.
11. Draw a chart you could use to keep track of the scores of all the league games played in one season.
12. Draw a chart you could use to keep track of the high, the low, and the noontime temperatures each day for a month.
13. On an automobile trip you read other license plates to see where they are from. Draw a chart you could use to keep track of the information.

*14. You have to place the order at the fast-food restaurant. There are five others in your group. Show how you could organize all the choices into one order.

KEEPING SHARP

Add, subtract, or multiply.

1. 458 + 165 623
2. 2478 + 3687 6165
3. 1284 + 567 + 2193 4044
4. 890 − 482 408
5. 4257 − 672 3585
6. 33 302 − 28 365 4937
7. 6 × 584 3504
8. 35 × 97 3395
9. 47 × 7068 332 196
10. 3315 − 1139 2176
11. 8 × 2805 22 440
12. 726 + 1464 2190
13. 24 × 345 8280
14. 13 964 + 19 063 33 027
15. 4000 − 1945 2055
16. 319 × 1486 474 034

65

RELATED ACTIVITIES

- Students may select an appropriate exercise from page 65 and carry out the activity to complete the prepared chart or survey form. These may be kept and used in preparing pictographs and bar graphs after pages 68 and 69 have been completed.

and milk. Have the students prepare the actual chart for Ex. 1 and 2.

Exercises: For these exercises, the students are not required to conduct the actual surveys or complete the charts they prepare; the emphasis is on preparing the charts. For this, careful consideration will have to be given to each situation. For instance, in Ex. 1, the students must consider the number of people running for each office. For Ex. 2, the number of songs in the survey must be considered, and the survey form must be designed to reveal the order of preference for first, second, and third choices.

Select a few of the exercises for the students to complete at this time. You may wish to let students select several for which they express an interest. Ex. 14 is starred because there is more than one way to solve the problem. For example, a survey chart can show the items of food listed by row and the names of the members in the group by column. Alternatively, each individual could write her/his order on a separate sheet, and these could be used to compile a single order.

Keeping Sharp: These exercises provide practice with skills in addition, subtraction, and multiplication.

Assessment

1. Draw a survey form you could use to find out whether more students in your class were born in Canada or in another country, and in which province or territory they were born if they were born in Canada.

Forms will vary. One example is given.

WHERE WERE YOU BORN?	
Born in Canada	
Not born in Canada	
WHERE WERE YOU BORN IN CANADA?	
Alta.	
B.C.	
Man.	
N.B.	

Pages 66-67

LESSON OUTCOME

Interpret information shown in a tally sheet and a check list

Vocabulary

tally

Organizing Information

Gwen asked schoolmates, "What is your favorite pastime?"
She marked a **tally** for each answer.

	Soccer	Stamp Collecting	Skating	Swimming	Television	Hockey	Hiking	Skateboarding	Reading	Bicycling	Baseball	Fishing
Girls	I	IIII	𝍸 I	𝍸 III	𝍸	𝍸 I	IIII	𝍸 II	𝍸 IIII	IIII	III	𝍸
Boys	𝍸 I	III	II	𝍸 III	𝍸 II	𝍸 IIII	IIII	𝍸	𝍸	𝍸 I	IIII	III

Then she completed this chart.

	Soccer	Stamp Collecting	Skating	Swimming	Television	Hockey	Hiking	Skateboarding	Reading	Bicycling	Baseball	Fishing
Indoor		✓	✓	✓	✓	✓			✓			
Outdoor	✓		✓	✓		✓	✓	✓		✓	✓	✓
Daytime	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
Nighttime		✓	✓	✓	✓	✓			✓			
Colder Weather		✓	✓		✓	✓			✓			
Warmer Weather	✓	✓		✓	✓		✓	✓	✓	✓	✓	✓
Noisy	✓			✓	✓	✓		✓			✓	
Quiet		✓	✓				✓		✓	✓		✓
Alone		✓	✓		✓			✓	✓	✓		✓
With Others	✓		✓	✓		✓	✓			✓	✓	

66

LESSON ACTIVITY

Using the Pages

- The previous lesson dealt with the preparation of a survey form or chart to assist in collecting information. In this lesson, the information from the survey is organized and interpreted.

  Discuss Gwen's survey chart and how a *tally* is used to record each answer. Ensure that the students understand the meaning of the symbol 𝍸. Explain that every fifth tally in a category is marked across the previous four to facilitate counting the tallies by fives.

  The chart beneath the tally sheet makes use of the headings for the columns of the tally sheet. For example, there is no check under "soccer" for "indoor" because soccer is not an indoor pastime. Have the students note the pastimes listed in the columns and the categories listed in the rows.

Working Together: These exercises suggest the kinds of questions for which the answers may be interpreted from the chart and/or tally sheet on page 66. For example, Ex. 1 requires searching in one row of the tally sheet for the appropriate pastime. Ex. 2 requires a similar procedure (twice) and also addition of two numbers. Ex. 3 is still more involved, requiring that two rows be examined simultaneously in order to arrive at the numbers to be added. This procedure is extended for three rows in Ex. 4. It is important to have students explain how they obtain their answers for Ex. 1-4. Use other similar exercises as required. The students may suggest questions to be answered from the chart.

Exercises: Ex. 1-10 concern the skill of interpreting the information displayed on page 66. For Ex. 11, the students are to prepare a tally sheet similar to the one shown at the top of page 66. Note that topics other than favorite team sports are suggested. This exercise should be continued for

Working Together

Use Gwen's chart.

1. How many girls said that their favorite pastime is skating? 6
2. How many boys and girls like skateboarding best? 12
3. How many boys like a pastime that can be shared with others? 39
4. How many boys and girls like an indoor pastime best? 72

Exercises

Use Gwen's chart.

1. How many girls like fishing best? 5
2. How many boys like hockey best? 9
3. How many boys and girls like watching television best? 12
4. How many boys and girls like bicycling or hiking best? 18
5. How many girls like a pastime that can be done in the daytime? 62
6. How many boys like a pastime that can be done at night? 34
7. How many boys and girls like a noisy pastime best? 69
8. How many girls like a quiet pastime that can be done alone? 28
9. How many boys like a pastime that can be done outdoors in colder weather? 11
10. Which quiet pastime is the favorite of the greatest number of boys and girls? Reading

Ask your schoolmates:

11. Make a list of team sports and find out their favorites. Show your results in order. Start with the sport that is the favorite of the greatest number of boys and girls.

Instead of team sports, you may wish to find out their favorite hobbies, school subjects, TV shows, books, animals, chores, holidays, or something else that is especially interesting to you.

Where is the food that you buy prepared?

1. Check labels in your kitchen. Make a tally chart.
2. Think of a good way to show your results to your classmates.

PROBLEM SOLVING

67

RELATED ACTIVITIES

• If students completed survey forms and charts as suggested in *Related Activities* on page T71, have them organize the information and prepare a list of relevant questions.

several days to enable the students to carry out their surveys. As an alternative, each student may prepare a survey form on a separate sheet of paper, showing her/his name at the top of the page. The sheets may then be circulated to enable the students to mark tallies on one another's sheets. Provide an opportunity for the results to be displayed and discussed.

Problem Solving: Some students may be surprised to learn that many foods purchased in cans have not been prepared in Canada. These exercises enable them to apply the concepts of this lesson in a problem-solving situation.

Assessment

Use Gwen's chart to answer these questions.

1. How many boys and girls like a quiet pastime? 55
2. How many girls like an indoor pastime that can be done with others? 20

Pages 68-69

LESSON OUTCOME

Interpret and draw a pictograph; interpret and draw a bar graph

Materials

graph paper (copies of page T397), colored paper and large sheets of paper (optional)

Vocabulary

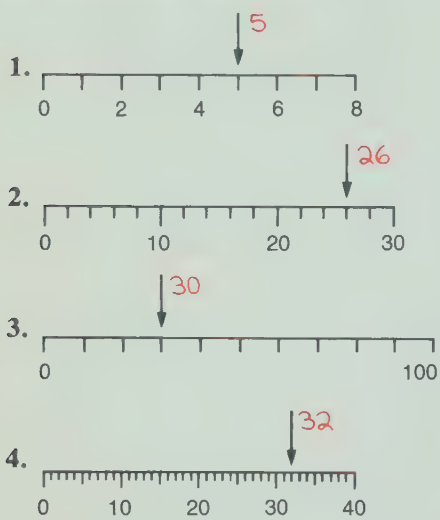
pictograph, bar graph, seconds, s

Prerequisite Skills

Read a scale

Checking Prerequisite Skills

Write the number for the point marked by the arrow on each scale.



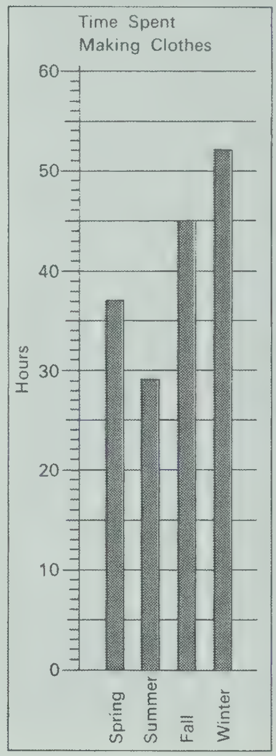
Pictographs and Bar Graphs

Tina counted the things she found in the sewing cabinet.

This **pictograph** shows how many pins, needles, buttons, and snaps she found.

Things in Our Sewing Cabinet		
Pins	/ / / / / / / / / / / / / / / / / /	18
Needles	/ / / /	4
Buttons	⊙⊙⊙⊙⊙⊙⊙⊙⊙⊙⊙⊙⊙⊙⊙⊙	16
Snaps	⊛⊛⊛⊛⊛⊛⊛⊛⊛⊛⊛⊛	12
Each picture stands for 5 items.		

This **bar graph** shows how much time Tina's parents spent making clothes.



Working Together

From the graphs, tell

1. how many snaps are in the sewing cabinet. 60
2. how many pins and needles there are. 110
3. how many more buttons than snaps there are. 20
4. how many hours were spent making clothes in the fall. 45
5. how many fewer hours were spent making clothes in the summer than in the spring. 8

Draw

6. another pictograph that shows how many pins, needles, buttons, and snaps Tina counted. In this graph let each picture stand for 10 items.

Draw a bar graph to show this information:

7.

Our Button Box			
red buttons	8	blue buttons	16
black buttons	13	white buttons	19

Graphs for Ex. 6 and 7 are shown on page T366.

68

LESSON ACTIVITY

Using the Pages

- Begin with a brief discussion of the photograph on page 69. Ask for ways in which the buttons are different and ways in which they are alike. Note that there are four different colors, three different sizes, and either two or four holes. Have students suggest other items that are found in a sewing cabinet. Then direct their attention to the top of page 68.
- Introduce the examples by reviewing that the previous two lessons involved collecting and organizing information, and that this lesson concerns displaying information in a pictorial form called a *graph*. A graph usually enables one to interpret information more easily than a chart or a survey form. Discuss each graph in turn, drawing attention to such aspects as the title of the graph, the heading for each row or column, the meaning of each symbol (in a pictograph), and the scale shown (in a bar graph). Relate the name of each kind of graph to its form. Have the students note that the symbols in the pictograph are aligned vertically, and that the bars in the bar graph have the same width and are equally spaced. Although the terms *horizontal* and *vertical* are not formally introduced until page 72, you may wish to introduce them here to describe the difference between the bar graph illustrated on page 68 and the one on page 69.

Working Together: Ex. 1-3 deal with interpreting the information shown in the pictograph. Ex. 4 and 5 deal with interpreting the bar graph. For Ex. 3, have students explain how the answer can be obtained from the pictograph without using subtraction. This emphasizes the advantage of lining up the symbols vertically. These exercises also provide an opportunity to contrast the two kinds of graphs presented here. For example, to answer Ex. 1, it is necessary to count by fives for the row of snaps in the pictograph. However, to answer a similar question for the bar graph (Ex. 4), it is necessary to align the top of the appropriate bar with the vertical scale and read the number indicated on the scale.

Graphs for Ex. 1-4 are shown on pages T366 and T367

Exercises

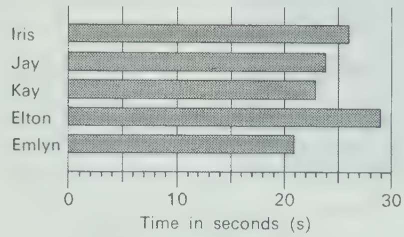
Draw a pictograph to show how many buttons there are of each

1. of 4 colors. 2. of 3 sizes.

Draw a bar graph to show how many buttons there are

3. of each of 3 sizes. 4. with two holes or with four holes.

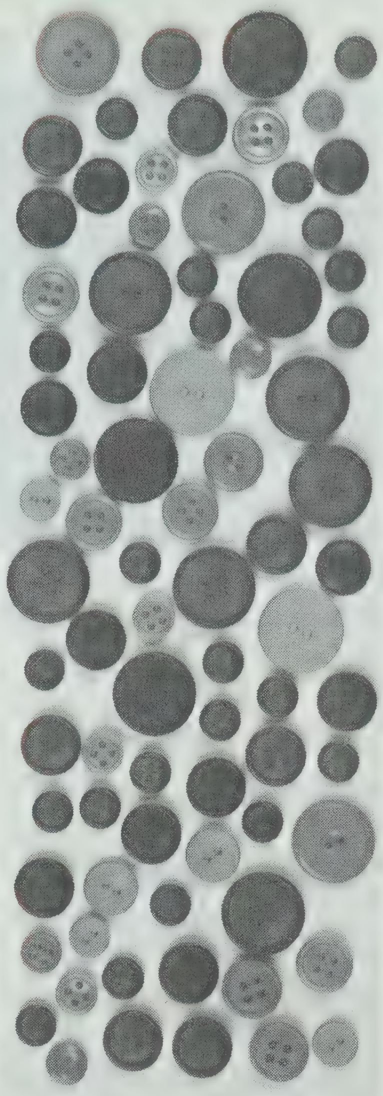
This graph shows the time each runner needed in a foot race.



5. How long did Jay take? 24 s
6. Who took the longest time? Elton
7. Who won the race? Emlyn
8. How many seconds did Kay finish ahead of Iris? 3
9. Which two runners finished closest together? Jay and Kay

Choose a group of people. Collect the information from them and draw a pictograph or a bar graph to show their favorite

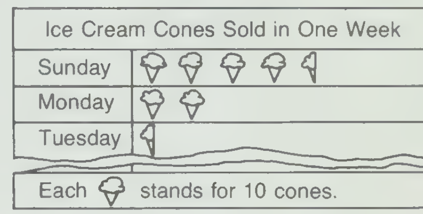
10. food, movie, job, flower, story, car, store, color, fruit, song, game, athlete, or something special that you would like to find out about.



69

RELATED ACTIVITIES

- Display the graphs completed for Ex. 10. Have the students who drew them prepare a few questions about each graph on a separate sheet of paper. Display the questions beneath the graphs and have students answer them. This will provide practice in interpreting graphs.
- Have students cut examples of bar graphs and pictographs from newspapers and magazines. Display them and help the students interpret the information shown.
- Have students interpret and prepare pictographs that involve halves of symbols, as shown in the following example.



Ex. 6 and 7 require the students to draw a pictograph and a bar graph. You may wish to have them work individually using copies of page T 397. However, the activity can be more appealing if students work in small groups to prepare larger graphs. They may use colored paper to cut symbols for the pictograph and bars for the bar graph. The symbols and bars can be pasted onto large sheets of paper. This activity can stimulate a discussion of important aspects such as how many of each symbol are required, what scale is appropriate, and how long the bars should be. It can also lead to a discussion of the advantages and disadvantages of each form of graph. When the groups have completed their graphs, display them and discuss their differences.

Exercises: Ex. 1-4 relate to the photograph on page 69. The students must decide how many objects each picture will represent for Ex. 1 and 2. Provide the students with graph paper (copies of page T 397) on which to draw their graphs. Ex. 10 should be worked on for several days. Have the students work individually or in small groups. It is recommended that they show the three stages of development to complete this exercise: collect the information in a prepared tally sheet or survey form; organize the information; and display the information in a graph.

Assessment

Draw a bar graph to show the favorite kinds of movies for Grade Five students at Robert's School.

1.

Kind of Movie	Number of Students
Western	24
Musical	16
Comedy	28
Detective	32
Adventure	29
Cartoon	11

A bar graph is shown on page T367.

Draw a pictograph to show the following information.

2.

Baseball Equipment			
bats	6	right-hand gloves	3
balls	18	left-hand gloves	12
catcher's masks	3	bases	6

A pictograph is shown on page T367.

Pages 70-71

LESSON OUTCOME

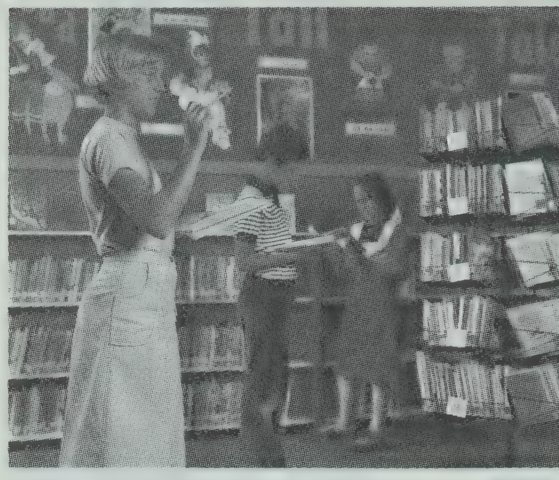
Interpret a line graph

Vocabulary

line graph

Reading Line Graphs

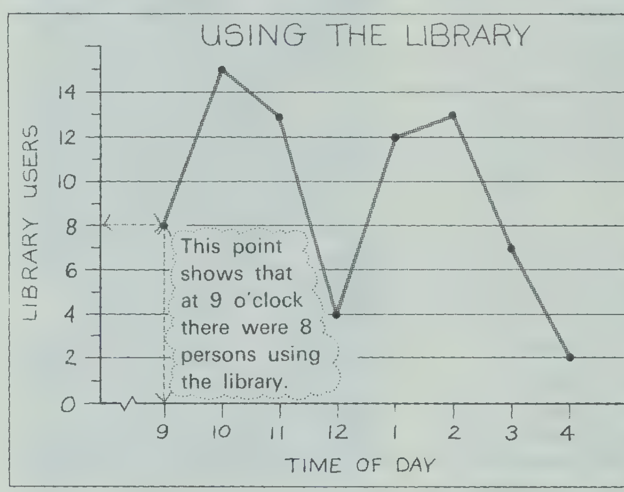
Tracy counted the number of persons who were using the school library at different times during the day.



Her chart looked like this.

9 o'clock	8
10 o'clock	15
11 o'clock	13
12 o'clock	4
1 o'clock	12
2 o'clock	13
3 o'clock	7
4 o'clock	2

She drew a **line graph** to show her information.



70

LESSON ACTIVITY

Using the Pages

- Review the names and characteristics of the two kinds of graphs (pictographs and bar graphs) presented in the previous lesson. Discuss the aspects of the graph displayed on page 70 with the students and note that it is neither a bar graph nor a pictograph. Introduce the term *line graph*. Students may suggest that the graph looks similar to a sequence of connected line segments.

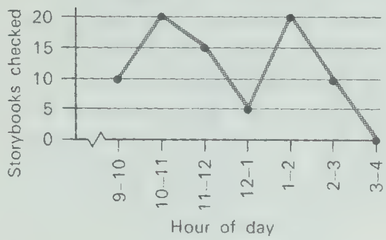
  Have a student read the statement above the photograph to introduce the example. Have the students note that Tracy counted the persons eight times during the day: on each hour, beginning at 9 o'clock. To emphasize that a graph must have a title, scales, and headings, ask what information is shown, what the numbers 9, 10, 11, . . ., 4 indicate, and what the numbers 0, 2, 4, . . ., 14 indicate. Draw particular attention to the portion of the scale between 0 and 9 for "Time of Day" and discuss the meaning of the break indicated. Have a student explain how to determine from the graph the number of users at 9 o'clock. Ask how many were using the library at 3 o'clock, at what time there was the least number of persons using the library, and other similar questions. Ask what the line segment joining the dots for the number of users at 9 o'clock and at 10 o'clock indicates. Lead the students to realize it shows that the number of persons using the library increased during this time. Have students note the sharp decline in the number of persons between 11 o'clock and 12 o'clock, indicated by the corresponding line segment. Ask them to suggest an explanation for the decline. Discuss other aspects of the graph in a similar manner. Summarize by noting that the line graph shows the trend or pattern for the use of the library during the day more clearly than the chart to the right of the photograph, hence, an advantage of showing the information in a graph.

Working Together: Before assigning the exercises, have students identify the title and the headings of the graph and explain the scales shown. Ensure that they understand the significance of a time shown as 9-10 for "Hour of day".

Working Together

This line graph shows the number of storybooks checked out each hour.

Number of Storybooks Checked Out Each Hour

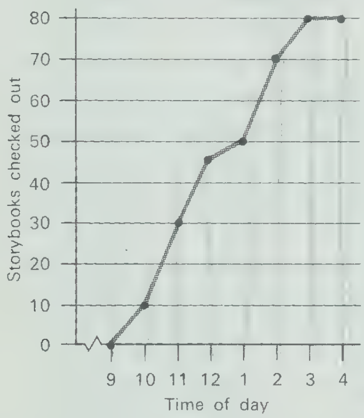


1. How many storybooks were checked out from 9 to 10? 10
2. How many storybooks were checked out from 1 to 2? 20
3. During which hour were 15 storybooks checked out? 11 - 12
4. During which hour were the fewest storybooks checked out? 12-1
5. Were more storybooks checked out from 11 to 12 or from 2 to 3? How many more? 11 - 12 5
6. During which hours were the same number of storybooks checked out? 9 - 10 and 2 - 3

Exercises

This line graph shows the total number of storybooks checked out during the day.

Total Number of Storybooks Checked Out During the Day



1. How many storybooks had been checked out by 11 o'clock? 30
2. How many storybooks had been checked out by 1 o'clock? 50
3. By what time had 45 storybooks been checked out? 12
4. By what time had 70 storybooks been checked out? 2
5. The library opened at 9 in the morning. During which hour in the morning were the most storybooks checked out? 10 - 11
6. During which hour in the morning were the fewest storybooks checked out? 9 - 10
7. During which hour in the afternoon were the most storybooks checked out? 1 - 2
8. During which hour in the afternoon were the fewest storybooks checked out? 3 - 4

71

RELATED ACTIVITIES

• Have students cut examples of line graphs from newspapers, reports, and magazines. Display these with the examples of pictographs and bar graphs. Help the students interpret the information shown in the line graphs. Contrast the three kinds of graphs displayed.

Discuss Ex. 1-6 and ask other similar questions as required. Have students suggest questions to be asked about the graph. Pay particular attention to Ex. 4 and 6, and have students suggest reasons to explain these situations.

Exercises: The graph in *Working Together* shows the number of storybooks checked out during each hour. Ensure that the students realize the different situation presented in this graph, as indicated by the title. Since the graph involves the total number of books, the trend of the graph is always upward; it must continually rise or remain at the same level. Note that the number of books checked out by 12 o'clock is 45 (Ex. 3). The students will need to read carefully in order not to miss the phrase "in the morning" for Ex. 5 and similar phrases for Ex. 6-8.

Assessment

This line graph shows the number of storybooks checked in each hour.

Number of Storybooks Checked in Each Hour

1. How many storybooks were checked in from 11 to 12? 5
2. During which hour were 15 storybooks checked in? 10 - 11
3. During which hour were the fewest storybooks checked in? 12 - 1
4. Were more storybooks checked in from 9 to 10 or from 1 to 2? How many more? from 9 to 10; 10

LESSON OUTCOME

Match points on a grid with ordered pairs of numbers

Materials

overhead projector and transparency showing a grid (optional); copies of page T 397 and a straight edge for each student

Vocabulary

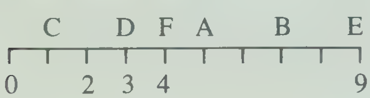
ordered pair, grid, plot a point, horizontal, vertical

Prerequisite Skills

Match whole numbers with points on a number line

Checking Prerequisite Skills

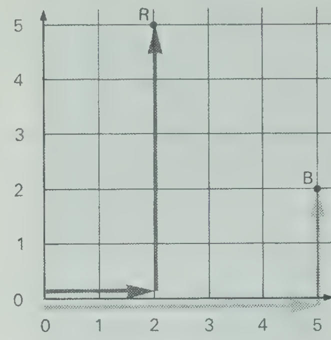
For this number line,



1. what point matches 3? D
2. what number names point B? 7

Ordered Pairs of Numbers and Points on a Grid

Each point on this grid can be matched with a pair of numbers.



To match point R with a pair of numbers, count over 2 and up 5.

Point R matches the number pair (2,5).

Point B matches (5,2).

(2,5) and (5,2) are the same pair of numbers, but their *order* is different.

(2,5) and (5,2) are **ordered pairs** of numbers.
(2,5) and (5,2) are *different* ordered pairs of numbers.

Each ordered pair of numbers can be matched with a point on a grid.

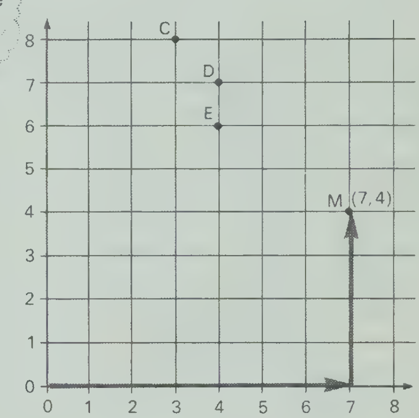
First, draw a **horizontal** number line and a **vertical** number line, starting from the same point.

To match the ordered pair (7,4) with a point on the grid, count over 7 and up 4.

(7,4) matches point M.

(4,7) is the same pair of numbers as (7,4), but it is a different *ordered* pair of numbers.

Find out which of the points C, D, or E, matches (4,7).

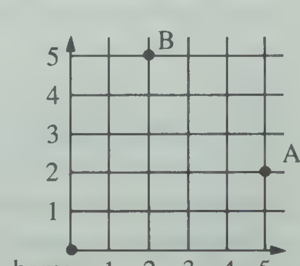


72

LESSON ACTIVITY

Before Using the Pages

- Establish the concept of ordered pairs of numbers through the following activity. Draw a large, square grid on the board, or display a grid using an overhead transparency prepared from copies of page T 391. Mark a dot and identify the position by printing the word "home" as shown. Tell the following story as you trace paths on the grid and mark dots. The story is about two brothers who were told how to find a hidden treasure by walking five blocks from home, turning the corner, and then walking two blocks. Trace a path for the first brother to reach point A by moving 5 units to the right and then 2 units up. Trace a path for the second brother to reach point B by moving 5 units up and 2 units to the right. Tell the students that the first brother found the treasure. Repeat for other examples so that the one who walks first to the right and then up always finds the treasure. Have students guess the rule for finding the treasure. Then write an ordered pair such as (3, 1) on the board as the clue and have a student describe how to locate another treasure from this clue. Include other examples such as (0, 4) and (6, 0).



Using the Pages

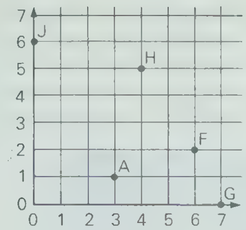
- The examples introduce the formal language and notation of ordered pairs of numbers and emphasize the procedure of counting *over* first and then *up*. Guide the students through the examples, drawing attention to new vocabulary and the manner of writing an ordered pair of numbers using parentheses. Emphasize the significance of the order of the two numbers, hence, the term *ordered pair* of numbers.

  Point out the two zeros where the horizontal and vertical number lines meet and have the students recall the point associated earlier with the word "home". Ask what ordered pair of numbers identifies this point.

Working Together

Complete.

		Count over	Count up
1.	point A	? 3	? 1
2.	point F	? 6	? 2
3.	point G	? 7	? 0



Give the ordered pair of numbers matching **4.** point H. (4,5) **5.** point J. (0,6)

Use graph paper. Draw a horizontal number line and a vertical number line, starting from the same point. Then plot each of these.

When you draw a dot for an ordered pair of numbers, you are **plotting** the point.

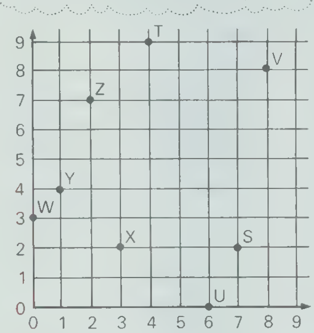
6. B(6,3) **7.** E(1,0) **8.** R(0,8)

Answers are given on page T367.

Exercises

Give the ordered pair of numbers that matches

1. point Z. (2,7) **2.** point Y. (1,4)
3. point X. (3,2) **4.** point W. (0,3)
5. point V. (8,8) **6.** point U. (6,0)
7. point T. (4,9) **8.** point S. (7,2)



Use graph paper. Draw a horizontal number line and a vertical number line, starting from the same point. Then plot each of these.

Answers are given on page T367.

9. A(3,5) **10.** B(0,7) **11.** C(5,3) **12.** D(1,4) **13.** E(2,2) **14.** F(4,0)
15. G(5,5) **16.** H(6,5) **17.** K(7,5) **18.** L(8,5) **19.** P(9,5) **20.** Q(8,6)
21. R(7,7) **22.** S(6,8) **23.** T(5,9) **24.** X(5,8) **25.** Y(5,7) **26.** Z(5,6)

KEEPING SHARP

Find the result. Work with the grouped numerals first.

1. (89 + 176) × 497 131 705
2. 5000 − (2974 + 1036) 990
3. (75 × 62) − 3895 755
4. 6215 − (4037 − 1859) 4037
5. 28 × (1410 − 684) 20 328
6. 40 512 − (23 × 1688) 1688
7. (3028 − 2996) × (593 + 1674) 72 544

RELATED ACTIVITIES

• Have students plot the points corresponding to ordered pairs of numbers that follow a simple pattern, for example, (0,0), (1,2), (2,4), (3,6), (4,8). Have them suggest three other ordered pairs that fit the given pattern and plot the points on the same grid.

• Label 26 points on a grid with capital letters, one point for each letter of the alphabet. Have students write the sequence of ordered pairs of numbers to identify the letters that spell their first and/or last names.

• Students having difficulty matching ordered pairs with points on a grid may find it helpful to walk on a large grid outlined in tape or chalk on the floor. They must always begin their trip from 0 (the point where the two number lines meet). The first number of the ordered pair indicates the number of units to walk "over" and the second number, the number of units to walk "up".

• A simple geoboard showing nails in a square array can be labeled to represent a grid. Students may work in pairs so that one student places a bead on a nail and the other names the ordered pair of numbers. Alternatively, a student may name an ordered pair of numbers, and then place the bead; the other may say "I agree" or "I disagree", giving reasons for the statement.

Working Together: The chart for Ex. 1-3 refers to the points on the grid at the right and emphasizes the order of the numbers in an ordered pair. This concept is applied in completing Ex. 4 and 5.

Give each student a copy of page T 397 and work with them to prepare a section of the grid for graphing. Have them prepare a section in the upper left portion of the graph paper by drawing and labeling two number lines from the same point. Discuss why a 10-by-10 grid would be large enough, by noting the numbers in the ordered pairs for Ex. 6-8. Introduce the phrase *plot a point* on a grid. Note that Ex. 7 and 8 deal with zero as one of the numbers in an ordered pair. Pay particular attention to these, because some students find such ordered pairs difficult to locate.

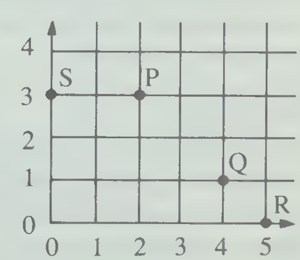
Exercises: To show Ex. 9-26, the students may prepare another section of the same graph paper used earlier. It would be beneficial to discuss the results of some of these exercises. For example, the points for Ex. 9 and 15-19 all belong to the same horizontal line, and the points for Ex. 11, 15, and 23-26 all belong to the same vertical line.

Keeping Sharp: The instructions indicate that students are to work with the grouped numerals first. It may be necessary to review the procedure using smaller numbers and demonstrate that without parentheses to indicate which operation is to be performed first, it is sometimes possible to obtain different results.

Assessment

Give the ordered pair of numbers that matches

1. point P. (2,3)
2. point Q. (4,1)
3. point R. (5,0)
4. point S. (0,3)



Use graph paper. Draw a horizontal number line and a vertical number line, starting from the same point. Then plot each of these. Plotted points are shown on page T367.

5. (3,4) **6.** (0,2) **7.** (1,0)

Pages 74-75

LESSON OUTCOME

Draw a line graph

Materials

a large sheet of graph paper or an overhead projector (optional); several sheets of graph paper (copies of page T 397 or T 398) and a straight edge for each student

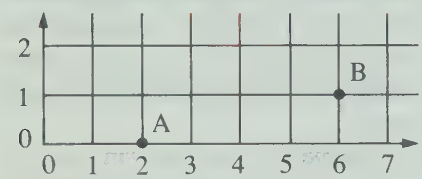
Prerequisite Skills

Match points on a grid with ordered pairs of numbers

Checking Prerequisite Skills

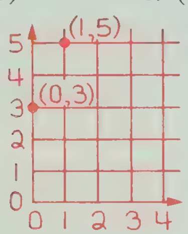
Give the ordered pair of numbers that matches

1. point A.(2,0) **2.** point B.(6,1)



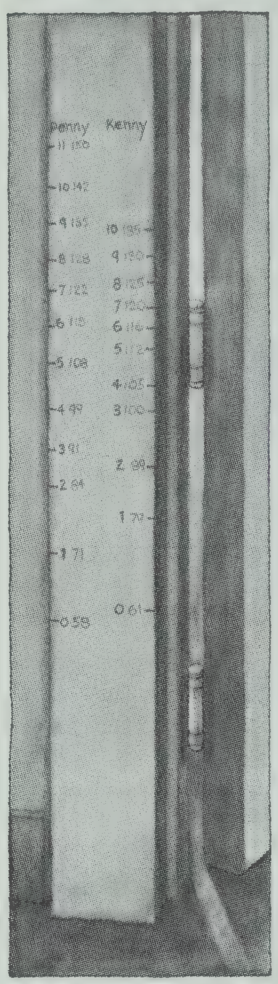
Use graph paper. Draw a horizontal number line and a vertical number line, starting from the same point. Then plot each of these.

3. (1,5) **4.** (0,3)



Drawing Line Graphs

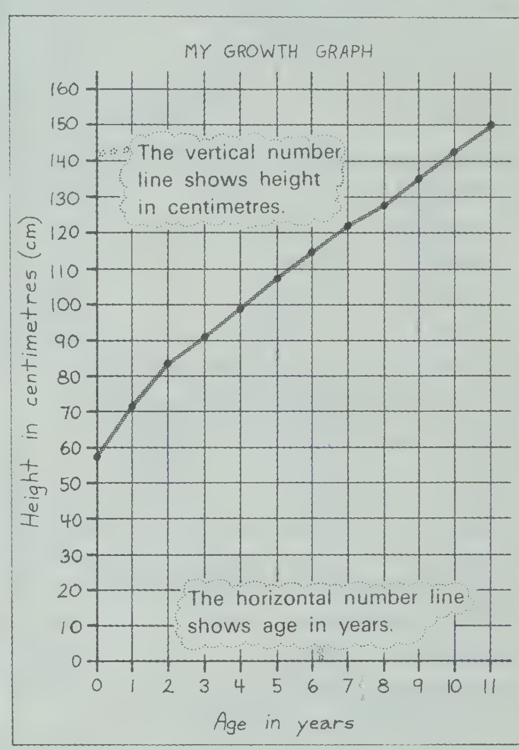
The information used for a line graph is often in the form of ordered pairs of numbers. To draw the line graph you must plot the points and then connect them.



Each year on their birthdays, Penny and Kenny have their heights marked on a door frame in their home.

The first number at each mark gives age. The second number gives height.

Penny drew a line graph to show how she has grown in 11 years.



74

LESSON ACTIVITY

Before Using the Pages

- Briefly review the work of the previous lesson by showing points on a grid and having the students name the corresponding ordered pairs of numbers. Also, write ordered pairs of numbers and have the students plot the points. Explain that one use for ordered pairs of numbers is to prepare line graphs.

Using the Pages

- Read the statements at the top of page 74 to introduce the situation and to learn what the numbers in the illustration identify. Ask what Penny's height was at age five and what Kenny's height was at age two. The height is measured in centimetres, as indicated on the graph. Ask what numbers indicate Penny's height at birth (0,58).

  Draw attention to the graph. Begin by pointing out the headings and the scales for the horizontal and vertical number lines and relate these to the numbers shown for Penny on the door frame. Then relate each point on the graph to an ordered pair of numbers derived from the illustration. Write the ordered pairs on the board: (0,58), (1,71), (2,84), (3,91), . . ., (11,150). Discuss how joining the points in sequence shows a line graph that gives an indication of Penny's height at any time over an eleven-year period. The line graph can also reveal periods of more rapid growth and those of slower growth.

Working Together: Ex. 1-4 lead students through the steps of preparing a line graph. Provide each student with a copy of page T 397 or T 398. Develop the graph on the board or on a large sheet of graph paper (or use an overhead projector) for Ex. 1-4 at the same time as the students prepare their own graphs. Discuss such aspects as appropriate scales and headings and interpreting the information in the chart to give ordered pairs of numbers. When the points have been marked on the graph, have the students join them in sequence. The points do not lie in one straight line, but a straight edge may be used to draw line segments between consecutive points.

Working Together

Use graph paper.

1. Start at a point and draw a horizontal number line. Mark it to show *hours*.
2. Start at the same point and draw a vertical number line. Mark it to show *kilometres*.
3. Plot the information shown in the chart.

Time (in hours)	Distance (in kilometres)
0	0
1	6
2	11
3	15
4	18

A line graph is shown on page T367.

4. Connect the points to make a line graph.

Exercises

Draw a line graph for each.

What do you think the line graphs that you draw might be showing?

1. Mark the horizontal number line to show *days*. Mark the vertical number line to show *centimetres*.

Time (in days)	Height (in centimetres)
0	0
3	1
6	3
9	5
12	7

2. Mark the horizontal number line to show *kilograms*. Mark the vertical number line to show *dollars*.

Mass (in kilograms)	Cost (in dollars)
2	4
3	6
5	10
9	18
12	24

3. Mark the horizontal number line to show *years*. Mark the vertical number line to show *kilograms*.

Time (in years)	Mass (in kilograms)
0	4
2	12
4	16
6	21
8	26
10	32

Gather the information and draw a line graph to show

4. the temperature during the daytime.
5. the high temperature each day for a month.

Answers will vary

Draw

*6. a Growth Graph for Kenny.

*7. the Growth Graphs for Penny and Kenny on the same grid.

Line graphs for Ex. 1, 2, 3, 6, and 7 are shown on page T367

75

RELATED ACTIVITIES

- Students who have kept a record at home of their heights over a period of time can draw their own "growth graphs".
- Temperature graphs are often used to compare temperatures for the same days of different years. When the line graph for Ex. 5 is completed, another line graph can be drawn on the same grid to show one of the following.

1. the high temperature each day for the same month in the previous year
2. the highest temperature ever recorded for each day of the same month
3. the lowest temperature ever recorded for each day of the same month

This activity can provide practice in collecting the necessary information (from newspapers, for example), organizing it, and displaying it in a graph.

- Provide, or challenge students to find, other real-life relationships. If necessary, provide the clue, or give a hint by example, that familiar everyday relationships often are expressed as *rates*. Examples include dollars per kilogram, metres per second, grams per litre, and dollars per hour. Relationships like these are *linear*. The points of their graphs are in one line. Have students generate and graph the ordered pairs for each relationship presented. Astute students will note that a solid line will give the graph for some relationships whereas points that are not joined but are in a line will give the graph for other relationships.

Exercises: Provide the students with a sheet of graph paper for each exercise. Encourage them to work carefully and neatly. Suggest that they study the numbers in each chart to help them choose appropriate scales for the horizontal and vertical number lines for Ex. 1-3. The information required for Ex. 4 and 5 may be measured by different students and recorded for everyone in a chart on the board. Thus, the graphs would be a long-term activity.

Ex. 6 is starred because it is necessary to look elsewhere to obtain the necessary information (see the illustration on page 74). Ex. 7 extends the concept of the lesson and provides an opportunity to discuss the advantages of showing both line graphs on the same grid. The two line graphs may be drawn in different colors. When the students have finished the exercises, discuss the question shown in the "thought cloud" above Ex. 2. Ex. 1 might suggest the rate of growth of a plant, for example.

Assessment

Draw a line graph.

1. Mark the horizontal number line to show *hours*. Mark the vertical number line to show *centimetres*.

Time (in hours)	Height (in centimetres)
0	30
1	27
2	23
3	19
4	16

A line graph is shown on page T367.

Page 76

OBJECTIVE

Collect, organize, and display information to solve a problem

Materials

graph paper (copies of pages T 397 and T 398) or large sheets of paper and construction paper

RELATED ACTIVITIES

- Interested students can complete Ex. 1-3 several times for different questions suggested on the page. They may also be motivated to carry out similar research for the "typical" teacher.

Collecting, Organizing, and Displaying Information

Is this the "typical" student?

In your school or class, what is the typical student like . . .

height? color of eyes?
size of feet? favorite clothes?
. . . and so on.

What does the typical student like to do . . .

in school? at home?
on Saturdays? with spare time?
. . . and so on.

What does the typical student like to eat . . .

for breakfast? from a lunch box?
kinds of fruit? kinds of snacks?
. . . and so on.

Where does the typical student come from . . .

birthplace? size of family?
which street? distance from school?
. . . and so on.

What does the typical student want to do . . .

next summer? as a teenager?
that's exciting? in an adult career?
. . . and so on.

Answers will vary.

PROBLEM SOLVING

1. Choose a question from this page or make up your own about the typical student.
2. Collect and organize the information you need to answer the question.
3. Display your results with a chart, a picture, or a graph.

76

LESSON ACTIVITY

Using the Page

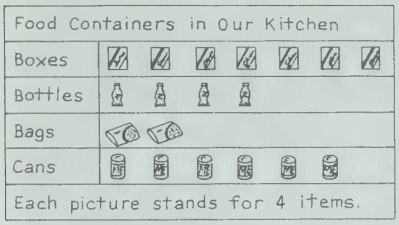
- These exercises offer an opportunity to apply the concepts and skills presented in this unit to problem-solving situations. Questions such as "What is a typical family?" are frequently researched by government and commercial agencies. Survey forms are mailed out for people to complete and return. The results are organized and displayed for interpretation. Frequently, the results are different from those that might be expected. A brief introduction of this nature can be provided before the students read the example on page 76.

  Discuss the photograph in relation to the question shown above it. The "typical" student is one who wears blue denim jeans, eats hamburgers, owns a 10-speed bicycle, plays softball, football, and rides a skateboard.

  Select one of the questions suggested in the example and develop it to indicate what would be required for Ex. 2 and 3. For example, the students can help one another to measure their heights in centimetres. The information can be collected and organized into intervals of 5 cm starting with the shortest height, namely, 121 cm – 125 cm, 126 cm – 130 cm, 131 cm – 135 cm, and so on. The information can be displayed in a bar graph to show the number of students in each interval. The bar of greatest length would reveal the approximate height of the "typical" student.

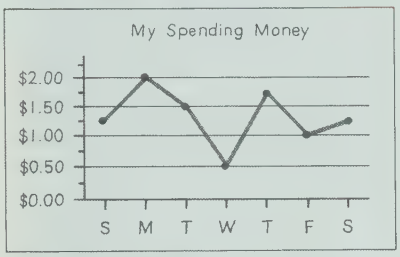
  Some students may enjoy working in groups of two or three, while others may prefer to work individually. The graphs and charts can be more effectively displayed if they are prepared on large sheets of paper with the use of construction paper for bars and pictograph symbols.

Checking Up



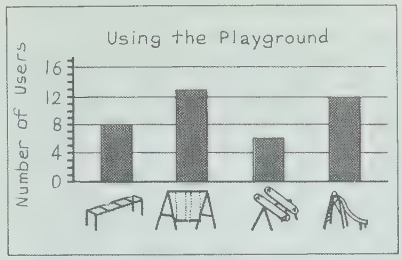
For this pictograph,

1. how many bags are in the kitchen? 8
2. of which kind of container is there the greatest number? boxes
3. how many bottles and cans are there together? 40
4. are there more bottles or cans? How many more? cans 8



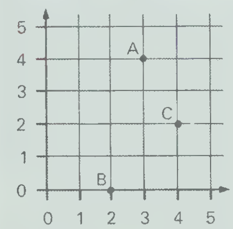
For this line graph,

9. on which day was there the greatest amount? Monday
10. how much was there on Tuesday? $1.50
11. on which day was there $1.75? Thursday
12. was there more money on Thursday or on Friday? Thursday How much more? $0.75



For this bar graph,

5. which piece of equipment is most popular? swings
6. which piece of equipment is least popular? seesaw
7. how many children played on the slide? 12
8. did more children play on the slide or monkey bars? How many more? slide 4



Name each point with an ordered pair of numbers.

13. A (3,4) **14.** B (2,0) **15.** C (4,2)

Use graph paper. Draw horizontal and vertical number lines starting from the same point. Then plot each of these.

16. P(3,1) **17.** S(0,4)
18. X(5,3) **19.** Y(1,2)

Answers are given below.

77

Page 77

OBJECTIVE

Demonstrate an understanding of the concepts and skills presented in this unit

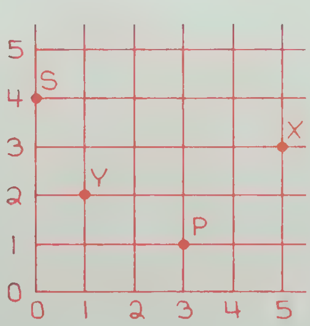
RELATED ACTIVITIES

- Students may play the game known as "Battleship", which is based on the concept of ordered pairs of numbers.
- Students may enjoy cutting strips of construction paper to match the distance around their neck, wrist, waist, hips, and ankle. The strips may be pasted onto a large sheet of paper to form bars of a bar graph and the results interpreted.
- Have students plot ordered pairs of numbers related to sums and products. Joining the points to form a line graph can suggest fractions (or even integers) as addends or factors. Three examples are given below.

1. The numbers in each ordered pair have a sum of 10: (0,10), (1,9), (2,8), . . ., (10,0).
2. The numbers in each ordered pair have a product of 24: (1,24), (2,12), (3,8), (4,6), . . ., (24,1). (The graph will suggest a curve, not a straight line.)
3. The second number in each ordered pair is twice the first number: (0,0), (1,2), (2,4), (3,6), ...

Skills	Exercises	Related Pages
Interpret a pictograph	1-4	T 74-T 75
Interpret a bar graph	5-8	T 74-T 75
Interpret a line graph	9-12	T 76-T 77
Match ordered pairs of numbers with points on a grid	13-19	T 78-T 79

Ex. 16-19



Comments

These exercises concentrate on the ability to interpret rather than to draw the three kinds of graphs (pictographs, bar graphs, and line graphs). Students having difficulty will need more opportunity to interpret simple graphs. Some practice may be required in reading scales of the kind shown in the bar graph and the line graph.

Students having difficulty with Ex. 13-19 may find it helpful to work with pegs on a pegboard that has been marked to show the horizontal and vertical axes. Have them place pegs for ordered pairs of numbers named by you. Have them name ordered pairs of numbers for pegs that you place on the pegboard.

Pages 78-79

OBJECTIVE

Demonstrate competence in addition, subtraction, and multiplication skills; solve related word problems

Checking Skills

Add.

1. 164 + 142 = 306
2. 235 + 365 = 600
3. 2894 + 1945 = 4839
4. 4936 + 1289 = 6225
5. 28 354 + 3 349 = 31 703
6. 58 567 + 37 874 = 96 441
7. $15 978 + 6 581 = $22 559
8. $29.64 + 43.87 = $73.51
9. 2584 + 168 + 1766 = 4518
10. $3498 + 564 + 2368 = $6430
11. 2 285 + 357 + 14 606 + 6 159 = 23 407
12. $126.82 + 75.93 + 121.67 + 98.74 = $423.16
13. 307 + 469 776
14. 567 + 253 820
15. 3379 + 276 3655
16. 1986 + 5126 7112
17. 26 585 + 23 751 50 336
18. 14 587 + 46 945 61 532
19. 352 + 2276 + 442 3070
20. 15 917 + 4 668 + 17 439 38 024
21. $3104 + $1866 $4970
22. $27.47 + $2.83 $30.30
23. $891 + $2377 + $1548 $4816
24. $25.98 + $6.87 + $12.78 $45.63

Subtract.

1. 335 − 173 = 162
2. 532 − 264 = 268
3. 1316 − 844 = 472
4. 2274 − 1775 = 499
5. 11 858 − 3 963 = 7895
6. 67 248 − 48 659 = 18 589
7. 4000 − 1785 = 2215
8. 20 000 − 16 026 = 3974
9. 7082 − 184 = 6898
10. 14 001 − 2 533 = 11 468
11. $743 − 589 = $154
12. $6000 − 2431 = $3569
13. $23.14 − 9.26 = $13.88
14. $95.00 − 54.24 = $40.76
15. 582 − 125 457
16. 3637 − 941 2696
17. 4281 − 3345 936
18. 15 359 − 9 769 5590
19. 26 371 − 18 398 7973
20. 32 113 − 29 526 2587
21. 7000 − 2073 4927
22. 40 050 − 1 698 38 352
23. $9150 − $1976 $7174
24. $40 500 − $4 147 $36 353
25. $12.55 − $2.87 $9.68
26. $20.00 − $9.26 $10.74

78

LESSON ACTIVITY

Using the Pages

- The exercises are arranged in four sections as follows.
  Section 1: Addition
  Section 2: Subtraction
  Section 3: Multiplication
  Section 4: Word problems

  The word problems are not grouped according to operation. The students will need to give careful consideration to each problem to determine the operation(s) to be used. Some problems involve more than a one-step solution (Ex. 7-9).
- In working through the exercises on these pages, students review the major skills in addition, subtraction, and multiplication that were encountered in previous units. You may wish to have them complete one section at a time, or you may prefer to assign several exercises from each section to be completed together. Remind the students that a statement should conclude the solution for each problem. Note that the students' competence in working with whole numbers is important in developing similar skills with decimals in Unit 6.

Multiply.

1. 193 × 7 = 1351
2. 2306 × 4 = 9224
3. 38 × 51 = 1938
4. 506 × 58 = 29 348
5. 2595 × 40 = 103 800
6. 1756 × 89 = 156 284
7. 149 × 461 = 68 689
8. 578 × 217 = 125 426
9. 1945 × 162 = 315 090
10. 4730 × 358 = 1 693 340
11. \$54 × 16 = \$864
12. \$425 × 45 = \$19 125
13. \$6.31 × 2 = \$12.62
14. \$28.47 × 87 = \$2476.89

15. 6 × 290 1740
16. 9 × 5484 49 356
17. 92 × 92 8464
18. 67 × 235 15 745
19. 50 × 8739 436 950
20. 73 × 2260 164 980
21. 261 × 766 199 926
22. 349 × 628 219 172
23. 193 × 3509 677 237
24. 345 × 1271 438 495
25. 28 × \$885 \$24 780
26. 365 × \$701 \$255 865
27. 84 × \$9.21 \$773.64
28. 6 × \$39.80 \$238.80

Solve.

1. The forestry workers planted 28 225 pine tree seedlings. They planted 24 790 fir tree seedlings. How many seedlings did they plant in all? 53 015
2. The pickle plant can pack 17 750 jars of pickles each hour. How many jars can it pack in an 8 h shift? 142 000
3. The store placed 23 500 books on sale. After one week, it still had 8767 of the books. How many had it sold? 14 733
4. Lila does 75 sit-ups each day. How many does she do in a year? 27 375
5. Mrs. Meyer won \$2000 in a contest. Taxes on her prize were \$637. How much did she have after taxes? \$1363
6. On three trips to the Northwest Territories, conservation officers counted 3760, 12 570, and 6225 moose. How many did they count in all? 22 555
7. 4185 tickets were sold in advance. 3648 people went to each of the 3 games. How many tickets were sold at the gate? 6759
8. Terry earned \$24.87 on his paper route and \$9.75 raking leaves. He spent \$28.75. How much did he have left? \$5.87
9. Mr. Ali earns \$775 each month. Mrs. Ali earns \$438 each month working part time. How much do the Ali's earn in a year? \$14 556

79

RELATED ACTIVITIES

- Choose related activities and games from preceding units for review and enrichment.
- Practice in extensions of basic addition facts can help improve speed and accuracy in column addition. Use exercises similar to the following.

Add.

1. 4 + 3, 34 + 3, 84 + 3
2. 6 + 9, 46 + 9, 76 + 9

- Addition of several one-digit numbers can be provided through both oral and written exercises. The work of addition extensions in the previous activity can be applied in this activity.
- Select several exercises and have students interpret the place value of digits for each numeral. Also, have them express numbers in different ways, as shown below.

Subtraction

Ex. 6 67 248:
1. 60 000 + 7 000 + 200 + 40 + 8
2. 6 ten thousands, 7 thousands, 2 hundreds, 4 tens, 8 ones
3. 67 thousand 248

Ex. 1 173:
1. 1 hundred 7 tens 3 ones
2. 17 tens 3 ones

- Select numbers from several exercises and have students round the numbers to a given place value. Have them estimate the sum (difference, product) for several exercises.

Unit 5 Overview

Division

At the beginning of this unit, the procedures involved in doing simple division by one-digit divisors are reviewed, with attention to the use of multiplication to determine the quotient and in the algorithm itself. Use of the algorithm is extended to three-digit dividends which require no regrouping, followed by several lessons with regrouping in dividends having up to five digits. For these first lessons, partial quotients have been stacked and zeros have been shown in the products of tens and hundreds. Then the standard algorithm is developed in two lessons. Finding an average is introduced as a practical application of division in everyday life. The lesson on the use of the calculator extends the use of keycharts which were introduced in Unit 3. The need for adjusting answers according to differences in situations is developed in the lesson on problem solving.

Prerequisite Skills

- complete the basic multiplication facts
- write the missing factor in a multiplication fact
- multiply a multiple of ten by a one-digit number, products to 90
- multiply a multiple of one hundred by a one-digit number, products to 900
- use multiplication to complete basic division facts
- regroup with numbers to 99 999

Unit Outcomes

- use multiplication to divide, divisors and quotients to 9, remainders
- divide a two-digit or a three-digit number by a one-digit number, no regrouping, remainder zero
- divide a two-digit number by a one-digit number, regrouping tens as ones, remainders
- divide a three-digit number by a one-digit number, one or two regroupings, remainders
- divide by a one-digit number with regrouping, dividends with up to five digits, remainders
- use a shorter form for division
- use the standard algorithm to divide by a one-digit number
- use division to find an average
- divide amounts of money by a one-digit number using the standard form
- solve word problems involving division
- prepare a keychart to show the order of pressing the keys $\boxed{+}$, $\boxed{-}$, $\boxed{\times}$, and $\boxed{\div}$ on a calculator to solve a problem
- give the most reasonable answer to a word problem

Background

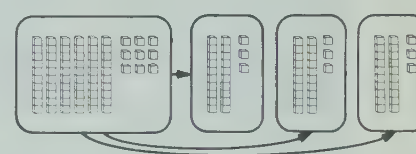
Of all the algorithms used in the four basic operations, the one for long division usually causes the most difficulty for students, probably because it requires a meaningful combination of skills in numeration and place value, in multiplication, and in subtraction. There are two types of relationships in division: the *partitive*, or sharing, concept relates to finding the size of each group when the number of groups is known, and the *quotitive*, or measurement, concept relates to finding the number of groups when the size of each group is known. The same algorithm is used for both types of division.

The most commonly used algorithm is sometimes known as the *distributive method* because it involves dividing each place value of the dividend in succession. It is important to think of place values in both the dividend and the quotient. Models are helpful in developing meaningful procedures, particularly in the partitive approach. Each stage of dividing the dividend results in a number for the quotient. At first, these numbers are stacked above the dividend in their proper places; later, single digits are written in the proper places of the quotient for each stage of dividing the dividend.

Partitive Division

3)$\overline{69}$ asks, "If 69, or 6 tens 9 ones, are shared among 3, how many will each get?"

```
  3 ┐
 20 ┘→ 23
3)69
  60    (3 × 2 tens)
   9
   9    (3 × 3 ones)
   0
```



2 tens 3 ones or 23 for each

Quotitive Division

3)$\overline{69}$ asks, "How many 3's are there in 69?"

```
  3 ┐
 20 ┘→ 23
3)69    How many 3's are there in 60?
  60    (3 × 20)
   9    How many 3's are there in 9?
   9    (3 × 3)
   0
```

It is relatively easy to divide two-place and three-place dividends when each place value is a multiple of the divisor as in 2)$\overline{468}$ and 3)$\overline{396}$. Expanded notation of the dividends reveals this clearly.

2)$\overline{400 + 60 + 8}$ 3)$\overline{300 + 90 + 6}$

However, when the first digit on the left of the dividend is less than the divisor and when there are remainders in any of the stages, a meaningful approach based on place value is required. It is necessary under these circumstances to interpret the dividends in more than one way. In A, the dividend 324 may be considered as 3 hundreds 2 tens 4 ones, but 3 (hundreds) ÷ 4 does not give a whole number as the quotient; 32 tens 4 ones is better because 32 (tens) ÷ 4 is possible. The stages of the procedure are 32 tens ÷ 4 = 8 tens (80), and 4 ones ÷ 4 = 1. In B, a similar renaming of the dividend 376 as 37 tens 6 ones is required in the first step, and a renaming of the remaining dividend, 2 tens 6 ones, as 26 ones is needed to complete the second step.

```
A     1 ┐              B     5 ┐
     80 ┘→ 81               70 ┘→ 75 R1
   4)324                  5)376
     320                    350
       4                     26
       4                     25
       0                      1
```

The transition from stacked quotients to single quotients requires emphasis on the place values represented by the individual digits in the quotients. At first, students need to know these values to find the products. As shown, the 6 in the quotient represents 6 tens and the product is recorded as 240.

$$\begin{array}{r l} 68 & \\ 4\overline{)275} & \\ \underline{240} & 4 \times 6 \text{ tens} = 24 \text{ tens, or } 240 \\ 35 & \\ \underline{32} & 4 \times 8 \text{ ones} = 32 \\ 3 & \end{array}$$

Later in the unit the students are shown how to perform the division in the standard algorithm without writing a zero (zeros) in the products when tens (hundreds) are involved in the quotient. Again, place values need to be emphasized so that the regrouping involved in the "bring down" step is carried out meaningfully. In the example shown, 25 hundreds are divided; the remainder of 1 hundred is combined with 6 tens of the dividend and renamed as 16 tens; then the remainder of 1 ten is combined with 4 ones and renamed as 14 ones.

$$\begin{array}{r l} 854 \text{ R2} & \\ 3\overline{)2564} & \\ \underline{24}\downarrow & 3 \times 8 \text{ (hundreds)} \\ 16 & 1 \text{ hundred } 6 \text{ tens (or 16 tens)} \\ \underline{15}\downarrow & 3 \times 5 \text{ (tens)} \\ 14 & 1 \text{ ten } 4 \text{ ones (or 14 ones)} \\ \underline{12} & 3 \times 4 \text{ (ones)} \\ 2 & \end{array}$$

Variations frequently occur in the price of an item, or in the time it takes to complete the same task a number of times, and it is usually of interest to know the *average* in such cases. For instance, the time required to drive to school on five successive days will vary because of a number of factors, but equalizing the longer and shorter times can provide one average length of time. The most common method of obtaining an average involves two operations: addition to find the sum of the numbers, and division of the sum by the number of addends. If the times in minutes for driving were 18, 23, 19, 22, and 18, the average would be found as shown. It should be pointed out that in some instances an addend may be zero, but it must be considered as an addend to obtain the correct divisor. For example, a student might have 2, 3, 0, 1, and 4 errors in mathematics on five days. In this case, the sum of 10 would be divided by 5, not 4, and the average would be 2.

$$\begin{array}{r} 18 \\ 23 \\ 19 \\ 22 \\ \underline{18} \\ 100 \end{array}$$

$100 \div 5 = 20$

Average time: 20 min

The lesson on problem solving in this unit draws attention to the fact that remainders in division in real life are treated differently according to the circumstances in which they arise. Students need to know both the how and the why of three methods of dealing with them:

(1) the remainder is ignored;

(2) the remainder is recorded as a whole number, as a decimal, or as a fraction;

(3) the remainder suggests increasing the quotient by 1.

On page 98, Ex. 4 illustrates the first method, Ex. 5 illustrates the second method, and Ex. 1 and 2 illustrate the third method.

Teaching Strategies

If the quotitive approach has been used in the early development of the division algorithm, the students may be accustomed to showing their work by "side stacking". It is advisable to make a gradual transition at this time to placing the quotients above the dividends, since this form is much easier to use in dividing decimals (Unit 12). Such a transition may be carried out in three stages as shown in C, D, and E. If, in moving from D to E, some students have more errors, it may be necessary for them to continue writing zeros in the products when the digits in the quotient represent hundreds and tens, as shown in D.

$$\begin{array}{l r l l r l r} \text{C} & & & \text{D} & & \text{E} & \\ & 5 & \rceil & & & & \\ & 40 & \rangle \!\leftarrow 145 & & & & \\ & 100 & \rfloor & & 145 & & 145 \\ & 3\overline{)435} & & & 3\overline{)435} & & 3\overline{)435} \\ & \underline{300} & & & \underline{300} & & \underline{3} \\ & 135 & & & 135 & & 13 \\ & \underline{120} & & & \underline{120} & & \underline{12} \\ & 15 & & & 15 & & 15 \\ & \underline{15} & & & \underline{15} & & \underline{15} \\ & 0 & & & 0 & & 0 \end{array}$$

Differences in achievement will probably be noted as students work through the lessons in this unit. As stated earlier, the division algorithm involves a variety of skills in which some students may encounter difficulty. It is important, therefore, for the results of each new lesson to be checked and to be diagnosed accurately for the causes of errors. If the errors seem to be of the same type, it may be necessary to reteach the lesson. If more attention must be given to place values, models of thousands, hundreds, tens, and ones are recommended. If the errors seem to be of several different types, the students should be grouped and provided with specific help according to their needs. Suggestions are offered in the lesson outlines and in the *Related Activities* to provide for students who need assistance, as well as for those who need extension and enrichment of the topics.

The lesson on the use of the calculator emphasizes making a keychart to solve a problem. This requires a careful study, not only of the numbers, but also of the relationships between them, which are indicated in the wordings of the problem situations. If calculators are not available in the classroom, the preparation of keycharts can still be of value to the students in developing problem-solving skills. Of the six problems presented on page 97, four are two-step problems. Without a calculator, a student must decide what numbers and what operation to use first and then what operation to use with that result and another number.

Materials

counters (optional)
models for thousands, hundreds, tens, and ones (optional)
large sheets of paper
calculators (optional)

Vocabulary

division	dividend	$\div$, $\overline{)\quad}$
divide	quotient	average
divisor	remainder	gram, g

LESSON OUTCOME

Use multiplication to divide, divisors and quotients to 9, remainders

Materials

counters (optional)

Vocabulary

division, divide, divisor, dividend, quotient, remainder, ÷, $\overline{)}$

Prerequisite Skills

Complete the basic multiplication facts; write the missing factor in a multiplication fact

Checking Prerequisite Skills

Multiply.

1. 3	**2.** 5	**3.** 6	**4.** 9
7	4	6	8
21	20	36	72

Complete.

5. $4 \times \square = 32$ 8

6. $9 \times \square = 36$ 4

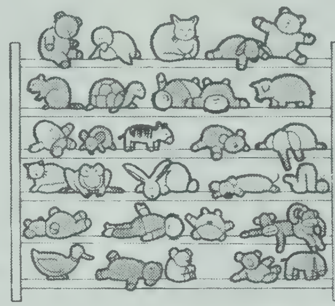
7. $7 \times \square = 35$ 5

8. $3 \times \square = 27$ 9

5 DIVISION

Using Multiplication to Divide

Angela had 6 shelves in her room for her 30 animals. If she wanted to keep the same number on each shelf, how many animals should she keep on each shelf?



Divide 30 by 6.

For $6\overline{)30}$,

think $6 \times \square = 30$

$6 \times 5 = 30$

Write $\begin{array}{r} 5 \\ 6\overline{)30} \end{array}$

There should be 5 animals on each shelf.

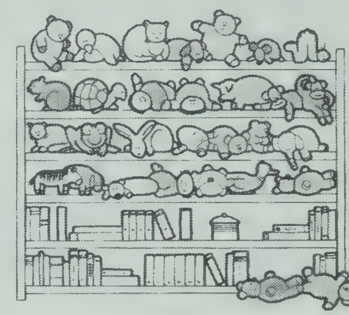
Later, when Angela needed two of her shelves for other things, she had just four shelves to use for her animals.

Divide 30 by 4.

For $4\overline{)30}$,

think $4 \times 7 = 28$

$4 \times 8 = 32$. . .too many!



Write $\begin{array}{r} 7 \\ 4\overline{)30} \\ 28 \\ \hline 2 \end{array}$

7 — animals for each shelf

28 — animals on shelves

2 — animals remaining

Angela can place 7 animals on each shelf and there will be 2 left over.

Here is another way to show the remainder in a division.

$\begin{array}{r} 7 \text{ R2} \\ 4\overline{)30} \\ 28 \\ \hline 2 \end{array}$

quotient — 7; remainder — R2; divisor — 4; dividend — 30

80

LESSON ACTIVITY

Before Using the Pages

- Present a few oral exercises, similar to the following, to prepare the students for the concept of division from the partitive (sharing) aspect. Some students may need to use counters or draw diagrams. Have students explain how they obtained their answers.
  1. There are 20 dots in 5 equal rows. How many dots are there in each row?
  2. With 28 cards and 4 players sharing them equally, how many cards are there for each player?
  3. If there are 24 players and 3 teams with the same number of players on each team, how many players are there on each team?

  Discuss the ways in which diagrams and counters can be used to help find the answers. The inefficiency of the method becomes apparent as the size of the numbers increases. Lead the students to suggest that thinking of multiplication is a more efficient approach; for instance, in the first exercise above, $5 \times \square = 20$ (5 times what number equals 20) is a related multiplication sentence.

  Ask students to show how to indicate that 28 shared equally by 4 is 7, using mathematical symbols.

$$\begin{array}{r} 7 \\ 4\overline{)28} \end{array} \text{ or } 28 \div 4 = 7$$

Ask for the name of the operation represented by the symbols ÷ and $\overline{)}$.

Using the Pages

- The worked examples review the partitive aspect of division and emphasize the use of multiplication for division. Division symbols and terminology as well as the concept of remainders are reviewed.

  Guide the students through the examples. Note that a division such as $6\overline{)30}$ is read "thirty divided by six". Pay particular attention to the format for the division $4\overline{)30}$ because it suggests the standard form for long division. Also mention that the remainder is shown to the right of the quotient. Emphasize the step of multiplying 4 and 7 to

Working Together

Answers are given on page T368

Give the multiples of each of these, from 1 × to 9 ×.

1. 2   2. 3   3. 4   4. 5   5. 6   6. 7   7. 8   8. 9

Each dividend is a multiple of the divisor. Complete the matching multiplication fact.

9. For 8)24 (3), use 8 × 3 = 24.

10. For 3)18 (6), use 3 × 6 = 18.

Find the quotient. Give the multiplication fact you use.

11. 2)10 5   2 × 5 = 10

12. 7)49 7   7 × 7 = 49

13. 48 ÷ 6 8   6 × 8 = 48

Give the greatest multiple of 7 that is less than the dividend.

Example: For 7)33, use 7 × 4 = 28.

14. 7)22   7 × 3 = 21

15. 7)53   7 × 7 = 49

Complete.

16. 7)38 5
35 ← 7 × 5
3

17. 6)40 6
36 ← 6 × 6
4

Find the quotient and the remainder. Give the multiplication fact you use.

18. 3)16 5 R1   3 × 5 = 15

19. 52 ÷ 6 8 R4   6 × 8 = 48

20. 7)26 3 R5   7 × 3 = 21

21. 28 ÷ 4 7 R0   4 × 7 = 28

22. 9)26 2 R8   9 × 2 = 18

Exercises

Write the six quotients for each row. Which row can you do the fastest?

1.	2)6 3	3)15 5	4)28 7	2)16 8	3)9 3	4)32 8
2.	18 ÷ 2 9	18 ÷ 3 6	18 ÷ 6 3	24 ÷ 3 8	24 ÷ 4 6	24 ÷ 8 3
3.	3)12 4	4)12 3	5)30 6	6)30 5	7)56 8	8)56 7
4.	16 ÷ 4 4	25 ÷ 5 5	36 ÷ 6 6	49 ÷ 7 7	64 ÷ 8 8	81 ÷ 9 9
5.	7)28 4	5)20 4	9)36 4	2)8 4	8)32 4	6)24 4
6.	48 ÷ 8 6	35 ÷ 7 5	72 ÷ 9 8	42 ÷ 7 6	72 ÷ 8 9	63 ÷ 9 7

Divide. Show the quotient and the remainder.

7. 6)33 5 R3   8. 2)15 7 R1   9. 8)37 4 R5   10. 7)43 6 R1   11. 4)19 4 R3   12. 8)79 9 R7
13. 5)39 7 R4   14. 9)71 7 R8   15. 6)40 6 R4   16. 3)11 3 R2   17. 7)61 8 R5   18. 9)52 5 R7
19. 43 ÷ 8 5 R3   20. 15 ÷ 4 3 R3   21. 47 ÷ 6 7 R5   22. 26 ÷ 3 8 R2   23. 39 ÷ 9 4 R3   24. 34 ÷ 4 8 R2
25. 21 ÷ 6 3 R3   26. 19 ÷ 2 9 R1   27. 20 ÷ 3 6 R2   28. 46 ÷ 5 9 R1   29. 53 ÷ 7 7 R4   30. 60 ÷ 8 7 R4

81

RELATED ACTIVITIES

• Students may benefit from writing families of related multiplication and division facts for given arrays. Copies of page T396 may be helpful for presenting the arrays.

3 × 4 = 12   3)12 (4)

4 × 3 = 12   4)12 (3)

• Prepare number wheels and tables from copies of page T390 to provide practice in writing missing factors and completing division facts.

÷	4
16	
28	
32	

• On a large sheet of paper, write the division exercise 4)30 with the terms for division as shown at the bottom of page 80. Display it for several days to reinforce the language and format of division.

• Help to familiarize the students with the language of division through oral exercises similar to the following, referring to the exercises on page 81.

Name the dividend in Ex. 17.
Name the divisor in Ex. 21.
Name the quotient in Ex. 25.
Name the remainder in Ex. 25.

show that there are 28 animals on the shelves. Have students explain the meaning of each of the terms *divisor*, *dividend*, *quotient*, and *remainder*.

Working Together: Ex. 1-8 provide a review of basic multiplication facts and can help students understand the concept in Ex. 9-13 that the dividend is a multiple of the divisor. Point out that this is also true for a division in which the remainder is zero. Note the appearance of the symbol ÷ in Ex. 13. Ex. 14-17 review the steps of dividing when the dividend is not a multiple of the divisor. The multiples listed for Ex. 1-8 are helpful in completing these exercises. For Ex. 18-22, the multiplication fact that is used can be shown to the right of the appropriate line in the division, as suggested in Ex. 16 and 17. This step is of particular importance in preparing for the next lesson.

Exercises: You may wish to have the students rule a page to show six rows and six columns and write the quotients in a chart for Ex. 1-6. For these, the emphasis is on rapid recall of related multiplication facts; thus, it is not necessary for them to copy the exercises. Ex. 2-5 are of particular interest and should be discussed orally after their completion for the following concepts. Ex. 2 demonstrates that for any given dividend, the greater the divisor, the smaller the quotient. The divisions of Ex. 3 taken in pairs involve a family of related multiplication and division facts, for example, 3)12 (4), 3 × 4 = 12, 4)12 (3), 4 × 3 = 12. For each division of Ex. 4, the divisor and the quotient are equal. For Ex. 5, the quotient for each division is 4.

You may wish to remind the students to write the divisions of Ex. 19-30 using the symbol).

Assessment

Write the quotient.

1. 28 ÷ 4 7   2. 42 ÷ 7 6   3. 36 ÷ 9 4

4. 5)45 9   5. 8)16 2

Divide. Show the quotient and the remainder.

6. 3)13 4 R1   7. 6)40 6 R4   8. 2)17 8 R1   9. 9)66 7 R3   10. 7)37 5 R2

Pages 82-83

LESSON OUTCOME

Divide a two-digit or a three-digit number by a one-digit number, no regrouping, remainder zero

Materials

models for hundreds, tens, and ones

Prerequisite Skills

Multiply a multiple of ten by a one-digit number, products to 90; multiply a multiple of one hundred by a one-digit number, products to 900; use multiplication to complete basic division facts

Checking Prerequisite Skills

Multiply.

1. 2×40 80 **2.** 5×10 50

3. 1×200 200 **4.** 2×300 600

Divide. Write the multiplication fact you use.

5. $\overset{2}{4\overline{)8}}$ $4 \times 2 = 8$ **6.** $\overset{1}{6\overline{)6}}$ $6 \times 1 = 6$ **7.** $\overset{3}{3\overline{)9}}$ $3 \times 3 = 9$

Sharing Hundreds, Tens, and Ones

Divide 648 by 2.

6 hundreds 4 tens 8 ones

For $2\overline{)648}$, share the 6 hundreds first.

Think

$2 \times 3 = 6$

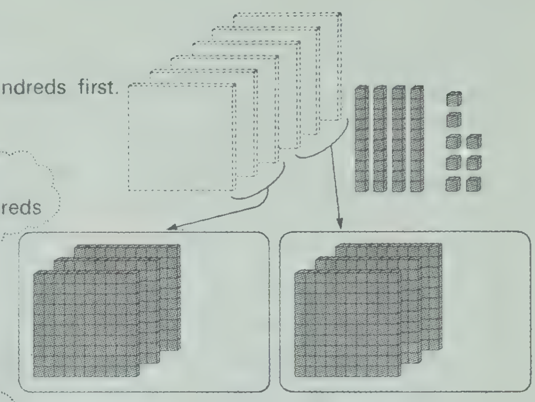
2×3 hundreds = 6 hundreds

$2 \times 300 = 600$

Write

$$\begin{array}{r} \boxed{300} \\ \boxed{2}\overline{)648} \\ \underline{\boxed{600}} \\ 48 \end{array}$$

still to share



Next, share the 4 tens.

Think

$2 \times 2 = 4$

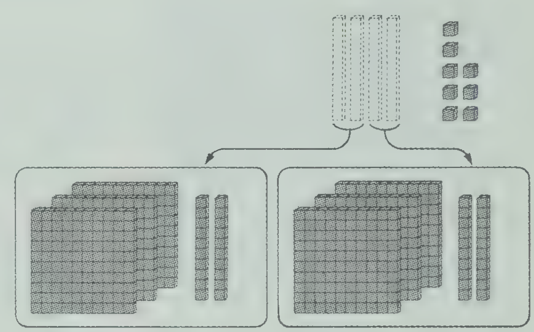
2×2 tens = 4 tens

$2 \times 20 = 40$

Write

$$\begin{array}{r} \boxed{20} \\ 300 \\ \boxed{2}\overline{)648} \\ \underline{600} \\ 48 \\ \underline{\boxed{40}} \\ 8 \end{array}$$

still to share



82

LESSON ACTIVITY

Before Using the Pages

- Since the division exercises on these pages do not involve regrouping, the steps of dividing each place value of the dividend can be seen in terms of basic division facts. For example, the basic division fact $\overset{2}{4\overline{)8}}$ is applied in the following extensions, and emphasis is still given to using multiplication to divide.

$\overset{2 \text{ tens}}{4\overline{)8 \text{ tens}}}$	$\overset{2 \text{ hundreds}}{4\overline{)8 \text{ hundreds}}}$
$\overset{20}{4\overline{)80}}$	$\overset{200}{4\overline{)800}}$

Working with models will help students to understand the steps that are written in the division algorithm. Write exercises such as $3\overline{)936}$, $2\overline{)842}$, and $5\overline{)55}$ on the board. Have students work in small groups sharing models of hundreds, tens, and ones. For instance, ask a group of three students to divide $3\overline{)936}$ by sharing 9 hundreds, 3 tens, and 6 ones. Direct them to share the hundreds first, then the tens, then the ones, explaining that establishing this order now will prepare them for divisions involving regrouping later. Encourage them to think of multiplication to assist in the process of sharing. For example, rather than share the 9 hundreds one by one, students can think, $3 \times 3 = 9$, 3×3 hundreds = 9 hundreds, and each of the three students may take three hundreds in one step. Help the students understand the division process by questioning them while they work. For example, ask them a series of questions similar to the following.

"How many hundreds are there to share?"
"How many hundreds will each receive?"
"What number does this represent?"
"How many hundreds have been shared?"
"What number does this represent?"
"What is left to share?"

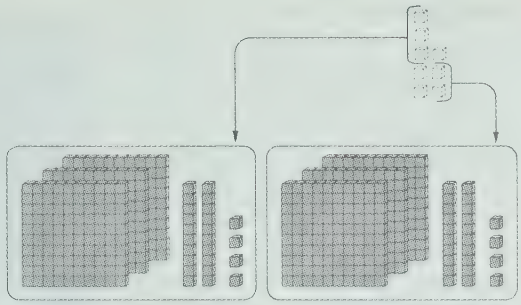
Use the same procedure to consider tens, and then ones.

Then, share the 8 ones.

Think $2 \times 4 = 8$

Write

$$\begin{array}{r} \boxed{4} \\ 20 \\ 300 \\ 2\overline{)648} \\ \underline{600} \\ 48 \\ \underline{40} \\ 8 \\ \underline{\boxed{8}} \\ 0 \end{array}$$



Add for the quotient.

$$\begin{array}{r} 4 \\ 20 \\ 300 \\ 2\overline{)648} \\ \underline{600} \\ 48 \\ \underline{40} \\ 8 \\ \underline{8} \\ 0 \end{array} \rightarrow 324$$

When 648 is divided by 2, the quotient is 324.

Working Together

How many hundreds, how many tens, and how many ones are named in each?

1. 402 4 hundreds 0 tens 2 ones
2. 720 7 hundreds 2 tens 0 ones
3. 86 0 hundreds 8 tens 6 ones

Divide.

4. $3\overline{)690}$ 230
5. $2\overline{)264}$ 132
6. $804 \div 4$ 201

Exercises

Divide.

1. $2\overline{)448}$ 224
2. $3\overline{)366}$ 122
3. $2\overline{)680}$ 340
4. $4\overline{)448}$ 112
5. $7\overline{)77}$ 11
6. $2\overline{)288}$ 144
7. $2\overline{)800}$ 400
8. $3\overline{)903}$ 301
9. $484 \div 4$ 121
10. $360 \div 3$ 120
11. $668 \div 2$ 334
12. $939 \div 3$ 313
13. $606 \div 6$ 101
14. $393 \div 3$ 131
15. $842 \div 2$ 421
16. $80 \div 4$ 20
17. $636 \div 3$ 212

Choose one operation sign and use it in each ▩.
Choose one operation sign and use it in each △.

How many ways can you choose the operations so that both paths give the same result? 4

$$\begin{array}{ccccc} 6 & \rightarrow & \boxed{}\ 3 & \rightarrow & ? \\ \downarrow \triangle\ 2 & & & & \downarrow \triangle\ 2 \\ ? & \rightarrow & \boxed{}\ 3 & \rightarrow & ? \end{array}$$

Operation signs
+
−
×
÷

▩	△
+	−
−	+
×	−
−	×

try this

RELATED ACTIVITIES

- Exercises similar to the following can help students with the sequence of steps in the written format.

$$\begin{array}{rl} \square & \\ \square & \\ \square & \\ 4\overline{)884} & \\ \underline{800} & \leftarrow 4 \times \square \\ 84 & \\ \underline{80} & \leftarrow 4 \times \square \\ 4 & \\ \underline{4} & \leftarrow 4 \times \square \\ 0 & \end{array}$$

Add to find the quotient.

- Demonstrate that multiplication can be used to check the quotient in a division exercise. Choose an exercise from page 83 and have students multiply the quotient and the divisor. If the product is equal to the dividend, the quotient is correct.

Ex. 10 $\begin{array}{r} 120 \\ 3\overline{)360} \end{array}$ $\begin{array}{r} 120 \\ \underline{\times \quad 3} \\ 360 \end{array}$

Using the Pages

- The worked example relates each step of the division to the sharing concept and emphasizes the use of multiplication for division. Emphasize that division is carried out place by place for each place value in turn from left to right. Question the students as you guide them through the example, paying particular attention to the numerals highlighted by red in each step. The sequence of questions might be as follows.
  "How many hundreds, tens, and ones are there for 648?"
  "What is shared first?"
  "What does 300 show?"
  "What numbers are multiplied to give 600?"
  "What is left to share?"
  "How is this number obtained?"
  "What is shared next?"
  Draw attention to the bracket and arrow that relate the partial quotients to the final quotient, 324.

Working Together: Ex. 1-3 review the place value of each digit in a numeral. Students apply that skill in dividing place by place in Ex. 4-6. Note that in Ex. 4 there are no ones to share, and in Ex. 6, there are no tens to share. For those exercises there will be just two partial quotients. Develop other similar exercises on the board as needed.

Exercises: The students will need to be cautioned to place their exercises carefully to allow space for writing partial quotients above a dividend.

Try This: The students will likely use a "guess and test" strategy. Help them organize the guesses they have tested to prevent repetition. These may be shown in a list like the following.

If ▩ is +, then △ may be _____.
If ▩ is −, then △ may be _____.
If ▩ is ×, then △ may be _____.
If ▩ is ÷, then △ may be _____.

Assessment

Divide.

1. $2\overline{)442}$ 221
2. $39 \div 3$ 13
3. $5\overline{)505}$ 101
4. $8\overline{)880}$ 110

Pages 84-85

LESSON OUTCOME

Divide a two-digit number by a one-digit number, regrouping tens as ones, remainders

Materials

models for tens and ones

Prerequisite Skills

Find the quotient and the remainder, divisors and quotients to 9; divide a two-digit number by a one-digit number, no regrouping; regroup with numbers to 99

Checking Prerequisite Skills

Find the quotient and the remainder.

1. $3\overline{)8}$ 2 R2
2. $5\overline{)7}$ 1 R2
3. $4\overline{)19}$ 4 R3
4. $3\overline{)29}$ 9 R2
5. $3\overline{)36}$ 12
6. $5\overline{)55}$ 11
7. $4\overline{)80}$ 20
8. $2\overline{)68}$ 34

Complete.

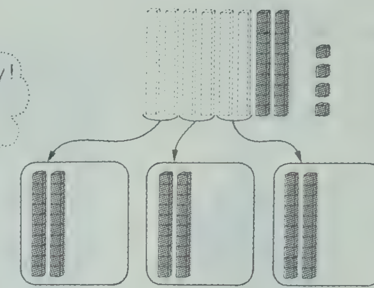
9. 1 ten 0 ones = 10 ones
10. 1 ten 9 ones = 19 ones
11. 2 tens 4 ones = 24 ones
12. 3 tens 0 ones = 30 ones

Regrouping Tens

Divide 84 by 3.
84 equals 8 tens 4 ones.

For $3\overline{)84}$, share the 8 tens first.

3 × 2 = 6, 3 × 3 = 9. . . too many!
3 × 2 tens = 6 tens
3 × 20 = 60



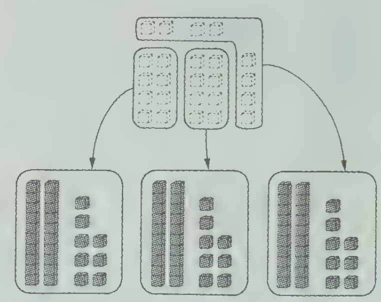
Write

$$\begin{array}{r} 20 \\ 3\overline{)84} \\ \underline{60} \\ 24 \end{array}$$

still to share

Think of the 2 tens 4 ones that remain as 24 ones. Then share the 24 ones.

3 × 8 = 24



Write

$$\begin{array}{r} 8 \\ 20 \\ 3\overline{)84} \\ \underline{60} \\ 24 \\ \underline{24} \\ 0 \end{array}$$

Add for the quotient.

$$\begin{array}{r} 8 \\ 20 \\ 3\overline{)84} \\ \underline{60} \\ 24 \\ \underline{24} \\ 0 \end{array} \quad 8 + 20 \rightarrow 28$$

When 84 is divided by 3, the quotient is 28.

84

LESSON ACTIVITY

Before Using the Pages

- Have the students use models of tens and ones to demonstrate $2\overline{)42}$. Ask one student to explain the procedure while another student reviews the written format on the board. In this way, each stage of the sharing can be associated with the corresponding step in the written form. Use a similar procedure for $3\overline{)42}$. Allow the students to discover how this example differs from the first one. Ask, for example, "How many tens are there?", "How many persons are sharing?", "Are there enough tens for each to have one ten? for each to have two tens?", and "What is a fair way to share the remaining ten and the two ones?" The students will likely suggest regrouping 1 ten as 10 more ones and sharing 12 ones.

  Have the students use models to complete other divisions such as $4\overline{)52}$ and $3\overline{)48}$. Include divisions that have remainders greater than zero, such as $2\overline{)31}$ and $5\overline{)62}$.

If the division $2\overline{)42}$ is on the board, the students may be able to suggest the steps for writing the division $3\overline{)42}$.

$$\begin{array}{r} 1 \\ 20 \\ 2\overline{)42} \\ \underline{40} \\ 2 \\ \underline{2} \\ 0 \end{array} \; \rightarrow 21 \qquad \begin{array}{r} 4 \\ 10 \\ 3\overline{)42} \\ \underline{30} \\ 12 \\ \underline{12} \\ 0 \end{array} \; \rightarrow 14$$

Using the Pages

- The worked example relates each step of the division $3\overline{)84}$ to the concept of sharing, but emphasizes the use of multiplication in division. The red on the numerals and diagrams highlights each step of the division and helps to associate the mathematical computation with the sharing process.

Exercises

Before playing the game the 3 girls have to share the chips equally.

1. There are 81 blue chips. How many will each girl get? 27
2. There are 49 red chips. How many will each girl get? 16 How many will be left over? 1
3. There are 74 white chips. How many will each girl get? 24 How many will be left over? 2

Divide.

4. $3\overline{)54}$ 18 5. $2\overline{)37}$ 18 R1 6. $4\overline{)52}$ 13
7. $8\overline{)44}$ 5 R4 8. $6\overline{)82}$ 13 R4 9. $5\overline{)85}$ 17
10. $2\overline{)74}$ 37 11. $4\overline{)66}$ 16 R2 12. $7\overline{)98}$ 14

13. 99 ÷ 4 24 R3 14. 96 ÷ 2 48 15. 79 ÷ 3 26 R1
16. 72 ÷ 6 12 17. 76 ÷ 4 19 18. 53 ÷ 2 26 R1
19. 90 ÷ 8 11 R2 20. 87 ÷ 3 29 21. 97 ÷ 7 13 R6

You can multiply the divisor and the quotient, then add the remainder, to check your work.

× then +

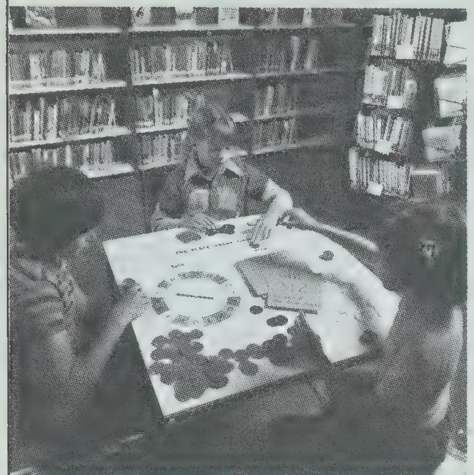
Example: For $4\overline{)99}$ 24 R3

multiply → 4 × 24 = 96

then add → 96 + 3 = 99

If your result does not match the dividend, there is a mistake in your work.

22. Check your work for Exercises 1 to 21.



Working Together

Give the first multiplication fact you can use to find the quotient.

Example: For $2\overline{)92}$, use $\boxed{2 \times 4 = 8}$.
2 × 4 tens = 8 tens, or 2 × 40 = 80.

1. $2\overline{)52}$ 2×2=4 2. $4\overline{)64}$ 4×1=4 3. $3\overline{)76}$ 3×2=6

Complete.

4. $6\overline{)84}$ (10 + 4 → 14)
60
24
24
0

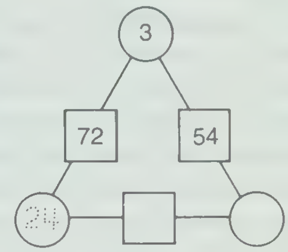
5. $3\overline{)70}$ (20 + 3 → 23 R 1)
60
10
9
1

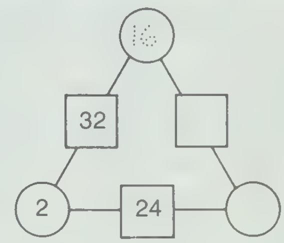
Divide.

6. $4\overline{)92}$ 23 7. 38 ÷ 2 19 8. $5\overline{)72}$ 14 R2

RELATED ACTIVITIES

- For further practice, you may wish to have students complete Ex. 1-12 on page 336.
- Have students practice multiplication and division by completing diagrams similar to the following on copies of page T 390. Factors are shown in the circles; products are shown in the squares.





Review with the students that division is carried out place by place, beginning with the greatest place value. Guide them through the solution, paying particular attention to the numerals highlighted by red. A sequence of questions similar to those suggested for the worked example on pages 82 and 83 may be used here. Take the opportunity to review the terminology of division by having students identify the quotient and the remainder and by having them give the names that identify 84 and 3 in the division.

Working Together: Before the students begin Ex. 1-3, discuss the example that precedes the exercises. Have students explain why 2 × 4 = 8 is the *first* multiplication fact to apply in sharing 9 tens between 2 (as opposed to 2 × 3 = 6 or 2 × 5 = 10, for example). This helps to explain the word "first" in the instruction for these exercises.

Ex. 4 and 5 provide partially completed divisions. Students should explain the steps that have been completed and then continue the division. Use other similar examples as required. For Ex. 6-8, the students are required to complete all the steps independently.

Exercises: A brief discussion of the photograph on page 85 can prepare the students for Ex. 1-3. For example, it appears that the color of a chip identifies its place value and that the chips are shared first to begin the game.

Draw the students' attention to the description of the use of multiplication to check division. Have a student show all the steps on the board for the division $4\overline{)99}$. Have another student explain and show how to use multiplication to check the division.

Assessment

Divide.

1. $3\overline{)72}$ 24 **2.** $4\overline{)65}$ 16 R1 **3.** $7\overline{)84}$ 12 **4.** $5\overline{)87}$ 17 R2

LESSON OUTCOME

Divide a three-digit number by a one-digit number, one or two regroupings, remainders

Materials

models for hundreds, tens, and ones

Prerequisite Skills

Find the quotient and the remainder, quotients and divisors to 9; divide a three-digit number by a one-digit number, no regrouping; divide a two-digit number by a one-digit number, regrouping, remainders; regroup with numbers to 999

Checking Prerequisite Skills

Find the quotient and the remainder.

1. $2\overline{)7}$ 3 R1	2. $8\overline{)51}$ 6 R3	3. $9\overline{)42}$ 4 R6
4. $6\overline{)50}$ 8 R2	5. $7\overline{)20}$ 2 R6	6. $5\overline{)36}$ 7 R1
7. $2\overline{)482}$ 241	8. $3\overline{)693}$ 231	9. $8\overline{)800}$ 100
10. $2\overline{)78}$ 39	11. $8\overline{)92}$ 11 R4	12. $7\overline{)81}$ 11 R4

Complete.

13. 5 hundreds 1 ten = 51 tens
14. 1 hundred 4 tens = 14 tens
15. 6 hundreds 6 tens = 66 tens
16. 2 hundreds 0 tens = 20 tens

Regrouping Hundreds, Regrouping Tens

When 4 persons share 520 building pieces equally, each will get more than 100 pieces.

To find the exact amount, divide 520 by 4.

When 6 persons share 520 building pieces equally, each will get fewer than 100 pieces.

To find the exact amount, divide 520 by 6.

520 = 5 hundreds 2 tens 0 ones

For $4\overline{)520}$, divide the 5 hundreds first.

4 × 1 = 4, 4 × 2 = 8 . . . too many!
4 × 1 hundred = 4 hundreds
4 × 100 = 400

Write

$$\begin{array}{r} 100 \\ 4\overline{)520} \\ \underline{400} \\ 120 \end{array}$$

Think of the 1 hundred 2 tens that remain as 12 tens. Then divide the 12 tens.

4 × 3 = 12
4 × 3 tens = 12 tens
4 × 30 = 120

Write

$$\begin{array}{r} 30 \\ \underline{100} \\ 4\overline{)520} \\ \underline{400} \\ 120 \\ \underline{120} \\ 0 \end{array} \quad \rightarrow 130$$

Add for the quotient.

no more to divide

When 4 share the building pieces equally, each gets 130 pieces.

For $6\overline{)520}$, there are not enough hundreds to give 1 hundred to each of the 6. Think of 5 hundreds 2 tens as 52 tens. Then divide the 52 tens.

6 × 8 = 48, 6 × 9 = 54 . . . too many!
6 × 8 tens = 48 tens
6 × 80 = 480

Write

$$\begin{array}{r} 80 \\ 6\overline{)520} \\ \underline{480} \\ 40 \end{array}$$

Think of the 4 tens 0 ones that remain as 40 ones. Then divide the 40 ones.

6 × 6 = 36
6 × 7 = 42 . . . too many!

Write

$$\begin{array}{r} 6 \\ \underline{80} \\ 6\overline{)520} \\ \underline{480} \\ 40 \\ \underline{36} \\ 4 \end{array} \quad \rightarrow 86\ \text{R4}$$

Add and write the remainder to finish.

When 6 share equally, each gets 86 pieces with 4 pieces left over.

86

LESSON ACTIVITY

Before Using the Pages

- Have students use models of hundreds, tens, and ones to find the quotients for exercises similar to the following. Remind them to share the hundreds first, then the tens, then the ones, and to think of multiplication to help in the sharing. The exercises suggested below are selected so that students deal first with no regrouping, then with one regrouping, and finally with two regroupings.

1. $3\overline{)693}$	2. $4\overline{)852}$
3. $2\overline{)520}$	4. $3\overline{)243}$
5. $2\overline{)532}$	6. $2\overline{)333}$

Have several students explain the regrouping that was needed to carry out the sharing process for each exercise. Some students may be able to write the divisions and compare the results with those found by sharing models.

Using the Pages

- There are two worked examples and each involves the same dividend, 520. For $4\overline{)520}$, there will be one regrouping; for $6\overline{)520}$, there will be two regroupings.

Begin with a brief discussion of the photograph on page 87. Some students may have a set of building pieces similar to the ones shown in the photograph. Point out that four students appear to be sharing the pieces and that two other students appear to be observing. Ask if each person would receive more pieces if four persons were sharing or if six persons were sharing. Then have students read the two problems at the top of page 86.

Guide the students through the development of $4\overline{)520}$ first and then of $6\overline{)520}$. Have students help to explain the steps. Note that for $6\overline{)520}$, there are not enough hundreds for each of 6 groups to have at least 1 hundred. In using the models, students would regroup 5 hundreds as 50 more tens. In the division algorithm, the corresponding procedure is given by the statement, "Think of 5 hundreds 2 tens as 52 tens."

Working Together

Give the first multiplication fact you can use to find the quotient.

Example: For 6)468, use $\boxed{6 \times 7 = 42}$, 6 × 7 tens = 42 tens, or 6 × 70 = 420.

1. 2)938 2 × 4 = 8 2. 7)546 7 × 7 = 49 3. 4)902 4 × 2 = 8

Complete.

```
   7
  40
 200
4. 3)741
   600 ← 3 × 200
   141
   120 ← 3 × 40
    21
    21 ← 3 × 7
     0
```

Add to find the quotient.

Divide.

5. 5)890 178 6. 512 ÷ 8 64 7. 6)808 134 R4

Exercises

Divide.

1. 4)348 87 2. 3)449 149 R2 3. 7)875 125
4. 9)306 34 5. 8)704 88 6. 2)553 276 R1
7. 6)341 56 R5 8. 7)999 142 R5 9. 5)380 76

10. 663 ÷ 5 132 R3 11. 152 ÷ 8 19
12. 702 ÷ 6 117 13. 394 ÷ 7 56 R2
14. 168 ÷ 3 56 15. 190 ÷ 2 95
16. 666 ÷ 8 83 R2 17. 503 ÷ 4 125 R3
18. 711 ÷ 9 79 19. 824 ÷ 6 137 R2

How many building blocks will each get

20. when 735 are divided among 3? 245
21. when 735 are divided among 5? 147
22. when 735 are divided among 8? 91 R7

Remember, you can multiply the divisor and quotient, then add the remainder to check a division.

RELATED ACTIVITIES

- For further practice, you may wish to have students complete Ex. 13-20 on page 336.
- Have students divide a given three-digit dividend by each of the numbers 1 to 9 in turn and observe the decreasing quotients.

1)392 = 392	2)392 = 196	3)392 = 130 R2
4)392 = 98	5)392 = 78 R2	6)392 = 65 R2
7)392 = 56	8)392 = 49	9)392 = 43 R5

- Students who are experiencing difficulties may find it helpful to complete exercises similar to the following.

1. 4)9 2 R1	4)90 22 R2	4)94 23 R2
2. 3)21 7	3)210 70	3)216 72
3. 6)8 1 R2	6)80 13 R2	6)804 134

Working Together: Ex. 1-3 deal with selecting the first multiplication fact that can be used to find the quotient. An example is provided. Ex. 4 helps students follow the sequence of steps in the process of long division. Students who may not be ready for Ex. 5-7 will benefit from other exercises similar to Ex. 4. The divisions in Ex. 1-3 may be used for this purpose.

Exercises: Ensure that the students place their exercises to allow space for showing the "stacked" quotient above the dividend for each division. Draw attention to the reminder in the "thought cloud" at the bottom of page 87. If you prefer, name specific exercises for the students to check.

Ex. 20-22 provide an opportunity to develop the concept that for a constant dividend the quotient decreases as the divisor increases. Ask the students to predict which division will give the greatest quotient and which the least quotient for Ex. 20-22 before they begin the exercises.

Assessment

Divide.

1. 6)726 121 2. 4)893 223 R1 3. 5)360 72 4. 9)843 93 R6

LESSON OUTCOME

Divide by a one-digit number with regrouping, dividends with up to five digits, remainders

Materials

models for thousands, hundreds, tens, and ones (optional)

Prerequisite Skills

Find the quotient and the remainder, quotients and divisors to 9; divide a three-digit number by a one-digit number; regroup with numbers to 99 999

Checking Prerequisite Skills

Find the quotient and the remainder.

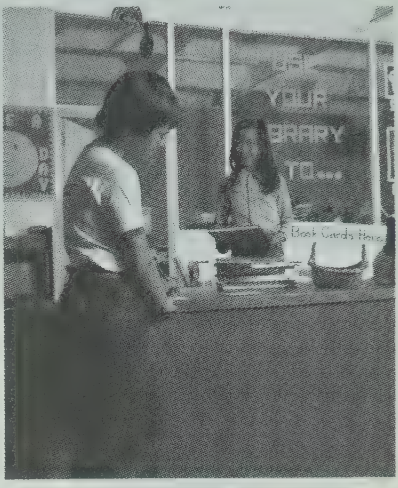
1. $4\overline{)17}$ 4 R1
2. $8\overline{)12}$ 1 R4
3. $6\overline{)52}$ 8 R4
4. $3\overline{)29}$ 9 R2
5. $7\overline{)40}$ 5 R5
6. $9\overline{)83}$ 9 R2
7. $6\overline{)945}$ 157 R3
8. $3\overline{)728}$ 242 R2
9. $9\overline{)872}$ 96 R8

Complete.

10. 1 thousand 1 hundred = 11 hundreds
11. 3 thousands 2 hundreds = 32 hundreds
12. 2 ten thousands 7 thousands = 27 thousands
13. 5 ten thousands 6 thousands = 56 thousands

Dividing by a One-Digit Number

Billy's library books have a total of 1134 pages. They must be returned in 3 weeks. If Billy reads about the same number of pages each week, how many pages would he have to read each week to finish the books?



Divide 1134 by 3.

For $3\overline{)1134}$, there are not enough thousands for 1 thousand pages to be read each week. Think of 1 thousand 1 hundred as 11 hundreds. Then divide the 11 hundreds.

$3 \times 3 = 9$, $3 \times 4 = 12$. . . too many!
3×3 hundreds = 9 hundreds
$3 \times 300 = 900$

Write
$$\begin{array}{r} 300 \\ 3\overline{)1134} \\ \underline{900} \\ 234 \end{array}$$

Think of the 2 hundreds 3 tens 4 ones that remain as 23 tens 4 ones. Then, divide the 23 tens.

$3 \times 7 = 21$, $3 \times 8 = 24$. . . too many!
3×7 tens = 21 tens
$3 \times 70 = 210$

Write
$$\begin{array}{r} 70 \\ 300 \\ 3\overline{)1134} \\ \underline{900} \\ 234 \\ \underline{210} \\ 24 \end{array}$$

Think of the 2 tens 4 ones that remain as 24 ones. Then divide the 24 ones.

$3 \times 8 = 24$

Write
$$\begin{array}{r} 8 \\ 70 \\ 300 \\ 3\overline{)1134} \\ \underline{900} \\ 234 \\ \underline{210} \\ 24 \\ \underline{24} \\ 0 \end{array} \rightarrow 378$$

Add for the quotient.

Billy would have to read 378 pages each week to finish the books.

88

LESSON ACTIVITY

Before Using the Pages

- The division algorithm for one-digit divisors is extended in this lesson for dividends with up to five digits. The procedure developed for two-digit and three-digit dividends is the same for greater dividends: divide place by place from left to right, regrouping as required, and using multiplication and subtraction to divide.

  Begin by working with the students to complete divisions with four-digit dividends and five-digit dividends with no regrouping. Use examples similar to the following.

  $2\overline{)4268}$ $3\overline{)69\ 396}$ $4\overline{)44\ 848}$

  Then develop exercises that present a similar approach, and have students explain how the exercises are similar and how they differ.

  $4\overline{)280}$ $4\overline{)2800}$ $4\overline{)28\ 000}$

  $3\overline{)471}$ $3\overline{)4715}$ $3\overline{)47\ 159}$

For each example, emphasize using basic multiplication facts to determine the digits of the quotient. For example, for $3\overline{)47\ 159}$, $3 \times 1 = 3$ leads to 3×1 ten thousand = 3 ten thousands; $3 \times 5 = 15$ leads to 3×5 thousands = 15 thousands, and so on.

$$\begin{array}{r} 5\ 000 \\ 10\ 000 \\ 3\overline{)47\ 159} \\ \underline{30\ 000} \\ 17\ 159 \\ \underline{15\ 000} \\ 2\ 159 \end{array}$$

Using the Pages

- Establish why division is used to find the answer to the problem. (The number in each of three equal groups is required.) Guide the students in a discussion of the worked example by asking questions about the steps in the division.

Working Together

Complete.

1. $4\overline{)7324}$ — 1, 30, 800, 1000 — 1831

4000 ← 4 × 1000
3324
3200 ← 4 × 800
124
120 ← 4 × 30
4
4 ← 4 × 1
0

Add to find the quotient.

2. $9\overline{)3207}$ — 6, 50, 300 — 356

2700 ← 9 × 300
507
450 ← 9 × 50
57
54 ← 9 × 6
3

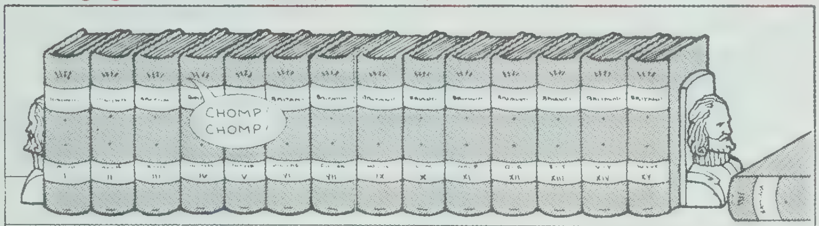
Divide.

3. $3\overline{)7401}$ 2467
4. 27 832 ÷ 8 3479
5. $6\overline{)1726}$ 287 R4

Exercises

Divide.

1. $3\overline{)5244}$ 1748
2. $2\overline{)7257}$ 3628 R1
3. $7\overline{)217}$ 31
4. $4\overline{)24\,690}$ 6 172 R2
5. $8\overline{)3600}$ 450
6. $5\overline{)878}$ 175 R3
7. $5\overline{)24\,465}$ 4 893
8. $2\overline{)94}$ 47
9. $8\overline{)15\,915}$ 1 989 R3
10. $6\overline{)7409}$ 1234 R5
11. $6\overline{)40\,734}$ 6 789
12. $3\overline{)1078}$ 359 R1
13. $7\overline{)1740}$ 248 R4
14. $9\overline{)3573}$ 397
15. $4\overline{)1520}$ 380
16. 2850 ÷ 8 356 R2
17. 17 830 ÷ 2 8915
18. 80 ÷ 3 26 R2
19. 56 110 ÷ 9 6234 R4
20. 419 ÷ 7 59 R6
21. 2250 ÷ 6 375
22. 23 659 ÷ 4 5914 R3
23. 216 ÷ 8 27
24. 13 000 ÷ 5 2600
25. 16 650 ÷ 9 1850



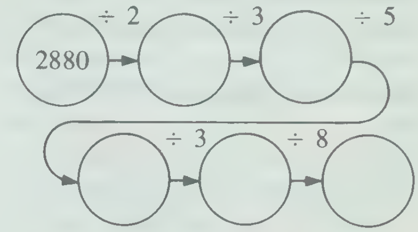
Bookworm started from outside the front cover of Volume I and tunneled its way to outside the back cover of Volume XV. Each volume is 4 cm (centimetres) thick. How far did Bookworm tunnel? 48 cm

PROBLEM SOLVING

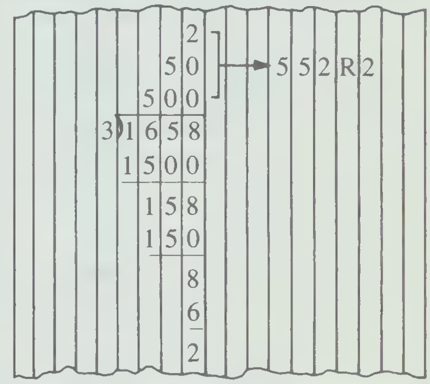
89

RELATED ACTIVITIES

- Have students use division to complete the following chain from left to right. They may use multiplication to check their work from right to left.



- Some students may find it helpful to use squared paper for division to help align the digits in their correct places. As an alternative, have them turn their lined paper sideways and use the vertical lines to separate the place values.



- For any division that has zero as a remainder, a division number sentence may be written.

7401 ÷ 3 = 2467

Have students write division sentences for such exercises in Ex. 1-25 on page 89.

Then draw their attention to the statement at the bottom of page 88 that concludes the solution.

Working Together: Exercises similar to Ex. 1 and 2 focus on the multiplications used at each stage of the division and help students to think through the procedure. Some students may find it helpful to show the solutions to Ex. 3-5 in a similar way.

Exercises: You may wish to have the students use multiplication (and addition) to check a selection of their division exercises. Also, remind them to place their exercises so that there is space for "stacked" quotients.

Problem Solving: Although multiplication is useful in solving the problem, caution the students to read the words of the problem carefully (*outside* the *front* cover, *outside* the *back* cover). Some students may want to use books to represent the situation. Note, also, that although there are 15 volumes, Volume VIII is not in the row between the book ends but is on its side, apart from the other books.

Assessment

Divide.

1. $3\overline{)1980}$ 660
2. $6\overline{)1035}$ 172 R3
3. $7\overline{)99\,652}$ 14 236
4. $5\overline{)12\,163}$ 2 432 R3

Pages 90-91

LESSON OUTCOME

Use a shorter form for division

Prerequisite Skills

Divide by a one-digit number

Checking Prerequisite Skills

Divide.

1. $3\overline{)174}$ 58
2. $5\overline{)509}$ 101 R4
3. $4\overline{)6324}$ 1581
4. $6\overline{)8230}$ 1371 R4

A Shorter Form for Division

Gordie and Carmen shared 175 dominoes equally to see who could build the faster domino course. How many dominoes are they using for each course?

Divide 175 by 2.

For $2\overline{)175}$, there are not enough hundreds so that both Gordie and Carmen get 1 hundred. Think of 1 hundred 7 tens as 17 tens. Then divide the 17 tens.

$2 \times 8 = 16$
2×8 tens = 16 tens
$2 \times 80 = 160$

Longer Form | Shorter Form

Write
$$\begin{array}{r} 80 \\ 2\overline{)175} \\ \underline{160} \\ 15 \end{array}$$
or
$$\begin{array}{r} 8 \\ 2\overline{)175} \\ \underline{160} \\ 15 \end{array}$$

Think of the 1 ten 5 ones that remain as 15 ones. Then divide the 15 ones.

$2 \times 7 = 14$

Write
$$\begin{array}{r} 7 \\ 80 \\ 2\overline{)175} \\ \underline{160} \\ 15 \\ \underline{14} \\ 1 \end{array}$$
or
$$\begin{array}{r} 87 \text{ R1} \\ 2\overline{)175}\phantom{\text{ R1}} \\ \underline{160}\phantom{\text{ R1}} \\ 15\phantom{\text{ R1}} \\ \underline{14}\phantom{\text{ R1}} \\ 1\phantom{\text{ R1}} \end{array}$$

Add and write the remainder to finish the longer form.

$$\begin{array}{r} 7 \\ 80 \\ 2\overline{)175} \\ \underline{160} \\ 15 \\ \underline{14} \\ 1 \end{array} \rightarrow 87 \text{ R1}$$

Write the remainder here and the work is done.

Each course has 87 dominoes. The falling dominoes that first reach the 1 domino left over belong to the faster course.

90

LESSON ACTIVITY

Before Using the Pages

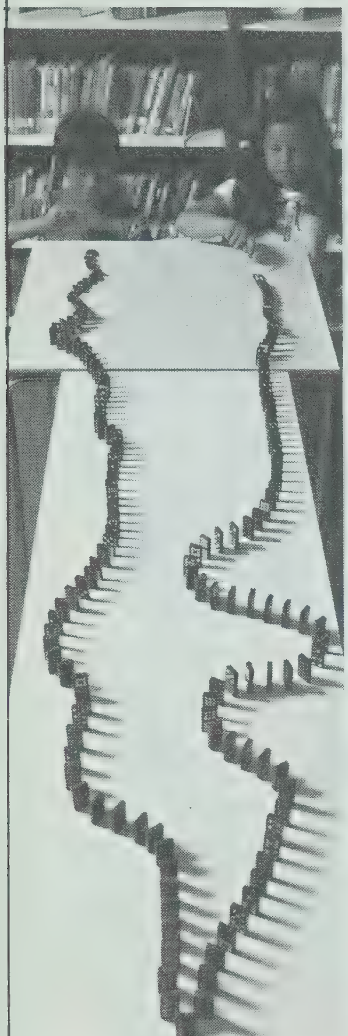
- This lesson presents the first stage in the transition toward the standard form for division. Students are encouraged to write the quotient in one line by writing the digit for each place value of the quotient. They will likely suggest this procedure if you imply that all the written work previously shown for deriving the quotient is not necessary.

$$\begin{array}{r} 2 \\ 40 \\ 100 \\ 3\overline{)426} \\ \underline{300} \\ 126 \\ \underline{120} \\ 6 \\ \underline{6} \\ 0 \end{array} \qquad \begin{array}{r} \boxed{1}\boxed{4}\boxed{2} \\ 3\overline{)4\ 2\ 6} \\ \underline{3\ 0\ 0} \\ 1\ 2\ 6 \\ \underline{1\ 2\ 0} \\ 6 \\ \underline{6} \\ 0 \end{array} \begin{array}{l} \\ \\ \leftarrow 3 \times \square \text{ hundred} \\ \\ \leftarrow 3 \times \square \text{ tens} \\ \\ \leftarrow 3 \times \square \text{ ones} \\ \\ \end{array}$$

Complete a division exercise on the board and then ring the "stacked" quotient, asking whether there is a way to reduce the amount of written work and show the final quotient. Have students suggest ways of thinking of the steps in the division that explain the shorter form.

Using the Pages

- Begin by having a student read the word problem at the top of page 90 and briefly discuss the photograph on page 91. Then ask the students to observe the two forms shown on page 90 for the division $2\overline{)175}$. Ask them to explain how the two forms differ. Emphasize that the shorter form involves less written work in the quotient, but the rest of the division format is the same. The shorter form shows the quotient 87 in standard form right away, whereas the longer form shows the expanded form of 87, (80 + 7), arranged vertically. Emphasize the need to place each digit of the quotient in the corresponding place above the dividend. Draw the students' attention to the concluding statement on page 90 and the explanation of the remainder 1.



Working Together

Complete.

1. $\begin{array}{r} 87 \\ 7\overline{)609} \\ 560 \\ \hline 49 \\ 49 \\ \hline 0 \end{array}$ 2. $\begin{array}{r} 674 \text{ R}3 \\ 8\overline{)5395} \\ 4800 \\ \hline 595 \\ 560 \\ \hline 35 \\ 32 \\ \hline 3 \end{array}$ 3. $\begin{array}{r} 234 \\ 4\overline{)936} \\ 800 \\ \hline 136 \\ 120 \\ \hline 16 \\ 16 \\ \hline 0 \end{array}$

Divide. Use the shorter form.

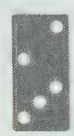
4. $6\overline{)5628}$ 938 5. 7296 ÷ 2 3648 6. $7\overline{)15\,076}$ 2 153 R5

Exercises

Divide. Use the shorter form.

1. $3\overline{)5490}$ 1830 2. $7\overline{)22\,876}$ 3 268 3. $6\overline{)407}$ 67 R5
4. $5\overline{)2882}$ 576 R2 5. $8\overline{)2797}$ 349 R5 6. $4\overline{)29\,700}$ 7425
7. $2\overline{)8536}$ 4268 8. $9\overline{)24\,740}$ 2 748 R8 9. $3\overline{)13\,728}$ 4 576
10. 1970 ÷ 6 328 R2 11. 8863 ÷ 3 2954 R1 12. 4851 ÷ 9 539
13. 99 ÷ 5 19 R4 14. 25 170 ÷ 6 4195 15. 7430 ÷ 2 3715
16. 20 679 ÷ 4 5169 R3 17. 5400 ÷ 8 675 18. 40 000 ÷ 7 5714 R2

The face of a domino has 2 parts.
If each part can have 0 to 6 dots,



1. how many different faces are possible for a domino? 28

How would you build a domino course

2. that branches into 3 paths? Answers will vary; for example, 28 dominoes, 9 dominoes in 2 paths, 10 in another
3. that crosses itself? Answers will vary; for example, 7 dominoes in each branch
4. that shows your initials? Answers will vary.

PROBLEM SOLVING

91

RELATED ACTIVITIES

- For further practice, you may wish to have students complete Ex. 21-28 on page 336.
- Some domino sets include dominoes for which there are up to nine dots on each half of the face. Ask students to find out the number of different domino faces possible under these conditions.
- In *Related Activities* on page T 5, it is suggested that cards be prepared for showing the standard form for a number when the cards overlap and the expanded form when they are spread apart. These cards will also be useful for helping students to understand the shorter form for writing the quotient. The appropriate cards may be displayed and overlapped as the quotient is determined.

$$\begin{array}{r} 1698 \\ 4\overline{)6792} \\ 4000 \\ \hline 2792 \\ 2400 \\ \hline 392 \\ 360 \\ \hline 32 \\ 32 \\ \hline 0 \end{array}$$

1	6	9	8

Working Together: Partially completed divisions help students to think through the steps of the shorter form for division as they complete the steps. Use other similar exercises as required. As the students work, remind them to place the digits with care so that those with the same place values are aligned.

Exercises: Since the shorter form for division reduces the "stacked" quotient to one line, the students do not need now to be so concerned about the spacing of exercises on the page.

Problem Solving: The emphasis in Ex. 1 is on the number of different faces possible for a domino. If students draw diagrams to find the solution, encourage them to find another way of deriving the answer, which is 28. An organized approach might be to consider the number of possibilities showing 0 dots on one part, the number of new possibilities for which there is 1 dot on one part, 2 dots on one part, and so on. The answers to Ex. 2-4, of course, may vary. Division can be helpful. For instance, in Ex. 2, $3\overline{)28}$ can suggest 3 paths with 9 dominoes in each, and 1 domino left over at the end of the course.

Assessment

Divide. Use the shorter form.

1. $2\overline{)395}$ 197 R1 2. $5\overline{)6745}$ 1349 3. $6\overline{)8000}$ 1333 R2 4. $4\overline{)26\,392}$ 6 598

LESSON OUTCOME

Use the standard algorithm to divide by a one-digit number; use division to find an average

Materials

models for hundreds, tens, and ones

Vocabulary

average, gram, g

Prerequisite Skills

Divide by a one-digit number

Checking Prerequisite Skills

Divide.

1. $7\overline{)847}$ 121
2. $3\overline{)2538}$ 846
3. $2\overline{)35\ 000}$ 17 500
4. $8\overline{)16\ 902}$ 2 112 R6

The Standard Form for Division

Lucy's parents and her two older sisters shared the driving on a trip. They traveled 2376 km and each drove about the same distance. About how far did each drive?

Divide 2376 by 4.

For $4\overline{)2376}$, there are not enough thousands so that each driver could drive 1 thousand kilometres. Think of 2 thousands 3 hundreds as 23 hundreds. Then divide the 23 hundreds.

4 × 5 = 20
4 × 5 hundreds = 20 hundreds

Write

$$\begin{array}{r} 5 \\ 4\overline{)2376} \\ \underline{20} \\ 3 \end{array}$$

You can save time by not writing these zeros.

Think of the 3 hundreds 7 tens that remain as 37 tens.

$$\begin{array}{r} 5 \\ 4\overline{)2376} \\ \underline{20} \\ 37 \end{array}$$

Then divide the 37 tens.

4 × 9 = 36
4 × 9 tens = 36 tens

Write

$$\begin{array}{r} 59 \\ 4\overline{)2376} \\ \underline{20} \\ 37 \\ \underline{36} \\ 1 \end{array}$$

Think of the 1 ten 6 ones that remain as 16 ones.

$$\begin{array}{r} 59 \\ 4\overline{)2376} \\ \underline{20} \\ 37 \\ \underline{36} \\ 16 \end{array}$$

Then divide the 16 ones.

4 × 4 = 16

Write

$$\begin{array}{r} 594 \\ 4\overline{)2376} \\ \underline{20} \\ 37 \\ \underline{36} \\ 16 \\ \underline{16} \\ 0 \end{array}$$

Lucy's mother, father, and two sisters each drove about 594 km.

92

LESSON ACTIVITY

Before Using the Pages

- This lesson completes the transition from the longer form to the standard form for division. Review that the amount of written work in the previous lesson was reduced by showing the quotient in one line. Demonstrate this by working $4\overline{)673}$ on the board (A). Show the students how the amount of written work can be reduced further by writing only those digits for the place value being considered at each step (B).

A

$$\begin{array}{r} 168\ \text{R1} \\ 4\overline{)673}\phantom{\ \text{R1}} \\ \underline{400}\phantom{\ \text{R1}} \\ 273\phantom{\ \text{R1}} \\ \underline{240}\phantom{\ \text{R1}} \\ 33\phantom{\ \text{R1}} \\ \underline{32}\phantom{\ \text{R1}} \\ 1\phantom{\ \text{R1}} \end{array}$$

B

$$\begin{array}{r} 168\ \text{R1} \\ 4\overline{)673}\phantom{\ \text{R1}} \\ \underline{4}\phantom{73\ \text{R1}} \\ 27\phantom{3\ \text{R1}} \\ \underline{24}\phantom{3\ \text{R1}} \\ 33\phantom{\ \text{R1}} \\ \underline{32}\phantom{\ \text{R1}} \\ 1\phantom{\ \text{R1}} \end{array}$$

C

← Divide 6 hundreds first.
← 4 × 1 hundred = 4 hundreds.
← Divide 27 tens.
← 4 × 6 tens = 24 tens.
← Divide 33 ones.
← 4 × 8 = 32 ones.

Erase the appropriate digits in A to obtain the form shown in B. Redevelop the exercise and use models to illustrate each step (C). Emphasize the importance of aligning the digits in the division process with those of the dividend. Thus, although the second step shows 2 hundreds and 7 tens, this is thought of as 27 tens. There is no need to write the 3 ones at this stage because ones are divided after the tens.

Using the Pages

- Introduce the word problem and discuss why division is used to find the solution. Note the symbol km for kilometres. Guide the students through the worked example. For instance, ask why the digit 5 of the quotient is written in the hundreds' place. Develop that the 3 hundreds that remain will need to be regrouped as tens and, together with the 7 tens, will be divided in the second step. This explains the need to show only the 7 tens (and not the 6 ones) in the second step. Explain that the arrows are not to be drawn in the solution: they are in the worked example to illustrate which digit of the dividend is required at each step.

Working Together

Complete.

1. 3)174 — quotient 5▒ (8); 15; 24; ▒ 24; 0

2. 9)756 — quotient 8▒ (4); 72; ▒ 36; ▒ 36; ▒ 0

3. 6)5084 — quotient 8▒▒ R ▒ (47, 2); 48; ▒ 28; ▒ 24; ▒ 44; ▒ 42; ▒ 2

Divide. Use the standard form.

4. 2)538 269 5. 1491 ÷ 7 213 6. 3)13 796 4598 R2

Exercises

Divide. Use the standard form.

1. 5)2410 482 2. 4)25 950 6487 R2 3. 2)1238 619
4. 7)672 96 5. 3)78 26 6. 4)8529 2132 R1
7. 9)85 929 9547 R6 8. 8)45 425 5678 R1 9. 3)5190 1730
10. 14 771 ÷ 2 7385 R1 11. 6730 ÷ 5 1346 12. 47 474 ÷ 6 7912 R2
13. 33 310 ÷ 7 4758 R4 14. 7524 ÷ 9 836 15. 6954 ÷ 6 1159
16. 19 761 ÷ 5 3952 R1 17. 636 ÷ 4 159 18. 3689 ÷ 8 461 R1

If an amount is thought of as being shared equally, the result is an **average**.

Example: If you go 20 km in 4 h, your average distance for each hour is 20 ÷ 4, or 5 km.

19. If 2376 km are traveled in 3 d, what is the average distance traveled each day? 792 km

What is the average?

20. 152 min to read 4 books 38 min
21. 875 g (grams) in 5 apples 175 g
22. 87 fish caught from 3 lakes 29 fish
23. 14 754 tickets for 6 shows 2459 tickets

In each case the remainder will be 0:

2) even number
3) sum of the digits is a multiple of 3
4) last 2 digits form a multiple of 4
5) last digit is 0 or 5
9) sum of the digits is a multiple of 9

Will the remainder be 0?

1. 2)736 yes 2. 2)1380 yes
3. 3)807 yes 4. 3)5967 yes
5. 4)316 yes 6. 4)9732 yes
7. 5)485 yes 8. 5)6730 yes
9. 9)252 yes 10. 9)8694 yes

Make up a three-digit number that gives a remainder of 0 when it is

11. divided by either 2 or 3. Answers will vary; for example, 102 or 132.
12. divided by any of 4, 5, or 9. Answers will vary; for example, 180 or 720.

try this

93

RELATED ACTIVITIES

• Number squares similar to the following may be assigned for practicing multiplication and division. You may wish to use copies of page T 391.

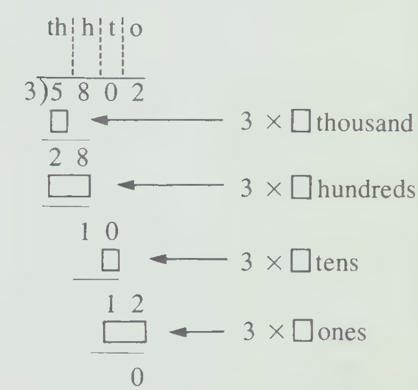
→ ×, ↓ ÷

24	12	288
3	2	
8		

→ ×, ↓ ÷

147	69	
3	3	

• Students having difficulty with the standard form for division may need practice with the multiplication steps. Exercises similar to the following will provide practice.



Working Together: Ex. 1-3 help students think through the steps of division using the standard form. Use other similar exercises as required. For Ex. 4-6, ask the students to determine the number of digits there will be in the quotient before they begin the division. For example, in Ex. 4, the first digit of the quotient will be in the hundreds' place and thus the quotient will have three digits. By the same reasoning, the quotient in Ex. 5 will also have three digits. Some students will find it helpful to turn their lined paper sideways and use the vertical lines to keep digits for the same place value properly aligned.

Exercises: If students have not previously encountered the concept of an *average*, discuss it with them before they begin Ex. 19-23. An example is provided on page 93. Note the symbols that are used in these exercises and direct the students to the table on page 342 if they need assistance.

Try This: The information provided enables students to determine whether the remainder in a division will be 0 for divisors of 2, 3, 4, 5, and 9. Ex. 11 and 12 suggest the concept of least common multiple in an informal manner. For instance, if a number is exactly divisible by both 2 and 3, as in Ex. 11, it must be exactly divisible by their least common multiple 6. Students will use different methods to solve these exercises; they should be given an opportunity to share their methods with others.

Assessment

Divide. Use the standard form.

1. 3)197 65 R2 2. 8)4523 565 R3 3. 7)9305 1329 R2 4. 4)36 863 9215 R3

What is the average?

5. 111 exercises completed in 3 min. 37

Pages 94-95

OBJECTIVE

Demonstrate competence in dividing by a one-digit number using the standard form; add first and then divide to find an average; solve related word problems

Materials

large sheets of paper

Practice

Divide. Use the standard form.

1. 3)5000 1666 R2	2. 6)5000 833 R2	3. 7)5000 714 R2	4. 8)5000 625	5. 9)5000 555 R5
6. 4)39 816 9 954	7. 6)39 816 6 636	8. 7)39 816 5 688	9. 8)39 816 4 977	10. 9)39 816 4 424
11. 7)861 123	12. 7)3192 456	13. 7)5523 789	14. 7)8638 1234	15. 7)39 746 5 678

16. 10 000 ÷ 9 1111 R1	17. 20 000 ÷ 9 2222 R2	18. 30 000 ÷ 9 3333 R3	19. 40 000 ÷ 9 4444 R4
20. 27 156 ÷ 4 6789	21. 33 945 ÷ 5 6789	22. 40 734 ÷ 6 6789	23. 47 523 ÷ 7 6789
24. 54 312 ÷ 8 6789	25. 63 168 ÷ 8 7896	26. 71 736 ÷ 8 8967	27. 77 424 ÷ 8 9678

Divide to find the missing factor.
Then multiply to check your work.

28. 3 × ▒ = 87 29   29. 5 × ▒ = 725 145   30. 8 × ▒ = 6864 858   31. 2 × ▒ = 5702 2851

32.	33.	34.	35.	36.	37.
983	479	76	3146	7236	1985
6	9	7	3	4	5
5898	4311	532	9438	28 944	9925

try this

Answers will vary.

Replace each ▒ with a digit to show a division. Do not use the same digit more than once in each exercise.

Example: For ▒)▒ with ▒ above, use 2)6 with 3 above.

1. 3)12   4
2. 2)68   34
3. 3)162   54
4. 2)618   309
5. 7)3192   456
6. 3)5082   1694
7. 3)20457   6819

The farther you go, the harder they get. How many can you do?

Make up a division exercise

Answers will vary.

38. with a quotient of 739.
3695 ÷ 5 = 739

39. with a divisor of 7 and a quotient of 168.
1176 ÷ 7 = 168

40. with a quotient of 568 and a remainder of 7.
4551 ÷ 8 = 568 R7

*41. to match each line of this chart, if possible.
Examples are given.

divisor	dividend	quotient	
even	even	even	2)8 4
even	even	odd	2)6 3
even	odd	even	—
odd	even	even	5)50 10
even	odd	odd	—
odd	even	odd	—
odd	odd	even	—
odd	odd	odd	7)49 7

94

LESSON ACTIVITY

Using the Pages

- Select one of Ex. 1-27 and complete it on the board with the students to review the steps of the standard form for division.

  Have the students read the instructions that precede Ex. 28. Ask them to explain in their own words what is required. If necessary, develop one of Ex. 28-37 on the board.

  The phrase "if possible" is significant in Ex. 41. This exercise is starred because the answers vary and for some, there is no solution. This can be understood better by considering the following multiplications and relating the division situations of Ex. 41 to this.

×	even number	odd number
even number	?	?
odd number	?	?

If remainders greater than zero are accepted, the following are possible solutions for the chart in Ex. 41.

divisor	dividend	quotient	
2	28	14	
2	14	7	
6	13	2 (R1)	*
3	18	6	
8	25	3 (R1)	*
5	36	7 (R1)	*
7	29	4 (R1)	*
9	27	3	

*If the condition that the remainder must be zero is imposed, then there is no solution.

- Discuss the example at the top of page 95 to prepare the students for the addition step that precedes dividing to find the average in Ex. 42-49.

You can add, then divide to find an average.

Example:

Lucy's diary shows the temperature at noon for each day of her family's vacation trip. What was the average temperature at noon for the week?

Add.

$$\begin{array}{r} 25 \\ 23 \\ 19 \\ 24 \\ 26 \\ 28 \\ \underline{30} \\ 175 \end{array}$$

Then divide by the number of days.

$$\begin{array}{r} 25 \\ 7\overline{)175} \\ \underline{14} \\ 35 \\ \underline{35} \\ 0 \end{array}$$

The average temperature for the week was 25°C.

Find the average.

42. Lucy's family drove 1282 km to their vacation spot and 1094 km back. What was the average distance each way? 1188 km

43. Lucy caught 6 fish: 394 g, 406 g, 428 g, 476 g, 489 g, and 513 g. What was their average mass? 451 g

44. Four pieces of string were 82 cm, 68 cm, 71 cm, and 75 cm long. What was their average length? 74 cm

45. On his school tests, Bob scored 89, 78, 95, 82, and 91. What was his average score on the tests? 87

46. During the week, it took Jack 25 min, 15 min, 19 min, 31 min, 41 min, 18 min, and 12 min to wash the dishes. What was the average length of time? 23 min

47. In the basketball games, Sue scored 42, 49, 38, 45, 18, 33, 41, and 38 points. What was her average number of points? 38

48. In the hockey games, Herb played for 35 min, 23 min, 37 min, 41 min, 34 min, 40 min, 44 min, 38 min, and 32 min. What was his average playing time? 36 min How many times were above average? 5

49. Fair attendance for one week was 5268, 2637, 3680, 4472, 2894, 5716, and 6945. What was the average attendance for each day? 4516 How many days were above the average? 3

RELATED ACTIVITIES

- For further practice, you may wish to have students complete Ex. 29-32 on page 336.
- Have students keep a record of the outdoor temperature at noon each day for a week and then find the average temperature. Discuss the most reasonable way to interpret the remainder if one occurs in the division (see *Giving the Most Reasonable Answer* on page 98).
- Discuss the meaning of terms such as "batting average" in baseball.

Try This: Multiplication exercises similar to these were encountered on page 49. Since there are different solutions, have the students share their solutions in the following way. Display a separate, large sheet of paper for each of Ex. 1-7. Invite each student in turn to write a solution on one sheet. Note that it is implied that the remainder is always zero. Remind the students that a digit may not be repeated in an exercise.

1. $\begin{array}{r} \square \\ \square\overline{)\square\square} \end{array}$ $\qquad \begin{array}{r} 8 \\ 3\overline{)24} \end{array}$ $\qquad \begin{array}{r} 3 \\ 6\overline{)18} \end{array}$

$\begin{array}{r} 8 \\ 4\overline{)32} \end{array}$

OBJECTIVE

Divide amounts of money by a one-digit number using the standard form; solve related word problems

RELATED ACTIVITIES

- The list for the game ''Shopping Spree'' described on page T377 can now include such statements as ''From now on, every item you purchase will be reduced in price. Divide the regular price by two to find the reduced price,'' and ''Your shopping limit is now one-half of the original amount.''

Practice

Amounts of money are divided just like whole numbers.

Examples:

\$ 79	\$ 1 684	\$2.43	\$ 68.42
5)\$395	8)\$13 472	3)\$7.29	7)\$478.94
35	8	6	42
45	5 4	1 2	58
45	4 8	1 2	56
0	67	09	2 9
	64	9	2 8
	32	0	14
	32		14
	0		0

Divide.

1. 4)\$304 \$76
2. 2)\$70 \$35
3. 5)\$160 \$32
4. 6)\$4068 \$678
5. 9)\$33 858 \$ 3 762
6. 8)\$9.84 \$1.23
7. 5)\$5.75 \$1.15
8. 3)\$24.81 \$8.27
9. 4)\$19.92 \$4.98
10. 7)\$31.08 \$4.44
11. \$72 ÷ 3 \$24
12. \$320 ÷ 5 \$64
13. \$6482 ÷ 7 \$926
14. \$448 ÷ 8 \$56
15. \$7.00 ÷ 4 \$1.75
16. \$85.41 ÷ 9 \$9.49
17. \$9.26 ÷ 2 \$4.63
18. \$77.70 ÷ 6 \$12.95

Anu, Robert, and Michael earned money shoveling snow.

19. They were paid \$2.35, \$2.25, \$2.75, \$1.75, and \$2.45 for the sidewalks they shoveled. How much did they earn in all? \$11.55
20. What was the average amount they earned for each job? \$2.31
21. They shared the money equally. How much did each earn? \$3.85
22. They spent 4 h in the morning and 3 h in the afternoon looking for work and shoveling. What was the average amount they earned for each hour they spent? \$1.65

*23. Tony earned \$5.25 for shoveling 3 walks. He earned \$1.35 for a fourth walk. Was his average for 3 walks greater than or less than his average for 4 walks?
His average for 3 walks was \$1.75.
His average for 4 walks was \$1.65.
His average for 3 walks was greater than his average for 4 walks.

96

LESSON ACTIVITY

Using the Page

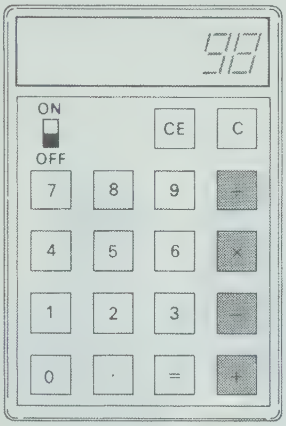
- Ask students to read the amounts of money shown as quotients and dividends in the examples at the top of the page. Emphasize that the division process for money is the same as for whole numbers, but it is necessary to include the symbol \$ and, in some cases, a decimal point to separate dollars and cents. Ask the students to place these symbols where they appear in the quotient before beginning a division.
- Remind the students to read carefully to determine the operations required for solving Ex. 19-23. For Ex. 22, some students may find the average amount earned in one hour by one boy rather than the average amount earned in one hour by the three boys. Either answer is acceptable. Ex. 23 is starred because its solution involves more than one step.

Answers may vary.

1. 1261 $\boxed{+}$ 1574 $\boxed{+}$ 928 $\boxed{+}$ 1089 $\boxed{+}$ 1213 $\boxed{\div}$ 5 $\boxed{=}$ 1213 2. 365 $\boxed{+}$ 366 $\boxed{+}$ 365 $\boxed{+}$ 365 $\boxed{=}$ 1461

Keycharts and the $\boxed{+}$, $\boxed{-}$, $\boxed{\times}$, and $\boxed{\div}$ Keys

Keycharts can be written across or down.



98 $\boxed{+}$ 146 $\boxed{+}$ 179 $\boxed{\div}$ 3 $\boxed{=}$ 141

Yvonne had scores of 98, 146, and 179 for 3 games of bowling. What was her average score?

98
$\boxed{+}$
146
$\boxed{+}$
179
$\boxed{\div}$
3
$\boxed{=}$
141

3. 774 $\boxed{\div}$ 9 $\boxed{=}$ 86

4. 277 $\boxed{\times}$ 4 $\boxed{+}$ 302 $\boxed{\div}$ 5 $\boxed{=}$ 282

Add, then divide by 3.

5. 4025 $\boxed{\div}$ 7 $\boxed{\times}$ 3 $\boxed{=}$ 1725

6. 750 $\boxed{\div}$ 6 $\boxed{-}$ 84 $\boxed{=}$ 41

Keycharts can be different for different calculators. For the calculator you use, are these keycharts correct for finding the average of 98, 146, and 179?

Make a keychart that shows how to solve each of these.

1. What is the average distance: 1261 km, 1574 km, 928 km, 1089 km, and 1213 km?
2. How many days are there from the beginning of the year 1979 to the end of the year 1982?
3. Pearl needs to cut 9 pieces of string, all the same length, from a piece that is 774 cm long. How long can each piece be?
4. Mrs. Hoy earned $277 for each of 4 weeks. She earned $302 for a fifth week. What were her average earnings for each week?
5. *4025 names were listed in 7 columns with the same number in each column. How many names were in the last 3 columns?
6. *Mr. Gibbs shared a $750 prize with 5 others. He has spent $84 of his share. How much prize money does he have left?

If you are using a calculator, be sure to make keycharts that will work for your machine.

Can you solve each problem?

Calculator

97

Page 97

OBJECTIVE

Prepare a keychart to show the order of pressing the keys $\boxed{+}$, $\boxed{-}$, $\boxed{\times}$, and $\boxed{\div}$ on a calculator to solve a problem

Materials

calculators (optional)

RELATED ACTIVITIES

- If calculators are available, have the students use them to complete the keycharts for the exercises on page 97.

LESSON ACTIVITY

Using the Page

- This lesson extends the work on page 61 by introducing the operation key for division. Also, keycharts are presented in a vertical arrangement as well as in the familiar horizontal arrangement.
- The students may be able to read the example and proceed with the exercises without an introductory discussion. The amounts of money encountered will involve a whole number of dollars, and thus, no decimal points are needed. Since calculators vary, it is essential to relate the exercises on this page to the particular kind of calculator used in the classroom.

  For Ex. 2, students will have to consider whether one of these years is a leap year. Although the computation involved in Ex. 5 and 6 is not difficult, a certain skill in reasoning is required. For Ex. 5, the number of names is the same for any three columns; for Ex. 6, if Mr. Gibbs shares with five others, his share is found by dividing $750 by 6.

Page 98

OBJECTIVE

Give the most reasonable answer to a word problem

RELATED ACTIVITIES

- Encourage students to write word problems similar to those on the page for others to solve. The pictures in this unit may suggest topics for the word problems.

Giving the Most Reasonable Answer

Cynthia has 13 oranges for herself and 3 friends. What is the fairest way to share the oranges?

The fairest way to share the oranges would be to separate the orange left over into 4 equal parts and share them too.

Use division to solve each of these. Tell what you would do with the remainder to get the most reasonable answer.

Answers will vary.

1. Greg has 1000 apples that he must wrap for trays that hold 6 each. How many trays will he need for the 1000 apples? 167
2. Each car can carry 5 people. 82 people are going. How many cars are needed? 17
3. 75 535 people attended 9 games. What was the average number for each game? 8393
4. Each batch of cookies uses 2 eggs. How many batches could be made with 13 eggs? 6
5. The 8 boys earned $34 that they wanted to share equally. How much should each boy get? $4.25
6. Irma took 25 min to read 4 pages. What was her average time for reading a page? 6 min 15 s
7. Matt plans to read the same number of pages each day for a week. There are 200 pages. How many should he read each day? 29

PROBLEM SOLVING

98

LESSON ACTIVITY

Using the Page

- In abstract division, the remainder "remains". In practical situations that involve division, the remainder is sometimes rounded up, rounded down, or simply ignored. The word problems on this page present a number of practical situations and ask students to suggest a reasonable way to deal with the remainder in each situation. These problems can motivate considerable discussion. For instance, in Ex. 5, the division process can continue beyond $8\overline{)\$34}$ by showing the dividend as $34.00. In Ex. 6, some students may express 25 min as 1500 s and find the quotient for the division $4\overline{)1500}$.

Checking Up

Divide.

1. 5)35 7	**2.** 4)24 6	**3.** 9)36 4	**4.** 7)49 7
5. 3)29 9 R2	**6.** 2)17 8 R1	**7.** 6)20 3 R2	**8.** 8)52 6 R4
9. 2)46 23	**10.** 4)84 21	**11.** 3)936 312	**12.** 2)624 312
13. 3)78 26	**14.** 5)90 18	**15.** 2)91 45 R1	**16.** 6)70 11 R4
17. 4)268 67	**18.** 7)903 129	**19.** 9)529 58 R7	**20.** 5)683 136 R3
21. 3)2022 674	**22.** 8)17 560 2195	**23.** 6)7970 1328 R2	**24.** 7)45 071 6438 R5
25. 5)$860 $172	**26.** 9)$6525 $ 725	**27.** 3)$5.94 $1.98	**28.** 8)$34.80 $ 4.35
29. 42 ÷ 6 7	**30.** 18 ÷ 3 6	**31.** 30 ÷ 4 7 R2	**32.** 61 ÷ 9 6 R7
33. 36 ÷ 3 12	**34.** 86 ÷ 2 43	**35.** 484 ÷ 4 121	**36.** 264 ÷ 2 132
37. 91 ÷ 7 13	**38.** 71 ÷ 4 17 R3	**39.** 734 ÷ 2 367	**40.** 851 ÷ 3 283 R2
41. 470 ÷ 5 94	**42.** 342 ÷ 8 42 R6	**43.** 6308 ÷ 4 1577	**44.** 9280 ÷ 7 1325 R5
45. 4473 ÷ 9 497	**46.** 4343 ÷ 6 723 R5	**47.** 31 285 ÷ 5 6257	**48.** 60 209 ÷ 8 7526 R1
49. $336 ÷ 4 $84	**50.** $5201 ÷ 7 $743	**51.** $8.94 ÷ 6 $1.49	**52.** $89.55 ÷ 9 $9.95

Solve.

53. The 4 children shared 176 peanuts equally. How many peanuts did each child get? 44

54. The seed packet said that the seeds would produce fruit in 91 d. How many weeks is this? 13

55. Mr. Tompkins earned the same amount of money each day for 5 d. His paycheck was $370. How much did he earn each day? $74

56. The 8 children shared $18.80 equally. How much money did each child get? $2.35

57. When 520 building blocks are shared equally by 3 children, how many will each child get? 173 How many will be left over? 1

58. When 175 dominoes are shared equally by 6 children, how many will each child get? 29 How many will be left over? 1

59. The 7 boys stood on the scale. It showed 238 kg. What was the average mass of each boy? 34 kg

60. The girls measured their heights to be 128 cm, 143 cm, 138 cm, and 147 cm. What was their average height? 139 cm

99

Page 99

OBJECTIVE

Demonstrate an understanding of the concepts and skills presented in this unit

RELATED ACTIVITIES

- Students may enjoy practicing division by playing the game "Lucky Zero" described on page T 378.
- Because division is the inverse of multiplication, it is recommended that multiplication and division facts be practiced together. Have the students write families of multiplication and division facts as shown.

4 × 7 = 28	28 ÷ 4 = 7
7 × 4 = 28	28 ÷ 7 = 4

×	80	3)240 80
3	240	
×	30	8)240 30
8	240	

Skills	Exercises	Related Pages
Use multiplication to divide, divisors and quotients to 9, remainders	1-8, 29-32	T 88-T 89
Divide by a one-digit number, no regrouping	9-12, 33-36	T 90-T 91
Divide a two-digit number by a one-digit number, regrouping	13-16, 37, 38	T 92-T 93
Divide a three-digit number by a one-digit number, regrouping	17-20, 39-42	T 94-T 95
Divide by a one-digit number with regrouping, dividends with four or five digits	21-24, 43-48	T 96-T 97
Divide amounts of money	25-28, 49-52	T 104
Divide to find an average	59, 60	T 100-T 101 T 102-T 103
Solve division problems	53-58	

Comments

Use of the division algorithm involves the following skills. It is important to determine whether most errors are a result of weakness in one particular skill, so that remedial assistance may then be planned and provided.

1. Recall basic multiplication facts
2. Relate multiplication and division
3. Recall the sequence of steps in the division algorithm
4. Subtract with regrouping
5. Interpret place value in numerals
6. Find and adjust trial estimates for a quotient

Students having difficulty with the standard form for division may benefit from using models to find a quotient, sharing place by place and regrouping as needed.

Exercises of the following type can reinforce place-value concepts.

$\boxed{1}$ 4 2	:	1 hundred	4 tens	2 ones
$\boxed{1\ 4}$ 2	:		$\boxed{1\ 4}$ tens	2 ones
$\boxed{1\ 4\ 2}$	:			$\boxed{1\ 4\ 2}$ ones

Unit 6 Overview

Decimals

A review of one-place decimals leads to a study of two-place and three-place decimals with attention to the place values represented. Students read and write decimals in numerals and in words. Comparison of two decimals with up to three decimal places precedes a lesson on ordering three or more decimals. Rounding to the nearest whole number is presented for decimals with up to three decimal places, and to the nearest tenth for two-place and three-place decimals. Skills in addition and subtraction of decimals to thousandths are developed and applied in solving related word problems. The lesson on the use of the calculator involves decimals and money. For it students are required to identify problem situations which are additive or subtractive and then to prepare appropriate keycharts for their solutions. The problem-solving skill in this unit deals with the organization of schedules for games. Skills in addition and subtraction of whole numbers and amounts of money are maintained in a *Keeping Sharp* feature, but these skills are also used in the same operations with decimals in the latter part of the unit.

Prerequisite Skills

- identify wholes that show ten equal parts
- compare and order whole numbers
- round whole numbers
- add whole numbers with regrouping
- subtract whole numbers with regrouping

Unit Outcomes

- read and write numerals and words for one-place, two-place, and three-place decimals
- interpret place value in one-place, two-place, and three-place decimals
- compare two decimals showing tenths, hundredths, and thousandths
- order decimals showing tenths, hundredths, and thousandths
- round one-place, two-place, and three-place decimals to the nearest whole number
- round two-place and three-place decimals to the nearest tenth
- add decimals with regrouping, one to four addends with the same number of decimal places (to thousandths)
- subtract decimals with regrouping, subtrahends and minuends with the same number of decimal places (to thousandths); use addition to check subtraction
- demonstrate the ability to use a calculator for additive and subtractive situations involving decimals and amounts of money
- organize information for the solution of a problem

Background

In the strictest sense of the word, "decimals" usually refers to the numerals for decimal fractions such as 0.4 and 5.6. For simplicity in this series, the word "decimals" is used when speaking of the rational numbers as well as the numerals.

Decimals are being used more and more widely in everyday life, especially with the introduction of the metric system of measurement. In this unit, decimals are associated with a variety of athletic activities to indicate measurements of length, time, and mass, and are also used in recording competitive scores. An appreciation of decimals and the number values that they represent depends on a thorough understanding of the place-value structure of our numeration system.

The Hindu-Arabic numeration system is a base-ten (decimal) system with the digits 1 to 9 to represent counting numbers and the digit 0 to serve as a placeholder. It should be mentioned that 0 is more than a placeholder; it is also the digit to represent the empty set, or to represent the whole number that is less than 1. But, in discussing numeration, 0 is primarily a placeholder. In a decimal system a digit in a numeral represents values which are powers of ten. In Unit 1 these powers were examined for whole numbers, and it was seen that in considering the places to the left of ones, the successive place values are ten times as great each time.

hundred thousands	ten thousands	thousands	hundreds	tens	ones
$10 \times 10\,000$	10×1000	10×100	10×10	10×1	1
10^5	10^4	10^3	10^2	10^1	10^0

For numbers less than one, the place-value structure may be extended to the right and here, too, each place has a value in terms of powers of ten. Proceeding to the right the value of each place is one-tenth the value on the left. In this structure, ones occupy the central position and are flanked on both sides in a balanced pattern by tens and tenths, by hundreds and hundredths, and by thousands and thousandths.

thousands	hundreds	tens	ones	tenths	hundredths	thousandths
10^3	10^2	10^1	10^0	10^{-1}	10^{-2}	10^{-3}

In decimal notation a decimal point separates the whole number part from the decimal part and the word "and" is read for it. The last place on the right is named to indicate the decimal value. For example, 4.75 is read as "four and seventy-five hundredths", and 3.205 is read as "three and two hundred five-thousandths". The practice of reading decimals by merely naming the digits separately, such as "three point two zero five", which does not state the value of the final place, should be discouraged. Note that a 0 is written in the ones' place of a decimal which represents a number less than one. The zero is usually not read, but it does ensure that the decimal point is not overlooked and that the digits which follow are interpreted correctly. The decimal 0.67, for instance, is read "sixty-seven hundredths".

As with whole numbers, attention for rounding decimals needs to be focused on two places: on the place to which the number is to be rounded and on the next place to the right. If this next place has any of the digits 0, 1, 2, 3, or 4, the number is rounded down; that is, the digit in the desired place of rounding remains the same, and all the digits to the right are ignored. On the other hand, if the digit in the next place is 5, 6, 7, 8, or 9, the number is rounded up; that is, the digit in the desired place of rounding is increased by one and all the digits to the right are ignored. Thus, 2.475 rounded to the nearest tenth becomes 2.5; 2.475 rounded to the nearest whole number becomes 2.

Comparing and ordering numbers involves a study of their numerals place by place from left to right. It is not necessary to

continue such a study from the greatest to the least place values, but only until a difference is noted. In the three examples shown, there are no differences in the tens' place and in the ones' place, but in the tenths' place, 4 is greater than 3, so 12.425 is the greatest of the three numbers. Then, to compare 12.375 and 12.370, there is no difference in the tenths' and hundredths' places, but 5 is greater than 0 in the thousandths' place, so 12.375 is greater than 12.370. Ordering the three numbers is now comparatively easy: 12.370, 12.375, 12.425 from least to greatest.

12.375
12.370
12.425

Addition and subtraction of decimals uses the same skills as for whole numbers since the base-ten structure of the numerals is merely extended to the right of ones. Regrouping in the two operations involves similar renaming on a 10-for-1 basis. In addition, the regrouping proceeds from right to left: 10 thousandths = 1 hundredth, 10 hundredths = 1 tenth, and 10 tenths = 1 one. In subtraction, the regrouping occurs in the opposite direction and 1 in each place value equals 10 in the next place value to the right.

Teaching Strategies

It is recommended that models to represent ones, tenths, and hundredths be available for each student to manipulate in the early lessons of the unit. Larger demonstration models should be used to represent decimals, while students use their own smaller models.

For comparing and ordering decimals, a vertical arrangement of the numerals, one above the other, is usually better than a horizontal one. Students should be encouraged to use a vertical arrangement to help them in making comparisons of decimals.

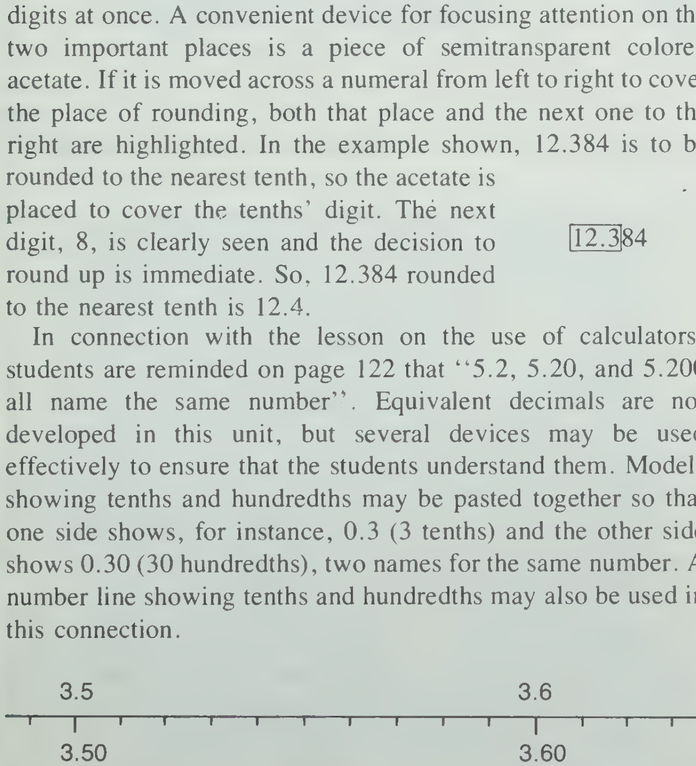
2.764 ○ 2.749

2.7$\boxed{6}$4
2.7$\boxed{4}$9

Rounding numbers is a comparatively easy skill, but some students may have difficulty because they try to consider all the digits at once. A convenient device for focusing attention on the two important places is a piece of semitransparent colored acetate. If it is moved across a numeral from left to right to cover the place of rounding, both that place and the next one to the right are highlighted. In the example shown, 12.384 is to be rounded to the nearest tenth, so the acetate is placed to cover the tenths' digit. The next digit, 8, is clearly seen and the decision to round up is immediate. So, 12.384 rounded to the nearest tenth is 12.4.

$\boxed{12.3}$84

In connection with the lesson on the use of calculators, students are reminded on page 122 that "5.2, 5.20, and 5.200 all name the same number". Equivalent decimals are not developed in this unit, but several devices may be used effectively to ensure that the students understand them. Models showing tenths and hundredths may be pasted together so that one side shows, for instance, 0.3 (3 tenths) and the other side shows 0.30 (30 hundredths), two names for the same number. A number line showing tenths and hundredths may also be used in this connection.



A place-value chart is also useful for showing the equivalent values of a decimal.

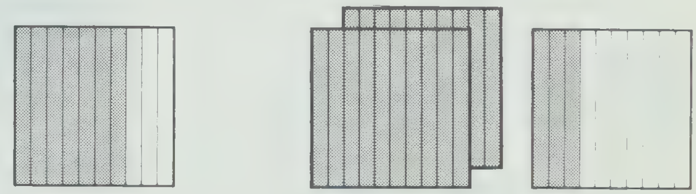
ones		tenths	hundredths	thousandths	
2	.	4			2 and 4 tenths
2	.	4	0		2 and 40 hundredths
2	.	4	0	0	2 and 400 thousandths

If there are only a few calculators available, it will be necessary to group the students and to begin the lesson earlier in the unit. If calculators are not available, students will not appreciate the value of the exercises which ask for the number of times the keys must be pressed; but they can benefit from solving the problems on page 123 using their usual methods.

Models for Ones, Tenths, and Hundredths

To represent one-place decimals such as 0.7 and 2.3, make white models and blue models using copies of page T 394. To show 0.7, use a white model and color 7 of the 10 strips blue (A). To show 2.3, display 2 blue whole models and color 3 of the 10 strips blue on a white model (B). Cut a few blue models into tenth strips. These may be placed on white models to show regrouping of 10 tenths as 1 whole (or vice versa). Also, students can place tenth strips over hundredths to illustrate equivalent decimals such as 0.2 and 0.20.

A B



0.7 2.3

To represent two-place decimals such as 0.34 and 1.76, make white models and blue models using copies of page T 395. To show 0.34, use a white model and color 34 of the 100 squares blue.

Materials

models for ones, tenths, and hundredths as described above
unmarked metre sticks and a metre stick marked in decimetres
a large blue model for one whole showing tenths on one side and the undivided (unmarked) whole on the reverse side
models for 1 thousand, 10 hundreds, 10 tens, and 10 ones (optional)
a large blue model for one whole showing hundredths on one side and an unmarked whole on the reverse side
tenth strips to match the demonstration model
two copies of page T 389 for each student
a large blue model for one whole showing hundredths with one of the hundredth squares marked to show 10 thousandths
newspapers, catalogs, and magazines
calculators (optional)
copies of page T 397

Vocabulary

decimal, decimal point	display
one-place decimal	Canada Fitness Awards
tenths	Award of Excellence
hundredths	individual medley
two-place decimal	butterfly
thousandths	backstroke
three-place decimal	breaststroke
bracket	freestyle

Pages 100-101

LESSON OUTCOME

Read and write numerals and words for one-place decimals; interpret place value in one-place decimals

Materials

a large blue model for one whole showing tenths on one side and the undivided (unmarked) whole on the reverse side; models for ones and tenths prepared with copies of page T 394 as described on page T 109; models for 1 thousand, 10 hundreds, 10 tens, and 10 ones (optional); unmarked metre sticks and a metre stick marked in decimetres (see the illustration on page 100)

Vocabulary

decimal, decimal point, one-place decimal, tenths

Prerequisite Skills

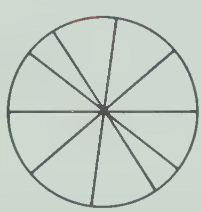
Identify wholes that show ten equal parts

Checking Prerequisite Skills

Which of these shows ten equal parts?

1. yes

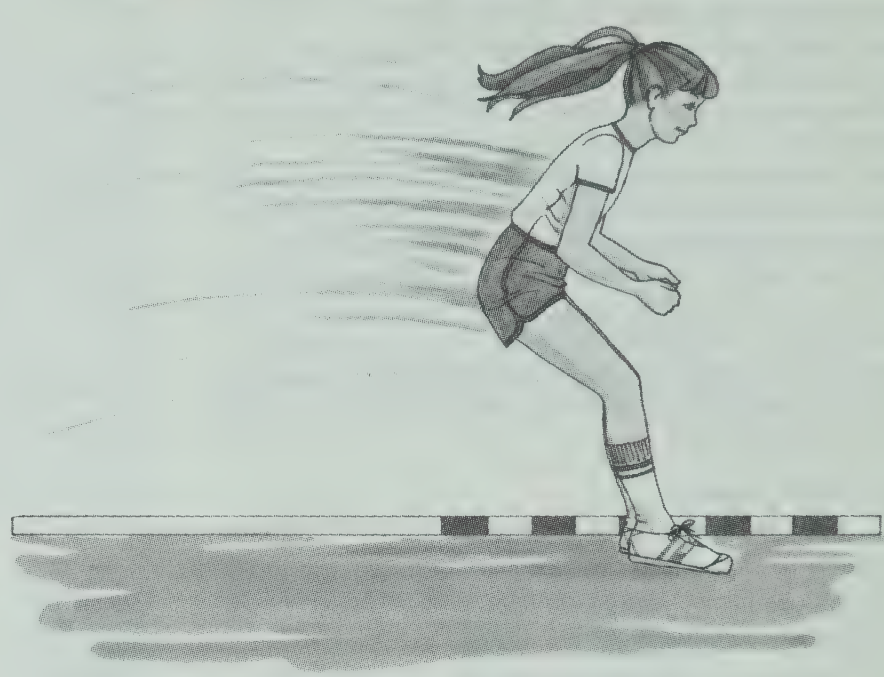
2. no



6 DECIMALS

Using Decimals to Show Tenths

Erin is trying the standing long jump in the Canada Fitness Awards tests. How far did she jump?



Erin jumped past 1 metre stick and 4 of 10 equal parts of another.

1 m (metre) and 4 of 10 equal parts of another metre is 1.4 m.

Erin jumped 1.4 m.
1.4 is a **decimal**.
The **.** is a **decimal point**.

When you read a decimal, say "and" for the decimal point.
Erin jumped one *and* four-tenths metres.

A decimal with one digit to the right of the decimal point is a **one-place decimal**.

A one-place decimal shows how many wholes and how many tenths of another whole there are.

100

LESSON ACTIVITY

Before Using the Pages

- Introduce the concept of decimals by discussing the need for numerals other than whole numbers.
- Display the unmarked side of the large blue whole model and tell the students that it represents one whole. Then display the reverse side and show them that the same whole is marked into 10 equal parts. They may be able to suggest that each part is named *one-tenth*. Display models of tenths less than one and have students name the number of tenths.

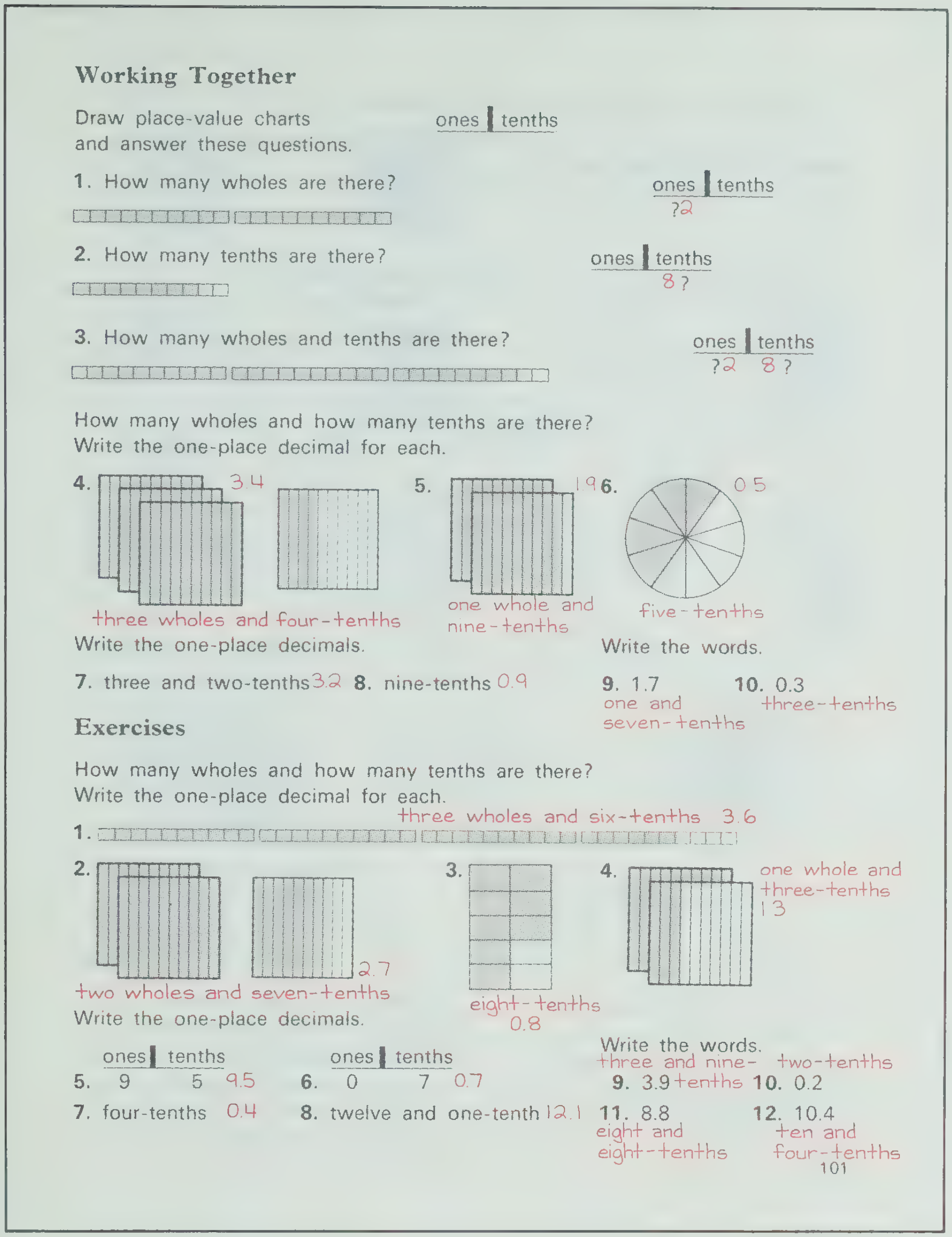
  Review the place-value names *thousands*, *hundreds*, *tens*, and *ones*. Have students recall that 1 thousand can be thought of (regrouped) as 10 hundreds, 1 hundred as 10 tens, and 1 ten as 10 ones. It follows logically in our numeration system that one whole (or 1) can be thought of in terms of 10 equal parts, and the name *tenths* describes the parts.

  Refer to a place-value chart and emphasize that each place from right to left has ten times the value of the place on its right, and each place from left to right has one-tenth the value of the place on its left. For example, 1 hundred = 10 tens and 1 ten = 10 ones. Similarly, 1 ten is one-tenth of 1 hundred and 1 one is one-tenth of 1 ten. Therefore, to show tenths, it is necessary to move one place to the right of ones. Since ones (and places to the left) are whole numbers, and since tenths (and places to the right) are less than whole numbers, a dot (decimal point) is used to separate them. Thus, 4 tenths may be shown by a 0 in the ones' place to represent no whole numbers and a 4 in the tenths' place, separated by a decimal point (0.4).

  Ask students how to show 9 tenths, 1 and 4 tenths, 3 and 8 tenths, and other similar one-place decimals. A place-value chart showing ones and tenths may be used, but the decimal notation using a decimal point should be emphasized before the students refer to page 100.

Using the Pages

- Read the title of the lesson on page 100 and have the students



Working Together

Draw place-value charts and answer these questions. ones | tenths

1. How many wholes are there? ones | tenths ?2

2. How many tenths are there? ones | tenths 8 ?

3. How many wholes and tenths are there? ones | tenths ?2 8 ?

How many wholes and how many tenths are there?
Write the one-place decimal for each.

4. 3.4 three wholes and four-tenths
5. 1.9 one whole and nine-tenths
6. 0.5 five-tenths

Write the one-place decimals.

7. three and two-tenths 3.2
8. nine-tenths 0.9

Write the words.

9. 1.7 one and seven-tenths
10. 0.3 three-tenths

Exercises

How many wholes and how many tenths are there?
Write the one-place decimal for each.

1. three wholes and six-tenths 3.6
2. 2.7 two wholes and seven-tenths
3. eight-tenths 0.8
4. one whole and three-tenths 1.3

Write the one-place decimals.

	ones	tenths	
5.	9	5	9.5
6.	0	7	0.7

7. four-tenths 0.4
8. twelve and one-tenth 12.1

Write the words.

9. 3.9 three and nine-tenths
10. 0.2 two-tenths
11. 8.8 eight and eight-tenths
12. 10.4 ten and four-tenths

101

RELATED ACTIVITIES

- Have students use models to represent the numbers in Ex. 7-10 of *Working Together*.
- Ask students to count by tenths to nine-tenths and write the decimals on the board. Ask what comes after nine-tenths and then ask for a simpler way to express ten-tenths. Emphasize that ten-tenths is the same as one whole. Then develop the relationships 20 tenths = 2 wholes, 30 tenths = 3 wholes, and so on. Have students copy and complete patterns similar to the following.

0.5	__	__
0.6	__	__
0.7	3.0	__
__	3.1	12.2
__	3.2	12.3
__	__	12.4
1.1	__	__

- Prepare a work sheet for which students match decimals with their word names.

three and one-tenth	3.0
three-tenths	1.3
three and five-tenths	13.3
one and three-tenths	0.3
three and zero-tenths	3.1
three and three-tenths	5.3
thirteen and three-tenths	3.5
five and three-tenths	3.3

observe the word "ten" in the word "tenths". Discuss the difference in meaning of the two words. Introduce the example, explaining the Canada Fitness Awards for those who are not familiar with them.

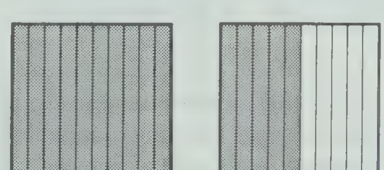
Guide the students through the example, paying particular attention to the terms *decimal*, *decimal point*, and *one-place decimal*. Emphasize that a decimal is read using the word *and* for the decimal point. Write the decimals 1.7 and 0.3 on the board and have students read the numerals, explaining the meaning of the 1 in 1.7 and the 0 in 0.3. Tell the students that such decimals as 0.3 may be read as "three-tenths" or as "zero and three-tenths". Point out the hyphen in the word name for 1.4 on page 100.

Working Together: Ex. 1-6 deal with writing a decimal for a given diagram, emphasizing that the ones are considered first and then the tenths. This establishes the order of writing the corresponding numeral from left to right and helps students to remember the zero for zero wholes as in Ex. 6 and 8. It also helps them to place the decimal point correctly. Draw attention to the black, vertical bar separating ones and tenths in the headings of the place-value charts (Ex. 1-3). This serves as a reminder for the position of the decimal point in the numeral.

Exercises: Remind the students that a hyphen is used in the word name of a decimal. Ensure that the students place the decimal point correctly and do not show a raised dot.

Assessment

Write the one-place decimal for each.

1.  1.5

2.  2.3

3. nine-tenths 0.9

4. seven and four-tenths 7

5. ones | tenths: 3 | 2 — 3.2

Write the words.

6. 4.8 four and eight-tenths
7. 0.6 six-tenths
8. 10.1 ten and one-tenth

Pages 102-103

LESSON OUTCOME

Read and write numerals and words for two-place decimals; interpret place value in two-place decimals

Materials

a large blue model for one whole showing hundredths on one side and an unmarked whole on the reverse side (the model must be the same size as the demonstration model used for tenths in the preceding lesson), tenth strips to match the demonstration model, models for ones and hundredths prepared with copies of page T 395 as described on page T 109

Vocabulary

hundredths, two-place decimal

Prerequisite Skills

Read and write decimal notation for tenths

Checking Prerequisite Skills

Write the decimals.

1. five-tenths 0.5
2. two and three-tenths 2.3
3. eleven and seven-tenths 11.7

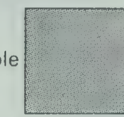
Write the words.

4. 10.4 ten and four-tenths
5. 0.1 one-tenth

Using Decimals to Show Hundredths

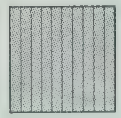
A decimal shows how many wholes and how many parts of another whole there are.

1 whole



ones
1

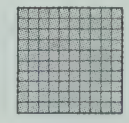
1 whole with 10 equal parts



ones	tenths
1	0

1.0 or 10 tenths

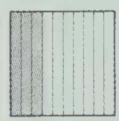
1 whole with 100 equal parts



ones	tenths	hundredths
1	0	0

1.00 or 100 hundredths

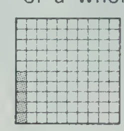
3 of 10 equal parts of a whole



ones	tenths
0	3

0.3 or 3 tenths

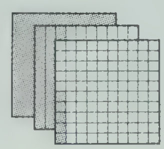
5 of 100 equal parts of a whole



ones	tenths	hundredths
0	0	5

0.05 or 5 hundredths

1 whole and 3 tenths 5 hundredths

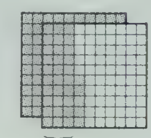


$$\begin{array}{r} 1 \\ 0.3 \\ 0.05 \\ \hline 1.35 \end{array}$$

ones	tenths	hundredths
1	3	5

or

1 and 35 hundredths



3 tenths = 30 hundredths

or **1.35** 1 and 35 hundredths

The decimal 1.35 stands for 1 whole and 35 of 100 equal parts of another whole.

It also shows that there are 135 hundredths in all.

A decimal with two digits to the right of the decimal point is a **two-place decimal**.

A two-place decimal shows how many wholes and how many hundredths of another whole there are.

102

LESSON ACTIVITY

Before Using the Pages

- Display the unmarked side of the large whole model and ask what it represents. Display the reverse side and have the students observe that there are 100 equal parts. They may be able to suggest that each part is named *one-hundredth*. Demonstrate that one-hundredth is part of the whole just as one-tenth is, but one-hundredth is a smaller part than one-tenth. Display models of hundredths (less than one and greater than one) and have students name the numbers represented.
- Place tenth strips on the demonstration model of hundredths to show that one-tenth is the same as ten-hundredths, two-tenths is the same as twenty-hundredths, and so on. Place tenth strips on a model for 47 hundredths, for example, to show that it is the same as 4 tenths 7 hundredths. Use other examples as required.
- Refer to a place-value chart and remind the students that each place to the right has one-tenth the value of the place on its left. Review that "tenths of ones" are located one place to the right of ones and separated from them by a decimal point.

thousands	hundreds	tens	ones	tenths

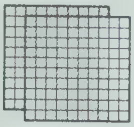
Ask where "tenths of tenths" (hundredths) are located. The students will probably suggest one more place to the right, making two places to the right of ones. Ask them to show, for example, 67 hundredths, 30 hundredths, 3 hundredths, 1 and 45 hundredths, 2 and 70 hundredths, 4 and 4 hundredths.

Using the Pages

- The examples on page 102 present hundredths in terms of part of the whole and in terms of place value. Read the title of the lesson on page 102 and point out the word "hundred" in the word "hundredths". Discuss the difference in meaning of the two words. Then guide the students through the examples. The place-value chart is

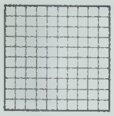
Working Together

How many wholes and how many hundredths are there?

1. 

ones	tenths	hundredths
1	7	5

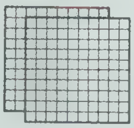
or 1 one and 75 hundredths

2. 

ones	tenths	hundredths
0	3	6

or 0 ones and 36 hundredths

Write the two-place decimal for each of these.

3.  1.40

4.

ones	tenths	hundredths
0	0	5

0.05

5. one and seven-hundredths 1.07

6. 12 and 25 hundredths 12.25

Write using words.

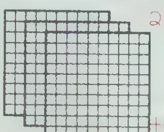
7. 3.59 three and fifty-nine hundredths

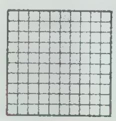
8. 0.67 sixty-seven hundredths

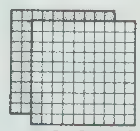
9. 11.01 eleven and one-hundredth

Exercises

How many wholes and how many hundredths are there?
Write the two-place decimal for each.

1  2.42 two ones and forty-two hundredths

2.  0.66 zero ones and sixty-six hundredths

3.  1.03 ~~one~~ one and three-hundredths

Write the two-place decimals.

4.

ones	tenths	hundredths
2	0	9

2.09

5.

ones	tenths	hundredths
0	2	5

0.25

6. 10 and 75 hundredths 10.75

7. three and six-hundredths 3.06

Write using words.
8 seven and thirty-three hundredths or 7 and 33 hundredths
9. four and four-hundredths or 4 and 4 hundredths

Example: 4.12 can be written "four and twelve-hundredths" or "4 and 12 hundredths".

8. 7.33 **9.** 4.04 **10.** 13.99 **11.** 5.10 **12.** 0.02 **13.** 21.12

10 thirteen and ninty-nine hundredths or 13 and 99 hundredths
11 five and ten-hundredths or 5 and 10 hundredths
12 two-hundredths or 2 hundredths
13 twenty-one and twelve-hundredths or 21 and 12 hundredths

103

RELATED ACTIVITIES

- Have students use models to represent the numbers in a selection of exercises from page 103.
- Have students complete charts similar to the following to show different names for the same number.

1 whole	10 tenths	100 hundredths
2 wholes	20 tenths	200 hundredths

1 tenth	=	10 hundredths
2 tenths	=	20 hundredths
3 tenths	=	30 hundredths

0.1 = 0.10
0.2 = 0.20
0.3 = 0.30

- Work sheets similar to the following provide practice in writing and interpreting two-place decimals.

Word name	o t h	Decimal
62 hundredths	0 6 2	0.62
	1 0 3	
9 hundredths		
		4.18

extended to the right so that 1 whole is interpreted as 1 one, then as 1 one 0 tenths, and eventually as 1 one 0 tenths 0 hundredths. Three-tenths is reviewed as a one-place decimal, that is, 0.3. Five-hundredths is introduced as a *two-place decimal*. Emphasize the zero in the tenths' place. Without the zero to show 0 tenths, the numeral becomes 5 tenths, not 5 hundredths. The relationship between tenths and hundredths is illustrated for 1.35. In all, there are three interpretations given for 1.35:

1 one 3 tenths 5 hundredths
1 and 35 hundredths
135 hundredths

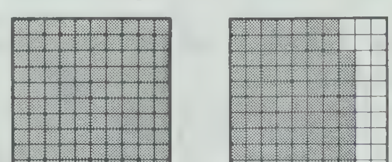
Summarize the concepts by reading the statements at the bottom of page 102.

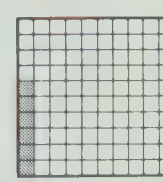
Working Together: Ex. 1 and 2 deal with place value for two-place decimals and the way in which they are read. Ex. 3-6 concern writing two-place decimals. Students having difficulty with Ex. 5 may find it helpful to think of a place-value chart similar to the one in Ex. 4. Use other examples as required.

Exercises: You may need to remind students to write a zero to the left of the decimal point for numbers less than one, as for Ex. 2.

Assessment

Write the two-place decimal for each.

1.  1.78

2.  0.06

3.

ones	tenths	hundredths
4	0	5

4.05

4.

ones	tenths	hundredths
0	3	2

0.32

5. twelve and seven-hundredths 12.07

6. 8 and 18 hundredths 8.18

Write using words.

7. 0.01 one-hundredth

8. 16.16 sixteen and sixteen-hundredth

Pages 104-105

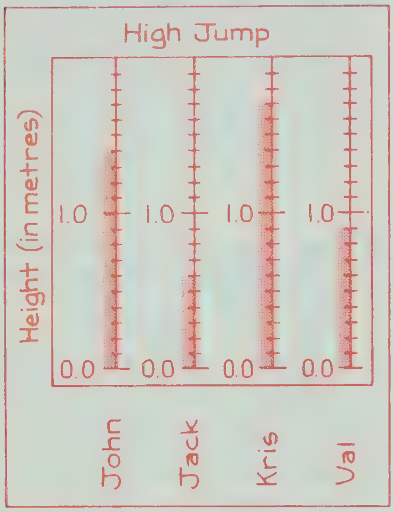
OBJECTIVE

Demonstrate competence in reading and writing decimal tenths and hundredths, and in interpreting place value in one-place and two-place decimals

Materials

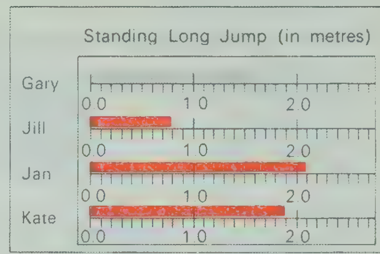
two copies of page T389 for each student

3.



Practice

Draw and color number lines to show how far each student jumped.



1.

Standing Long Jump			
Gary	1.6 m	Jill	0.8 m
Jan	2.1 m	Kate	1.9 m

Answer is given on page T115.

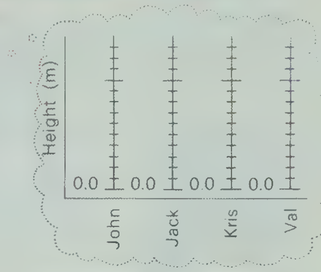
2.

Running Long Jump	
Mark	3.2 m
Theresa	2.9 m
Ian	3.5 m
Edward	4.1 m

Answer is given at the left.

3.

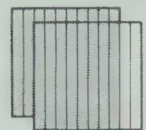
High Jump	
John	1.4 m
Jack	0.6 m
Kris	1.7 m
Val	0.9 m

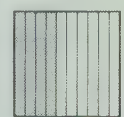


Write a one-place decimal for each of these.

4. 

1.3

5. 
1.8

6. 
0.4

7.

ones	tenths
2	7

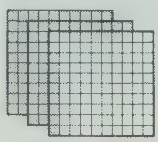
2.7

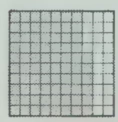
8.

ones	tenths
0	2

0.2

Write a two-place decimal for each of these.

9. 
2.08

10. 
0.76

11.

ones	tenths	hundredths
3	5	0

3.50

12.

ones	tenths	hundredths
0	0	9

0.09

Write the decimals.

13. three-tenths 0.3 **14.** eleven-hundredths 0.11 **15.** one and five-tenths 1.5

16. three and nineteen-hundredths 3.19 **17.** 12 and 80 hundredths 12.80

18. 20 and 2 hundredths 20.02 **19.** 2 and 0 tenths 2.0

Write using words.

20. 0.15 fifteen-hundredths **21.** 15.06 fifteen and six-hundredths

22. 0.7 seven-tenths **23.** 100.3 one hundred and three-tenths

Complete.

24. In 1.7, the 7 stands for 7 ? . tenths

25. In 1.07, the 7 stands for 7 ? . hundredths

104

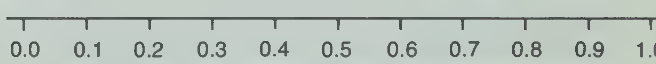
LESSON ACTIVITY

Before Using the Pages

- Draw a number line on the board and have students label the points for whole numbers. Demonstrate that the segments between these points can be divided into tenths to locate points for decimal tenths. Have students label several points for decimal tenths.

Emphasize that whole numbers have decimal names. For example, 1 and 1.0 are different names for the number one, and either numeral is satisfactory when labeling a number line. The situation often determines which numeral is used.

Draw another number line on the board showing the interval from 0 to 1 marked into ten equal parts. Use a decimal tenth to label each point.



Ask how points for decimal hundredths can be located on a number line. Lead the students to suggest that it is necessary to divide the segments between tenths into ten equal parts. Point out that in following the scale of the number line on the board, the procedure would be rather difficult. Draw and label another number line with a more appropriate scale for showing hundredths. (Note that this lesson will not require students to mark hundredths on a number line.)

Kathy Kreiner of Canada won an Olympic Gold Medal in the Giant Slalom.

Replace words with numerals, where possible, and write each sentence.

Example: Write "*two* minutes *three and forty-five hundredths* seconds" as "2 min 3.45 s".

Kathy's winning time was 1 min 29.13 s.

26. Kathy's winning time was *one* minute *twenty-nine and thirteen-hundredths* seconds.

27. The second-best time was 1 min *29 and 25 hundredths* seconds.

The second-best time was 1 min 29.25 s.

28. Kathy's time was *twelve-hundredths* of a second faster than the second-best time.

29. In an earlier Olympics, Canada's Nancy Greene won a gold medal in this event. Her time was 1 min *51 and 97 hundredths* seconds.

A slalom ski can be as long as 1.75 m.

30. A slalom ski can be as long as *one and seventy-five hundredths* metres.

A trick ski can be as short as 0.65 m

31. A trick ski can be as short as *sixty-five hundredths* of a metre.

The race was won by 0.07 s.

32. The race was won by *seven-hundredths* of a second.

Another skier took 1.01 s longer.

33. Another skier took *one and one-hundredth* seconds longer.

28 Kathy's time was 0.12 s faster than the second-best time

29. In an earlier Olympics, Canada's Nancy Greene won a gold medal in this event. Her time was 1 min 51.97 s

105

RELATED ACTIVITIES

- Ask students to find examples of records in sports events for which the numbers are decimals. These may be discussed and the numbers interpreted and compared.
- Have students prepare a list of situations that involve tenths. For example, gasoline is purchased by the litre and the gauge on the gasoline pump registers whole litres and tenths of a litre. A similar list may be prepared for hundredths. Students may cut pictures from magazines and catalogs to accompany the list. These may be displayed for reference.
- Provide exercises similar to the following to reinforce the concept of place value in decimals and whole numbers.

What does each 3 mean?

1. 4.13	2. 413
3. 41.3	4. 34.1
5. 43.1	6. 14.3
7. 341	8. 3.14

- Have students write decimals using the symbol \$ for amounts of money expressed as cents.

1¢ = \$ 0. 0 1

25¢ = \$ __. __ __

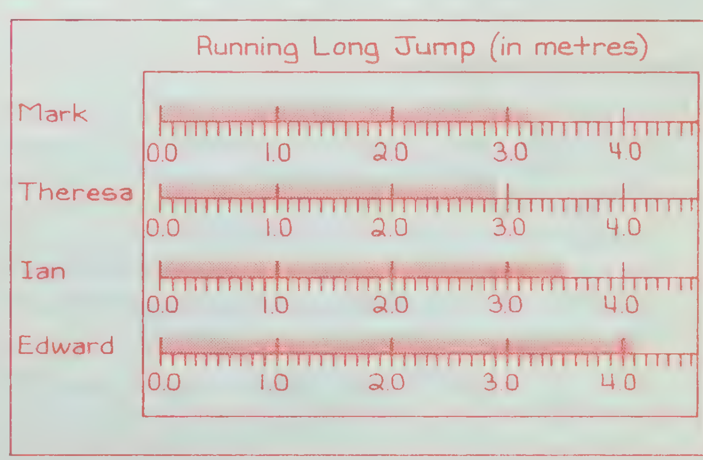
163¢ = \$ __. __ __

Using the Pages

- These exercises review the concepts of the previous two lessons and involve the use of the number line. Since Ex. 1-3 require marking twelve number lines, each student will need two copies of the number lines on page T 389. Note that since the first number line on page T 389 is marked in whole numbers, the students will need to adapt these numerals by showing the decimal points and the necessary zeros to the left of the decimal points. Ex. 3 suggests the use of vertical number lines. Have the students turn their pages and label the lines vertically.

Before the students begin the exercises on page 105, discuss the photograph at the top of the page and the example below the photograph. Introduce "slalom" and "trick ski", since these words appear in Ex. 30 and 31.

2.



LESSON OUTCOME

Read and write numerals and words for three-place decimals; interpret place value in three-place decimals

Materials

a large blue model of one whole showing hundredths with one of the hundredth squares marked to show 10 thousandths (use a copy of the 10-by-10 grid on page T 391)

Vocabulary

thousandths, three-place decimal

Prerequisite Skills

Read and write decimal notation for tenths and hundredths

Checking Prerequisite Skills

Write the decimals.

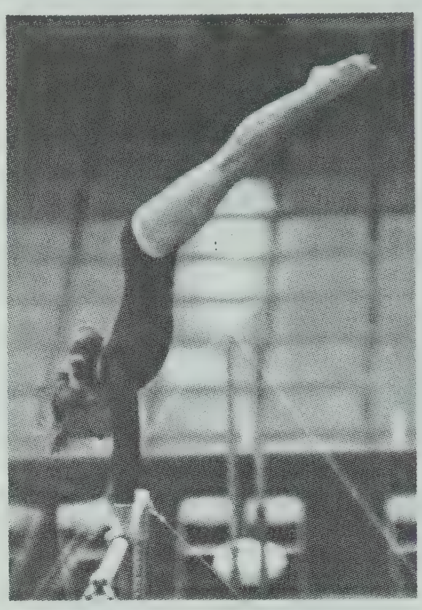
1. eight and two-tenths 8.2
2. four and twenty-hundredths 4.20
3. seventeen and three-hundredths 17.03

Write using words.

4. 0.96 ninety-six hundredths
5. 42.9 forty-two and nine-tenths
6. 56.09 fifty-six and nine-hundredths
7. 5.40 five and forty-hundredths

Using Decimals to Show Thousandths

Elfi Schlegel's All-Around score at the Canadian National Gymnastic Championships was 75.775.



75.775 has three digits to the right of the decimal point. It is a **three-place decimal**. The three decimal places are named in this chart.

thousands	hundreds	tens	ones	tenths	hundredths	thousandths
		7	5	7	7	5

To read any decimal, use the place name of its last digit.

75.7 is "75 and 7 *tenths*".

75.77 is "75 and 77 *hundredths*".

75.775 is "75 and 775 *thousandths*".

Remember that the word "and" is used for the decimal point.

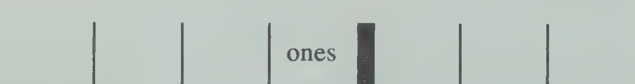
Elfi's All-Around score was 75 and 775 thousandths.

106

LESSON ACTIVITY

Before Using the Pages

- Draw lines on the board for a place-value chart with seven columns and write "ones" in the middle column, using white chalk. Label the remaining columns during the following discussion, using different colors of chalk to highlight the "symmetry" of the columns on either side of the ones' column. For example, "tens" and "tenths" would be shown in the same color, "hundreds" and "hundredths" in another color, and so on.



Ask for the name of the place value to the left of the ones' place and then for the place value to the right of the ones' place, labeling each in turn. Similarly, ask for and write the names of the place values to the left of the tens' place and to the right of the tenths' place. Ask for and write the name of the place value to the left of the hundreds' place. Ask what the name of the place to the right of the hundredths' place probably is. Students will likely suggest *thousandths*.

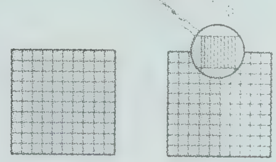
When the seven columns have been named, review the relationships among the places. Have students recall that 1 thousand equals 10 hundreds, 1 hundred equals 10 tens, and so on, to 1 one equals 10 tenths. At this point, remind the students that amounts less than one are represented. Continue with 1 tenth equals 10 hundredths, 1 hundredth equals one-tenth of 1 tenth, and 1 thousandth equals one-tenth of 1 hundredth. Ask students to explain the relationship between 1 hundredth and 10 thousandths.

Demonstrate the relationships for place values to the right of the ones' place as follows. From a large model of one whole, cut a strip for one tenth. From the tenth strip, cut a square for one hundredth. From the hundredth square, cut a strip for one thousandth. Ask students how many places to the right of the decimal point are needed to show thousandths.

Working Together

1 thousandth is
1 of 1000 equal parts of a whole.
How many wholes and how many thousandths are there?

1 thousandth is 1 of 10 equal parts of 1 hundredth.



1.

ones	tenths	hundredths	thousandths
1	4	9	1

or 1 one and 491 thousandths

2.

ones	tenths	hundredths	thousandths
2	0	3	8

or 2 ones and 38 thousandths

Write the three-place decimal for each of these.

3. 4 and 825 thousandths 4.825
4. 42 and 29 thousandths 42.029
5. five-thousandths 0.005

Write using words.

6. 0.375 three hundred seventy-five thousandths
7. 2.097 two and ninety-seven thousandths
8. 15.008 fifteen and eight-thousandths

Exercises

Write the three-place decimals.

1. one hundred twenty-five thousandths 0.125
2. 6 and 200 thousandths 6.200
3. one hundred and twenty-five thousandths 100.025
4. 5 and 18 thousandths 5.018

Write using words.

5. 3.590 three and five hundred ninety-thousandths
6. 3.509 three and five hundred nine-thousandths
7. 3.059 three and fifty-nine thousandths
8. 0.359 three hundred fifty-nine thousandths

Think of a place-value chart. Tell what each 5 means.

tens	ones	tenths	hundredths	thousandths

Example: In 23.456, the 5 means 5 hundredths.

9. 1.568 5 tenths
10. 5.033 5 ones
11. 11.005 5 thousandths
12. 0.057 5 hundredths
13. 50.505 5 tens 5 tenths 5 thousandths

Replace words with decimals, where possible, and write each sentence.

14. Elfi scored *18 and 975 thousandths* points on the balance beam.
Elfi scored 18.975 points on the balance beam.

15. In the floor exercises, Elfi missed a perfect score of 20 by *875 thousandths* of a point.
In the floor exercises, Elfi missed a perfect score of 20 by 0.875 point.

107

RELATED ACTIVITIES

- Have students ring different names for the same number as shown.
18.4: 18.04, 18.40, 18.004, 18.400
13.02: 13, 13.2, 13.020, 13.002
17.1: 17.01, 17.100, 17.10, 17.001
- The third activity in *Related Activities* on page T 113 can be adapted for three-place decimals having up to five digits.
- Prepare three sets of cards: set A to show tenths (''one-tenth'' to ''nine-tenths'' on one side and ''0.1'' to ''0.9'' on the other); set B to show hundredths (''ten-hundredths'' to ''ninety-hundredths'' on one side and ''0.10'' to ''0.90'' on the other); and set C to show thousandths (''100 thousandths'' to ''900 thousandths'' on one side and ''0.100'' to ''0.900'' on the other). Have students show three names for the same number by matching one card from each set using either side of a card as shown.

seven-tenths

seventy-hundredths

0.700

0.5

fifty-hundredths

500 thousandths

Using the Pages

- The example introduces the *three-place decimal* and illustrates that thousandths are useful in naming scores in competitive sports. Particular attention is given to reading a decimal according to the name of the place value of the last digit on the right. Read the example and discuss it with the students. Emphasize the manner of reading the decimals 75.7, 75.77, and 75.775.

Working Together: Ex. 1 and 2 deal with thousandths as part of a whole and in terms of place value. Relate the digits in the place-value chart of Ex. 1 to the diagrams to the right of the chart. The diagrams show 1 whole, 4 tenths (40 hundredths), 9 hundredths, and 1 thousandth for 1.491. Three-place decimals for which there are 0 tenths and/or 0 hundredths should be given careful attention (Ex. 2, 4, 5, 7, and 8). Models and place-value charts are helpful in explaining such examples. Have the students sketch diagrams to illustrate the number in Ex. 2, for example, and interpret the place value of each digit in the numerals for Ex. 6-8. Emphasize that in reading a decimal, the name of the place value of the last digit on the right is used.

Exercises: The word ''and'' is the significant word in each of Ex. 1-4 because it indicates the position of the decimal point. Note the phrase ''where possible'' in the instructions preceding Ex. 14 and 15. The numeral 20 in Ex. 15 may be left as a whole number or it may be expressed as 20.000.

Assessment

Write the three-place decimal for each.

1. six and one hundred twenty-four thousandths 6.124

2. three hundred five-thousandths 0.305

3. four and four-thousandths 4.004

Write using words.

4. 2.620 **5.** 0.043 **6.** 8.009 **7.** 3.165

4. two and six hundred twenty-thousandths
5. forty-three thousandths
6. eight and nine-thousandths
7. three and one hundred sixty-five thousandths

Tell what each digit means.

8. 67.403
6 tens 7 ones 4 tenths 0 hundredths 3 thousandths

Page 108

LESSON OUTCOME

Compare two decimals showing tenths, hundredths, and thousandths

Materials

models for ones, tenths, and hundredths

Prerequisite Skills

Compare whole numbers; interpret place value in three-place decimals

Checking Prerequisite Skills

Which is greater,

1. 207 or 270? 270
2. 3560 or 3506? 3560
3. 10 457 or 4 750? 10 457

Think of a place-value chart.
Tell what each 4 means.

4. 3.462 4 tenths
5. 41.063 4 tens
6. 9.384 4 thousandths
7. 16.24 4 hundredths

RELATED ACTIVITIES

- As an extension of the lesson, have students compare two numbers for which one of the two is a whole number and the other a decimal.

3.15 ◯ 3
7 ◯ 6.908
14 ◯ 14.001

Comparing Decimals

Al scored 18.4 on the rings and 18.375 on the parallel bars.

In which event did he have a better score?

18.4 is the same as 18.400.

Study the digits from the left.

18.400 } show the same number of tens
18.375 } and the same number of ones.

18.400 shows 4 tenths.
18.375 shows 3 tenths.

4 is greater than 3, so
18.400 is greater than 18.375.

4 > 3, so
18.400 > 18.375.

Al had the better score on the rings.

Working Together

Write each as a two-place decimal.

1. 0.6 0.60
2. 2.1 2.10

Write each as a three-place decimal.

3. 1.7 1.700
4. 3.05 3.050

Compare the whole numbers and the digits in the decimal places. Tell which decimal is greater.

5. 5.647
5.674
5.674 > 5.647
6. 13.305
13.35
13.35 > 13.305

Use >, <, or = to make a true statement.

Example: 1.36 is less than 1.63, so 1.36 < 1.63.

7. 1.205 ◯ 1.052 >
8. 0.09 ◯ 0.8 <

Exercises

Use >, <, or = to make true statements.

1. 2.07 ◯ 2.070 =
2. 28.1 ◯ 21.8 >
3. 1.236 ◯ 12.36 <
4. 1.58 ◯ 1.85 <
5. 25.3 ◯ 25.003 >
6. 0.6 ◯ 0.600 =
7. 7.6 ◯ 7.56 >
8. 3.750 ◯ 3.75 =
9. 5.74 ◯ 54.7 <
10. 15.3 ◯ 15.30 =
11. 0.408 ◯ 0.48 <
12. 10.54 ◯ 10.457 >

108

LESSON ACTIVITY

Before Using the Page

- Review with the students that such numerals as 2.5 and 2.50 are names for the same number. Use models to show, for instance, that 5 tenths, 50 hundredths, and 500 thousandths are equal parts of a whole. Use a place-value chart to show that for 2.5, there are 2 ones in the ones' place, 5 tenths in the tenths' place, 0 hundredths in the hundredths' place, and 0 thousandths in the thousandths' place.

ones	tenths	hundredths	thousandths
2	5	0	0

Using the Page

- The example demonstrates that in order to compare two decimals, they must be considered in terms of the same number of decimal places. Then the comparison can be considered place by place, from left to right, as in comparing two whole numbers. In discussing the example with the students, some of them may suggest that 18.375 should be greater than 18.4, because 375 is greater than 4. It is important to demonstrate that 4 tenths is the same as 400 thousandths, which is greater than 375 thousandths. Then explain that it is unnecessary to compare the numbers beyond the tenths' place, since 4 tenths is greater than 3 tenths.

Working Together: Ex. 1-4 deal with showing the same number of decimal places for two numbers. Use other similar exercises as required before continuing with Ex. 5-8.

Exercises: Note that for exercises such as Ex. 2 and 3, the comparisons need not involve the digits to the right of the decimal point, only the whole number digits.

Assessment

Use >, <, or = to make true statements.

1. 3.05 ◯ 3.050 =
2. 17.4 ◯ 13.9 >
3. 8.3 ◯ 8.29 >
4. 0.206 ◯ 0.26 <
5. 11.8 ◯ 11.80 =
6. 21.004 ◯ 21.4 <

Ordering Decimals

Who had the greatest score on the parallel bars?

Al scored 18.375

Beau scored 18.55

Cy scored 18.5

To compare 18.375, 18.55, and 18.5,
first think of each as a three-place decimal.

18.375 = 18.375
18.55 = 18.550
18.5 = 18.500

Then, start from the left and look at the digits place by place.

18.375	18.375	18.375
18.550	18.550	18.550
18.500	18.500	18.500
same same	18.375 is the least.	18.550 is the greatest.

Beau had the greatest score on the parallel bars.

Turn the page for more work with ordering decimals.

LESSON OUTCOME

Order decimals showing tenths, hundredths, and thousandths

Vocabulary

bracket

Prerequisite Skills

Order whole numbers; write one-place and two-place decimals as equivalent three-place decimals

Checking Prerequisite Skills

List from least to greatest.

1. 1975, 1988, 1963, 1936

2. 26 815, 26 005, 26 847, 6 815, 26 800

Write each as a three-place decimal.

3. 0.4 0.400 **4.** 13.8 13.800

5. 6.07 6.070 **6.** 1.23 1.230

1. 1936, 1963, 1975, 1988

2. 6 815, 26 005, 26 800, 26 815, 26 847

LESSON ACTIVITY

Before Using the Pages

- Review the comparison of two decimals, for example, 12.2 and 12.224, place by place from left to right. Emphasize the importance of thinking of each number in terms of the same number of decimal places. Then tell the students that today's lesson involves comparing and ordering more than two decimals.

Using the Pages

- Have a student explain in her/his own words what is meant by the title *Ordering Decimals*. The worked example shows a situation that requires the ordering of three decimals. Guide the students through the example. Emphasize that lining up the numerals vertically so that the decimal points are aligned helps to visualize each as a three-place decimal. Note that the left-to-right comparison for the three given numbers results in finding the least number first.

  It may be necessary to review with some students why the comparison is carried out in a left-to-right order. Present an example using whole numbers and another using decimals, such that no digits are shown in one or two places at the left.

1. ____ _6_ _9_
   ____ _7_ _4_

2. ____ . ____ _4_ _2_
   ____ . ____ _1_ _7_

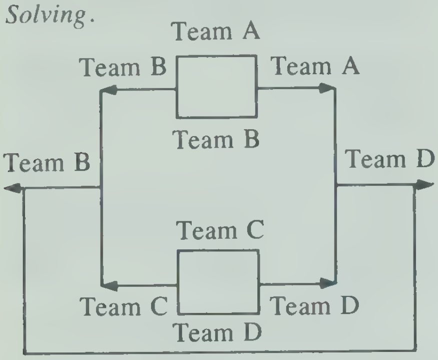
The students should conclude that the most important digits for comparing numbers are those on the left.

Working Together: Ex. 1 and 2 deal with showing the same number of decimal places in the numerals for the given numbers. This skill is applied in ordering numbers in Ex. 3 and 4. Note that the order for listing the numbers is different for these two exercises.

Exercises: Some students may prefer to write the numerals in vertical form with the decimal points aligned, prior to ordering the numbers.

RELATED ACTIVITIES

- Students may be interested in the following extension of the method shown for the example in *Problem Solving*.



Ask what they think of this type of arrangement for a tournament and when it would be preferable to the one shown on page 110. They may adapt their answers to Ex. 1-4 for the above procedure.

- Students can practice ordering two-place decimals using numerals for amounts of money shown in catalogs, magazines, newspapers, cash register tapes, and so on.

Working Together

Write the numerals so that they show the same number of decimal places.

1. 0.300 0.770 1.150 0.600
2. 0.790 0.821 0.500 0.200

List from greatest to least.

3. 0.23, 0.203, 2.2, 0.3
2.2, 0.3, 0.23, 0.203

List from least to greatest.

4. 1.5, 1, 0.515, 1.51
0.515, 1, 1.5, 1.51

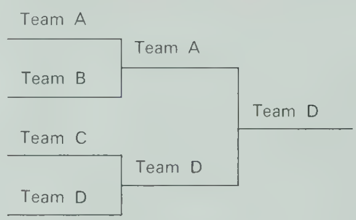
Exercises

List from greatest to least.

1. 2.18, 2.81, 2.09
2.81, 2.18, 2.09
2. 19.7, 1.97, 197, 0.197
197, 19.7, 1.97, 0.197
3. 1.2, 2.012, 1.201, 1.21
2.012, 1.21, 1.201, 1.2
4. 0.6, 0.64, 0.46, 0.646
0.646, 0.64, 0.6, 0.46

List from least to greatest.

5. 5.747, 5.745, 5.774
5.745, 5.747, 5.774
6. 2.3, 2.23, 2.203, 3.02
2.203, 2.23, 2.3, 3.02
7. 0.491, 0.05, 0.50, 0.150
0.05, 0.150, 0.491, 0.50
8. 10.408, 10.84, 12.4, 10.4
10.4, 10.408, 10.84, 12.4



This *bracket* shows how 4 teams were paired in the soccer tournament. It shows Team A defeated Team B, and Team D defeated Team C. Then Team D defeated Team A for the championship.

1. Make a bracket that will pair 8 teams for a softball tournament.
2. 15 students are entered in the wrist-wrestling tournament. Make a bracket for the 15.
3. 17 students are entered in the table-tennis tournament. Two of them will play at a time. Make a bracket for the 17.
4. 25 students are entered in the dominoes tournament. Three students will play in each game. Each game will have one winner. Make a bracket for this tournament.

PROBLEM SOLVING

Winning teams will vary, but the diagrams will be similar to those given on page T368.

110

Problem Solving: Organizational skills are required in situations that involve arranging tournaments in competitive sports. The example provided outlines a competition involving four teams. As the games were completed, the winning teams were shown on the corresponding branches of the *bracket*. For Ex. 1-4, you may prefer to have the students decide the winning members of each round so that the brackets will be completely labeled. Note that the students will have to consider how to adapt an odd number of teams or players for a tournament in Ex. 2-4. The usual procedure is that one person (team) does not play in the first round (he/she/the team receives a "bye"), but enters the competition with the winners from another round.

Assessment

List from greatest to least.

1. 0.25, 0.35, 0.3, 0.2 0.35, 0.3, 0.25, 0.2
2. 5.60, 5.69, 5.96, 5.5 5.96, 5.69, 5.60, 5.5

List from least to greatest.

3. 0.691, 0.069, 0.009, 0.006 0.006, 0.009, 0.069, 0.691
4. 3.57, 3.7, 3.578, 3.58 3.57, 3.578, 3.58, 3.7

Practice

Write the decimals.

1. twenty and five-tenths 20.5
2. 205 thousandths 0.205
3. two hundred and five-thousandths 200.005
4. 2 and 5 hundredths 2.05

Write using words.

5. 5.180 five and one hundred eighty-thousandths
6. 0.518 five hundred eighteen-thousandths
7. 5.018 five and eighteen-thousandths
8. 500.8 five hundred and eight-tenths

Tell what each 3 means.

9. 0.123 3 thousandths
10. 1.234 3 hundredths
11. 12.34 3 tenths

Tell what each 0 means.

12. 3.06 0 tenths
13. 0.730 0 ones, 0 thousandths
14. 1.005 0 tenths, 0 hundredths

Complete.

15.	one-place decimal	0.7	1.6 ?	0.3 ?	10.5		
16.	two-place decimal	0.70	1.60	0.30 ?	10.50 ?	2.57	1.36 ?
17.	three-place decimal	? 0.700	? 1.600	0.300	? 10.500	? 2.570	1.360

Use >, <, or = to make true statements.

18. 15.85 ● 15.580 >
19. 4.600 ● 4.6 =
20. 6.083 ● 6.308 <
21. 3.842 ● 37.42 <
22. 12.50 ● 12.050 >
23. 20.02 ● 20.020 =

List from greatest to least.

24. 8.723, 8.732, 8.727, 8.372
8.732, 8.727, 8.723, 8.372
25. 0.23, 2.03, 0.023, 2.3
2.3, 2.03, 0.23, 0.023

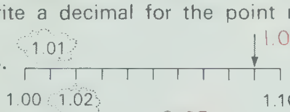
List from least to greatest.

26. 11.11, 11.101, 11.011, 11.1
11.011, 11.1, 11.101, 11.11
27. 1.043, 0.134, 0.143, 1.034
0.134, 0.143, 1.034, 1.043

Copy and complete each pattern.

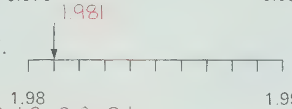
28. 1.06, 1.07, 1.08, ▒, ▒, ▒
29. 1.6, 1.7, 1.8, ▒, ▒, ▒
30. 2.041, 2.031, 2.021, ▒, ▒, ▒
31. 20.4, 20.3, 20.2, ▒, ▒, ▒
32. 0.356, 0.357, 0.358, ▒, ▒, ▒
33. 3.504, 3.503, 3.502, ▒, ▒, ▒

Write a decimal for the point marked with an arrow.

34. 1.09


35. 0.076


36. 3.05


37. 1.981


28. 1.09, 1.10, 1.11
29. 1.9, 2.0, 2.1
30. 2.011, 2.001, 1.991
31. 20.1, 20.0, 19.9
32. 0.359, 0.360, 0.361
33. 3.501, 3.500, 3.499

Page 111

OBJECTIVE

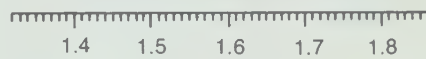
Demonstrate competence in reading and writing numerals and words for decimals to thousandths, in interpreting place value to thousandths, and in comparing and ordering decimals

RELATED ACTIVITIES

- Students having difficulty with Ex. 34-37 will benefit from more experience in labeling number lines to show tenths, hundredths, and thousandths. Use appropriate number lines from copies of page T 389 and have students mark dots to locate points for decimals.

1. Mark these points on the number line.

A 1.7	B 1.69	C 1.71
D 1.55	E 1.43	F 1.77



2. Mark these points on the number line.

G 4.01	H 4.010	I 4.009
J 4.019	K 4.021	L 4.006

LESSON ACTIVITY

Using the Page

- Review the instructions for any group of exercises for which the format may be new. For instance, the completed chart for Ex. 15-17 will show names for the same number in a column. Discuss the reason for the shaded portions in the last two columns of the chart for Ex. 15.

  Some students may have difficulty completing the patterns for Ex. 28-33. They may think of the corresponding whole number patterns for assistance, as shown below for Ex. 28.

106	107	108	109	110	111	112
1.06	1.07	1.08	1.09	____	____	____

These patterns help to prepare students for addition and subtraction of decimals.

Pages 112-113

LESSON OUTCOME

Round one-place, two-place, and three-place decimals to the nearest whole number; round two-place and three-place decimals to the nearest tenth

Materials

newspapers or catalogs

Prerequisite Skills

Round whole numbers; interpret place value in three-place decimals

Checking Prerequisite Skills

Round to the nearest ten.

1. 19 20 **2.** 135 140 **3.** 199 200

Round to the nearest hundred.

4. 249 200 **5.** 750 800 **6.** 387 400

Tell the meaning of each digit.

7. 14.036 1 ten
4 ones
0 tenths
3 hundredths
6 thousandths

Rounding Decimals

Canada's Arnie Boldt set a world record of 1.90 m in the high jump event. Is 1.90 closer to 1 or to 2?

To *round* a decimal to the nearest whole number,

ones	tenths	hundredths
this place		

check the digit in the tenths place.

ones	tenths	hundredths
	this place	

If the digit in the tenths place is 5, 6, 7, 8, or 9, round up. If it is 0, 1, 2, 3, or 4, round down.

For 1.90, the digit in the tenths place is 9.

Round 1.90 up.

1.90 rounded to the nearest whole number is 2.

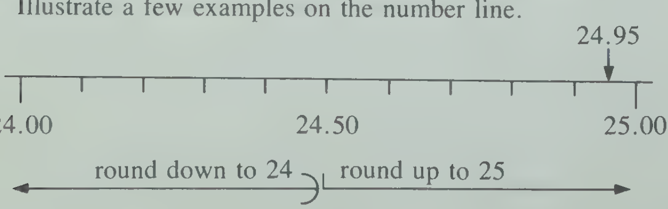
The high jump bar that Arnie Boldt jumped was about 2 m above the ground.

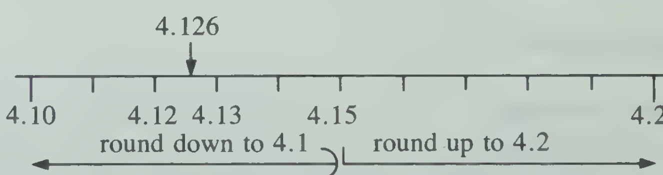
112

LESSON ACTIVITY

Before Using the Pages

- Choose an item advertised for sale in a newspaper or a catalog, for example, a pair of skates having a price of $24.95. Tell the students that in speaking of the price paid for an object, we often tell *about how much* the object cost rather than quote the exact price. Write $24.95 on the board and have a student name an approximate price for the pair of skates. The suggestion will likely be "about $25.00". Say that $24.95 *rounded to the nearest whole number of dollars* is $25.00. Explain that decimals can be rounded, for example, to the nearest whole number or to the nearest tenth. Illustrate a few examples on the number line.





- Say, "I am thinking of a number, but I am not going to show you all the digits." Write 3.8_ _ on the board and ask, "Can you round my number to the nearest whole number?" Use different digits in place of the 8 in the tenths' place and repeat the question. Ask why it is not necessary to know the digits in the hundredths' and thousandths' places. Develop similar exercises for rounding a decimal to the nearest tenth, for example, 3.82_.

Using the Pages

- The worked example demonstrates that the rule developed for rounding whole numbers to a given place (see page 11) also applies in rounding decimals to a given place. By observing the digit to the right of the given place, it can be determined whether to round up or round down.

Working Together

When rounding to $\boxed{\text{this place}}$, first check the digit in (this place).

ones	tenths	hundredths	thousandths
$\boxed{\ }$	( )		
	$\boxed{\ }$	( )	

1. When rounding to the nearest one, check the digit in the __?__ place. tenths
2. When rounding to the nearest tenth, check the digit in the __?__ place. hundredths

If the digit you check is 5, 6, 7, 8, or 9, round up.
If the digit you check is 0, 1, 2, 3, or 4, round down.

Would you round down or up to the nearest whole number?

3. 6.125 down
4. 2.5 up

Would you round down or up to the nearest tenth?

5. 0.68 up
6. 1.349 down

Round to the nearest whole number.

7. 6.125 6
8. 2.5 3

Round to the nearest tenth.

9. 0.68 0.7
10. 1.349 1.3

Exercises

Round to the nearest whole number.

1. 2.4 2   2. 3.7 4   3. 5.5 6   4. 10.2 10   5. 2.37 2
6. 8.69 9   7. 12.45 12   8. 0.902 1   9. 5.499 5   10. 9.723 10

Round to the nearest tenth.

11. 2.46 2.5   12. 6.02 6.0   13. 1.35 1.4   14. 0.54 0.5   15. 3.98 4.0
16. 1.907 1.9   17. 8.089 8.1   18. 0.095 0.1   19. 10.748 10.7   20. 1.250 1.3

KEEPING SHARP

Add.

1. $\begin{array}{r} 1644 \\ \underline{528} \end{array}$ 2172
2. $\begin{array}{r} 28\,572 \\ \underline{36\,946} \end{array}$ 65 518
3. 2 576 + 11 528 14 104
4. 384 + 576 + 402 1362
5. 716 + 1839 + 52 2607
6. \$392 + \$7168 + \$3453 \$11 013
7. \$1.48 + \$3.68 + \$5.88 \$11.04

Subtract.
Add to check.

Example: $\begin{array}{r} 453 \\ \underline{-\ 298} \\ 155 \end{array}$ → $\begin{array}{r} 155 \\ \underline{+\ 298} \\ ? \end{array}$

This sum should match the first number in the subtraction.

8. $\begin{array}{r} 315 \\ \underline{83} \end{array}$ 232
9. $\begin{array}{r} 2136 \\ \underline{439} \end{array}$ 1697
10. 8374 − 3959 4415
11. 30 103 − 4 117 25 986
12. \$2000 − \$1562 \$438
13. \$38.43 − \$18.68 \$19.75

113

RELATED ACTIVITIES

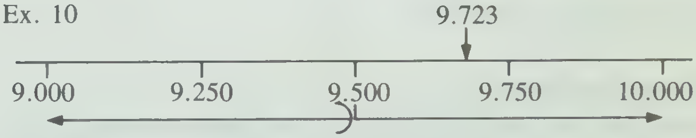
- Ask students to report examples of the use of rounded numbers that they have experienced outside the classroom.
- Students can practice rounding numbers to the nearest whole number of dollars for amounts of money shown in catalogs, newspapers, or cash register tapes.
- Have students round the numbers in Exercises 8-10 and 16-20 to the nearest hundredth. Then have them round each result to the nearest whole number. (Ex. 8-10) or the nearest tenth (Ex. 16-20). Let them compare their answers with the correct answers for the exercises. Point out the incorrect results (6 in Ex. 9 and 10.8 in Ex. 19) that may occur when rounding is carried out one place at a time for two or more places.

A discussion of the illustration may reveal that some students were not aware of the fact that there are competitive sports for physically handicapped persons. Arnie Boldt was able to clear 1.90 m in the high jump event in the 2nd National Amputee Championships at the University of Alberta in Edmonton in 1977. A height of 1.90 m to the nearest whole number of metres is 2 m. Students may recall that 2 m is about the height of a door.

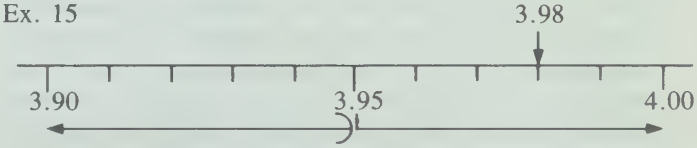
Working Together: Ex. 1 and 2 extend the rule for rounding to a given place to include decimals. Knowing how to apply the rule is examined in Ex. 3-6. The rule is then applied in Ex. 7-10. Remind the students that if the digit to the right of the given place is 5, it is agreed that the number is rounded up (Ex. 4 and 8). Ensure that the students do not show more places than required in the rounded numeral. For example, in Ex. 8, the answer is 3, not 3.0.

Exercises: Some students may need assistance with Ex. 10 and 15. Rounding these numbers results in changing the digit to the left of the given place. Number lines can help to explain the results.

Ex. 10



Ex. 15



Keeping Sharp: These exercises review skills in addition and subtraction and the use of addition to check subtraction of whole numbers. Similar skills will be presented for decimals in the following lessons.

Assessment

Round to the nearest whole number.

1. 4.7 5   2. 7.395 7   3. 9.81 10

Round to the nearest tenth.

4. 1.25 1.3   5. 10.729 10.7   6. 0.93 0.9

Pages 114-115

OBJECTIVE

Demonstrate competence in rounding decimals to the nearest tenth or to the nearest whole number, and in comparing decimals

Materials

newspapers and magazines

Vocabulary

Canada Fitness Awards, Award of Excellence

Practice

Round to the nearest whole number.

1. 3.5 4
2. 4.98 5
3. 12.009 12
4. 9.59 10
5. 6.05 6
6. 2.455 2

Round to the nearest tenth.

7. 2.24 2.2
8. 0.816 0.8
9. 6.009 6.0
10. 5.255 5.3
11. 3.09 3.1
12. 1.97 2.0

Complete.

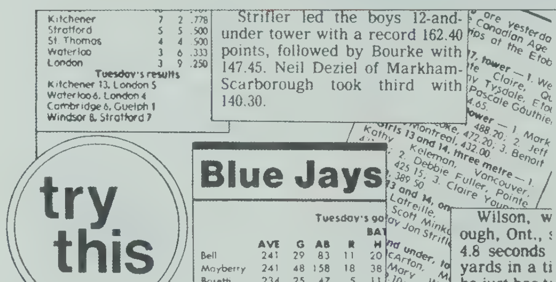
		Nearest tenth	Nearest whole number
13.	3.48	? 3.5	? 3
14.	10.905	? 10.9	? 11
15.	1.53	? 1.5	? 2
16.	7.059	? 7.1	? 7
17.	19.96	? 20.0	? 20

Use >, <, or = to make true statements.

18. 8.4 ● 8.084 >
19. 2.065 ● 2.056 >
20. 1.06 ● 10.6 <
21. 10.606 ● 10.66 <
22. 3.050 ● 3.05 =
23. 0.32 ● 0.032 >

Use the chart at the top of the next page to help you answer these.

24. Curt did 45 speed sit-ups. What level did he reach? better than 3
25. Curt ran the sprint in 9.5 s. What level did he reach? below 1
26. Debbie's time was 11.75 s in the shuttle run. What level did she reach? better than 2



Use old newspapers and magazines.

1. Collect clippings that show how decimals are used in sports.
2. Make up a problem for a classmate using the information in your collection.

114

LESSON ACTIVITY

Using the Pages

- Begin with a discussion of the awards in the photograph on page 115. The bronze award, for example, is presented to those achieving Level 1 or better in four tests. Discuss the requirements shown for each level of the tests in the chart. Ask questions such as "If a ten-year-old girl ran the sprint in nine and two-tenths seconds, what level did she reach?" Discuss the fact that 9.2 s refers to Level 1, because it is not fast enough for Level 2. Similarly, note that a time of 9.8 s is too slow even for Level 1. The number 0 may be used to designate a score too slow for Level 1. Questions of this nature are encountered in Ex. 24-26. The chart above Ex. 27 presents the same kind of exercise, since each score must be translated into the corresponding level to complete Ex. 27. The information obtained in Ex. 27 is then used in Ex. 28.

Try This: Provide the students with newspapers and magazines, particularly sports magazines and the sports section of newspapers. Have them search for clippings to complete Ex. 1 over a period of several days. The information can then be used for Ex. 2. Other sources of suitable information are the *Guinness Book of World Records* and the *World Almanac*.

CANADA FITNESS AWARDS

Levels of Achievement			
		10-year-old	
Test	Level	boy	girl
Sprint	3	7.8 s	8.0 s
	2	8.2 s	8.4 s
	1	9.1 s	9.4 s
Speed Sit-ups (in one minute)	3	42	38
	2	35	31
	1	29	23
Flexed Arm Hang	3	68 s	56 s
	2	49 s	31 s
	1	16 s	9 s
Shuttle Run	3	11.0 s	11.6 s
	2	11.8 s	12.2 s
	1	12.5 s	12.9 s
Standing Long Jump	3	1.65 m	1.57 m
	2	1.52 m	1.45 m
	1	1.40 m	1.32 m
Distance Run	3	64 s	65 s
	2	67 s	69 s
	1	74 s	78 s

AWARD OF EXCELLENCE
Level 3 or better in each test

GOLD
Level 3 or better in four tests

SILVER
Level 2 or better in four tests

BRONZE
Level 1 or better in four tests

This chart shows how Curt, Debbie, and three classmates performed in the Canada Fitness Awards tests.

	Abner	Betsy	Curt	Debbie	Ellen
Sprint	8.2s	7.9s	9.5s	8s	8.25s
Speed Sit-ups	20	38	45	20	32
Flexed Arm Hang	50s	60s	60s	55s	35s
Shuttle Run	11.0s	11.5s	10.9s	11.75s	13s
Standing Long Jump	1.5m	1.61m	1.7m	1.44m	1.6m
Distance Run	65s	65s	64s	69.5s	80s

*27. Make a chart that shows the levels reached by each of the five students in the tests.

	Abner	Betsy	Curt
Sprint	2	3	0
Speed Sit-ups	0	3	
Flexed Arm Hang	2		

Curt took 9.5 s in the sprint. That was even too slow for Level 1.

A chart is shown below

*28. Which award did each earn?

28. Abner: Silver
Betsy: Award of Excellence
Curt: Gold
Debbie: Bronze
Ellen: Silver

115

27.

	Abner	Betsy	Curt	Debbie	Ellen
Sprint	2	3	0	3	2
Speed Sit-ups	0	3	3	0	2
Flexed Arm Hang	2	3	2	2	2
Shuttle Run	3	3	3	2	0
Standing Long Jump	1	3	3	1	3
Distance Run	2	3	3	1	0

RELATED ACTIVITIES

- Some students may be involved in finding or improving their own levels of achievement in the Canada Fitness Awards tests. If not, this lesson may motivate them to do so.
- Adapt the exercises of *Try This* for the results obtained and records set by students at your school track meet.

Pages 116-117

LESSON OUTCOME

Add decimals with regrouping, one to four addends with the same number of decimal places (to thousandths)

Vocabulary

individual medley, butterfly, backstroke, breaststroke, freestyle

Prerequisite Skills

Add whole numbers with regrouping; interpret place value in three-place decimals

Checking Prerequisite Skills

Add.

1. $\begin{array}{r} 143 \\ 178 \\ \hline 321 \end{array}$ **2.** $\begin{array}{r} 3047 \\ 968 \\ \hline 4015 \end{array}$ **3.** $\begin{array}{r} 9234 \\ 345 \\ 3456 \\ 67 \\ \hline 13102 \end{array}$

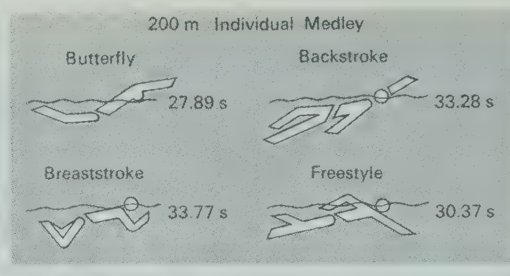
Tell the meaning of each digit.

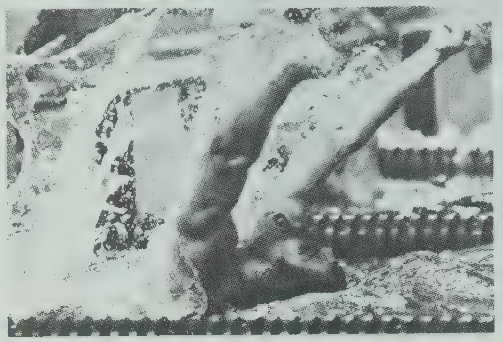
4. 23.652
2 tens
3 ones
6 tenths
5 hundredths
2 thousandths

5. 127.03
1 hundred
2 tens
7 ones
0 tenths
3 hundredths

Adding Decimals

The chart shows the time it took Canada's Graham Smith to swim 50 m using each stroke. His total time for the 200 m race was a world record. What was his total time?





Add 27.89, 33.28, 33.77, and 30.37.

Line up the decimals. Add hundredths and regroup.

$$\begin{array}{r} \scriptstyle 3 \\ 27.89 \\ 33.28 \\ 33.77 \\ 30.37 \\ \hline 1 \end{array}$$

Add tenths and regroup.

$$\begin{array}{r} \scriptstyle 2\,3 \\ 27.89 \\ 33.28 \\ 33.77 \\ 30.37 \\ \hline 31 \end{array}$$

Add ones and regroup.

$$\begin{array}{r} \scriptstyle 1\,2\,3 \\ 27.89 \\ 33.28 \\ 33.77 \\ 30.37 \\ \hline 5\ 31 \end{array}$$

Add tens.

$$\begin{array}{r} \scriptstyle 1\,2\,3 \\ 27.89 \\ 33.28 \\ 33.77 \\ 30.37 \\ \hline 125.31 \end{array}$$

Place the decimal point in the sum in line with those in the addends.

Graham Smith swam the 200 m in 125.31 s.

There are 60 s in 1 min. 125.31 s equals 2 min 5.31 s.

116

LESSON ACTIVITY

Before Using the Pages

- Have the students find the sum of two whole numbers as shown in A. Write the same addition exercise with decimal addends as shown in B. Ask students to interpret place value in each whole number addend and in the corresponding decimal addend. Then have them suggest what the sum of the decimal addends is without adding. Extend this concept for exercises as shown in C and D.

A $\begin{array}{r} 1628 \\ +\ 3947 \\ \hline 5575 \end{array}$ B $\begin{array}{r} 162.8 \\ +\ 394.7 \\ \hline \end{array}$ C $\begin{array}{r} 16.28 \\ +\ 39.47 \\ \hline \end{array}$ D $\begin{array}{r} 1.628 \\ +\ 3.947 \\ \hline \end{array}$

Using the Pages

- The worked example demonstrates that the procedure used to add whole numbers also applies to addition of decimals. The addition is carried out from right to left, regrouping as needed.

Have students read the names of the four swimming strokes shown in the illustration at the top of page 116. Ask them to explain the term "Individual Medley". Ask which swimming stroke is demonstrated in the photograph. Then have a student read the word problem at the top of the page. Note that the word "total" indicates the need for addition to solve the problem.

Have students note that the decimal addends are arranged in vertical form so that hundredths are aligned, tenths are aligned, and so on. As a result, the decimal points are also aligned. This fact can be applied in lining up addends for which the number of digits is not the same, as in 107.6 and 14.5. (In *Starting Points in Mathematics 5*, examples such as 1.076 + 14.5 are not encountered.)

Lead the students through the steps of adding and regrouping. Pay particular attention to placing the decimal point in the sum. Ask the students how adding decimals is different from adding whole numbers. They may suggest, for example, that a decimal point is shown in each addend and in the sum.

Working Together

Line up the decimals in vertical form.

1. 22.54 + 17.67 + 23.61   22.54 17.67 23.61
2. 1.607 + 2.308 + 11.905   1.607 2.308 11.905
3. 17.1 + 16.9 + 22.7   17.1 16.9 22.7
4. 7.05 + 6.98 + 7.17   7.05 6.98 7.17

Add by following the steps.

5.
```
  2.569
  3.472
  6.041
```
Add thousandths and regroup.
Add hundredths and regroup.
Add tenths and regroup.
Add ones.

6.
```
  3.047
  2.968
  6.015
```
Add and regroup.
Add and regroup.
Add and regroup.
Add.

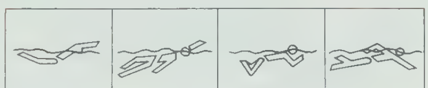
Exercises

Add.

1. 14.3 + 17.8 = 32.1	2. 12.5 + 27.6 = 40.1	3. 86.9 + 43.8 = 130.7	4. 35.6 + 76.7 = 112.3	5. 14.5 + 107.6 = 122.1
6. 6.45 + 3.97 = 10.42	7. 7.39 + 6.57 = 13.96	8. 4.23 + 3.98 = 8.21	9. 5.29 + 4.87 = 10.16	10. 17.74 + 8.34 = 26.08
11. 14.609 + 27.098 = 41.707	12. 27.639 + 82.476 = 110.115	13. 86.306 + 29.577 = 115.883	14. 92.064 + 18.679 = 110.743	15. 25.279 + 5.791 = 31.070

16. 2.607 + 9.703 + 5.725   18.035
17. 12.15 + 13.79 + 26.42   52.36
18. 30.1 + 407.2 + 6.8 + 29.2   473.3
19. 1.609 + 2.324 + 5.637   9.570

The chart shows the time taken to swim 50 m using each stroke.



Tony	28.79 s	31.46 s	36.85 s	31.90 s
Francis	29.55 s	31.27 s	35.48 s	32.88 s
Joey	28.35 s	32.17 s	36.43 s	31.94 s
Chip	31.22 s	32.80 s	33.49 s	31.65 s

20. Who used the least time for the first two strokes? Tony
21. What was the total time for each swimmer?
22. For the 200 m race, who was first? Who was second? Who was third?

The winner is the swimmer who takes the least time.

21 129.00 s (Tony)
129.18 s (Francis)
128.89 s (Joey)
129.16 s (Chip)

22 1st Joey
2nd Tony
3rd Chip

117

The sum obtained names the total time in seconds, which is 125.31 s. Have students explain why this is the same as 2 min 5.31 s.

Working Together: Ex. 1-4 provide practice in lining up decimal addends in vertical form. Emphasize that the place values will be aligned when the decimal points are aligned. Ex. 5 and 6 deal with the right-to-left order of adding and regrouping. For practice in reading decimals, ask students to read addends and sums aloud.

Exercises: The students will need to select information from the chart on page 117 to solve Ex. 20 and 21. The answers to Ex. 22 depend on the results obtained in Ex. 21. Note that the concepts of comparing and ordering decimals are applied in Ex. 20 and 22.

RELATED ACTIVITIES

- For further practice in adding decimals, you may wish to have students complete selected exercises from Ex. 1-27 on page 333.
- Have students complete exercises involving whole numbers and the corresponding exercises involving decimals.

1. 326 + 1097
2. 32.6 + 109.7
3. 3.26 + 10.97
4. 0.326 + 1.097

- Prepare a work sheet with exercises similar to the following. Ask students to round addends to the nearest whole number and add to estimate the sums.

1. 24.93 + 5.07
2. 39.162 + 9.857
3. 0.914 + 4.213
4. 55.59 + 44.41

Assessment

Add.

1. 7.24 + 3.78 = 11.02
2. 18.53 + 1.62 + 5.47 = 25.62
3. 1.402 + 3.616 + 0.147   5.165
4. 16.5 + 403.1 + 27.8 + 1.4   448.8

UNIVERSITY LIBRARY

Pages 118-119

LESSON OUTCOME

Subtract decimals with regrouping, subtrahends and minuends with the same number of decimal places (to thousandths); use addition to check subtraction

Materials

models for ones, tenths, and hundredths (optional)

Prerequisite Skills

Subtract whole numbers with regrouping; interpret place value in three-place decimals

Checking Prerequisite Skills

Subtract.

1. 752 195 557	**2.** 404 277 127
3. 13 645 8 397 5 248	**4.** 5000 326 4674

Tell the meaning of each digit.

5. 1.234 1 one 2 tenths 3 hundredths 4 thousandths	**6.** 12.34 1 ten 2 ones 3 tenths 4 hundredths

Subtracting Decimals

The women's world record for skating 500 m is 40.80 s. Sylvia Burka holds the Canadian record of 42.15 s. How many seconds more is her time than the world record?

Subtract 40.80 from 42.15.

Line up the two decimals. Show the greater number first. Subtract hundredths.

$$\begin{array}{r} 42.15 \\ \underline{40.80} \\ 5 \end{array}$$

Regroup 2 ones 1 tenth as 1 one 11 tenths. Subtract tenths.

$$\begin{array}{r} \scriptstyle 1\ 11 \\ 4\cancel{2}.\cancel{1}5 \\ \underline{40.80} \\ 35 \end{array}$$

Subtract ones.

$$\begin{array}{r} \scriptstyle 1\ 11 \\ 4\cancel{2}.\cancel{1}5 \\ \underline{40.80} \\ 1.35 \end{array}$$

Place the decimal point in line with the others.

The subtraction is complete. It can be checked by adding 1.35 and 40.80.

$$\begin{array}{r} \scriptstyle 1 \\ 40.80 \\ \underline{1.35} \\ 42.15 \end{array}$$

If this number did not match the first number used in the subtraction, there would be a mistake in the work.

Sylvia Burka's Canadian record is 1.35 s more than the world record.

118

LESSON ACTIVITY

Before Using the Pages

- Ask the students to think of a two-place decimal. Have two students write their decimals on the board. Ask which of the two decimals is greater. Then ask how to find out how much greater it is. They will likely suggest using subtraction. Let the students try the subtraction on their own. Have a few students show their work on the board.

Using the Pages

- The worked example demonstrates that the procedure used to subtract whole numbers also applies to subtraction of decimals. The subtraction is carried out from right to left, regrouping as required.

  Ask a student to read the word problem at the top of page 118. Mention that the words "How many more" indicate the use of subtraction. Explain that as in the previous lesson, when the two numbers are arranged in vertical form for subtraction, the decimal points are aligned. This results in corresponding place values being aligned.

  Lead the students through the steps of subtracting and regrouping, noting the position of the decimal point in the difference. Then have them perform the addition steps aloud to check the subtraction. Read the statement at the bottom of the page. Summarize the procedure by concluding that subtracting decimals is similar to subtracting whole numbers.

Working Together: Ex. 1-5 present partially completed subtractions. It is important to have students explain the completed steps in presenting their work on the board. Ex. 5 may need particular attention. For the regrouping shown, 9 wholes are interpreted as 900 hundredths. When 1 hundredth is regrouped as 10 more thousandths, there are 899 hundredths. The use of models will help to illustrate this. Also, the corresponding whole number subtraction may be shown to help students relate the procedure to decimals.

Working Together

Complete each subtraction.

1. 4.35 − 2.91 = 1.44
2. 2.514 − 1.392 = 1.122
3. 14.20 − 9.52 = 4.68
4. 5.423 − 1.876 = 3.547
5. 9.005 − 7.597 = 1.408

Complete the first subtraction using whole numbers.
Then complete each of the others using decimals.

6. 6352 − 1894 = 4458
7. 6.352 − 1.894 = 4.458
8. 635.2 − 189.4 = 445.8
9. 63.52 − 18.94 = 44.58

10. 3000 − 1143 = 1857
11. 300.0 − 114.3 = 185.7
12. 30.00 − 11.43 = 18.57
13. 3.000 − 1.143 = 1.857

Subtract.

14. 5.1 − 3.3 = 1.8
15. 4.55 − 0.98 = 3.57
16. 8.703 − 2.760 = 5.943
17. 3.933 − 2.965 = 0.968
18. 2.063 − 0.674 = 1.389
19. 5.003 − 4.286 = 0.717

Exercises

Subtract.

1. 18.5 − 12.9 = 5.6
2. 75.2 − 19.5 = 55.7
3. 30.6 − 14.7 = 15.9
4. 4.34 − 3.58 = 0.76
5. 38.16 − 19.41 = 18.75
6. 15.63 − 9.95 = 5.68
7. 6.06 − 3.77 = 2.29
8. 7.623 − 1.495 = 6.128
9. 3.605 − 2.914 = 0.691
10. 15.205 − 7.325 = 7.880
11. 3.004 − 1.927 = 1.077
12. 6.000 − 1.814 = 4.186
13. 125.78 − 54.63 71.15
14. 31.4 − 29.7 1.7
15. 8.127 − 3.029 5.098
16. 97.035 − 8.174 88.861
17. 0.87 − 0.69 0.18
18. 5.000 − 2.394 2.606

Subtract.
Add to check.

Example: 5.42 − 1.79 = 3.63 → 3.63 + 1.79 = ?

This sum should match the first number in the subtraction.

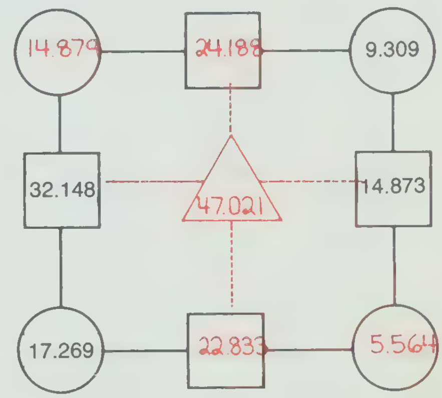
19. 9.3 − 2.5 = 6.8
20. 2.10 − 0.21 = 1.89
21. 51.23 − 37.94 = 13.29
22. 25.00 − 5.06 = 19.94
23. 4.321 − 3.836 = 0.485
24. 3.000 − 0.647 = 2.353

119

RELATED ACTIVITIES

• For further practice in subtracting decimals, you may wish to have students complete selected exercises from Ex. 28-59 on page 333.

• Use copies of page T 390 to assign practice in addition and subtraction as indicated in the diagram below. Addends are shown in the circles and sums are shown in the squares. When the diagram is completed, have students add the numbers in opposite squares of the diagram. They will discover that, if their work is correct, the two sums are the same. Ask students to draw a triangular shape as shown and write this number in it.



  8 9 9 15
  ~~9 0 0 5~~
− 7 5 9 7

Ex. 6-13 emphasize the similarity in subtracting whole numbers and subtracting decimals. Attention is directed toward interpreting the numerals of corresponding exercises and writing the corresponding differences. Thus, when the difference is found for the whole numbers in Ex. 10, the same sequence of digits is used to write the differences for Ex. 11-13. The students are required only to place the decimal point in each difference. You may wish to have the students use addition to check their subtractions for some of Ex. 14-19.

Exercises: Remind the students to align the decimal points when writing Ex. 13-18 in vertical form.

Assessment

Subtract. Add to check.

1. 6.32 − 4.87 = 1.45
2. 8.074 − 2.984 = 5.090
3. 4.000 − 1.234 = 2.766
4. 3.40 − 0.43 2.97
5. 32.41 − 21.97 10.44

OBJECTIVE

Demonstrate competence in adding and subtracting decimals with regrouping; solve related word problems

Practice

Add.

1.	2.	3.	4.	5.	6.
17.5	9.21	4.29	2.357	4.697	91.88
3.7	9.84	1.93	0.944	6.355	8.27
21.2	19.05	6.22	3.301	11.052	100.15

Subtract.

7.	8.	9.	10.	11.	12.
24.2	5.13	13.16	4.611	5.551	40.00
5.8	2.36	9.44	3.762	1.856	20.38
18.4	2.77	3.72	0.849	3.695	19.62

Subtract. Add to check.

13.	14.	15.	16.	17.	18.
68.3	8.25	32.30	9.445	10.34	1.004
29.7	4.39	5.65	4.486	8.47	0.746
38.6	3.86	26.65	4.959	1.87	0.258

Which result is better? How much better?

19. **100 m Run**
10.58 s
11.06 s
10.58 s by 0.48 s

20. **High Jump**
1.89 m
1.98 m
1.98 m by 0.09 m

21. **100 m Swim**
57.28 s
60.05 s
57.28 s by 2.77 s

22. **Weightlifting**
208.2 kg
212.8 kg
212.8 kg by 4.6 kg

23. **Gymnastics**
67.925
76.295
76.295 by 8.370

24. **Batting Average**
0.318
0.333
0.333 by 0.015

Read the directions carefully.
Use the three numbers shown. → | 1.064 3.000 1.604 |

25. Add the greatest and least numbers. Subtract the other. 2.460

26. Subtract the least from the greatest number. Add the other. 3.540

27. Add the least number to the difference of the other two. 2.460

28. Subtract the least number from the sum of the other two. 3.540

29. From the greatest number subtract the difference of the other two. 2.460

30. Add the greatest number to the difference of the other two. 3.540

31. From the greatest number subtract the sum of the other two. 0.332

32. Subtract the least number from the difference of the other two. 0.332

120

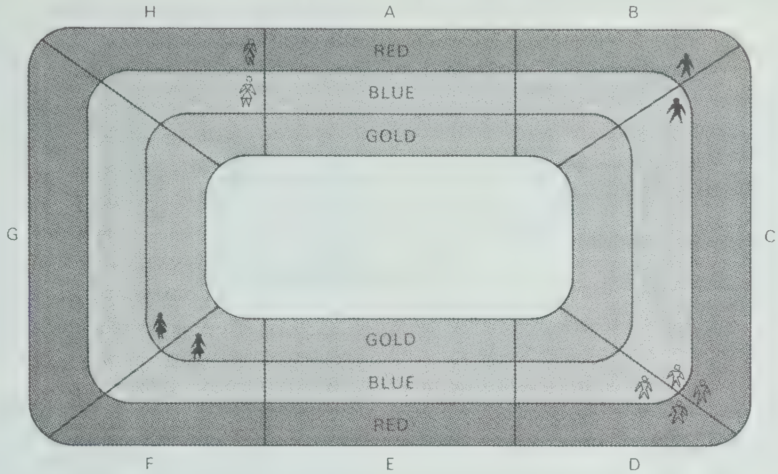
LESSON ACTIVITY

Using the Pages

- Begin with a discussion of the illustration at the top of page 121. Have students describe the seating arrangement of the arena. There are eight sections identified by the letters A to H, and three colors in each section, red, blue, and gold. Ask questions such as "What is the price of a seat in Section B of the gold area for an adult?" "for a ten-year-old child?" "for an eleven-year-old child?" Draw attention to the symbols drawn in the arena to represent persons. Note that these symbols appear in Ex. 35-38.

  The exercises on these two pages review not only addition and subtraction of decimals, but also include comparing and ordering decimals (Ex. 19-32). The language of addition and subtraction is given particular attention in Ex. 25-32, in which students must interpret such words as *difference*, *sum*, *least*, and *greatest*. Ex. 39 and 40 are starred because results will vary according to the situation selected by each student.

ARENA SEATING AND TICKET PRICES



	A	B	C	D	E	F	G	H
GOLD	$5.75	$4.75	$4.15	$4.75	$5.75	$4.75	$4.15	$4.75
BLUE	$4.40	$3.80	$3.50	$3.80	$4.40	$3.80	$3.50	$3.80
RED	$3.45	$3.15	$2.75	$3.15	$3.45	$3.15	$2.75	$3.15
Standing Room Only (behind all seating)			$2.20	Children (10 and under)		$0.75 off the price of each ticket		

Solve.

33. Which costs more, a blue seat in Section A or a gold seat in Section C? How much more? blue in A $0.25

34. How much do two gold seats in Section F cost? $9.50

35. How much did and pay for their adult tickets? $8.90

36. How much did and pay for their adult tickets? $6.95

37. How much did and pay for their adult tickets? $6.65

38. How much did the four pay for their adult tickets? $13.20

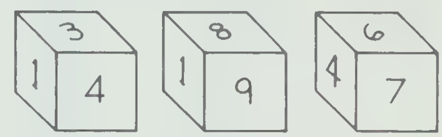
˙39. Where would you, a friend, and a parent sit? How much would your tickets cost?
Answers will vary

˙40. Where could you, a friend, and a parent sit if you have only $10 to spend for tickets?
Answers will vary, for example, blue B, C, D, F, G, or H if 10 or under, any red seat

121

RELATED ACTIVITIES

• Students may play a game using three dice: one marked 1, 2, 3, 4, 5, 6; one marked 1, 2, 3, 7, 8, 9; and one marked 4, 5, 6, 7, 8, 9. The players take turns tossing the dice. The numbers shown may be interpreted as a three-place decimal in different ways, using zeros as necessary for place holders. Each player adds the first two numbers he/she tosses and interprets. Thereafter, the numbers are added to the previous sum. The player whose sum is closest to a previously agreed upon limit, such as 90.000, without exceeding it is the winner.



0.386	3.860	38.600
0.368	3.680	36.800
0.836	8.360	83.600
0.683	6.830	68.300

For a simpler version of the game, have students use the digits indicated by the dice in place values only to the right of the decimal point. Some examples are shown in the first column above. Adjust the limit accordingly.

The game may also be played by beginning with the limit and subtracting successive tosses of the dice. The player who comes closest to zero is the winner.

UNIVERSITY LIBRARY

Pages 122-123

OBJECTIVE

Demonstrate the ability to use a calculator for additive and subtractive situations involving decimals and amounts of money

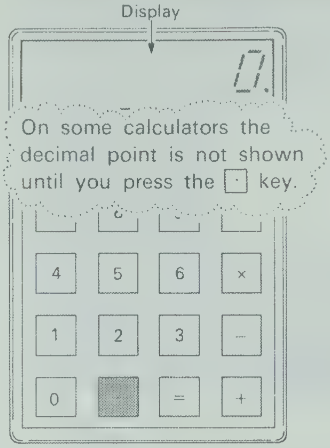
Materials
calculators (optional), copies of page T 397

Vocabulary
display

The [·] Key on a Calculator

The displays on many calculators always show a decimal point.

Display



The [·] key lets you put the decimal point anywhere you want it.

Press, and most displays show

Press	Display
3	3.
9	39.
8	398.

Press, and most displays show

Press	Display
3	3.
·	3.
9	3.9
8	3.98

When using a calculator, how many times must you press the keys to enter each of these values?

For decimals less than 1, like 0.259, the first 0 is already displayed. The [0] key does not have to be pressed.

For zeros at the end of a decimal, on the right side of the decimal point, the [0] key does not have to be pressed.

Remember, 5.2, 5.20, and 5.200 all name the same number.

For amounts of money, just the decimal is entered. There is no [$] key.

1. 2.64 4	**2.** 35.6 4
3. 6.381 5	**4.** 457 3
5. 0.259 4	**6.** 10.64 5
7. 3.07 4	**8.** 0.008 4
9. 5.20 3	**10.** 7.180 4
11. 4.00 1	**12.** 60.0 2
13. 80 2	**14.** 0.70 2
15. $4.98 4	**16.** $10.52 5
17. $25 2	**18.** $6.50 3
19. $0.25 3	**20.** $0.90 2

122

LESSON ACTIVITY

Using the Pages

- This is an appropriate time to introduce the decimal key on a calculator. Note that on most calculators, the decimal point is displayed automatically to the right of the ones' place when whole numbers are entered.

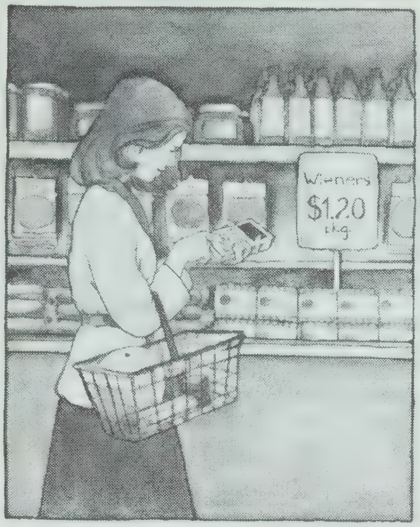
  Point out the different displays for the entries 398 and 3.98. To enter 398, three keys must be pressed. To enter 3.98, four keys must be pressed. Discuss the displays that would be obtained by pressing the same keys in a different sequence. Point out that 39.8 requires four keys and 0.398 also requires four keys, not five as might be supposed. Attention is drawn to this in the "thought cloud" related to Ex. 5. Before the students begin the exercises, it would be beneficial to discuss the important features suggested in the "thought clouds" for Ex. 9 and 15. However, if students are using calculators to help them answer Ex. 1-20, let them consider the suggestions independently before you discuss them.

  The illustration on page 123 suggests a practical use of the calculator. The problems on this page all relate to purchasing food. Emphasize that entries are to be made using as few keys as possible, if the calculator is to be used efficiently. Thus, in the example at the top of page 123, only three keys need to be pressed to display a numeral for $1.20.

  The exercises on these two pages concern the number of times keys must be pressed, not the sequence of pressing the keys. However, if the students prepare keycharts for the exercises, they will find it easier to explain how they obtained their results. Copies of page T 397 will be helpful in preparing keycharts, as illustrated for Ex. 24.

[·] [4] [5] [+] [1] [+] [·] [8] [=] ____

There are 9 keys to press.



To enter $1.20, it is enough to press

The display will show

Key	Display
1	1.
.	1.
2	1.2

Remember, 1.2 and 1.20 name the same number.

How many times must you press the keys to solve each of these with a calculator?

Don't forget to use the $\boxed{+}$, $\boxed{-}$, and $\boxed{=}$ keys when they are needed.

21. Wieners cost $1.20. Hamburger costs $2.05. How much do they cost together? 9

22. Potatoes cost $1.98. Tomatoes cost $1.15. How much do they cost together? 10

23. Hamburger costs $2.05. Hamburger rolls cost $0.49. How much do they cost together? 9

24. Mustard costs $0.45. Ketchup costs $1.00. Relish costs $0.80. How much do they cost together? 9

25. Apples cost $1.80. A melon costs 79¢. How much do they cost together? 8

26. One can of beans costs 38¢. Another costs 50¢. Chips cost 89¢. How much do they cost together? 9 for cents, 11 for dollars

27. Hot-dog rolls cost 49¢. How much do the potatoes, tomatoes, rolls, wieners, hamburger, mustard, ketchup, relish, apples, melon, beans, and chips cost together? 55

Making keycharts can help you with these exercises.

Calculator

Answers are given at the right for those students who wish to solve the problems.

RELATED ACTIVITIES

- Ask students to prepare exercises similar to Ex. 21-27 for others to complete. They may select items advertised in newspapers.
- Let students use calculators to check their answers for exercises on pages 117 and 119.
- Have students play the game "Shopping Spree" described on page T 377, using calculators to find cumulative totals (or amounts remaining from their original limits). Ensure that they keep a written account of prices of items purchased so that their work may be checked for accuracy.

21. $3.25
22. $3.13
23. $2.54
24. $2.25
25. $2.59
26. $1.77
27. $13.97

UNIVERSITY LIBRARY

Page 124

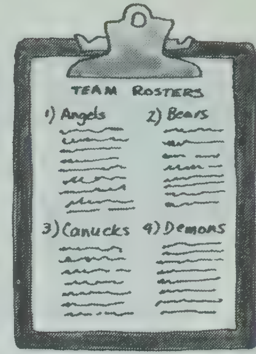
OBJECTIVE

Organize information for the solution of a problem

RELATED ACTIVITIES

- If students in your class are involved in team sports within the school, have them help to prepare the schedule for the tournament. You may prefer to have students prepare a schedule for an imaginary tournament.

Organizing Information



1-4.

Place \ Day	1	2	3	4	5	6
1	AB	BD	AD	CD	AC	BC
2	CD	AC	BC	AB	BD	AD

Four teams are ready to play.

Think about where and when the teams would play in your community.

1. Make up a schedule so that any team plays each of the other teams two times.
2. On your schedule show when each game will be played.
3. On your schedule show where each game will be played.
4. Is it possible for two games to be played at the same time? If so, show this in your schedule.

Possible schedules for Ex. 5 and 6 are given on page T368.

Two more teams, the Elves and the Flyers, are formed.

5. Make up a schedule so that any team plays each of the other teams two times. Show when and where each game will be played.

The Giants and Hawks are the seventh and eighth teams to be formed.

6. Make up a schedule so that any team plays each of the other teams one time. Show when and where each game will be played.

If you have two or more places for playing, try to make your schedule so that

A. no team plays in the same place twice in a row, and

B. each team plays in each place at least one time.

PROBLEM SOLVING

124

LESSON ACTIVITY

Using the Pages

- On page 110, the *Problem Solving* feature involved organizing information by pairing teams to compete in a tournament. Any two teams played each other only once, and a bracket was drawn to illustrate the pairing. On this page, similar problems are encountered, but teams meet one another twice in competition. A different system for presenting the schedule will have to be developed.

  Discuss various ways of organizing the schedules and how these ways depend on situations that must be considered. For example, if only one place is available for playing the games, they must be scheduled at different times. The number of times that games can be scheduled in one day depends on the length of time required for each game, on whether the day is a school day, and whether the teams would play more than one game each day. If several places are available, whether some or all of them are used at the same time would influence the schedule.

Show how a schedule can have the form of a list as follows.

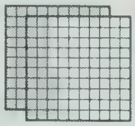
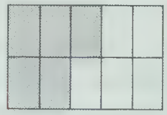
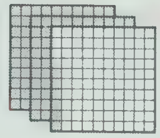
Day 1: Team A plays team B at place 1.
Team C plays team D at place 2.
Day 2: Team A plays team C at place 1.
Team B plays team D at place 2.

It is obvious that such a list is a cumbersome method for showing a schedule. A chart can show the information more concisely. For the following chart, the letters A, B, C, and D indicate the teams, the symbols P1 and P2 indicate the places, and the numerals indicate the days. Have the students interpret the chart, and then encourage them to make suggestions and explore their own ideas for making the schedules.

Team \ Team	A	B	C	D
A		1 / P1	2 / P1	3 / P2
B	4 / P2		3 / P1	2 / P2
C	5 / P2	6 / P1		1 / P2
D	6 / P2	5 / P1	4 / P1	

Checking Up

Write a decimal for each of these.

1.  1.25
2.  0.5
3.  2.06

4. 

5. 

	ones	tenths	hundredths	
6.	0	8	9	0.89

	ones	tenths	hundredths	thousandths	
7.	7	0	0	4	7.004

8. 5 and 7 hundredths 5.07
9. 20 and 125 thousandths 20.125

Tell what each 3 means.

10. 1.375 3 tenths
11. 2.630 3 hundredths
12. 0.543 3 thousandths

Write each as a one-place decimal.

13. 6.300 6.3
14. 0.50 0.5

Write each as a two-place decimal.

15. 1.7 1.70
16. 1.030 1.03

Write each as a three-place decimal.

17. 4.4 4.400
18. 2.01 2.010

Use >, <, or = to make true statements.

19. 2.38 ● 2.6 <
20. 1.83 ● 1.830 =
21. 0.1 ● 0.095 >

List from greatest to least.

22. 0.689, 0.757, 0.692, 0.325
0.757, 0.692, 0.689, 0.325

List from least to greatest.

23. 5.07, 6.2, 5.007, 6.021
5.007, 5.07, 6.021, 6.2

Round to the nearest whole number.

24. 6.45 6
25. 8.5 9
26. 2.085 2

Round to the nearest tenth.

27. 6.45 6.5
28. 3.728 3.7
29. 4.095 4.1

Add.

30. 4.7 + 8.3 = 13.0
31. 1.43 + 3.97 = 5.40
32. 13.42 + 6.89 = 20.31
33. 3.698 + 0.384 = 4.082
34. 5.735 + 0.849 + 3.621 = 10.205

Subtract.

35. 7.0 − 2.9 = 4.1
36. 6.14 − 5.96 = 0.18
37. 13.02 − 2.19 = 10.83
38. 3.525 − 1.929 = 1.596
39. 1.002 − 0.989 = 0.013

125

Page 125

OBJECTIVE

Demonstrate an understanding of the concepts and skills presented in this unit

RELATED ACTIVITIES

- For further practice, you may wish to have the students complete the exercises on page 333 that were not completed earlier.
- Have students write the word names for numbers that show the same sequence of digits with the decimal point in different positions.

417 41.7 4.17 0.417

This activity may be conducted orally, rather than in writing. If a magnet will adhere to the chalkboard, write a sequence of digits on the board and place a magnet in different positions to represent the decimal point. Ask students to read the numerals.

- The ability to interpret place value in decimals is important for developing the concepts of multiplying and dividing decimals. Provide exercises, similar to the following, which require students to write different names for the same number.

4 tenths 6 hundredths = _____ hundredths

3 ones 2 tenths = _____ tenths

54 tenths = _____ tenths 4 hundredths

1 hundredth 5 thousandths = _____ thousandths

Skills	Exercises	Related Pages
Write decimals to match diagrams	1-3	T 110-T 113
Read a scale marked in decimal hundredths or thousandths	4, 5	T 121
Write decimals	6-9	T 112-T 117
Interpret place value in decimals	10-12	T 110-T 117
Compare decimals	13-21	T 118
Order decimals	22, 23	T 119-T 120
Round decimals	24-29	T 122-T 123
Add decimals	30-34	T 126-T 127
Subtract decimals	35-39	T 128-T 129

Comments

If students have difficulty with Ex. 13-18, provide practice with models of tenths and hundredths. Tenth strips may be placed on models of hundredths to help understand, for example, that 1.7 (Ex. 15) names the same number as 1.70.

Models are also helpful for students having difficulty with examples in which there are 0 tenths in a two-place decimal, and 0 tenths and/or 0 hundredths in a three-place decimal. Represent decimals using models and show each decimal in a place-value chart.

ones	tenths	hundredths
1	0	4

To help students with exercises similar to Ex. 4 and 5, ask them to name the decimals aloud for each point (3 and 60 hundredths, 3 and 61 hundredths, 3 and 62 hundredths, . . .) (40 thousandths, 41 thousandths, 42 thousandths, . . .).

Unit 7 Overview

Measurement

This unit deals with metric measurements of length in terms of metres, centimetres, and millimetres, and of area in terms of square centimetres and square metres. The relationship between centimetres and metres is established using the numbers 100 and 0.01, between millimetres and centimetres using the numbers 10 and 0.1, and between millimetres and metres using the numbers 1000 and 0.001. Conversion of measurements from one unit to another is accomplished by renaming them, part by part, rather than by multiplication. The perimeter of a polygon is presented as the total length of its sides, before the shorter methods of finding perimeters of squares and rectangles are developed. Area is approached through counting the square units and parts of units which are covered by shapes composed of various polygons. The simplified technique for finding the area of a rectangle is then developed. Two short lessons reveal that rectangles having the same perimeter are not necessarily the same shape, nor are those with the same area. Two *Problem Solving* features challenge students further in the topics of perimeter and area, and the lesson on page 145 shows how models may be used to discover the relationship between the diameter and the circumference of a circle. In addition to the *Checking Up* lesson at the end of the unit, a *Checking Skills* page of exercises and word problems is provided to assess skills in multiplication.

Prerequisite Skills

- write decimals to express tenths, hundredths, and thousandths
- add decimal tenths
- multiply whole numbers
- identify shapes that are square and shapes that are rectangular
- write a multiplication sentence for an array

Unit Outcomes

- measure length in centimetres and in metres
- identify the centimetre as one-hundredth of a metre
- express metres as centimetres and centimetres as metres
- measure length in millimetres
- identify the millimetre as one-tenth of a centimetre and one-thousandth of a metre
- identify a diameter and a radius of a circle
- express lengths given in one unit in terms of another unit using decimals, for metres, centimetres, and millimetres
- use addition to find the perimeter of a shape the sides of which are measured in millimetres or in centimetres
- use multiplication to find the perimeter of a square
- use multiplication and addition to find the perimeter of a rectangle; solve related problems
- find area in square centimetres by counting whole units and half units
- use multiplication to find the area of a rectangular shape in square centimetres
- use multiplication to find the area of a rectangular shape in square centimetres and in square metres; solve related problems
- show a rectangular shape having a given perimeter
- show a rectangular shape having a given area
- use models to help solve problems

Background

The metric system was introduced in France in the latter part of the eighteenth century by a committee which was formed to develop a rational system of measurement. As a result, the *metre*, *litre*, and *kilogram* were created as standard units. Since then, more and more countries have adopted the system and, in 1960, agreements were reached concerning the various units and their symbols. It has been called the *International System of Units*, or SI (from the French name, Le Système International d'Unités).

Prior to the adoption of the metric system, most measurement systems lacked rational structure. Units of measurement were often developed from parts of the body, such as the hand and the foot, and varied according to local customs and adaptations. Conversions from one unit to another involved such diverse factors as 2, 3, 4, $5\frac{1}{2}$, 12, 16, and 5280. The metric system, on the other hand, is based on the simple and logical decimal system, the same as our base-ten numeration system. Units are related by factors such as 10, 100, and 1000, and conversion is easily accomplished from one unit to another by multiplication and division. Computation is, therefore, much easier with metric measures. Furthermore, the units for length, capacity, and mass are interrelated.

The decimal nature of the metric system permits prefixes denoting powers of ten to be used with any base unit. For some types of measurement many different terms are used, while in others only a few are commonly used. The usual prefixes are shown in the chart at the right. For linear measure, the most common units are kilometre (km), metre (m), centimetre (cm), and millimetre (mm). Since these units are not consecutive in the chart, conversions sometimes involve powers of ten, rather than merely ten. For example, centimetre and millimetre are consecutive, so converting from one to the other involves multiplying by 10 or by 0.1 (dividing by 10). However, centimetre and metre are not consecutive; they are two places apart, and are related by the factor 100 (10^2). The relationships between millimetre and metre and between kilometre and metre involve the factor 1000 (10^3). Metric measures of area are derived from linear units. A square with sides 1 cm long encloses an area of 1 square centimetre (1 cm^2). Other common units for area are the square metre (m^2) and the square kilometre (km^2).

kilo	—	1000
hecto	—	100
deca	—	10
_____	—	1
deci	—	0.1
centi	—	0.01
milli	—	0.001

A reference and source book which contains ideas for activities that may be used in studying metric measurement is *World of Metric* by Sharon E. Odegard, published by Ginn and Company.

As soon as students learn more than one unit in each kind of measurement they are faced with the need to select the most appropriate unit for the situation. For example, the centimetre is satisfactory for measuring many small objects, such as pieces of paper or ribbon, and small boxes; but for measuring the width of a house, the length of a fence, or the depth of a lake, the metre is more suitable. The next difficulty that students encounter is one of precision in measurement. For example, an object said to be 2 cm in length may have a length greater than 2 cm but less than 3 cm. It is frequently necessary to name the length of any object to the nearest unit, by stating that its length is *about* a certain

number of units. Estimation involves similar approximation using the word *about*, but it is even less precise because the number of units is not determined by any standard measuring device. Only direct experience in measuring objects with standard units can provide any degree of expertise in estimating with measures.

Generalizations are drawn through an inductive-deductive process which involves an examination of specific items. These are then tested and applied in new situations. An educated person has much of her/his knowledge in the form of generalizations. The *Keeping Sharp* feature on page 131 looks at place values in factors and products and several important generalizations are formed, namely, that the product of tens and tens is hundreds, and that the product of tens and hundreds is thousands. These generalizations for whole numbers are introduced at this time because multiplication with decimal factors in Unit 8 will be more easily understood with such a background.

Teaching Strategies

The lessons which deal with linear measurement include a number of activities both in the preliminary work and in the exercises in the textbook. Since these may entail considerable movement about the classroom, it may be advisable to divide the class into groups and to schedule the lessons in a manner which will reduce the amount of activity at one time. Such scheduling may have one or two groups begin the measuring activities while others proceed to work in other subjects or to practice exercises with whole-number operations.

In the lessons, converting measurements from one unit to another is presented in steps, such as in the following.

$$\begin{aligned} 425 \text{ cm} &= 400 \text{ cm} + 25 \text{ cm} \\ &= 4 \text{ m} + 0.25 \text{ m} \\ &= 4.25 \text{ m} \end{aligned}$$

$$\begin{aligned} 1.375 \text{ m} &= 1000 \text{ mm} + 375 \text{ mm} \\ &= 1375 \text{ mm} \end{aligned}$$

Some of the better students may be able to make such changes by repositioning the decimal points, but this is beyond the expected level of achievement for this book.

In connection with the topic of perimeter there is a danger of over-simplification of the rule. It is incorrect to say that perimeter is found by adding the length and the width and multiplying by two. This rule applies only to rectangles. It is, therefore, important that students think of perimeter first as being the distance around a polygon, (*peri* means around, *meter* means to measure), and that shorter methods of finding perimeters of certain polygons depend upon the number and lengths of their sides. For example, the perimeter of an equilateral triangle may be found by multiplying the length of one side by 3, but the perimeter of a scalene triangle (sides of unequal length) may be found only by adding the lengths of the three sides.

It is important in teaching the lessons on finding the area of a rectangle to emphasize that the linear unit for measuring the sides determines the square unit for the area. If the sides are measured in centimetres, the area can be calculated directly in square centimetres by multiplying the dimensions. Similarly, if metres are used to measure the sides, multiplying the dimensions gives the area in square metres. The students do not meet any situations in this book where the two dimensions are given in different units; however, they should realize, if this emphasis is given in the lessons, that square units are possible only if the two dimensions are given in the same unit.

The lessons on pages 142 and 143 draw attention to the fact that rectangles with the same perimeter may have different dimensions, and that rectangles with the same area may also have different dimensions. Geoboards and dot paper are recommended so that students may experience these facts on a concrete level. After these lessons have been completed, it may be worthwhile to challenge some of the students to consider them on an abstract level. For example, if the perimeter of a rectangle is 20 units, the sum of the length and the width is 10 units, and the two addends are, therefore, 1 and 9, 2 and 8, 3 and 7, 4 and 6, or 5 and 5. Similarly, if the area of a rectangle is 24 square units, the dimensions are two factors with a product of 24, namely, 1 and 24, 2 and 12, 3 and 8, or 4 and 6. Logical thinking such as this is useful in solving many problems on an abstract level.

Results of the exercises on page 146 should be examined carefully to determine whether any topics require reteaching and/or additional work. The results of exercises on page 147 should also receive close scrutiny to determine specific needs of individual students before they begin Unit 8 (multiplying decimals).

Materials

metre sticks, centimetre ruler and a red pencil for each student; objects for which the lengths are to be measured

objects described in Ex. 12-16 and 18 on page 129 (domino, button, roll of tape, pencil sharpener, Canadian coins, round face of a watch) or other similar objects; ruler marked in centimetres and in millimetres for each student; metre sticks marked in centimetres and in millimetres

5 rectangular sheets of paper for each student; irregular two-dimensional shapes having from three to six sides

objects described in Ex. 9-11 on page 133 (Canadian flag, shoe, telephone dial)

copies of page T 397 for each student; a straight edge for each student; newspapers containing pictures; cards for finding area; geoboards and rubber bands, or copies of page T 396

three-dimensional shapes similar to those illustrated on pages 142 and 143 (optional)

tacks, string, objects that suggest circles, large sheets of paper for preparing charts

Vocabulary

metre, m	radius
centimetre, cm	diameter
millimetre, mm	perimeter
nearest centimetre	square
nearest tenth of a centimetre	rectangle
nearest hundredth of a metre	area
square centimetre, cm^2	length, width
square metre, m^2	circumference

LESSON OUTCOME

Measure length in centimetres and in metres; identify the centimetre as one-hundredth of a metre; express metres as centimetres and centimetres as metres

Materials

metre sticks, centimetre ruler, and a red pencil for each student; objects for which the lengths are to be measured

Vocabulary

metre, m, centimetre, cm, nearest centimetre, nearest hundredth of a metre

Prerequisite Skills

Write decimals to express hundredths

Checking Prerequisite Skills

Write each as a decimal.

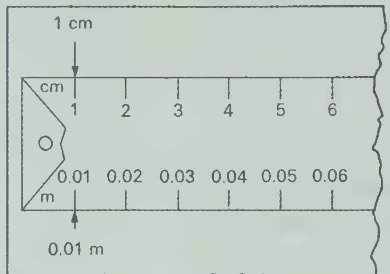
1. 34 hundredths 0.34
2. 50 hundredths 0.50
3. 1 and 17 hundredths 1.17
4. 2 and 8 hundredths 2.08

7 MEASUREMENT

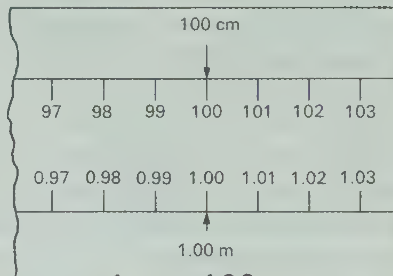
Measuring with Metres or Centimetres

A centimetre is one-hundredth of a metre.

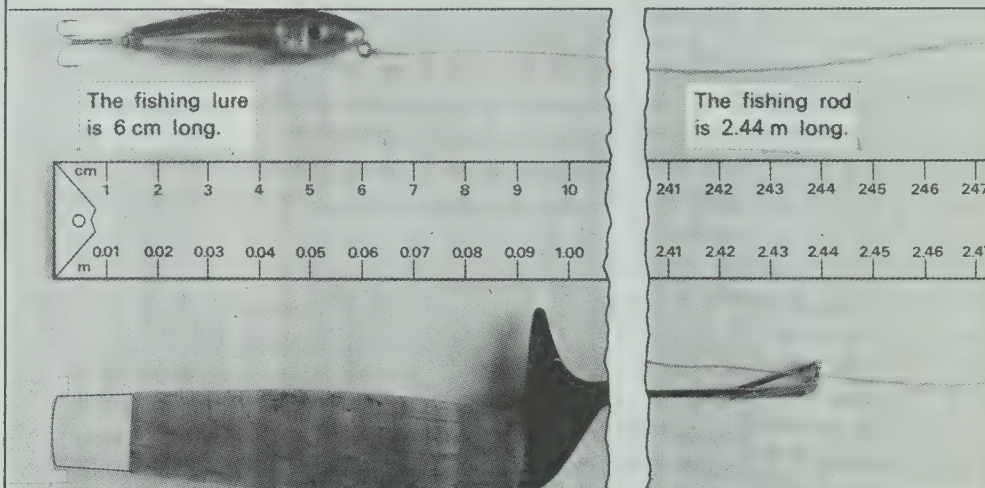
There are 100 cm in one metre.



1 cm = 0.01 m



1 m = 100 cm



1 cm = 1 hundredth of a metre
6 cm = 6 hundredths of a metre

6 cm = 0.06 m

The fishing lure is 6 cm or 0.06 m long.

2.44 m = 2 m and 0.44 m
2.44 m = 200 cm and 44 cm

2.44 m = 244 cm

The fishing rod is 2.44 m or 244 cm long.

126

LESSON ACTIVITY

Before Using the Pages

- Reacquaint students with the metre and the centimetre as units of length. Display a metre stick. Ask students to identify it and describe its use. Have them use metre sticks to measure (to the nearest metre) the length of a display board and the distance from the floor to the top of the display board. Have them find a length that is about 1 m, for example, the distance from a doorknob to the floor. Then discuss why the metre is unsuitable for measuring such lengths as the width of a chalkboard brush, the length of an eraser, the length of a pencil, or the height of a jar of paste.

  Lead the students to suggest the centimetre as a more appropriate unit of length and have them observe the centimetre markings on their own centimetre rulers. Have them state the length of their rulers and use the rulers to measure objects such as those named above. Explain that the ruler must be aligned carefully with the object being measured. (Some rulers have a mark for 0 cm while others do not.) Ask whether there is a relationship between the length of a metre and the length of a centimetre. Show students the centimetre markings on a metre stick. If the metre stick is unmarked, have them place centimetre rulers end to end to recall that 1 m is equal to 100 cm. Write the sentence 1 m = 100 cm on the chalkboard. Then write 1 cm = _____ m and have students suggest a number that will make a true statement. Ask the students to find an object that measures about 1 cm, for example, the width of a thumbtack.

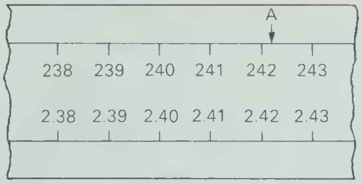
Using the Pages

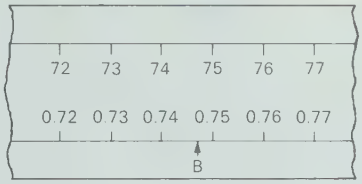
- Have students read the introductory statements at the top of page 126. Point out the use of a decimal to state the relationship 1 cm = 0.01 m. Ask how the scale shows lengths of 2 cm, 5 cm, 97 cm, and 102 cm in metres. Then ask how the scale shows such lengths as 0.04 m, 0.99 m, 1.00 m, and 1.03 m in centimetres. Emphasize that the scale shows two names for each mark and each names the same length.

Working Together

The top edge of this measuring tape is marked in centimetres. The bottom edge is marked in metres.

1. For point A, read the tape to the nearest centimetre. 242
2. For point B, read the tape to the nearest hundredth of a metre. 0.75





3. For point A, read the tape to the nearest hundredth of a metre. 2.42
4. For point B, read the tape to the nearest centimetre. 75

Complete.

5.

cm	79	? 300	125	? 33
m	? 0.79	3.60	? 1.25	0.33

Measure the length and complete.

6.

	cm	m
the chalkboard	?	?

Answers will vary

Exercises

Complete.

	cm	m
1.	250	? 2.50
2.	? 147	1.47
3.	95	? 0.95

Write each sentence. Change measurements in metres to centimetres and measurements in centimetres to metres.

4. The wallpaper in the roll was 9 m long and 0.53 m wide. The wallpaper in the roll was 900 cm long and 53 cm wide
5. The pane of glass was 240 cm long and 120 cm wide. The pane of glass was 2.40 m long and 1.20 m wide

Measure to the nearest centimetre or to the nearest hundredth of a metre. Write your measurement in red. Then use black and write the measurement in the other unit.

Answers will vary

		cm	m
6.	the length of your classroom	?	?
7.	the width of this book	?	?
8.	the height of your desk	?	?
9.	how high you can reach	?	?
10.	the distance from the door to the teacher's desk	?	?

127

RELATED ACTIVITIES

• Have students measure and record the lengths of objects from both the school and the home environment. As they become proficient in measuring, have them estimate lengths before they measure them and compare their estimates with the actual lengths. The results may be shown in a chart as follows. For lengths that are measured and recorded in one unit, ask students to express the same length in the other unit.

	Estimate		Measurement	
	cm	m	cm	m
height of door				
width of door				
length of bed				

• Ask students to help one another measure their heights. Record the measurements in centimetres and in metres on a large chart. Have students order the numbers from least to greatest and then group them into categories such as 1.30 m - 1.34 m, 1.35 m - 1.39 m, 1.40 m - 1.44 m, and so on. Have them prepare bar graphs to display the information. Discuss the results shown in the graphs. This activity may be extended to include the heights of all the students in Grade 5 at your school.

Ask students to study the second illustration on page 126 to note that a scale measuring up to 247 cm is suggested. Have them give the length of the fishing lure in centimetres, suggest a name for the same length in metres, and check their answer with the solution shown at the bottom of the page. Discuss the length of the fishing rod in a similar manner. (Note that the white portion of the handle of the fishing rod is aligned with the end of the scale.)

Working Together: For Ex. 1-4, discuss the meaning of the terms *nearest centimetre* and *nearest hundredth of a metre*. Emphasize that the scales show two ways of naming points A and B, one as centimetres and one as hundredths of a metre. Have students suggest how a length named in centimetres can be renamed in metres and vice versa. This will help them in completing Ex. 5. Use other similar exercises as required. For Ex. 6, have some students measure and record the length in metres and then write the length in centimetres. Have other students use the reverse procedure and compare the results.

Exercises: For Ex. 1-3, some students may need to write statements similar to those shown in the "thought clouds" at the bottom of page 126. Note that for Ex. 6-10, the lengths are measured and recorded using the more appropriate unit (metres or centimetres) and are then expressed in terms of the other unit.

Assessment

Measure to the nearest centimetre.

1. ________ 8 cm
2. ________ 6 cm

Complete.

	cm	m
3.	34	0.34
4.	174	1.74
5.	8	0.08
6.	130	1.30

Pages 128 - 129

LESSON OUTCOME

Measure length in millimetres; identify the millimetre as one-tenth of a centimetre and one-thousandth of a metre; identify a diameter and a radius of a circle

Materials

objects described in Ex. 12-16, 18, or other similar objects; a ruler marked in centimetres and in millimetres for each student; metre sticks marked in centimetres and in millimetres

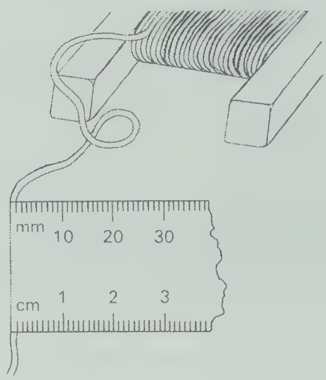
Vocabulary

millimetre, mm, radius, diameter

Measuring with Millimetres

How thick is Mindy's fishing line?

Millimetres are used for measuring small things or for measuring more exactly than with centimetres.



Mindy's fishing line is about 1 mm (millimetre) thick.

A millimetre is one-tenth of a centimetre.

$$1 \text{ mm} = 0.1 \text{ cm}$$

There are 10 mm in a centimetre.

$$1 \text{ cm} = 10 \text{ mm}$$

There are 10 mm in a centimetre and 100 cm in a metre. So, there are 100 × 10 mm, or 1000 mm, in a metre. A millimetre is 0.001 m.

128

LESSON ACTIVITY

Before Using the Pages

- Ask students to indicate whether the centimetre or the metre is the more appropriate unit for measuring the lengths of various objects, for example, the height of a door, the width of a driveway, the length of a shoe. Then name such lengths as the thickness of a dime, the width (diameter) of a nail, and the thickness of a piece of string. Students will likely suggest that neither unit is appropriate for these examples and that a smaller unit is required. They may be able to suggest the *millimetre* as a smaller unit. Have them observe the millimetre markings on their own rulers and use the millimetre scale to measure such lengths as the width of a pencil, the thickness of an eraser, or the length of their smallest finger. They may be able to find an object about 1 mm in thickness, for example, a dime or a paper clip. Ask what the relationship between the length of a millimetre and the length of a centimetre is. Have the students compare the two scales on their rulers to note the relationships: 10 mm = 1 cm, 20 mm = 2 cm, 30 mm = 3 cm, and so on, for multiples of 10 mm. Write the following sentences on the board and have students suggest numbers that will make true statements.

  1 cm = _____ mm

  1 mm = _____ cm

Using the Pages

- Begin with a brief discussion of the photograph on page 128. Some students may have had similar experiences with tangled fishing line on a fishing trip. Ask students to read the statements above and below the photograph. Have them note that the thickness of the fishing line is about 1 mm. Point out the use of a decimal in the sentence 1 mm = 0.1 cm. Ask the students to determine the number of millimetres in one metre, since there are 10 mm in 1 cm. Have them explain how they arrived at their answer. Show the students the centimetre and millimetre scales on a metre stick to verify the statement 1000 mm = 1 m. Write the

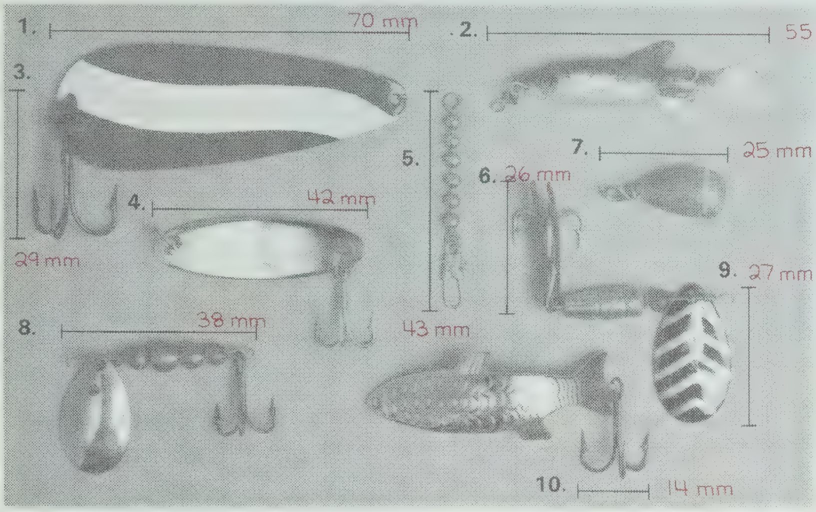
Working Together

Measure to the nearest millimetre.

1. ——— 14 mm 2. ——— 18 mm 3. the sharpened end of your pencil Answers will vary

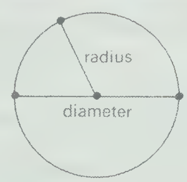
Exercises

Measure to the nearest millimetre.



A **radius** of a circle is a line segment with one end point at the center of the circle and the other end point on the circle.

A **diameter** of a circle is a line segment having both end points on the circle and containing the center.



Measure, to the nearest millimetre, the diameter of

11. the circle shown above. 32 mm
12. the dots on a domino. Answers will vary
13. a button. Answers will vary
14. a roll of tape. Answers will vary.
15. each hole in a pencil sharpener. Answers will vary.
16. each kind of Canadian coin. penny - 19 mm, nickel - 21 mm, dime - 17 mm, quarter - 24 mm

Measure, to the nearest millimetre, the radius of

17. the circle shown above. 16 mm
18. the face of a round watch. Answers will vary.

129

RELATED ACTIVITIES

- Have students help to prepare a large chart summarizing the units of length in this and the previous lesson. Display the chart for several days to reinforce the spelling of the words, the symbols, and the relationships among the units. If the length or the width of the chart is greater than 1 m, ask students to trace a metre stick onto it and show the different scales.

metre, m
1 m = 100 cm
1 m = 1000 mm
centimetre, cm
1 cm = 0.01 m
1 cm = 10 mm
millimetre, mm
1 mm = 0.1 cm
1 mm = 0.001 m

- Have students measure the thickness of their mathematics text in millimetres. Then have them find, without measuring, the height of a stack of 15 of the books, for example. Ask the students to suggest similar activities using centimetres and metres.

Measure	Calculate
the length of one paper clip	the length of a chain of 350 paper clips
the width of one ceiling tile	the width of the ceiling
the height of one step	the height of the staircase

following sentences on the board and have students complete them.

1 m = _____ mm

1 mm = _____ m

Working Together: Remind the students to align their rulers carefully with the line segments in Ex. 1 and 2 and with the lead portion of their sharpened pencil for Ex. 3.

Exercises: For Ex. 1-10, line segments are given to indicate the length to be measured for each object. Ensure that the students align their rulers with the line segments, not with the objects.

Before the students begin Ex. 11-18, discuss the terms *diameter* and *radius*. Draw circles on the board and mark the centre of each with a dot. Have students use a straight edge to draw different radii and diameters for each circle.

Provide the students with the objects described in Ex. 12-16 and 18 or use other similar objects. Caution them to take care in reading the scale, because millimetre markings are very small.

Assessment

Measure to the nearest millimetre.

1. ____________________ 32 mm

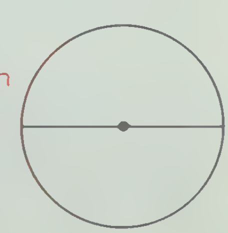
2. ______________________________ 50 mm

3. _____ 8 mm

4. ________________ 27 mm

Measure to the nearest millimetre,

5. the diameter of the circle. 26 mm
6. the radius of the circle. 13 mm



Pages 130-131

LESSON OUTCOME

Express lengths given in one unit in terms of another unit using decimals, for metres, centimetres, and millimetres

Materials

metre sticks marked in centimetres and in millimetres; a ruler marked in centimetres and millimetres and a red pencil for each student

Prerequisite Skills

Write decimals to express tenths, hundredths, and thousandths

Checking Prerequisite Skills

Write each as a decimal.

1. 65 hundredths 0.65
2. 6 and 5 tenths 6.5
3. 5 hundredths 0.05
4. 6 thousandths 0.006
5. 30 hundredths 0.30
6. 27 and 15 hundredths 27.15
7. 165 thousandths 0.165
8. 3 and 14 thousandths 3.014

Metres, Centimetres, and Millimetres

Greg needs a new spoke for his bicycle.

1. 1.25 m = 1 m + 0.25 m
   = 100 cm + 25 cm
   = 125 cm
2. 267 mm = 200 mm + 67 mm
   = 20 cm + 6.7 cm
   = 26.7 cm
3. 6 m = 600 cm
4. 4 cm = 40 mm
5. 6.3 cm = 63 mm
6. 3.75 m = 3 m + 0.75 m
   = 3000 mm + 750 mm
   = 3750 mm
7. 305 cm = 3.05 m
8. 2364 mm = 2.364 m
9. 58 cm = 0.58 m

He measured one spoke to be 31 cm long.

How many millimetres is 31 cm?

1 cm = 10 mm

31 cm = 310 mm

Each spoke is 310 mm long.

How many metres is 31 cm?

1 cm = 0.01 m

31 cm = 0.31 m

Each spoke is 0.31 m long.

Working Together

Use this chart for the exercises, if needed.

1 cm = 10 mm	1 mm = 0.1 cm
1 m = 100 cm	1 cm = 0.01 m
1 m = 1000 mm	1 mm = 0.001 m

Give each length in millimetres.

4. 4 cm **5.** 6.3 cm **6.** 3.75 m

Give each length in centimetres.

Example: 3.84 m = 3 m + 0.84 m
3.84 m = 300 cm + 84 cm
3.84 m = 384 cm

1. 1.25 m **2.** 267 mm **3.** 6 m

Give each length in metres.

7. 305 cm **8.** 2364 mm **9.** 58 cm

130

LESSON ACTIVITY

Before Using the Pages

- Have students recall the three relationships: 1 m = 100 cm, 1 m = 1000 mm, and 1 cm = 10 mm. Write the sentences on the board and then review the same relationships using decimals: 1 mm = 0.001 m, 1 cm = 0.01 m, and 1 mm = 0.1 cm. Draw a line segment 42 cm long on the board. Ask students to measure the line segment in centimetres. Write the measurement beside the segment. Have them express this length in millimetres and measure to confirm the result. Emphasize that these are two ways of naming the same length.

420 mm
42 cm

Use a similar procedure for other line segments. Then have students suggest how a length measured in one of the units can be expressed in terms of the other unit, without measuring.

- Instruct the students to draw a line segment having a length of 12 cm. Ask them to express this length as part of a metre. Have them express the length in millimetres and measure to confirm the result. Use other similar examples. Then ask students to suggest how a length named in metres can be expressed as centimetres and as millimetres without measuring.

0.12 m
12 cm
120 mm

Using the Pages

- Discuss the situation presented in the photograph. In order to purchase a new spoke, Greg must find the length of spoke that is needed. Have the students note that the length was measured in centimetres. Since 1 cm is the same as 10 mm, develop that 31 cm must be the same as 310 mm. The students may verify this on a metre stick marked in centimetres and millimetres. Similarly, since 1 cm is one-hundredth of a metre, develop that 31 cm is thirty-one

RELATED ACTIVITIES

- Emphasize that although lengths may be expressed in different units, there is probably a preferred unit in which to measure a length. Have students name the preferred unit (metres or centimetres) for measuring such lengths as the height of a room, the depth of a shoe box, the depth of a swimming pool, the width of a diving board, and the distance from the classroom to the school office. (This last distance may be measured using a trundle wheel.) The examples given in Ex. 19-26 should be discussed from the point of view of a preferred unit of measurement.
- Have students measure the diameter and the radius of a 33 rpm record, a 45 rpm record, and, if possible, a 78 rpm record. Ask them to measure, also, the diameter of the hole in the center of a 33 rpm record and a 45 rpm record.
- Discuss the meaning of the prefix "cent" in *centimetre* and "milli" in *millimetre* to help students relate each unit to the metre.

Exercises

Give each length in centimetres.

1. 5 m 500 cm
2. 870 mm 87.0 cm
3. 9.3 m 930 cm
4. 8 mm 0.8 cm
5. 48 mm 4.8 cm
6. 0.5 m 50 cm

Give each length in millimetres.

7. 17 cm 170 mm
8. 4.5 m 4500 mm
9. 18.3 cm 183 mm
10. 283 cm 2830 mm
11. 92 m 92 000 mm
12. 6.7 cm 67 mm

Give each length in metres.

13. 4674 mm 4.674 m
14. 1800 cm 18.00 m
15. 14 cm 0.14 m
16. 670 mm 0.670 m
17. 183 cm 1.83 m
18. 7 mm 0.007 m

Choose a unit and measure. Write the measurement in red. Then use black and write the measurement in each of the other units.
Answers will vary.

		mm	cm	m
19.	the width of your desk	?	?	?
20.	your height	?	?	?
21.	a giant step	?	?	?
22.	the length of a hair from your head	?	?	?
23.	the width of your classroom	?	?	?
24.	the length of the nail on your little finger	?	?	?
25.	the distance from your desk to the door	?	?	?
26.	the width of one of your front teeth	?	?	?

Study these multiplications. Then complete each of these.

A. 20 × 30 = 600 — 1. 2 tens × 3 tens = 6 ? hundreds

B. 600 × 40 = 24 000 — 2. 6 hundreds × 4 tens = 24 ? thousands

C. 50 × 800 = 40 000 — 3. 5 tens × 8 hundreds = 40 ? thousands

For multiplying, in general,

4. tens × tens = ? hundreds
5. hundreds × tens = ? thousands
6. tens × hundreds = ? thousands

Find the product of each without using paper or pencil.

7. 40 × 40 = 1600
8. 400 × 20 = 8000
9. 60 × 700 = 42 000
10. 700 × 30 = 21 000
11. 20 × 50 = 1000
12. 900 × 10 = 9000

How many times will the digit 0 appear in each product?

13. 300 × 50 (15 000) 3
14. 70 × 700 (49 000) 3
15. 50 × 60 (3000) 3

How many digits will there be in each product?

16. 91 × 79 (7189) 4
17. 312 × 24 (7488) 4
18. 476 × 92 (43 792) 5

KEEPING SHARP

131

hundredths of a metre, or 0.31 m. Summarize by saying it is often necessary to express a measurement given in one unit of length in terms of another unit of length.

Working Together: At this time, the procedure for expressing a measurement in terms of more than one unit of length does not involve applying rules for multiplication or division by 10, 100, or 1000. Emphasis is on understanding the relationships and using a series of steps to convert from one unit of length to another. These are shown in the example above Ex. 1-3. The chart to the left of the example summarizes the relationships between pairs of units. Help students relate each exercise to a sentence in the chart, to develop the steps.

Exercises: Have students refer to the chart on page 130 and to marked metre sticks as needed for assistance. Some students may be able to write answers without having to write the intermediate steps. Note that two colors are required for writing answers to Ex. 19-26.

Keeping Sharp: These exercises help students to develop an efficient approach to multiplication when factors are a multiple of ten from 10 to 90 or a multiple of one hundred from 100 to 900. The product can be seen as an extension of a basic multiplication fact, and the number of zeros in the product is determined by the number of zeros in the factors. This is summarized in Ex. 4-6 after the concepts are explored in Ex. 1-3. The concept is then applied in Ex. 16-18 in finding the number of digits in a product before multiplication is performed. This is a helpful device for estimating products. Ask students to check their answers for Ex. 16-18 by multiplying.

Assessment

Complete.

	mm	cm	m
1.	430	43	0.43
2.	1500	150	1.5
3.	260	26	0.26

	mm	cm	m
4.	1320	132	1.32
5.	1300	130	1.3
6.	2000	200	2

LESSON OUTCOME

Use addition to find the perimeter of a shape the sides of which are measured in millimetres or in centimetres

Materials

a ruler marked in millimetres and in centimetres and five rectangular sheets of paper for each student, irregular two-dimensional shapes having from three to six sides

Vocabulary

perimeter, nearest tenth of a centimetre

Prerequisite Skills

Add decimal tenths; measure line segments in centimetres and in millimetres

Checking Prerequisite Skills

Add.

1. 3.4 + 1.7 + 2.5 + 0.9 8.5

2. 6.8 + 6.8 + 4.7 + 4.7 23.0

Use an unmarked straight edge.

3. Draw two line segments.

Answers will vary for Ex. 3-5.

Use a millimetre ruler.

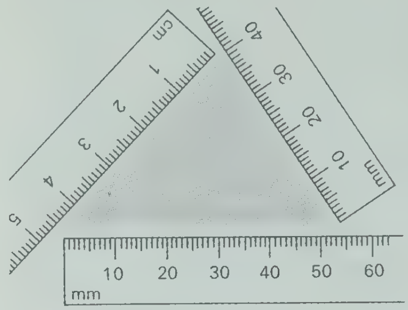
4. Measure the first line segment you drew.

Use a centimetre ruler.

5. Measure the second line segment you drew, to the nearest centimetre.

Finding the Perimeter

The sides of the shape are 52 mm, 39 mm, and 45 mm long.



The sides of the shape are 5.2 cm, 3.9 cm, and 4.5 cm long.

Add to find the distance around the shape.

In millimetres:	In centimetres:
52	5.2
39	3.9
45	4.5
136	13.6

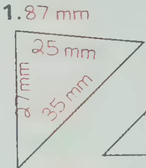
The distance around a shape is its **perimeter**.

The perimeter of the shape is 136 mm or 13.6 cm.

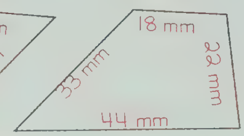
Exercises

Measure each side in millimetres. Then add to find the perimeter.

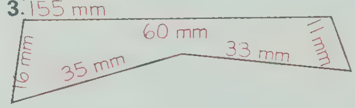
1. 87 mm



2. 117 mm



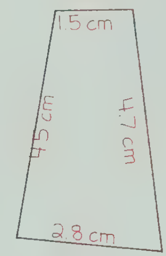
3. 155 mm



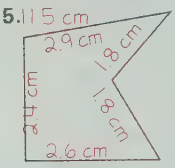
Measure each side in centimetres. Then add to find the perimeter.

Measure to the nearest tenth of a centimetre.

4. 13.5 cm



5. 11.5 cm

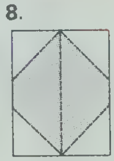


Fold a sheet of paper. Then find the perimeter of the shaded part. Answers will vary.

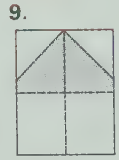
6.

7.

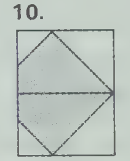
8.



9.



10.



132

LESSON ACTIVITY

Before Using the Page

- Provide each student with an irregular shape having from three to six sides. Direct them to find the distance around the shape. Discuss the procedures they use to do this. Ask if anyone recalls the mathematical term that means "the distance around".

Using the Page

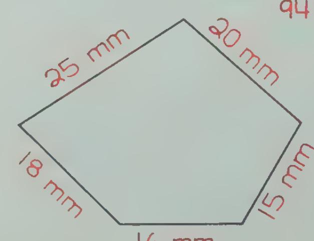
- Draw the students' attention to the spelling of the word *perimeter*, noting the "er" ending. Explain that the prefix "peri" means "around" and "meter" means "measure". Have a student identify the blue shape. Ask what must be found before the perimeter of the triangle can be determined. Have the students note the scales shown on the three rulers in the illustration. (One is marked in centimetres.) Discuss how addition is used to find the perimeter, and pay particular attention to the units of length for the two sums shown. Point out that for the ruler marked in centimetres, the length is read to the *nearest tenth of a centimetre* (4.5 cm). Emphasize that each length must be expressed in the same unit before addition is performed.

Exercises: Draw attention to the unit of length specified for Ex. 1-3 and Ex. 4 and 5. The choice of unit is left to each student for Ex. 6-10. Give each student five rectangular sheets of paper for Ex. 6-10. You may wish to demonstrate the folding technique for some of these.

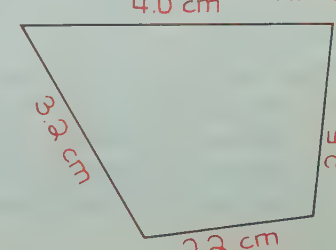
Assessment

For each shape, measure each side and find the perimeter.

1. Measure in millimetres. 94 mm

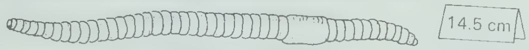


2. Measure in centimetres. 11.9 cm



Practice

Each length is given in one unit. Complete the chart to show the length in each of the units.

1. 

2. 

3. 

	mm	cm	m
1.	145	14.5	0.145
2.	5640	564	5.64
3.	240	24.0	0.240

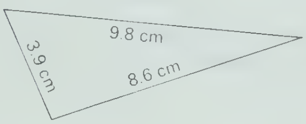
Measure each line segment in one unit. Complete the chart to show the length in each of the units.

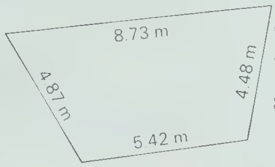
4. ______________________________

5. ____

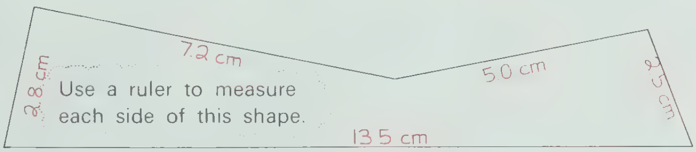
	mm	cm	m
4.	105	10.5	0.105
5.	150	15.0	0.150

Complete the chart to show perimeter in each unit.

6. 

7. 

	mm	cm	m
6.	223	22.3	0.223
7.	23 500	2350	23.50
8.	310	31.0	0.310

8. 

Choose a unit and measure. Write your measurement in red. Then use black and write the measurement in each of the other units.
Answers will vary

		mm	cm	m
9.	the length of a Canadian flag	?	?	?
10.	the greatest width of your shoe	?	?	?
11.	the diameter of a telephone dial	?	?	?
12.	the perimeter of your classroom	?	?	?

When you use centimetres, measure to one decimal place.

When you use metres, measure to two decimal places.

133

LESSON ACTIVITY

Using the Page

- Review the instructions for each group of exercises with the students to ensure that they understand what is required. Pay particular attention to the statements shown in the "thought clouds" for Ex. 8-12. It may be necessary to review the meaning of "diameter" in Ex. 11.

  Have the students work in small groups for Ex. 9, 11, and 12. One group can begin with these exercises and then continue with the rest of the page, while another group takes its place for Ex. 9, 11, and 12.

Page 133

OBJECTIVE

Demonstrate competence in measuring length (in metres, centimetres, and millimetres), in finding the perimeter of a shape, and in expressing measurements given in one unit of length in terms of another unit of length

Materials

metre sticks marked in centimetres and millimetres; a ruler marked in centimetres and millimetres and a red pencil for each student; objects described in Ex. 9-11

RELATED ACTIVITIES

- Have students tape large sheets of paper together to form a region wider and longer than their own width and height. Have them work in pairs to trace one another's outline on the paper. Have them use string to match the outline drawn and find their own perimeter. Ask them to express their perimeter in metres, in centimetres, and in millimetres. The outlines may be retained for the activity described in *Related Activities* on page T 149.
- Have students use pieces of string to find the perimeter of objects with curved edges, such as a wastebasket and the face of a dial clock.

LESSON OUTCOME

Use multiplication to find the perimeter of a square; use multiplication and addition to find the perimeter of a rectangle; solve related word problems

Materials

ruler marked in centimetres and millimetres for each student

Vocabulary

square, rectangle

Prerequisite Skills

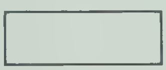
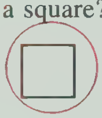
Multiply whole numbers; identify shapes that are square and shapes that are rectangular

Checking Prerequisite Skills

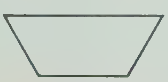
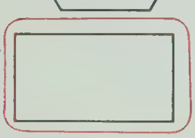
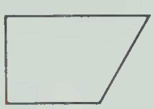
Multiply.

1. 4 × 175 700 2. 2 × 37 74
3. 2 × 396 792 4. 4 × 58 232

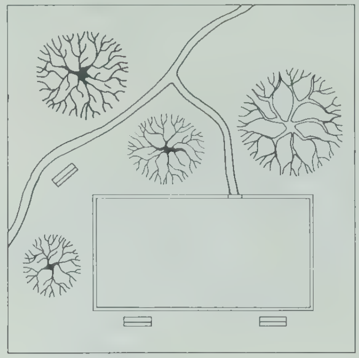
Which of these shapes is a square?

5. 
6. 
7. 
8. 

Which of these shapes is a rectangle?

9. 
10. 
11. 
12. 

Finding the Perimeter of a Square or a Rectangle



The city park covers a square block. Each side is 118 m long. How far is it around the park?

To find the perimeter,

add	or	multiply.
118		118
118		× 4
118		472
118		
472		

The perimeter of the city park is 472 m.

The skating rink has the shape of a rectangle. What is its perimeter?

There are two sides of 75 m each and two sides of 40 m each.

Add

75 + 40 + 75 + 40 = 230

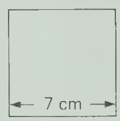
or multiply, then add.

(2 × 75) + (2 × 40) = 230

The perimeter of the rink is 230 m.

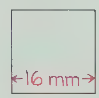
Working Together

Complete.

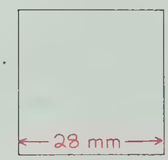
1. 

Perimeter is 4 × 7 cm, or 28 cm.

Measure and complete.

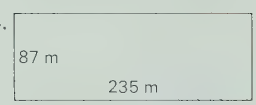
2. 

Perimeter is 4 × 16 mm, or 64 mm.

3. 

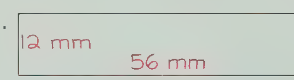
Perimeter is 4 × 28 mm, or 112 mm.

Complete.

4. 

Perimeter is 2(× 87 m) + (2 × 235 m) or 644 m.

Measure and complete.

5. 

Perimeter is 2(× 12 mm) + (2 × 56 mm) or 136 mm.

134

LESSON ACTIVITY

Before Using the Pages

- Review that all four sides of a square have the same length and opposite sides of a rectangle have the same length. Also, have the students recall that both a square and a rectangle have four right angles. Have students point out shapes in the classroom that suggest squares and rectangles, for example, ceiling tiles, floor tiles, windows, bricks, and desk tops. Have them identify sides of equal length.

Using the Pages

- Direct the students' attention to the illustration at the top of page 134. Have them identify the square shape and the rectangular shape. Ask a student to read the statements to introduce the situation. Have students explain the two methods shown for finding the perimeter of the city park. For the skating rink, ask students to point to the two sides that represent lengths of 40 m and the two that represent lengths of 75 m. Discuss the two procedures for finding the perimeter of the skating rink. Have students perform the multiplication and addition steps aloud. Point out that multiplication is useful in finding the perimeter of a rectangular or a square shape.

Working Together: For each of Ex. 1-5, have students name the shape and explain why multiplication can be used in finding the perimeter. Have them show their work for the addition and multiplication steps.

Exercises: You may wish to have the students draw diagrams to represent the shapes for Ex. 7-12. For Ex. 10, the students need to know that the picture in a 16 mm slide is a square with each side 16 mm long. For Ex. 12, the students must find the path traced out when a batter completes a home run and the distance between successive bases.

Keeping Sharp: These exercises reinforce and extend the work of multiplication from the *Keeping Sharp* feature on page 131.

Exercises

Find the perimeter for each square or rectangle.

1. 275 m — 1100 m

2. 2 cm, 10 cm — 24 cm

3. 17 km, 26 km — 86 km

4. 25 mm — 100 mm

Use a ruler to measure the sides of these shapes.

5. 42 mm, 37 mm — 158 mm

6. 74 m, 587 m — 1322 m

What is the perimeter

7. of a rectangle that is 18 cm long and 7 cm wide? 50 cm

8. of a square with each side 3 m long? 12 m

9. of a rectangle that is 19 mm wide and 23 mm long? 84 mm

*10. of the picture in a slide made from 16 mm film? 64 mm

Solve.

11. A yard in the shape of a rectangle has one side 15 m long and another side 18 m long. How many metres of fencing are needed for the four sides? 66 m

*12. In baseball, how far must a batter travel around the bases after hitting a home run? About 110 m

KEEPING SHARP

Tell how many digits there will be in each product. Then multiply.

1. 43 × 21 — 3 — 903
2. 306 × 32 — 4 — 9792
3. 58 × 74 — 4 — 4292
4. 216 × 89 — 5 — 19 224
5. 275 × 42 — 5 — 11 550

Multiply.

6. 17 × 52 — 884
7. 286 × 34 — 9724
8. 394 × 712 — 280 528
9. 2639 × 865 — 2 282 735

135

RELATED ACTIVITIES

- Have each student use a straight edge to draw four rectangular shapes on a sheet of paper. Ask students to exchange papers and find the perimeter of each shape by first measuring the lengths of appropriate sides.
- Have students find the perimeter of such objects as their mathematics text books, rectangular attribute pieces, and centimetre rulers.
- Provide students with copies of pages T382-T385. Have them measure the sides and find the perimeter of each of the polygons. You may wish to have them measure the diameter of each of the circles on page T383.

Assessment

Find the perimeter of each square or rectangle.

1. 25 mm, 60 mm — 170 mm

2. 120 m — 480 m

3. 168 cm — 672 cm

4. 85 m, 175 m — 520 m

Solve.

5. What is the perimeter of a rectangle that is 16 cm wide and 34 cm long? 100 cm

6. What is the perimeter of a square with each side 5 m long? 20 m

Pages 136-137

LESSON OUTCOME

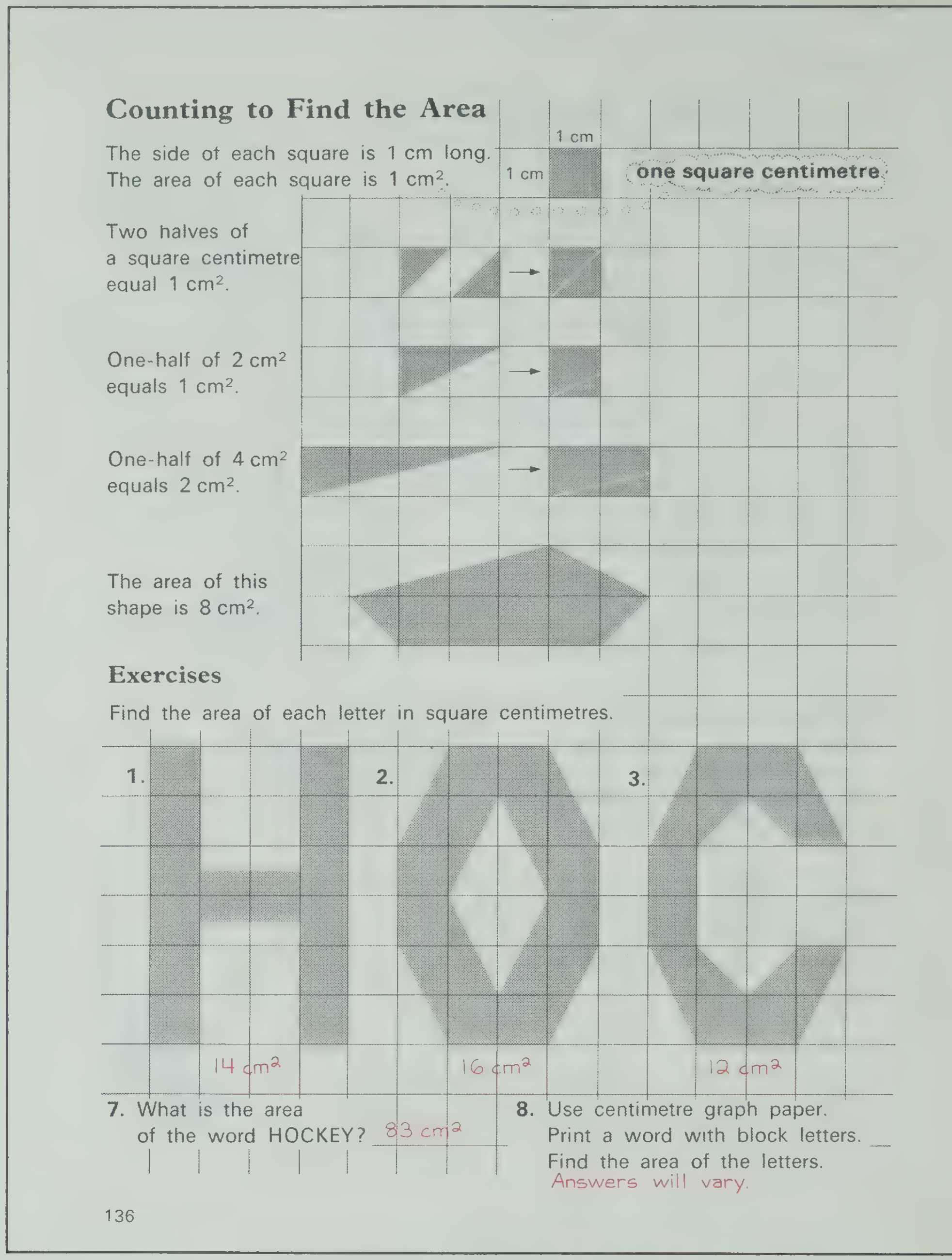
Find area in square centimetres by counting whole units and half units

Materials

copies of page T 397 and a straight edge for each student; newspapers containing pictures; cards for finding area (Ex. 10); geoboards and rubber bands, or copies of page T 396

Vocabulary

area, square centimetre, cm^2



Counting to Find the Area

The side of each square is 1 cm long.
The area of each square is 1 cm^2.

1 cm
1 cm
one square centimetre

Two halves of a square centimetre equal 1 cm^2.

One-half of 2 cm^2 equals 1 cm^2.

One-half of 4 cm^2 equals 2 cm^2.

The area of this shape is 8 cm^2.

Exercises

Find the area of each letter in square centimetres.

1. 14 cm^2
2. 16 cm^2
3. 12 cm^2

7. What is the area of the word HOCKEY? 83 cm^2

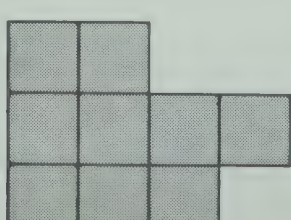
8. Use centimetre graph paper. Print a word with block letters. Find the area of the letters. Answers will vary.

136

LESSON ACTIVITY

Before Using the Pages

- Give each student a copy of page T 397. Instruct them to cut out shapes having boundaries that follow the grid lines. Have them paste the shapes on a sheet of paper and, beside each shape, write the number of squares (square centimetres) for that shape. If you wish, have them color inside each shape to reinforce the concept of a region, as opposed to the perimeter or boundary of a shape.



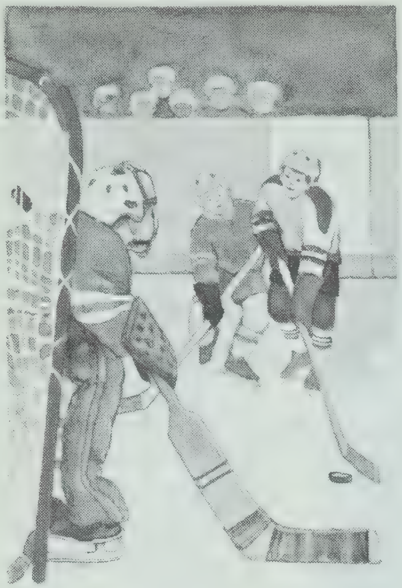
9 squares

Using the Pages

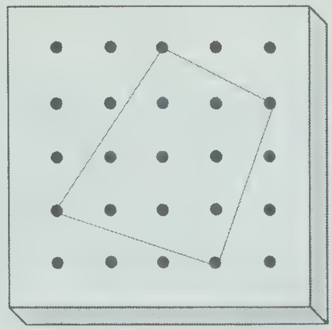
- Ask a student to read the title of the lesson on page 136. Draw attention to the word *area*. Ask what was counted to find the area of each shape cut out in the preliminary activity. Have the students notice that the squares are the same size (1 cm by 1 cm) as those in the grid on page 136. Emphasize that each square of those dimensions has an area of *one square centimetre*, which is written 1 cm^2.

  Point out that most shapes do not follow grid lines. Draw attention briefly to the letters that spell the word ''hockey'' at the bottom of pages 136 and 137. Some of the shapes in those letters are suggested in the examples at the top of page 136. Discuss each of the examples in turn and ask students to demonstrate them. For instance, have them cut a rectangle having an area of 4 cm^2 (4 cm by 1 cm) from the graph paper used earlier and cut along a diagonal to obtain two halves, each 2 cm^2 in area but triangular in shape. One of the halves can then be cut as shown and the two parts placed together to reveal two squares.

  Ask students why 8 cm^2 is the area of the last shape.



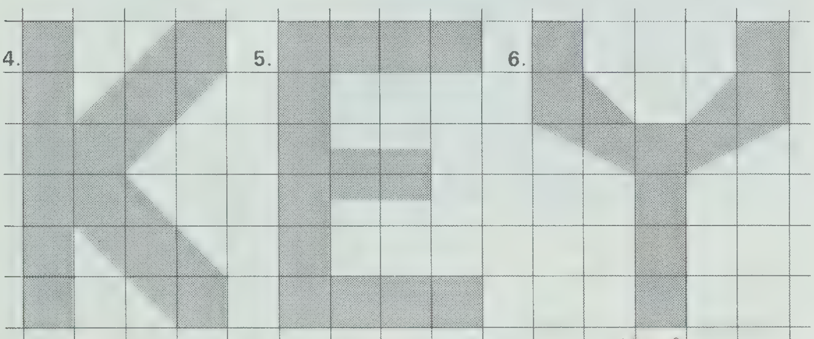
Make any four-sided shape on a geoboard, dot paper, or graph paper.



Tell how you could find the number of square units inside the shape.

Answers will vary.

PROBLEM SOLVING



4. 16 cm² 5. 14 cm² 6. 11 cm²

9. Cut a picture from a newspaper. Place the picture on centimetre graph paper. Find its area.
Answers will vary.

10. Place centimetre graph paper on a card. Find the area of the card.
Answers will vary.

137

RELATED ACTIVITIES

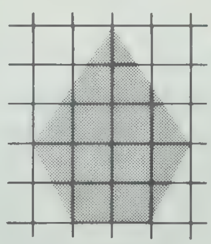
- If students carried out the activity described in *Related Activities* on page T 145, ask them to now find the area of their outline by cutting and pasting copies of page T 397 to cover the region enclosed by their outline. They can then count the squares to obtain an approximate measure of the area.
- Have students trace around various objects on centimetre graph paper and find an approximate measure of the area enclosed.
- Some students may enjoy creating shapes on geoboards and finding the area of each shape, as indicated in the *Problem Solving* feature.

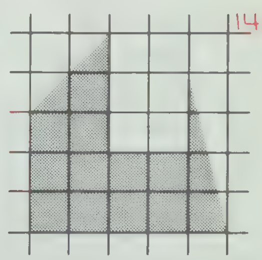
Exercises: Some students may need to trace the letters for Ex. 1-6 and mark parts of each letter in finding the area. For these exercises and for Ex. 8-10, provide the students with copies of page T 397. Remind them to use a straight edge to draw the block letters for Ex. 8. Have newspapers available for Ex. 9 and cards, such as used greeting cards, for Ex. 10.

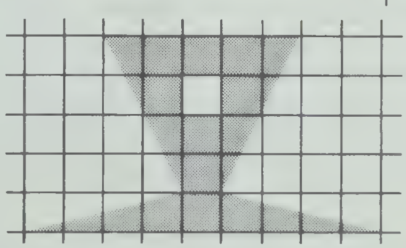
Problem Solving: Provide students with geoboards and rubber bands and ask them to copy the shape shown. If geoboards are not available, use copies of page T 396 or T 397; or let students use more than one device to solve the problem. They will encounter parts of units in this problem, although the total area is a whole number of square units.

Assessment

Find the area of each shape in square centimetres.

1. 
12 cm²

2. 
14 cm²

3. 
16 cm²

Pages 138-139

LESSON OUTCOME

Use multiplication to find the area of a rectangular shape in square centimetres

Materials
copies of page T 397 and a straight edge for each student

Prerequisite Skills
Write a multiplication sentence for an array; multiply whole numbers

Checking Prerequisite Skills
Write a multiplication sentence to match each array.

1. 
$4 \times 7 = 28$

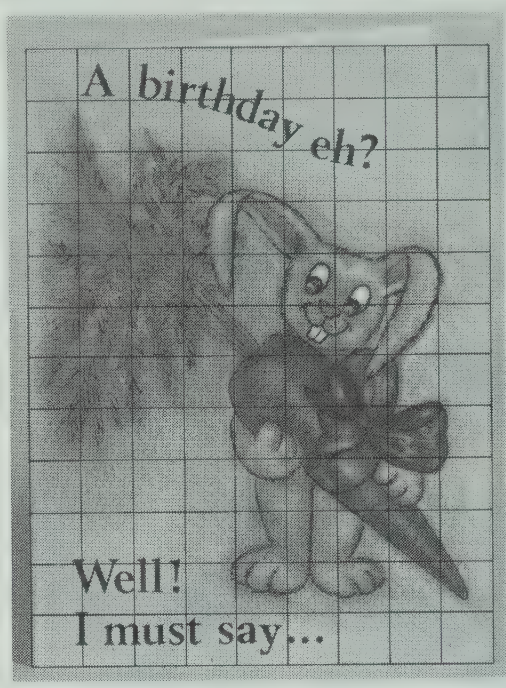
2. 
$5 \times 3 = 15$

Multiply.

3. 7×63 441
4. 12×35 420
5. 43×43 1849
6. 29×50 1450

Finding the Area of a Rectangle

The birthday card has the shape of a rectangle. For the area of the front of the card, find the number of square centimetres inside the rectangle.



Counting the squares shows there are

108 cm^2

inside the rectangle.

Multiplying is a faster way than counting to find the area.

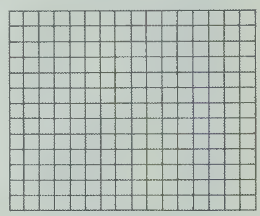
There are 12 rows of squares with 9 in each row.

$12 \times 9 = 108$

The area of the front of the card is 108 cm^2.

Working Together

For the small squares in this rectangle, →



1. how many rows are there? 13
2. how many are in each row? 16
3. how many are there in all? 208

What is the area of a rectangle having

4. 16 rows of square centimetres with 8 in each row? 128 cm^2
5. 24 rows of square centimetres with 37 in each row? 888 cm^2

138

LESSON ACTIVITY

Using the Pages

- Have a student read the title of the lesson on page 138. Emphasize that all the shapes in this lesson are rectangular. Ask a student to read the statements above the illustration. Greeting cards such as birthday cards come in many different sizes, but are usually rectangular in shape. Centimetre grid lines have been shown on the face of the card. Ask how to find its area. Lead the students to suggest that using multiplication is a more efficient approach than counting single squares. Ask why multiplication is useful in this example, whereas it was not useful in the examples on page 136. Summarize that a rectangular shape can show rows of squares with the same number of squares in each row.

Working Together: The sequence in Ex. 1-3 shows that it is necessary to know the number of rows of squares (square units) and the number of squares in each row to find the area of a rectangular shape. Then multiplication can be used. Ex. 4 and 5 illustrate that the area of a rectangle can be found without reference to a diagram, if the appropriate information is provided. Note that Ex. 4 and 5 involve the standard unit, square centimetres, whereas Ex. 1-3 do not.

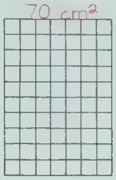
Exercises: Point out that each small square in Ex. 1-3 *represents* 1 cm^2, whereas the grid lines on the envelope for Ex. 8-10 form centimetre squares. Note that students can use their answers for Ex. 8 and 9 and subtraction to answer Ex. 10. Some students may recognize that the shapes for Ex. 3 and 7 are squares. This suggests the concept that will be developed later in Unit 9; namely, that a square is a particular kind of rectangle.

Provide the students with copies of page T 397 on which to show their answers for Ex. 11-13. If necessary, review how to plot ordered pairs of numbers before assigning these exercises. Ex. 13 will challenge some students since the rectangle does not match lines of the graph paper. Have students share their methods of solution with the rest of the class.

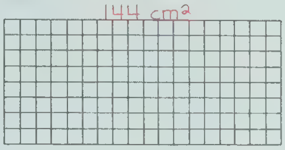
Exercises

Each small square represents 1 cm^2.
What is the area of each these?

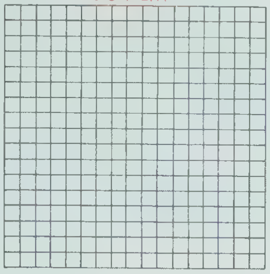
1. 70 cm^2



2. 144 cm^2



3. 289 cm^2

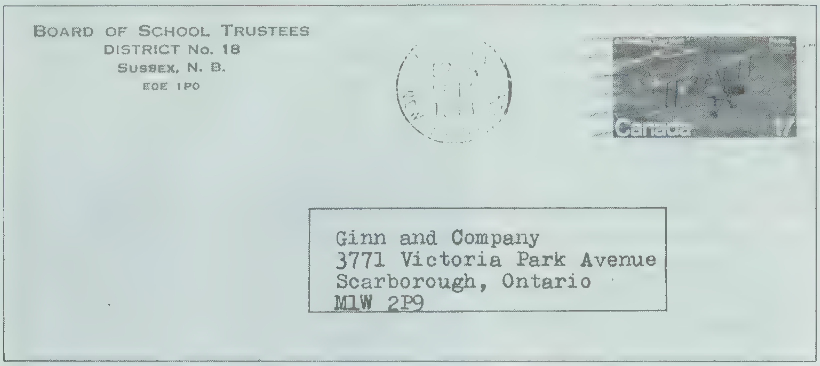


What is the area of each rectangle?

4. 7 rows of square centimetres, 19 in each row 133 cm^2
5. 26 rows of square centimetres, 4 in each row 104 cm^2
6. 65 rows of square centimetres, 87 in each row 5655 cm^2
7. 48 rows of square centimetres, 48 in each row 2304 cm^2

What is the area of

8. the envelope? 112 cm^2
9. its label? 14 cm^2
10. the part of the envelope not covered by the label? 98 cm^2



Use graph paper with centimetre squares. Draw a horizontal number line and a vertical number line starting from the same point. Find the area of the rectangle

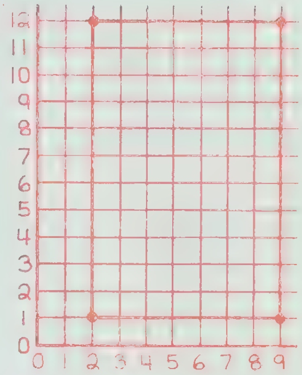
11. with corners at (2,1), (9,1), (9,12), and (2,12). 77 cm^2
12. with corners at (0,0), (17,0), (17,15), and (0,15). 255 cm^2
*13. with corners at (4,0), (13,12), (9,15), and (0,3). 75 cm^2

139

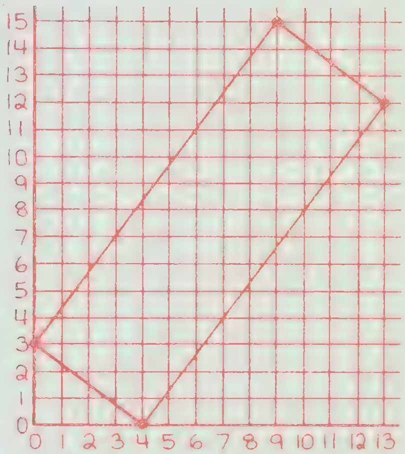
RELATED ACTIVITIES

- Provide students with several used greeting cards of various sizes. Have them trace a card on centimetre graph paper and find the area.
- Ask students to create exercises similar to Ex. 11-13 for other students to complete.

11.



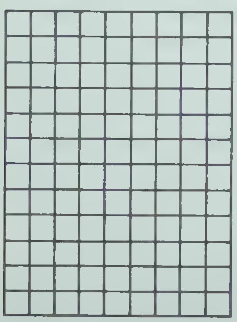
13.



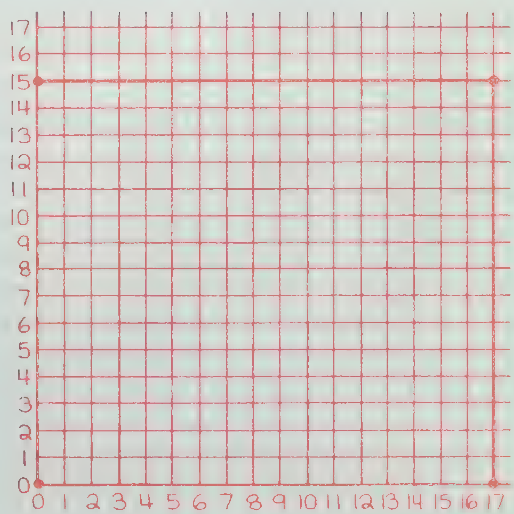
Assessment

Each small square represents 1 cm^2. What is the area?

1. 108 cm^2



12



What is the area of each rectangle?

2. 8 rows of square centimetres, 37 in each row 296 cm^2
3. 42 rows of square centimetres, 35 in each row 1470 cm^2

Pages 140-141

LESSON OUTCOME

Use multiplication to find the area of a rectangular shape in square centimetres and in square metres; solve related word problems

Materials

centimetre ruler for each student, metre sticks

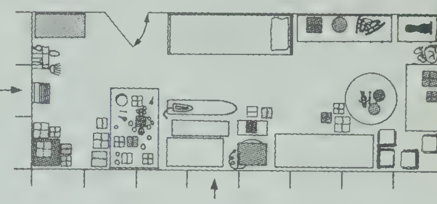
Vocabulary

length, width, square metre, m^2

Finding the Area of a Rectangle by Using Its Length and Width

What is the area of the storeroom floor?

The number of units along this side of the rectangle (its width) shows how many rows of square units there are in the rectangle.



The number of units along this side of the rectangle (its length) shows how many square units there are in each row.

There are 3 rows of square units with 8 in each row.

$3 \times 8 = 24$

For the storeroom, each square is a square metre.

The area of the storeroom floor is 24 m^2 (square metres).

Working Together

If you were to use the centimetre marks along the sides to draw squares on this picture,

1. how many rows of squares would be in your picture? 5
2. how many squares would be in each row? 9
3. how many square centimetres would be in your picture? 45
4. What is the area of the rectangle? 45 cm^2

What is the area of a rectangle

5. 12 cm long and 8 cm wide? 96 cm^2
6. 38 m long and 24 m wide? 912 m^2

140

LESSON ACTIVITY

Before Using the Pages

- Have the students recall that certain units of length are more appropriate than others for measuring long distances. For example, metres are more appropriate than centimetres or millimetres for measuring the width of the classroom. Then show that square centimetres are appropriate for expressing the area of a greeting card, for example, but not for the area of the floor or the chalkboard. Ask what size square would be an appropriate unit for the area of the floor. Lead the students to suggest a square having sides 1 m long. Introduce the term *square metre* and the symbol m^2. Arrange metre sticks on the floor in the shape of a square to enable the students to visualize 1 m^2. Use metre sticks to mark a rectangular area of, for example, 8 m^2.

Using the Pages

- Have students identify the diagram at the top of page 140 as the plan of a storeroom. Ask them to identify the length and the width of the room. Relate this diagram to the one below it showing the grid lines. Emphasize that the grid lines represent a metre grid. Ask whether the area of the storeroom floor, 24 m^2, can be found from the first diagram alone. Have students explain how it can be done. Draw attention to the symbol m^2 for square metres in 24 m^2. Point out that usually the word *width* refers to the shorter side of a rectangle and *length* refers to the longer side. Ask what kind of rectangle has the same length and width.

Working Together: Have students identify the marked side representing the length of the photograph and the marked side representing the width. Ex. 1-4 demonstrate that it is not necessary to show the centimetre squares in a rectangle in order to find the area. Ex. 5 and 6 show that the area can be found when the length and the width of a rectangle are known. Have the students draw diagrams to represent the rectangles in Ex. 5 and 6 and mark the length and width with the given dimensions. Emphasize that these diagrams

Exercises

Find the area of each rectangle.

1. 29 m, 29 m — 841 m²

2. 22 cm, 17 cm — 374 m²

3. 19 m, 34 m — 646 m²

4. 4 cm, 7 cm — 28 cm²

5. 6 cm, 5 cm — 30 cm²

Use a ruler to measure the sides of these shapes.

Complete.

	Length	Width	Area
6.	18 cm	9 cm	? 162 cm²
7.	26 m	5 m	? 130 m²
8.	14 m	14 m	? 196 m²
9.	32 cm	28 cm	? 896 cm²
10.	66 m	44 m	? 2904 m²
11.	50 cm	35 cm	? 1750 cm²
12.	37 cm	37 cm	? 1369 cm²
13.	125 m	125 m	? 15 625 m²

The four └'s that you find elsewhere on this page are the corners of a rectangle.

14. Use a ruler to help you find the area of this rectangle. 408 cm²

Solve.

15. Mr. Soo plans to paint a wall. The wall, in the shape of a rectangle, is 3 m high and 4 m wide. How many square metres are to be painted? 12

16. A square picture hanging on a wall is 25 cm on each side. How much of the wall is covered? 625 cm²

17. Charlene fenced in some land in the shape of a rectangle. It was 6 m long and 5 m wide. How much land did she fence? 30 m² How much fencing did she need? 22 m

*18. The piece of metal used for a pipe was in the shape of a rectangle 2 m long and 30 cm wide. What was its area? 6000 cm²

141

RELATED ACTIVITIES

- Have students measure and use multiplication to find the area of such objects as envelopes, greeting cards, sheets of paper, book covers, and snapshots. Ask them to measure the edges of rectangular regions of the school and the schoolyard in metres and use multiplication to find the area of each region.
- Have students investigate how many centimetre squares are needed to form a larger square with an area of 1 m².
- Ask students to draw "floor plans" for rooms by following the grid lines on centimetre graph paper. Have them give the area of each room if each square represents 1 m².

represent rectangles with the given dimensions and will help them to visualize the situation presented.

Exercises: Note that although both centimetres and metres are encountered in these exercises, there is no attempt to relate square metres and square centimetres. Also, with the exception of Ex. 18 (starred), the length and the width of each rectangle are given in the same unit of measurement. Most students will likely measure in centimetres rather than in millimetres for Ex. 4, 5, and 14. Either unit is acceptable, of course. This may lead some students to compare square millimetres and square centimetres. Observe the students as they work and question those who measure the four sides of a shape. Ask whether or not that is always necessary.

Assessment

Find the area of each rectangle.

1. 9 m, 10 m — 90 m²

2. 19 cm, 24 cm — 456 cm²

Complete.

	Length	Width	Area
3.	17 cm	11 cm	187 cm²
4.	43 m	21 m	903 m²
5.	65 m	65 m	4225 m²

Solve.

6. A picture is 50 cm wide and 75 cm long. What is the area of the picture? 3750 cm²

Page 142

LESSON OUTCOME

Show a rectangular shape having a given perimeter

Materials

geoboards and rubber bands, or copies of page T 396; copies of page T 397; three-dimensional shapes similar to those illustrated in the *Problem Solving* feature on pages 142 and 143 (optional)

Prerequisite Skills

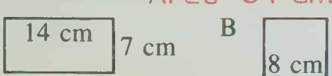
Find the perimeter and the area of a rectangular shape

Checking Prerequisite Skills

For each of the following shapes,

1. find the perimeter.
2. find the area.

Perimeter: 32 cm
Area: 64 cm2

A 

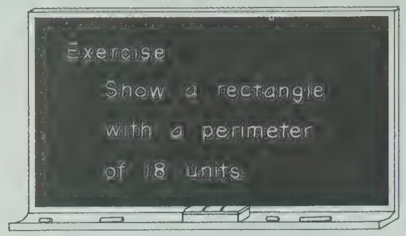
Perimeter: 42 cm
Area: 98 cm2

RELATED ACTIVITIES

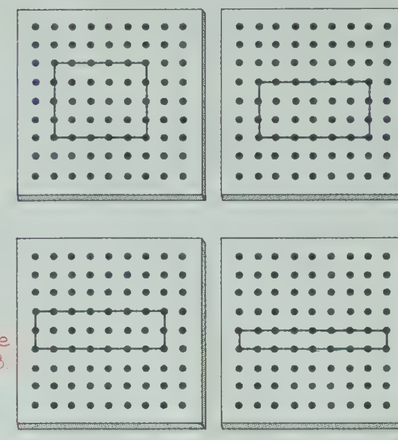
- Have students complete a table to show the whole number dimensions of all the possible rectangles having a perimeter of 24 cm. Have them write the results in a chart and find the area of each shape, to determine which shape has the greatest area.

  Ask them to repeat the activity using perimeters of 8 cm and 16 cm to discover whether a square will have the greatest area.

Rectangles Having a Given Perimeter



The students used geoboards.



Exercises

Use a geoboard, dot paper, or graph paper. Show two rectangles each having a perimeter of

The dimensions possible are given on page T368

1. 8 units.
2. 12 units.
3. 24 units.
4. 16 units.

What is the area of each rectangle you made with a perimeter of

See the chart given for Ex 1-4 on page T368

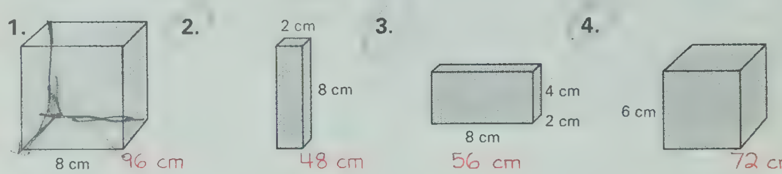
5. 8 units?
6. 12 units?
7. 24 units?
8. 16 units?

Solve.

9. What is the area of a square that has a perimeter of 20 cm? 25 cm2

*10. One side of a rectangle is twice the length of another side. The perimeter of the rectangle is 36 cm. What is the area of the rectangle? 72 cm2

Find the total length of all the edges for each of these eight shapes.



1. 96 cm — all squares
2. 48 cm — squares on top and bottom
3. 56 cm — all rectangles, no squares
4. 72 cm — four square sides

142

LESSON ACTIVITY

Before Using the Page

- Give each student a copy of page T 397. Instruct them to cut out a rectangular shape having a perimeter of 14 cm.

Using the Page

- Draw attention to the exercise on the chalkboard illustrated on page 142. Have students verify that each rectangle on the illustrated geoboards has a perimeter of 18 units. Show the dimensions of each in a chart. In summary, show that the four rectangles have the same perimeter. Ask whether they also have the same area.

Exercises: Provide the students with geoboards and rubber bands, or copies of page T 396 or T 397. If they are using geoboards, the shapes for Ex. 1-4 should be copied onto dot paper or graph paper. Have them mark the number of units in the length and the width of each rectangle and write the perimeter and the area beside each for Ex. 1-8. When the students have finished the exercises, summarize the results in a chart.

Perimeter	Width	Length	Area
8 units	1 unit	3 units	3 square units

Many students will likely use a "guess and test" approach for Ex. 10. Have students explain the method they used.

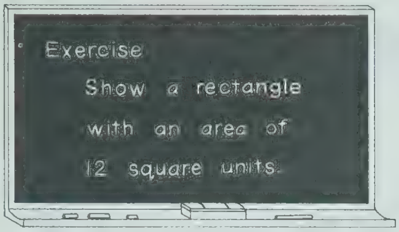
Problem Solving: The directive above Ex. 1 refers to the eight exercises that are shown for this feature on pages 142 and 143. If similar shapes are available, these may be of help to some students. Although there is another task to perform for each exercise (see the directive at the bottom of page 143), you may wish to have the students consider it in the following lesson.

Assessment

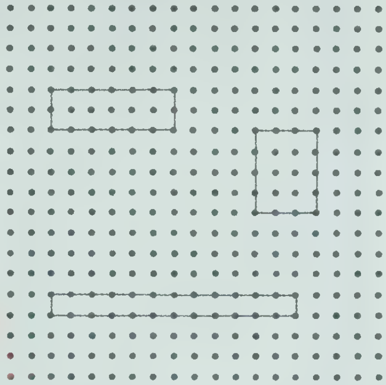
1. On centimetre graph paper, show two rectangles each having a perimeter of 16 cm. Answers will vary.
2. What is the area of each rectangle you made?

1 cm by 7 cm (7 cm2) 3 cm by 5 cm (15 cm2)
2 cm by 6 cm (12 cm2) 4 cm by 4 cm (16 cm2)

Rectangles Having a Given Area



The students used dot paper.



Exercises

Use centimetre dot paper or graph paper. Show two rectangles each having an area of

The dimensions possible are given on page T368.

1. 4 cm².
2. 16 cm².
3. 24 cm².
4. 30 cm².

What is the perimeter of each rectangle you made with an area of

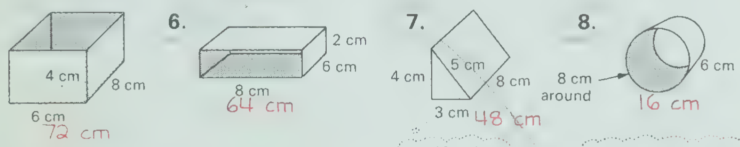
See the chart given for Ex 1-4 on page T368

5. 4 cm²?
6. 16 cm²?
7. 24 cm²?
8. 30 cm²?

Solve.

9. What is the perimeter of a square that has an area of 36 cm²? 24 cm

*10. One side of a rectangle is 8 cm longer than another side. Its area is 48 cm². What is the perimeter of the rectangle? 32 cm



open at the top

open at front and back

half of a rectangle on each end

open at front and back

PROBLEM SOLVING

Find the number of square centimetres on the outside of each of these eight shapes.

1 384 cm²   2 72 cm²   3 112 cm²   4 216 cm²
5 160 cm²   6 120 cm²   7 108 cm²   8. 48 cm²

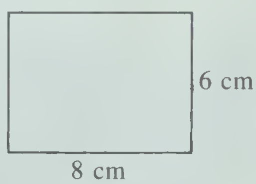
LESSON ACTIVITY

Using the Page

- Have the students contrast the exercise illustrated on the chalkboard on this page with the one on page 142. Emphasize that on this page the area of the rectangle, 12 square units, is given, whereas on the previous page, the perimeter is given. Have them verify that each of the three rectangles illustrated on the dot paper has an area of 12 square units. Ask whether the rectangles also have the same perimeter.

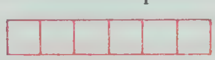
Exercises: Provide the students with copies of page T 396 or T 397. Use a procedure similar to the one for the previous lesson, summarizing the results in a chart. Have students explain their method of solution for Ex. 9 and 10.

Problem Solving: This is a continuation of the exercises begun on page 142. Their purpose is to reinforce the concepts of length, perimeter, and area. Ex. 8 is of particular interest. If the shape is considered a paper tube, students can imagine it cut once so that it can be placed on a flat surface as a rectangle measuring 6 cm by 8 cm.

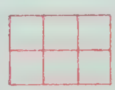


Assessment

1. On centimetre graph paper, show two rectangles each having an area of 6 cm².
2. What is the perimeter of each rectangle you made?



Perimeter : 14 cm



Perimeter : 10 cm

Page 143

LESSON OUTCOME

Show a rectangular shape having a given area

Materials

copies of page T 396 or T 397 for each student

Prerequisite Skills

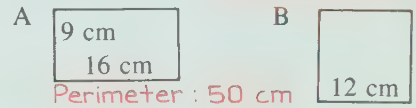
Find the perimeter and the area of a rectangular shape

Checking Prerequisite Skills

For each of the following shapes,

1. find the perimeter.
2. find the area.

Perimeter : 48 cm
Area : 144 cm²



Perimeter : 50 cm
Area : 144 cm²

RELATED ACTIVITIES

- Adapt the activity from the preceding page to determine which shape has the least perimeter for an area of 16 cm², for example. The activity may be repeated for an area of 36 cm².

Area	Length	Width	Perimeter
16 cm²	1 cm	16 cm	34 cm
16 cm²	2 cm	8 cm	

Page 144

OBJECTIVE

Demonstrate competence in finding the perimeter and the area of a rectangular shape and a square shape; solve related word problems

Materials

centimetre ruler for each student

RELATED ACTIVITIES

- Have the students complete a chart showing the whole number dimensions (in centimetres) of rectangles having perimeters of 24 cm. Have them plot points on a grid using the two dimensions in each case as an ordered pair and observe the pattern.

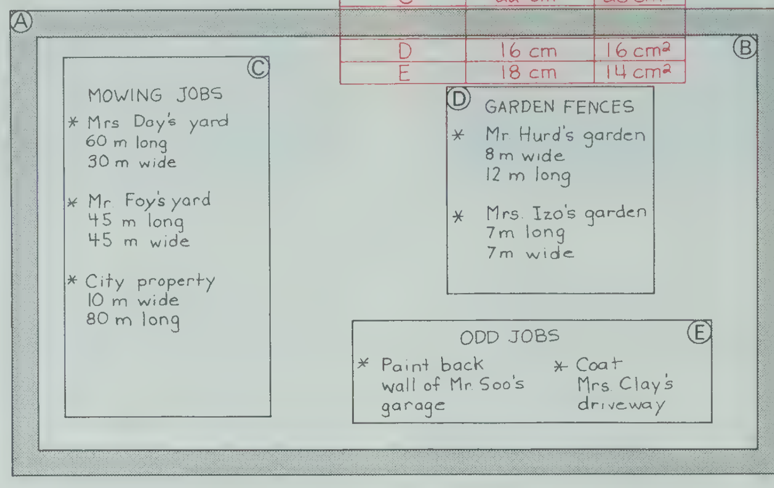
width	length
1	11
2	10
3	
11	

- Have the students complete a chart showing the whole number dimensions (in centimetres) of rectangles having areas of 36 cm². Have them plot points on a grid using the dimensions as ordered pairs and observe the pattern.

width	length
1	36
2	18
3	
36	

Practice

Rectangle	Perimeter	Area
A	48 cm	135 cm²
B	44 cm	112 cm²
C	22 cm	28 cm²
D	16 cm	16 cm²
E	18 cm	14 cm²



Use the job board above.

1. Mr. Hurd's garden has the shape of a rectangle. How much fencing is needed for the four sides? 40 m

2. Mrs. Izo's garden has the shape of a square. How much fencing is needed for the four sides? 28 m

3. Mrs. Clay's driveway has the shape of a rectangle 6 m wide and 18 m long. It costs $4.25 to coat each square metre. How much will the job cost? $459.00

4. Each lawn to be mowed has the shape of a rectangle. Which mowing job would you choose? Why? I would choose the city property because it has the least area.

5. There are five rectangles in the picture above. Measure with a centimetre ruler to find the perimeter and the area of each.

Complete.

	6.	7.	8.	9.	*10.	*11.	*12.
Length	16 m	35 cm	50 cm	25 m	9 cm	? 30 m	? 12 cm
Width	13 m	27 cm	48 cm	25 m	? 8 cm	2 m	? 5 cm
Perimeter	? 58 m	? 124 cm	? 196 cm	? 100 m	34 cm	? 64 m	34 cm
Area	? 208 m²	? 945 cm²	? 2400 cm²	? 625 m²	? 72 cm²	60 m²	60 cm²

144

LESSON ACTIVITY

Using the Page

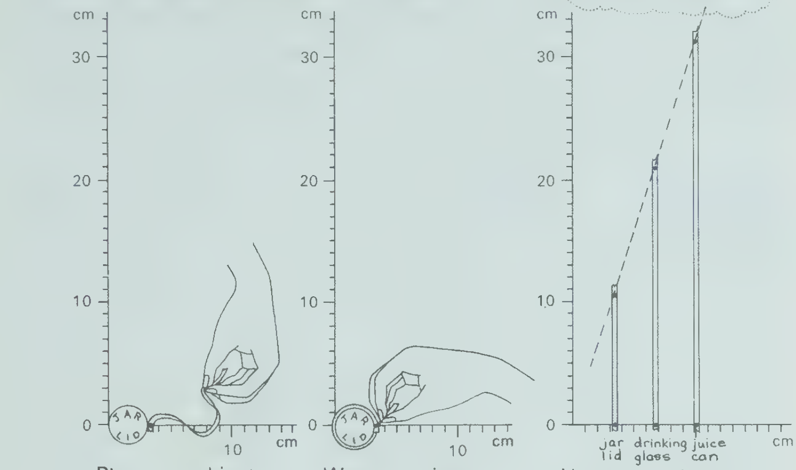
- Before the students begin, discuss the illustration. Have them read the items shown on the job board. Draw attention to the five capital letters that identify the five rectangles in the diagram. The students will need to measure the dimensions of the rectangles for Ex. 5. For Ex. 1-4, the students must decide whether the problem involves the concept of perimeter or area. Provide assistance as needed. Note that answers may differ for Ex. 4. Presumably, the lawn having the least area is the most desirable choice in terms of effort needed to perform the task. However, a student wishing to earn more money may choose a lawn with greater area.

  Ex. 10 and 11 are starred because their solutions involve more than two steps and the use of division. Ex. 12 is more challenging than the others. Students will likely use a "guess and test" procedure for it.

Working with a Model

Tacks and string can be used to make a chart that shows distances around round objects.

"Distance around" is also called **circumference**.



Place an object and mark its diameter.

Wrap a string around for the circumference.

Unwrap and tack up the string.

1. Make a chart like those above using objects that suggest large circles.

pots, pans, wastebasket, wheels, plates, clock, bottles

2. Make a chart like those above using objects that suggest small circles.

coins, thumbtacks, rings, cans, lids and caps, toy wheels, buttons

3. How could you use one of your charts to find the circumference of a soap bubble?

4. How could you use one of your charts to find the circumference of the hole in a doughnut?

5. How could you use one of your charts to find the diameter of the circle you could make with a string 50 cm long?

See the comments in Related Activities.

PROBLEM SOLVING

145

Page 145

OBJECTIVE

Use models to help solve problems

Materials

tacks, string, objects that suggest circles, large sheets of paper for preparing charts

Vocabulary

circumference

RELATED ACTIVITIES

- The completed charts can be used to find the circumference or the diameter of other circular objects. The line joining the tops of the strings can be extended as needed. Assign exercises similar to Ex. 3-5. For example, in Ex. 11 on page 133, the diameter of a telephone dial was measured. To find the circumference of the telephone dial using the chart, a vertical line segment is drawn from the horizontal scale at the point for the diameter of the dial. It ends at the line drawn to join the tops of the strings. The length of the vertical line represents the circumference of the dial and can be read from the vertical scale.

  Another observation that can be made from the charts is that the circumference of an object is about three times the measure of its diameter.

LESSON ACTIVITY

Using the Page

- The example illustrates how circular objects can be used to develop a graph in an investigation of the relationship between the circumference and the diameter of a circular object. Begin by displaying some of the objects named in the "thought clouds" below Ex. 1 and 2 and have students explain how they are alike. They may suggest that each object is round. Demonstrate that tracing around each object gives a circular shape, and introduce the word *circumference* for the distance around a circular shape. Ask how the circumference of a circular shape can be measured, and have students demonstrate this by using string or ribbon for one or two objects. Review the meaning of the term *diameter*.

  Draw attention to the illustration at the top of page 145 and discuss the procedure shown. An object is placed on the chart so that its diameter can be marked along the horizontal scale. Afterward, a string is wrapped around the object and then cut, giving the circumference of the object. The string is then tacked or taped to the chart, so that one end of the string meets the horizontal scale at the mark for the diameter. The procedure is repeated for several circular objects. The third diagram suggests that a straight line can be drawn to join the tops of the strings.

  Have the students work in small groups to prepare the centimetre scales on large sheets of paper and to carry out the procedure for two charts for Ex. 1 and 2. When the two charts are completed, have the students continue with Ex. 3-5. Display and discuss the charts (see *Related Activities*).

Page 146

OBJECTIVE

Demonstrate an understanding of the concepts and skills presented in this unit

Materials

ruler marked in centimetres and millimetres and a copy of page T 397 for each student

RELATED ACTIVITIES

- Students having difficulty relating units of length may benefit by completing patterns in tables similar to the following.

metres	1	1.5	9.5
centimetres	100	150	950
millimetres	1000	1500	9500

metres	0.05	0.10	0.45
centimetres	5	10	45
millimetres	50	100	450

metres	0.175	0.275	1.175
centimetres	17.5	27.5	117.5
millimetres	175	275	1175

Checking Up

Measure to the nearest centimetre.

1. ______________________________ 13 cm
2. ______________ 7 cm

Measure to the nearest millimetre.

3. __ 7 mm
4. ______________ 53 mm

Give each length in centimetres.

5. 2.35 m 235 cm
6. 15 mm 1.5 cm

Give each length in millimetres.

7. 5.7 cm 57 mm
8. 3 m 3000 mm

Give each length in metres.

9. 150 cm 1.50 m
10. 475 mm 0.475 m

Complete.

	mm	cm	m
11.	3000	300	3
12.	250	25	0.25
13.	1670	167	1.67

Find the perimeter.

14. 24 mm, 62 mm — 172 cm

15. 36 mm, 36 mm — 144 mm

Find the area.

16. 18 m, 136 m — 2448 m²

17. 52 cm, 78 cm — 4056 cm²

Measure to find the perimeter.

18. 3 cm, 6 cm — 18 cm

Measure to find the area.

19. 4 cm, 4 cm — 16 cm²

Draw a rectangle

20. with a perimeter of 28 cm.
21. with an area of 36 cm².

Answers will vary.
The dimensions possible are given on page T159.

146

Skills	Exercises	Related Pages
Measure in centimetres	1, 2	T 138-T 139
Measure in millimetres	3, 4	T 140-T 141
Express lengths in different units, for metres, centimetres, and millimetres	5-13	T 142-T 143
Find the perimeter of a square or a rectangle	14, 15, 18	T 144-T 147
Find the area of a square or a rectangle	16, 17, 19	T 148-T 153
Draw a rectangle for a given perimeter	20	T 154
Draw a rectangle for a given area	21	T 155

Comments

Provide students with copies of page T 397 on which to show their answers for Ex. 20 and 21.

Students having difficulty with Ex. 1-4 may not be aligning their rulers correctly for each line segment. They may also require extra practice in reading the scales on a ruler for centimetres and millimetres. Students having difficulty with Ex. 5-13 may benefit from locating points for various lengths on two or three metre sticks placed end to end. For example, a length of 150 cm (Ex. 9) can be seen as 1.5 m.

For Ex. 14-17, ensure that students are not confusing the concepts of perimeter and area. Provide experiences in covering a surface with unit squares to consolidate the concept of area. Experiences in walking around marked sections of the gymnasium or schoolyard will help to strengthen the concept of perimeter.

Checking Skills

Multiply.

1. 50 × 30 = 1500
2. 600 × 80 = 48 000
3. 30 × 20 = 600
4. 200 × 70 = 14 000
5. 700 × 10 = 7000
6. 400 × 90 = 36 000
7. 50 × 40 = 2000
8. 800 × 700 = 560 000
9. 300 × 300 = 90 000
10. 600 × 500 = 300 000

Tell how many digits there will be in each product. Then multiply.

11. 23 × 42 — 3 — 966
12. 506 × 73 — 5 — 36 938
13. 84 × 68 — 4 — 5712
14. 799 × 19 — 5 — 15 181
15. 265 × 56 — 5 — 14 840

Multiply.

16. 34 × 3 = 102
17. 685 × 5 = 3425
18. 1459 × 4 = 5836
19. 46 × 36 = 1656
20. 74 × 72 = 5328
21. 703 × 40 = 28 120
22. 355 × 21 = 7455
23. 2874 × 64 = 183 936
24. 2890 × 27 = 78 030
25. 523 × 956 = 499 988
26. \$3276 × 158 = \$517 608
27. \$4.09 × 9 = \$36.81
28. \$5.97 × 85 = \$507.45
29. \$37.46 × 7 = \$262.22
30. \$93.85 × 36 = \$3378.60
31. 5 × 1354 6770
32. 49 × 637 31 213
33. 516 × 419 216 204
34. 79 × \$67 \$5293
35. 3 × \$7.29 \$21.87
36. 827 × \$40.88 \$33 807.76

Solve.

37. Jenny bought 3 packages of stamps with 275 stamps in each package. How many stamps did she buy? 825
38. The airplane flies 1536 km each trip. How far does it fly in 9 trips? 13 824 km
39. 48 boxes of chalk were ordered for the school. Each box has 144 sticks. How many sticks of chalk would there be? 6912
40. 175 persons each brought 12 cookies to the Cookie Sale. How many cookies were there for the Cookie Sale? 2100
41. Each of 54 people paid \$375 for a one-week bus tour. How much did they pay in all? \$20 250
42. Each folding chair for the school cost \$11.45. How much did 325 folding chairs cost? \$3721.25

147

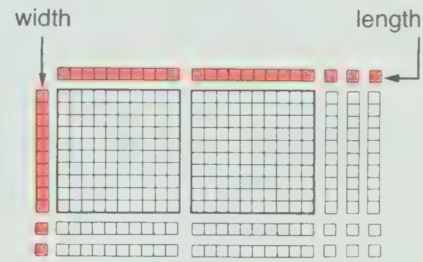
Page 147

OBJECTIVE

Demonstrate competence in multiplication skills; solve related word problems

RELATED ACTIVITIES

• This would be an appropriate time for students to relate the product of two two-digit factors to the area of a rectangle. Have them use base-ten Arithmetic Blocks (hundreds, tens, and ones), or centimetre graph paper. The following diagram illustrates that the product of 23 and 12 is 276.



Briefly, 2 tens and 3 ones are arranged to represent the length of a rectangle, and 1 ten and 2 ones are arranged to represent the width. The interior of the rectangle is then filled, using as many hundreds as possible, then tens, and finally ones. These are then counted to determine the product, in this case, 276, for 2 hundreds, 7 tens, and 6 ones.

LESSON ACTIVITY

Using the Page

• These exercises help to review and maintain skills in multiplying whole numbers and to prepare the students for applying these skills in multiplying decimals (Unit 8).

Ex. 1-15 are similar to those first presented in *Keeping Sharp* on page 131. Ex. 16-25 gradually increase in difficulty as the multipliers show from one to three digits. Ex. 26-30 and Ex. 34-36 involve amounts of money.

Remind the students to write concluding statements for Ex. 37-42.

Ex. 20
1 cm by 13 cm
2 cm by 12 cm
3 cm by 11 cm
4 cm by 10 cm
5 cm by 9 cm
6 cm by 8 cm
7 cm by 7 cm

Ex. 21
1 cm by 36 cm
2 cm by 18 cm
3 cm by 12 cm
4 cm by 9 cm
6 cm by 6 cm

Unit 8 Overview

Multiplying Decimals

This unit begins with a lesson to emphasize the similarities between the multiplication of whole numbers by whole numbers and the multiplication of decimals by whole numbers. Rounding decimal factors to whole numbers leads to estimating whole-number products and this is the basis for placing the decimal points in the products. Multiplication of a decimal by powers of ten (1000, 100, 10, 0.1, 0.01, and 0.001) is examined to discover how the digits move with respect to the position of the decimal point. This skill is then applied in converting from one metric unit of linear measurement to another. The generalization which was developed in Unit 7 — the product of tens and tens is hundreds — is now adapted to multiplication when the two factors both name tenths. Rounding the two decimal factors to whole numbers is again used to find the whole-number products and to place the decimal points. The lesson on the use of the calculator points out how the position of the decimal point can shift relative to the positions of the digits when one or more of the factors is a power of ten. The problem-solving skill presented in this unit is the ability to write an equation to represent the information given in a problem.

Prerequisite Skills

- complete the basic multiplication facts
- multiply two whole numbers
- interpret place value in numerals for whole numbers and decimals to thousandths
- express one metre as decimetres, centimetres, and millimetres, and vice versa
- use multiplication to find the area of a rectangular region in square metres
- round one-place decimals to the nearest whole number

Unit Outcomes

- multiply a one-place decimal to 0.9, a two-place decimal to 0.09, and a three-place decimal to 0.009 by a one-digit whole number
- multiply a decimal to thousandths by a one-digit whole number
- round decimal factors to the nearest whole number and multiply to estimate a product, then compare the estimate of the product with the exact product
- find the product of a decimal with up to three decimal places and a whole number
- round factors and multiply to estimate the product
- find the product of two factors when one factor is 1000, 100, 10, 0.1, 0.01, or 0.001
- multiply by 1000, 100, 10, 0.1, 0.01, or 0.001 to express a measurement given in one unit of length in terms of another unit of length
- find the product of two one-place decimals, factors to 0.9
- find the product of two one-place decimals, one factor to 0.9
- multiply two one-place decimals
- solve word problems involving multiplication with decimals
- find the product when one or more factors is a power of ten, using a calculator with a floating decimal point
- write an equation to represent the information in a word problem

Background

Since the structure of our numeration system is based on ten and powers of ten, operations with decimals are performed with the same basic facts and with the same underlying principles as with whole numbers. For example, the product of 7 and 9 is 63, and whether the 9 represents ones, tens, hundreds, thousands, tenths, hundredths, or thousandths, the 63 will always appear in the product. The place value of the digit 9 in each case is reflected in the placement of the product 63 to show the same value.

$$\begin{array}{r} 9 \\ \underline{7} \\ 63 \end{array} \quad \begin{array}{r} 90 \\ \underline{7} \\ 630 \end{array} \quad \begin{array}{r} 900 \\ \underline{7} \\ 6300 \end{array} \quad \begin{array}{r} 9\,000 \\ \underline{7} \\ 63\,000 \end{array} \quad \begin{array}{r} 0.9 \\ \underline{7} \\ 6.3 \end{array} \quad \begin{array}{r} 0.09 \\ \underline{7} \\ 0.63 \end{array} \quad \begin{array}{r} 0.009 \\ \underline{7} \\ 0.063 \end{array}$$

The distributive property of multiplication over addition is also evident when a decimal greater than one is multiplied, such as in 4 × 7.49. Each part of the factor 7.49 is multiplied by 4 and the three partial products are added, as shown in A and B. Regrouping (addition) may, of course, be performed at each step, as in C, but the distributive property also underlies this shortened procedure.

A
$$\begin{array}{r} 7 + 0.4 + 0.09 \\ \underline{\times \quad 4} \\ 28 + 1.6 + 0.36 = 29.96 \end{array}$$

B
$$\begin{array}{r} 7.49 \\ \underline{\times\ 4} \\ 0.36 \\ 1.60 \\ \underline{28.00} \\ 29.96 \end{array}$$

C
$$\begin{array}{r} {\scriptstyle 1\ 3} \\ 7.49 \\ \underline{\times\ 4} \\ 29.96 \end{array}$$

The commutative property of multiplication is also operative with decimals and the order of two factors may be changed, especially to make the multiplication easier. Either of two factors may be used as the multiplier, as shown in D and E.

D
$$\begin{array}{r} 0.7 \\ \underline{\times\ 243} \\ 21 \\ 280 \\ \underline{1400} \\ 170.1 \end{array}$$

E
$$\begin{array}{r} {\scriptstyle 3\ 2} \\ 243 \\ \underline{\times\ 0.7} \\ 170.1 \end{array}$$

The basic work in this unit is limited to two factors, but the lesson on page 167 involves more than two factors. Although the keycharts given in the lesson suggest a left-to-right use of the factors, any order of the same factors would yield the same product. This is because the associative property of multiplication applies to decimals as well as to whole numbers.

In Unit 7 the generalization was developed that the product of tens and tens is hundreds, and this is now extended to show that the product of tenths and tenths is hundredths. Models such as the one on page 161 are used to illustrate this. A place-value chart may also be used because of the symmetrical way in which ones are balanced on both sides by tens and tenths and by hundreds and hundredths. The place-value chart may be used to show the placement of products for whole-number factors and decimal factors. In the case of 3 tens × 2 tens, the product 6 is in the hundreds' place, one place to the left of the tens; in the case of 3 tenths × 2 tenths, the product 6 is in the hundredths' place, one place to the right of the tenths.

hundreds	tens	ones		tenths	hundredths
6	0	0	.		
		0	.	0	6

The movement to the left and to the right is even more evident when one factor is a power of ten. If these factors are greater than 1, the digits move to the left; if they are decimals, they move to the right. In both cases, the decimal point remains

fixed. Note how the digits move when 76 is multiplied by the factors at the left.

	10 000	1000	100	10	1		0.1	0.01	0.001
1				7	6				
10			7	6	0				
100		7	6	0	0				
1000	7	6	0	0	0				
1				7	6				
0.1					7	.	6		
0.01					0	.	7	6	
0.001					0	.	0	7	6

The number of places through which the digits move can be related to the number of zeros shown in the whole-number factor (10, 100, 1000), or to the number of decimal places shown in the decimal factor (0.1, 0.01, 0.001).

The lesson on the use of the calculator refers to a feature known as a "floating decimal point". Whereas in the previous method the decimal point remained in a fixed position and the digits moved, on a calculator display the digits remain in a fixed position and the decimal point moves. In either method, the relative positions of the digits and the decimal point are the same.

Students who have acquired skills in performing operations frequently experience difficulty in solving word problems. One approach to developing problem-solving ability involves the use of equations. To solve a word problem it is necessary to translate the structure of the situation into an equation by using the proper mathematical symbols. The equation can be structured to show the events either in the order in which they happened, or in the way in which the relationship is perceived between the known and the unknown quantities. Numerals are used to indicate what is known, and frames (□ , △) or letters (n, x, y) are used to represent what is unknown in equations, such as in 317 + □ = 405, n × 16 = 192, and 750 ÷ 15 = y. Situations in word problems are essentially instances of putting together, taking apart or comparing, or combinations of these, and each has its appropriate operation. The four basic number operations are related in several ways. Multiplication can be related to repeated addition, and division to repeated subtraction; the inverse relationships between addition and subtraction, and between multiplication and division are used frequently in solving equations.

addend + addend = sum　　　factor × factor = product
sum − addend = addend　　　product ÷ factor = factor

Solving for an unknown in an equation may be achieved directly or indirectly. In the latter case, either the same operation is used with a rearrangement of the known quantities, or an inverse operation is used.

Directly	Indirectly	
24 + 27 = n	78 − n = 33	6 × n = 252
51 = n	n = 78 − 33, or 45	n = 252 ÷ 6, or 42

Although there may be several equations for the same relationship, only one can actually be used directly to find the solution. For example, if the sum of two numbers is 33.6 and one addend is 14, there are four equations that can represent the situation, but only one that can be used.

14 + n = 33.6　　　33.6 − n = 14
n + 14 = 33.6　　　$\boxed{33.6 - 14 = n}$

Teaching Strategies

Competence in multiplying whole numbers is a prerequisite for multiplying with decimals, and any students who had difficulty with the work in Unit 3 may need review and practice in multiplying numbers with up to three digits by numbers with one, two, or three digits. If there is a group of such students, it may even be advisable to teach them as a separate group throughout this unit.

With regard to placing the decimal point in a product, emphasis should be placed on a meaningful approach as outlined in the suggestions for teaching the lessons. Rounding one or both factors to the nearest whole number and then multiplying them provides a reasonable estimate of the whole-number part of the product and helps in the placement of the decimal point. Another meaningful approach stresses the place values of the decimal parts of the factor or factors. For example, if a decimal naming tenths (hundredths) is multiplied by a whole number, the product must also show tenths (hundredths). Place values are also stressed in the generalization "the product of tenths and tenths is hundredths", and to show hundredths in a product there must be two decimal places. In contrast to these meaningful methods, which are both used in this unit, is the mechanical technique of adding the number of decimal places in the factors and using the sum as the number of decimal places in the product. The rule does work, but few who use it can tell why. Therefore, meaningful methods are to be preferred.

The suggestions for teaching the lessons recommend various materials that may be used to represent the multiplication of decimals on a concrete level, among which number lines and models showing ones, tenths, and hundredths are probably the most effective. Although the presentations in the book include illustrations and diagrams of these, it is much better if the students themselves have opportunities to use the materials.

Sometimes there is more than one way to think of the mathematical relationship represented in a problem and thus more than one equation may be written. For this reason, it is important that students be able to identify the nature of the given numbers, as well as of the unknown quantities, that is, whether they represent an addend or a sum, a factor or a product. For example, in 4 × n = 52, the factor 4 and the product 52 are known, and the unknown factor n may be found by division, as expressed by the equation 52 ÷ 4 = n.

If calculators are not available for the lesson on page 167, the students may enjoy completing the exercises mentally. In some cases, rearranging the factors makes the work easier, while in others merely grouping the factors is helpful.

Materials

five models for 7 tenths, three whole models, one model for 5 tenths (see pages T 108 and T 109)
a copy of page T 397 and a copy of page T 399 for each student, copies of page T 389 (optional)
flannel board and felt numerals (several for each of 0 to 9), or a magnetic board and magnetic numerals
metre sticks marked in tenths of a metre, models for hundredths or an overhead projector (optional)
models for tenths and hundredths
calculators (optional)

Vocabulary

floating decimal point　　　equation

LESSON OUTCOME

Multiply a one-place decimal to 0.9, a two-place decimal to 0.09, and a three-place decimal to 0.009 by a one-digit whole number

Materials

five models for 7 tenths, three whole models, one model for 5 tenths (see the comments on pages T 108 and T 109 for making decimal models)

Prerequisite Skills

Complete the basic multiplication facts

Checking Prerequisite Skills

Multiply.

1.	2.	3.	4.
4	5	8	7
7	6	3	7
28	30	24	49

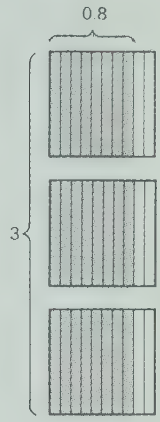
5. 9 × 6 54 6. 7 × 8 56 7. 8 × 9 72

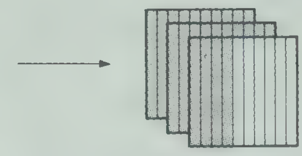
8 MULTIPLYING DECIMALS

Multiplying Decimals, to 0.9, 0.09, or 0.009, by One-Digit Whole Numbers

For 3 × 0.8, think of 3 groups of 8 tenths.

Multiply 3 and 0.8.

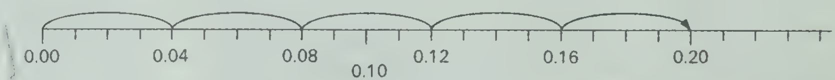




$3 \times 8 = 24$
3×8 tenths $= 24$ tenths
$3 \times 0.8 = 2.4$

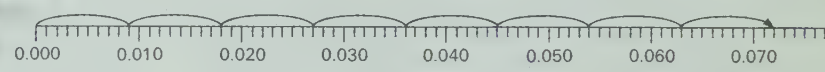
8 tenths	0.8
3	3
24 tenths	2.4

Multiply 5 and 0.04.



$5 \times 4 = 20$
5×4 hundredths $= 20$ hundredths
$5 \times 0.04 = 0.20$

4 hundredths	0.04
5	5
20 hundredths	0.20



Multiply 8 and 0.009.

$8 \times 9 = 72$
8×9 thousandths $= 72$ thousandths
$8 \times 0.009 = 0.072$

9 thousandths	0.009
8	8
72 thousandths	0.072

148

LESSON ACTIVITY

Before Using the Pages

- Name decimals and have students write the numerals on the board.

3 tenths	4 hundredths	9 thousandths
8 and 1 tenth	12 hundredths	45 thousandths

- Write decimals on the board and have students read the numerals.

  0.7 0.36 0.06 0.72 0.003 0.040

- Display five models of 7 tenths, one at a time. Ask what number is represented by each of the models so that the students understand they represent the same number. Group the models together and ask what number is represented (35 tenths). Ask for a simpler way to show 35 tenths and have a student regroup the models as 3 wholes and 5 tenths.
- Write the addition 0.7 + 0.7 + 0.7 + 0.7 + 0.7 = 3.5 on the board and ask for a shorter way to express the same idea. Lead the students to suggest the use of multiplication and the sentence 5 × 0.7 = 3.5. Use other similar examples as required.

Using the Pages

- The worked examples relate multiplication of a decimal by a whole number to equal groups and to jumps on the number line. The procedure is presented as an extension of basic multiplication facts.

  Discuss the procedure shown for the multiplication 3 × 0.8 = 2.4. Emphasize that the product, 24 tenths, is regrouped as 2 and 4 tenths, or 2.4.

  The number line is a suitable model for illustrating multiplication of hundredths and of thousandths. For each number line, have students read the numbers marked on the scale and state the number of jumps and the length of each jump. Ask students to explain in their own words how multiplying a decimal by a whole number is similar to multiplying two whole numbers.

Working Together

Complete each multiplication.

1. $4 \times 7 =$ 28
   4×7 tenths = 28 tenths
   $4 \times 0.7 =$ 2.8

2. $5 \times 3 =$ 15
   5×3 hundredths = 15 hundredths
   $5 \times 0.03 =$ 0.15

3. $7 \times 9 =$ 63
   7×9 thousandths = 63 thousandths
   $7 \times 0.009 =$ 0.063

4. $2 \times 0.3 =$ 0.6
5. $9 \times 0.06 =$ 0.54
6. 0.5×6 = 3.0
7. 0.09×8 = 0.72
8. 0.002×7 = 0.014

Exercises

Multiply.

1. 4×0.2 0.8
2. 7×0.8 5.6
3. 4×0.09 0.36
4. 8×0.004 0.032
5. 7×0.05 0.35
6. 3×0.04 0.12
7. 6×0.1 0.6
8. 8×0.007 0.056
9. 9×0.3 2.7
10. 5×0.006 0.030

11. 0.5×2 = 1.0
12. 0.07×5 = 0.35
13. 0.003×6 = 0.018
14. 0.8×9 = 7.2
15. 0.01×3 = 0.03
16. 0.9×9 = 8.1

Study the chart.

kilometre	metre	millimetre
1000 m	1 m	0.001 m
? kilolitre	litre	millilitre
? 1000 L	1 L	0.001 L
kilogram	gram	? milligram
1000 g	1 g	? 0.001 g

1. What do you think should go here?
2. What do you think should go here?
3. What do you think will complete the pattern in the top row of this chart?

1000 m	100 m	? 10 m	1 m	0.1 m	? 0.01 m	0.001 m
kilometre	? hectometre	? decametre	metre	decimetre	centimetre	millimetre

4. Find out the names that belong in the bottom row of the chart.

try this

Make a chart like the one above
5. for the litre.
6. for the gram.

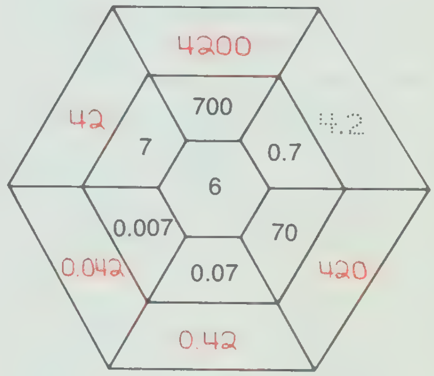
Answers are given below.

149

RELATED ACTIVITIES

- Have students complete multiplication exercises, similar to the following, in tables and diagrams on copies of page T 390.

×	0.08
4	0.32
7	0.56
6	0.48
3	0.24
9	0.72



Working Together: Ex. 1-3 help students relate the product of a whole number and a decimal to basic multiplication facts. Students who can interpret the decimal factor correctly will likely have little difficulty thinking of the product as a decimal with the same number of decimal places as the decimal factor. Ask students to give an oral explanation of Ex. 4-8, using the steps outlined in Ex. 1-3.

Exercises: Remind the students to note carefully the decimal factor in each exercise to determine the number of decimal places in the product.

Try This: In these exercises, students have an opportunity to summarize units of length and to understand the significance of a prefix used with the word *metre*. For example, the prefix "centi" implies one-hundredth and thus a centimetre is one-hundredth of a metre. The same prefixes are also applied to units of mass and capacity. For example, a centigram is a unit which is one-hundredth of a gram, and a centilitre is one-hundredth of a litre. Although certain units are used less frequently than others, students can appreciate the continuity of the metric system through exercises such as these. The students may refer to the table on page 342 for assistance.

Assessment

Multiply.

1. 3×0.2 0.6
2. 5×0.7 3.5
3. 8×0.09 0.72
4. 0.01×6 = 0.06
5. 0.003×5 = 0.015
6. 0.007×7 = 0.049

5.

1000 L	100 L	10 L	1 L	0.1 L	0.01 L	0.001 L
kilolitre	hectolitre	decalitre	litre	decilitre	centilitre	millilitre

6.

1000 g	100 g	10 g	1 g	0.1 g	0.01 g	0.001 g
kilogram	hectogram	decagram	gram	decigram	centigram	milligram

LESSON OUTCOME

Multiply a decimal to thousandths by a one-digit whole number

Prerequisite Skills

Multiply a whole number by a whole number less than 10; multiply a one-place decimal to 0.9, a two-place decimal to 0.09, and a three-place decimal to 0.009 by a whole number less than 10

Checking Prerequisite Skills

Multiply.

1. 24 × 3 = 72 **2.** 143 × 6 = 858 **3.** 1079 × 8 = 8632

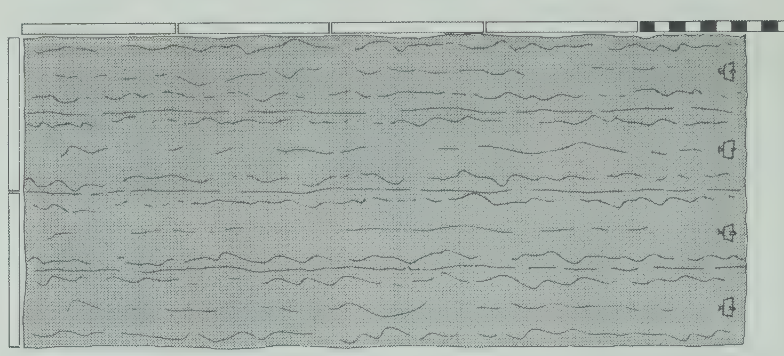
4. 5 × 320 1600 **5.** 7 × 2317 16 219

6. 2 × 0.06 0.12 **7.** 8 × 0.007 0.056

8. 0.4 × 4 = 1.6 **9.** 0.008 × 6 = 0.048

Multiplying Decimals by One-Digit Whole Numbers

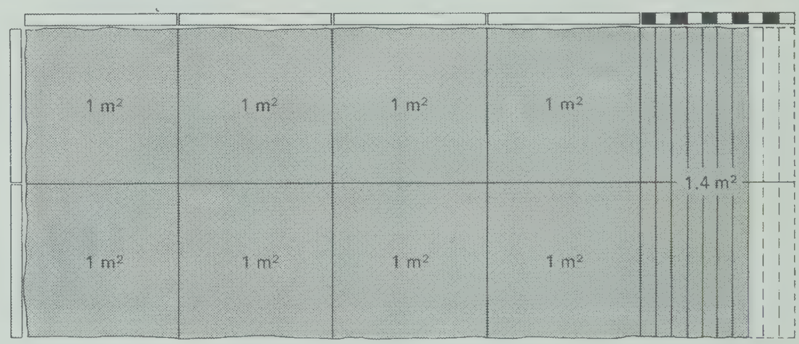
Wendy's flower garden is 4.7 m long and 2 m wide.
How many square metres are there in Wendy's garden?



To find the area of Wendy's garden in square metres, multiply 2 and 4.7.

$\begin{array}{r} {}^{1}\\ 4.7 \\ \underline{\times\ 2} \\ 4 \end{array}$	$\begin{array}{r} {}^{1}\\ 4.7 \\ \underline{\times\ 2} \\ 9\ 4 \end{array}$	$\begin{array}{r} 4.7 \\ \underline{\times\ 2} \\ 9.4 \end{array}$
2 × 7 tenths = 14 tenths or 1 one and 4 tenths.	2 × 4 ones = 8 ones. Another one makes 9 ones.	Place the decimal point in the product.

There are 9.4 m² in Wendy's garden.



150

LESSON ACTIVITY

Before Using the Pages

- Ask students to read and write decimals to thousandths, using a procedure similar to that described on page T 162 in *Before Using the Pages*.

  1 and 112 thousandths   2 and 15 thousandths
  10 and 7 hundredths   24 and 6 tenths

  6.439   243.6   35.53   8.009   14.05

- Ask students to write the following as decimals. Provide other similar examples as required.

  132 tenths   417 hundredths
  1415 thousandths   6045 thousandths

Using the Pages

- The worked example presents a situation in which it is necessary to multiply a decimal by a whole number. Point out the metre sticks that indicate the length and the width of Wendy's garden in the illustration. Note particularly the metre stick marked to show tenths of a metre. Have students name the length and the width in metres.

  Since it is necessary to find the area of a rectangular shape, multiplication will be used to solve the problem. Have the students observe that multiplying 2 and 4.7 is similar to multiplying 2 and 47, except that the product names a number of tenths. The statements shown in the "thought clouds" explain the steps of the procedure. The diagram at the bottom of the page helps to relate the multiplication to equal groups, namely, 2 groups of 4.7. There are 4 m² and 0.7 m² in each group. Discuss that 2 groups of 7 tenths is 14 tenths, or 1.4.

Working Together: Ex. 1-3 emphasize that multiplying a decimal by a whole number is similar to multiplying two whole numbers. The product, however, names a number of tenths (hundredths, thousandths) and thus requires a decimal point in the appropriate place. Have students describe similar steps in explaining their answers for Ex. 4-6. Use other similar exercises as required.

Working Together

Complete each multiplication.

1. 25 × 3 = 75 | 25 tenths × 3 = 75 tenths | 2.5 × 3 = 7.5

2. 53 × 4 = 212 | 53 hundredths × 4 = 212 hundredths | 0.53 × 4 = 2.12

3. 2375 × 7 = 16 625 | 2375 thousandths × 7 = 16 625 thousandths | 2.375 × 7 = 16.625

4. 6.9 × 8 = 55.2
5. 0.28 × 3 = 0.84
6. 1.035 × 6 = 6.210

Exercises

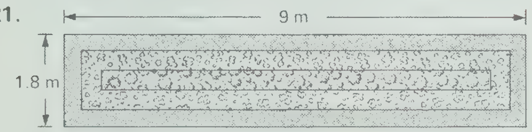
Multiply.

1. 2.5 × 9 = 22.5
2. 0.19 × 4 = 0.76
3. 0.006 × 3 = 0.018
4. 72.8 × 8 = 582.4
5. 0.708 × 6 = 4.248
6. 4.63 × 9 = 41.67
7. 0.4 × 8 = 3.2
8. 9.42 × 7 = 65.94
9. 0.07 × 9 = 0.63
10. 8.4 × 5 = 42.0
11. 63.95 × 6 = 383.70
12. 7.04 × 7 = 49.28

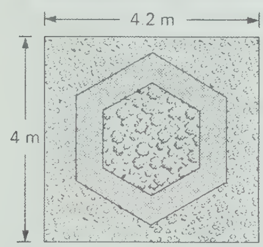
13. 5 × 0.179 0.895
14. 4 × 7.628 30.512
15. 2 × 0.096 0.192
16. 3 × 3.79 11.37
17. 2 × 5.8 11.6
18. 7 × 0.68 4.76
19. 5 × 2.065 10.325
20. 6 × 1.824 10.944

How many square metres are in each of these flower gardens?

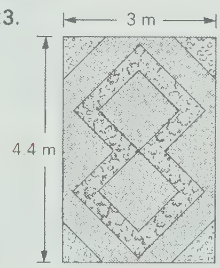
21. 16.2 m²



22. 16.8 m²



23. 13.2 m²



151

Exercises: Remind the students to show Ex. 13-20 in vertical form.

Assessment

Multiply.

1. 3.9 × 4 = 15.6
2. 6.25 × 6 = 37.50
3. 2.014 × 9 = 18.126

4. 5 × 3.127 15.635
5. 7 × 0.632 4.424
6. 8 × 0.049 0.392

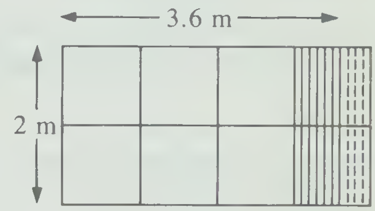
RELATED ACTIVITIES

- For further practice, you may wish to have the students complete some of Ex. 1-22 on page 335.
- Have students complete a multiplication exercise involving whole numbers. Then ask them to use their answer to help find products involving one decimal factor. Examples are given below.

$$\begin{array}{r}3062\\ \times\ 4\\ \hline\end{array}\qquad\begin{array}{r}30.62\\ \times\ 4\\ \hline\end{array}\qquad\begin{array}{r}3.062\\ \times\ 4\\ \hline\end{array}$$

$$\begin{array}{r}163\\ \times\ 5\\ \hline\end{array}\qquad\begin{array}{r}0.163\\ \times\ 5\\ \hline\end{array}\qquad\begin{array}{r}16.3\\ \times\ 5\\ \hline\end{array}$$

- Have students use decimetre squares (copies of page T 393) to represent square metres and decimetre strips (copies of page T 394) to represent tenths of a square metre. Have them paste the appropriate squares and strips on a sheet of paper to illustrate a garden having a length of 3.6 m and a width of 2 m, for example. Have them use multiplication to find the area of the garden.



$$\begin{array}{r}3.6\\ \times\ 2\\ \hline 7.2\end{array}$$

Area: 7.2 m²

LESSON OUTCOME

Round decimal factors to the nearest whole number and multiply to estimate a product, then compare the estimate of the product with the exact product

Prerequisite Skills

Multiply a whole number by a whole number less than 10; multiply a decimal to thousandths by a whole number less than 10

Checking Prerequisite Skills

Multiply.

1. 37 × 6 = 222
2. 814 × 9 = 7326
3. 4508 × 7 = 31 556
4. 4 × 392 1568
5. 2 × 9876 19 752
6. 4 × 7.43 29.72
7. 9 × 0.085 0.765
8. 9.6 × 7 = 67.2
9. 8.02 × 6 = 48.12
10. 4.805 × 8 = 38.440

RELATED ACTIVITIES

- Discuss with the students the meaning of such common advertising statements as
  "Buy one, get one free!"
  "Buy one at the regular price and get another for just 1¢!"
  "Two for the price of one!"

Have students write some examples for these and calculate the cost of buying certain quantities.

Estimating the Product

An estimate can help you remember where to place the decimal point in a product.

To estimate the product of 7 and 8.9, round 8.9 ⟶ 9 then multiply.

$$\begin{array}{r} 9 \\ \underline{7} \\ 63 \end{array}$$

For the exact product, multiply in the usual way.

$$\begin{array}{r} 8.9 \\ \underline{7} \\ 62.3 \end{array}$$

The estimate tells you that the product is about 63. A decimal point is needed here.

Working Together

Round to the nearest whole number.

1. 5.8 6
2. 7.49 7
3. 0.64 1
4. 3.057 3

Round to the nearest whole number and multiply to estimate the product. Then find the exact product.

5. 3.9 × 7 = 27.3 (28)
6. 6.094 × 4 = 24.376 (24)
7. 12.74 × 6 = 76.44 (78)

Exercises

Estimates may vary

Round to the nearest whole number and multiply to estimate the product. Then find the exact product.

1. 5.8 × 2 = 11.6 (12)
2. 6.93 × 5 = 34.65 (35)
3. 8.485 × 7 = 59.395 (56)
4. 6.8 × 6 = 40.8 (42)
5. 5.07 × 4 = 20.28 (20)
6. 4.644 × 8 = 37.152 (40)
7. 3.4 × 9 = 30.6 (27)
8. 6.37 × 5 = 31.85 (30)
9. 38.75 × 3 = 116.25 (117)
10. 4 × 7.19 28.76
11. 2 × 3.5 7.0
12. 8 × 9.2 73.6
13. 5 × 9.708 48.540
14. 3 × 5.098 15.294
15. 6 × 31.5 189.0
16. 9 × 9.91 89.19
17. 7 × 8.62 60.34

Estimates
10. 28 11. 8 12. 72 13. 50
14. 15 15. 192 16. 90 17. 63

Use the information shown in the picture on the next page to help you complete this chart.

		Estimated cost	Exact cost
18.	5 bags of grass seed	$25 ?	? $24.45
19.	8 bags of plant food	$16 ?	? $13.52
20.	6 cans of plant spray	$18 ?	? $17.34
21.	7 juniper bushes	$56 ?	? $55.65
22.	4 pear trees	$28 ?	? $29.00

LESSON ACTIVITY

Before Using the Page

- Write the following exercises on the board.

$$\begin{array}{r} 4.896 \\ \underline{\times \quad 3} \end{array} \qquad \begin{array}{r} 7.131 \\ \underline{\times \quad 6} \end{array} \qquad \begin{array}{r} 3.569 \\ \underline{\times \quad 8} \end{array} \qquad \begin{array}{r} 5.416 \\ \underline{\times \quad 2} \end{array}$$

Ask the students to estimate which product is closest to 30 without writing anything. Ask them to explain the procedure they used to find which product is closest to 30.

Using the Page

- The worked example demonstrates that rounding the decimal factor to the nearest whole number can help to estimate a product. The estimate of the product is an important aid in checking the location of the decimal point in the exact product. Discuss the reason for rounding 8.9 up to 9 rather than down to 8. (Note that only the decimal factor is rounded.) Write the multiplication example 7 × 8.9 on the board three times — one with the correct product and 6.23 and 0.623 for the other two products. Then have students relate the estimate of the product, 63, to the three products. Discuss from the standpoint of place value why 62.3 is correct and why the others are incorrect.

Working Together: Ex. 1-4 review the skill of rounding a decimal to the nearest whole number. The skill is applied in Ex. 5-7.

Exercises: Draw the students' attention to the photograph on page 153. Have them read the signs displayed to learn the prices of the various items. Point out that Ex. 18-22 on page 152 refer to some of these items.

Assessment

Round to the nearest whole number and multiply to estimate the product. Then find the exact product.

1. 6.7 × 3 = 20.1 (21)
2. 4.19 × 7 = 29.33 (28)
3. 6 × 2.335 14.010 (12)
4. 8 × 4.903 39.224 (40)

Practice

Round to the nearest whole dollar and multiply to estimate the cost. Then find the exact cost.

1. How much will 6 boxes of marigolds cost? $27 00 ($30)
2. How much will 4 Northern Spy apple trees cost? $27 80 ($28)
3. How much will 9 potted begonias cost? $11.61 ($9)
4. How much will 7 pots of mums cost? $17.43 ($14)
5. Garden gloves sell for $2.75 a pair. How much will 3 pairs cost? $8.25 ($9)
6. ★ How much is saved when 3 pairs of garden gloves are bought at a sale price of $1.98? $2.31 ($3)

7. $4.95 × 2 ($10) $9.90
8. $3.64 × 8 ($32) $29.12
9. $3.12 × 5 ($15) $15.60
10. $8.72 × 6 ($54) $52.32
11. $12.09 × 4 ($48) $48.36
12. $11.89 × 3 ($36) $35.67

13. 8 × $3.95 $31.60
14. 9 × $2.48 $22.32
15. 8 × $9.89 $79.12
16. 7 × $7.50 $52.50
17. 3 × 35¢ $1.05
18. 8 × 75¢ $6.00
19. 7 × 79¢ $5.53
20. 5 × 98¢ $4.90

Estimates
13. $32 14. $18 15. $80 16. $56
17. $0 18. $8 19. $7 20. $5

153

Page 153

OBJECTIVE

Demonstrate competence in estimating a product and multiplying decimals; solve related word problems

RELATED ACTIVITIES

- Have students search newspapers and catalogs for items advertised at sale prices. Ask them to estimate the savings on purchasing two to nine of any item and then find the exact amount saved on such a purchase. Discuss the implications of buying items advertised as sale items when such items are not really needed.
- Have students compare prices advertised as sale prices in one store with the regular prices for the same items in other stores.

LESSON ACTIVITY

Using the Page

- For each exercise, have the students estimate the cost before finding the exact cost. For Ex. 17-20, first have the students change the prices expressed in cents to dollars. Then the estimates for Ex. 18-20 can be found by the multiplications 8 × \$1, 7 × \$1, and 5 × \$1, respectively. However, in Ex. 17, in rounding to the nearest dollar, 35¢, or \$0.35, is rounded to \$0 and the estimate of the cost is 3 × \$0. Ask students to suggest an alternative procedure for estimating the cost. For example, they may think in terms of tenths of a dollar (dimes) and use 3 × 4 (tenths) = 12 (tenths) to arrive at \$1.20 as an estimate.

  Ex. 6 is starred because its solution involves more than one step and information must be obtained from Ex. 5 to solve the problem. Encourage students to solve this problem in more than one way and show different solutions on the board. For example, an estimate of the savings for each pair of gloves can be found from \$3 − \$2 = \$1 and thus about \$3 is saved on the purchase of 3 pairs of gloves. Some students may estimate the cost of 3 pairs on sale by multiplying (3 × \$2 = \$6) and then use this fact with the estimate of the cost (3 × \$3 = \$9) found in Ex. 5.

Pages 154-155

LESSON OUTCOME

Find the product of a decimal with up to three decimal places and a whole number; round factors and multiply to estimate the product

Prerequisite Skills

Find the product of two whole numbers; multiply a decimal to thousandths by a whole number less than 10

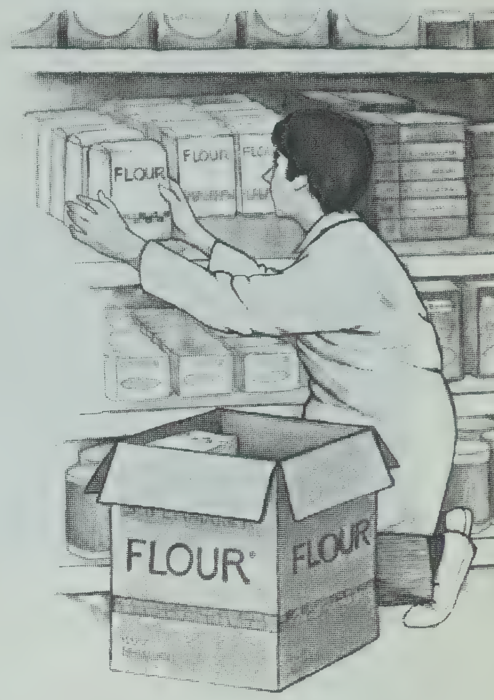
Checking Prerequisite Skills

Multiply.

1. 114 × 35 = 3990	**2.** 69 × 87 = 6003	**3.** 403 × 26 = 10 478
4. 216 × 139 = 30 024	**5.** 43 × 3184 = 136 912	
6. 25.4 × 8 = 203.2	**7.** 0.637 × 4 = 2.548	

Multiplying Decimals and Whole Numbers

The shelf can hold 200 kg safely. The clerk has 75 bags of flour. Each bag holds 2.5 kg. Can the shelf hold all the bags safely?



Multiply 75 and 2.5.

For the product

$$\begin{array}{r} 2.5 \\ \underline{75} \end{array}$$

you need to know how to multiply 5 and 25,

$$\begin{array}{r} {}^{2} \\ 2\ 5 \\ \underline{7\ 5} \\ 12\ 5 \end{array}$$

The decimal point would be here, but there is no need to show it yet.

and how to multiply 7 and 25.

$$\begin{array}{r} {}^{3} \\ 2\ 5 \\ \underline{7\ 5} \\ 12\ 5 \\ 175\ 0 \end{array}$$

The decimal point would be here, but there is no need to show it yet.

Then add and place the decimal point.

$$\begin{array}{r} 2.5 \\ \underline{75} \\ 12\ 5 \\ \underline{175\ 0} \\ 187.5 \end{array}$$

You can change the order of the factors to check your work.

$$\begin{array}{r} 75 \\ \underline{2.5} \\ 37\ 5 \\ \underline{150\ 0} \\ 187.5 \end{array}$$

If this result does not match the first result, there is a mistake in your work.

There are 187.5 kg of flour in all. The shelf can hold all the bags safely.

154

LESSON ACTIVITY

Before Using the Pages

- Write the following exercises on the board. Ask students to explain how to estimate each product. Have them write the estimates on the board as shown and then find the exact products.

	9.7 × 3	9.7 × 5	9.7 × 8	9.7 × 9
Exact product	29.1	48.5		
Estimate	30	50	80	90

For each exercise, discuss how the estimate helps to determine where the decimal point is located in the exact product. For example, for 3 × 9.7, discuss each of the possibilities (0.291, 2.91, 29.1, and 291) in relation to the estimate, 30. Have students carry out a similar procedure for the following exercises and see whether they can arrive at the exact products by estimating first. Leave space for writing the partial products.

	9.7 × 10	9.7 × 12	9.7 × 15	9.7 × 25
Partial products		19 4 97 0		
Exact product	97	116.4		
Estimate	100	120	150	250

Ask students to explain how they know where the decimal point must be placed for each of the exact products.

Using the Pages

- The worked example demonstrates that the product of a decimal and a whole number is found by the same series of steps as for the product of two whole numbers. Since both factors now have more than one digit, the process of multiplying involves partial products. Point out that the decimal point is not shown in the partial products but only in the final product. Preceding this lesson, the students have learned that the product of a whole number and a decimal tenth is a decimal tenth. This, as well as estimating

Working Together

Complete each multiplication.

1. 32 × 27 = 864 | 32 tenths × 27 = 864 tenths | 3.2 × 27 = 86.4

2. 143 × 18 = 2574 | 143 × 18 tenths = 2574 tenths | 143 × 1.8 = 257.4

3. 264 × 76 = 20 064 | 2.64 × 76 = 200.64

4. 718 × 36 = 25 848 | 0.718 × 36 = 25.848

5. 375 × 239 = 89 625 | 375 × 2.39 = 896.25

Multiply. Change the order of the factors, then multiply again to check your work.

6. 5.9 × 27 = 159.3
7. 324 × 1.76 = 570.24
8. 4.7 × 68 319.6
9. 94 × 3.25 305.50

Exercises

Multiply. Change the order of the factors, then multiply again to check your work.

1. 47 × 5.9 = 277.3
2. 3.69 × 16 = 59.04
3. 388 × 5.21 = 2021.48
4. 4.688 × 64 = 300.032
5. 264 × 0.7 184.8
6. 0.25 × 39 9.75
7. 74.4 × 76 5654.4
8. 188 × 0.073 13.724

Rounding can help you estimate products.

Example:

For the product of 38 and 2.13,

round 2.13 → 2
round 38 → 40
then multiply → 80

Multiply for the exact product.

2.13 × 38 = 80.94

Round and multiply to estimate the product. Then find the exact product.

9. 92 × 7.7 708.4 (720)
10. 5.51 × 74 407.74 (420)
11. 538 × 2.63 1414.94 (1500)
12. 38.9 × 21 816.9 (800)

Is this good thinking?

1. If Sue can run 1 km in 5 min, she can run 10 km in 50 min. no
2. If each bottle holds 1.5 L, then 100 of these bottles hold 150 L. yes
3. If 7 mm of rain fall in 1 h, then 70 mm will fall in 10 h. no
4. If 1 L of gasoline costs 22.9¢, 100 L will cost $22.90. yes
5. If the mass of 1 girl is 43 kg, the mass of 10 girls is 430 kg. no
6. If 52 players are on 10 teams, each team has 5.2 players. no

PROBLEM SOLVING

155

RELATED ACTIVITIES

• Exercises such as Ex. 5, 6, and 8 on page 155 may cause a little difficulty when the decimal factor is used as the multiplier. The zeros in the numerals do not need to be used in the multiplication process. Have students find the product, change the order of the factors, and multiply again to check their work.

Ex. 6

0.25	39
× 39	× 0.25
2 25	1 95
7 5	7 80
9.75	9.75

• Have the students find products by showing a partial product for each place value.

Ex. 6

0.25	
× 39	
0.45	(9 × 0.05)
1.8	(9 × 0.2)
1.50	(30 × 0.05)
6.0	(30 × 0.2)
9.75	

39	
× 0.25	
0.45	(0.05 × 9)
1.5	(0.05 × 30)
1.8	(0.2 × 9)
6.0	(0.2 × 30)
9.75	

the product, helps to explain the location of the decimal point in 187.5. Have students estimate the product by multiplying 3 and 75 to show that the whole number portion of the product is close to 225. Note that the product is also checked by changing the order of the factors and multiplying.

Working Together: Ex. 1-5 help students to relate multiplication with decimals to multiplication of whole numbers. Before the students begin Ex. 6-9, ask whether the product will be a one-place decimal, a two-place decimal, or a three-place decimal.

Exercises: The example above Ex. 9-12 demonstrates that each of the two factors in a multiplication may be rounded to estimate the product. Discuss the example with the students before assigning the exercises.

Problem Solving: Discuss these exercises in small groups so that the students can share opinions and give reasons for their answers.

Assessment

Multiply.

1. 32 × 4.3 = 137.6
2. 1.44 × 17 = 24.48
3. 2.616 × 58 = 151.728
4. 134 × 0.69 = 92.46

UNIVERSITY OF ALBERTA

Pages 156-157

OBJECTIVE

Demonstrate competence in finding the product of a decimal and a whole number; solve related word problems

Materials

a copy of page T 397 and a copy of page T 399 for each student, copies of page T 389 (optional)

Practice

Estimates
21. 640 22. 8000 23. 140 24. 500
25. $10 26. $360 27. $2100 28. $1600

Multiply. Change the order of the factors, then multiply again to check your work.

1. 58 × 2.9	**2.** 84.6 × 138	**3.** 2.58 × 89	**4.** 367 × 9.12	**5.** 43.25 × 29	**6.** 293 × 3.4
168.2	11 674.8	229.62	3347.04	1254.25	996.2

7. 35 × 19.6 686.0 **8.** 126 × 15.88 2000.88 **9.** 72 × 7.538 542.736 **10.** 251 × 4.177 1048.427

11. 14 × $5.49 $76.86 **12.** $2.31 × 66 $152.46 **13.** 218 × $7.34 $1600.12 **14.** $19.76 × 47 $928.72

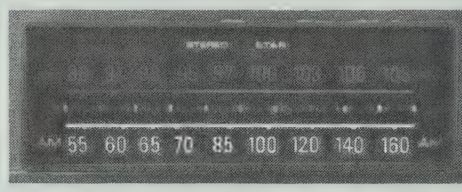
Round and multiply to estimate the product. Then find the exact product. Estimates may vary.

15. 2.3 × 6	**16.** 39.2 × 24	**17.** 849 × 1.3	**18.** 4.287 × 9	**19.** 3.429 × 71	**20.** 252 × 24.3
13.8 (12)	940.8 (800)	1103.7 (850)	38.583 (36)	243.459 (210)	6123.6 (5000)

21. 8 × 75.93 607.44 **22.** 384 × 16.28 6251.52 **23.** 7 × 19.7 137.9 **24.** 46 × 9.8 450.8

25. 5 × $1.93 $9.65 **26.** 64 × $5.61 $359.04 **27.** $7.17 × 283 $2029.11 **28.** 37 × $43.48 $1608.76

The "number lines" shown on radios help listeners find the stations they want.



Stations with names ending in FM are found along here.

The other stations are found along here.

Copy both "radio number lines" along the edges of a piece of paper. Mark your copies to show where to find these stations.

1. CFUN, Vancouver	1410	**2.** CFCN, Calgary	1060
3. CFMQ-FM, Regina	92.1	**4.** CKY, Winnipeg	580
5. CHUM-FM, Toronto	104.5	**6.** CINQ-FM, Montreal	99.3
		7. CKCW, Moncton	1220
		8. CHTN, Charlottetown	1190
		9. CHFX-FM, Halifax	96.1
		10. CJON, St. John's	930
		11. CFYK, Yellowknife	1340

try this

3. 9. 6. 5.
FM 88 91 93 95 97 100 103 106 108 FM

AM 55 60 65 70 85 100 120 140 160 AM
4. 10. 2. 8. 7. 11. 1.

LESSON ACTIVITY

Using the Pages

- You may wish to complete Ex. 2 and Ex. 16 on the board with the students to review the skills involved.
- Briefly discuss the photograph at the top of page 157 and have a student read the statement related to it. Draw the students' attention to the chart for Ex. 29. Discuss aspects of the chart to familiarize the students with the information shown. Ask, for example, what Laura's grandfather earned each hour in his fifth year and in his fortieth year, how many hours he worked each day in his twentieth year, and how many days he worked each week in his fiftieth year. Ask how to find how much he earned in one day, in one week, and in 52 weeks in his thirtieth year. Ask for another way to name a time period of 52 weeks. Provide students with copies of page T 399 on which to copy and complete the chart for Ex. 29.

  Give each student a copy of page T 397 on which to show the graph for Ex. 30. They may choose the information from one of the three rows they completed; that is, the amount earned in a day, a week, or 52 weeks. Discuss the headings to be used for the graph, suitable scales, and so on. Note that the students are reminded to round amounts of money before graphing them.

  Ex. 31 is starred and may be considered a long-term group project. Ask each student to contribute at least one row to the chart.

Try This: These exercises demonstrate an application of the number line. For some radios, stations are located by points on a scale and involve decimal numerals. To save space on the AM scale of a radio, the zero in the ones' place is usually omitted from each numeral. For example, the 55 on the scale refers to 550 kc (kilocycles). The radio shown is tuned just past 140 (1400 kc) and is, therefore, tuned to station CFUN, Vancouver (1410). You may wish to have students use copies of unmarked number lines from page T 389 for these exercises.

Laura's grandfather retired after 50 years with the company.

29. Complete this chart that shows how much he earned at different times in the 50 years.

	1st year	5th year	10th year	20th year	30th year	40th year	50th year
Amount earned each hour	$0.45	$0.76	$1.41	$3.18	$5.23	$7.37	$9.82
Hours worked in a day	11	10	10	9	8	8	8
Amount earned in a day	? $4.95	? $7.60	? $14.10	? $28.62	? $41.84	? $58.96	? $78.56
Days worked in a week	6	6	6	6	5	5	4
Amount earned in a week	? $29.70	? $45.60	? $84.60	? $171.72	? $209.20	? $294.80	? $314.24
Amount earned in 52 weeks	? $1544.40	? $2371.20	? $4399.20	? $8929.44	? $10 878.40	? $15 329.60	? $16 340.48

30. Draw a graph using the information from one of the rows you completed in the chart above. Graphs are shown below.

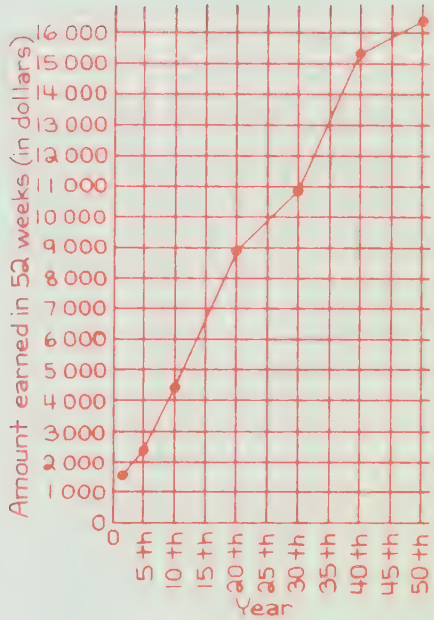
Round the amounts of money before you draw the graph.

*31. Find the information you need. Then complete a chart like this for jobs in your community. Answers will vary

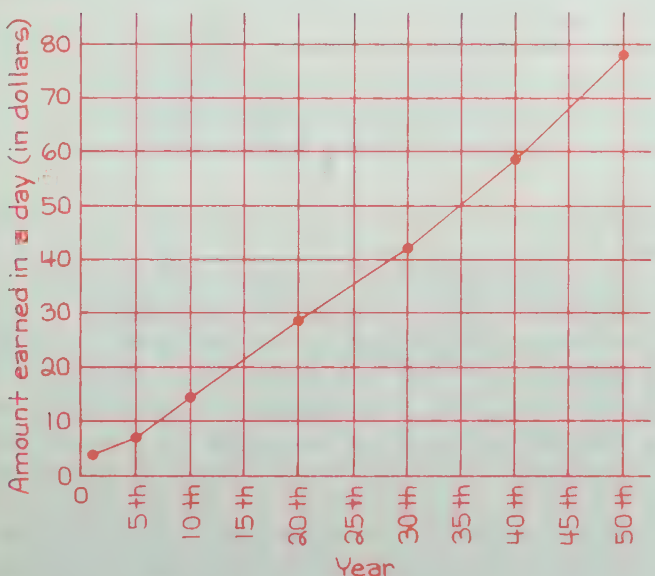
Job	Amount earned each hour	Hours worked in a day	Days worked in a week	Amount earned in a week
TV Repair	$6.28	8	5	$251.20
Sitter	$1.20			

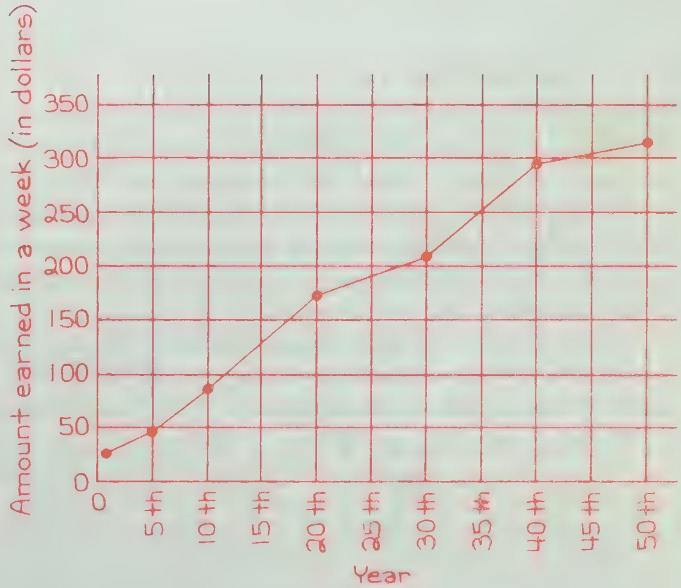
RELATED ACTIVITIES

- For further practice, you may wish to have students complete some of Ex. 23-46 on page 335.
- Have students extend the *Try This* feature to include favorite radio stations in their area.



30.





LESSON OUTCOME

Find the product of two factors when one factor is 1000, 100, 10, 0.1, 0.01, or 0.001

Materials

flannel board and felt numerals (several for each of 0 to 9), or a magnetic board and magnetic numerals

Prerequisite Skills

Interpret place value in numerals for whole numbers and decimals to thousandths; find the product of a whole number and a decimal

Checking Prerequisite Skills

What does each 4 mean?

1. 130 240 4 tens
2. 1302.40 4 tenths
3. 130.240 4 hundredths
4. 13 024 4 ones
5. 13.024 4 thousandths

Multiply.

6. 0.3 × 45 13.5
7. 1.364 × 27 36.828
8. 124 × 0.075 9.300

1000, 100, 10, 1, 0.1, 0.01, or 0.001 as a Factor

When you multiply a number and 10, 100, or 1000, the digits move to places with greater values in a place-value chart.

When you multiply a number and 0.1, 0.01, or 0.001, the digits move to places with lesser values in a place-value chart.

tens	ones				ten thousands	thousands	hundreds	tens	ones	tenths	hundredths	thousandths
2	8	×	1000	=	2	8	0	0	0			
2	8	×	100	=		2	8	0	0			
2	8	×	10	=			2	8	0			
2	8	×	1	=				2	8			
2	8	×	0.1	=					2 .	8		
2	8	×	0.01	=					0 .	2	8	
2	8	×	0.001	=					0 .	0	2	8

Working Together

Complete the chart. One factor is 576.

	When the other factor is	move the digits in 576	and the product is
1.	1000	3 places to the left	576 000
2.	100	2 ? places to the ? left	57?600
3.	10	1 ? place to the ? left	5?760
4.	1	0 ? places	?576
5.	0.1	1 ? place to the ? right	? 57.6
6.	0.01	2 ? places to the ? right	? 5.76
7.	0.001	3 ? places to the ? right	? 0.576

Multiply.

8. 35.9 × 100 3590
9. 0.001 × 4075 4.075
10. 10 × 26 260
11. 4 × 0.01 0.04
12. 1000 × 0.8 800
13. 100 × 0.1 10
14. 0.051 × 10 0.51
15. 0.1 × 3.5 0.35
16. 0.6 × 0.01 0.006

158

LESSON ACTIVITY

Before Using the Pages

- Prepare the flannel board to show a place-value chart from ten thousands to thousandths. Have students place felt numerals on the flannel board and reposition the numerals according to your instructions for the following activity. (The activity may be adapted for use with magnetic numerals on a magnetic board or the chalkboard, if magnets will adhere to it.)

  Write the following exercises on the board and have the students copy and complete them. Discuss the similarities and differences among the exercises.

  1. 46×1
  2. 46×10
  3. 46×100
  4. 46×1000
  5. 46×0.1
  6. 46×0.01
  7. 46×0.001

  Have a student place numerals in the place-value chart on the flannel board to show the first product, 46.

ten thousands	thousands	hundreds	tens	ones	tenths	hundredths	thousandths
			4	6			

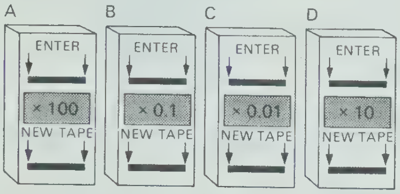
Have another student move the same numerals to show the product for 10 × 46 and describe the change in position and the need for showing 0 in the ones' place. Adjust the numerals to show 1 × 46. Then have a student show the product for 100 × 46 and describe the change. Adjust the numerals to show 1 × 46 and repeat the procedure until each product has been shown and described. For decimal multipliers, use a felt dot on the flannel board for the decimal point. You may wish to repeat the activity for a similar set of exercises using 146, for example, instead of 46. Ask the students to show the digits in the place-value chart without multiplying first using paper and pencil.

Exercises

Multiply.

1. 10 × 42 420
2. 0.01 × 520 5.2
3. 8 × 0.1 0.8
4. 27.64 × 100 2764
5. 1000 × 1.82 1820
6. 2.6 × 0.01 0.026
7. 100 × 0.008 0.8
8. 0.1 × 0.04 0.004
9. 0.001 × 6500 6.5
10. 0.63 × 10 6.3
11. 0.075 × 1000 75
12. 0.001 × 48 0.048
13. 0.1 × 7.6 0.76
14. 10 × 0.76 7.6
15. 0.03 × 100 3
16. 3 × 0.01 0.03

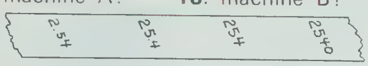
These machines change the numbers on each tape entered according to the rules shown.



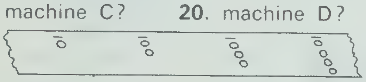
What will the new tape show when the tape below is entered into

machine A: 254, 2540, 25 400, 254 000

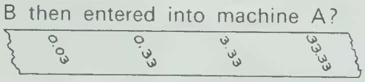
17. machine A?
18. machine B?



19. machine C?
20. machine D?



*21. machine B and the tape from B then entered into machine A?



*22. What results when a tape is entered into machine D and then its new tape entered into machine B?

18 machine B: 0.254, 2.54, 25.4, 254.0
19 machine C: 0.10, 1.00, 10.00, 100.00
20. machine D: 100, 1000, 10 000, 100 000
21 machine A: 0.3, 3.3, 33.3, 333.3

The names for different units of length all use "metre".

1 km (**kilo**metre)	= 1000 m
1 hm (**hecto**metre) =	100 m
1 dam (**deca**metre) =	10 m
1 m (metre) =	1 m
1 dm (**deci**metre) =	0.1 m
1 cm (**centi**metre) =	0.01 m
1 mm (**milli**metre) =	0.001 m

The names for different units of mass all use "gram" in the same way.

1. Make a chart, like the one above, for the units of mass.
A chart is shown on page T369
The chart would begin like this:

1 kg (kilogram) =	1000 g
1 h[illegible]gram) =	100 g

The names for different units of capacity all use "litre" in the same way.

2. Make a chart, like the ones above, for units of capacity.
A chart is shown on page T369
3. **Complete three charts like this, one for length, one for mass, and one for capacity.**

1 m = ? mm
1 m = ? cm
1 m = ? dm
1 m = 1 m
1 m = ? dam
1 m = ? hm
1 m = ? km

Charts are shown on page T369

try this

22 The final numbers will be the same as the original numbers

159

RELATED ACTIVITIES

- For further practice, you may wish to have students complete Ex. 47-54 on page 335.
- Prepare a set of numeral cards showing numbers such as 42, 3.61, 0.053, 18.21, and 124. Mark a die to show 1000, 100, 10, 0.1, 0.01, and 0.001. Have students toss the die, draw a numeral card, and find the product of the two numbers. If students play in groups of three, points may be awarded at the end of each round: two points for the greatest number, two points for the least number, and one point for the middle number. At the end of ten rounds, the player with the highest score is the winner.
- Use an overhead projector to demonstrate the product of a number and a power of ten. Write a numeral, for example, 3764, on one piece of acetate. Place it over another piece of acetate showing a place-value chart and a decimal point (A). Move the acetate sheet showing the numeral to the left or to the right over the place-value chart (B).

A

Th	H	T	O	t	h
		3	7.	6	4

B

Th	H	T	O	t	h
	3	7	6.	4	

Using the Pages

- The example presents a place-value chart showing all the products in sequence from 28 × 1000 to 28 × 0.001. Demonstrate that 28 means 2 tens 8 ones and that 28 × 1 results in no change to the place values of the digits. Then discuss the changes that do occur for each of the other examples.

Working Together: The completed chart will provide students with a summary of how each power of ten as a factor affects the digits of the other factor. The students should think of the decimal point being in a fixed position, and the digits moving to the left and to the right from the decimal point. Emphasize that factors such as 1000, 10, and 0.001 are special because our system of numeration is a base-ten system, and consecutive place values are related by the number ten. For this reason, they affect the digits of the other factor in a way which enables us to determine their products easily. Note that the product for Ex. 13 can be described in terms of either factor since each is a power of ten.

Exercises: Caution the students to keep the left-to-right order of the four numbers shown on the tapes for Ex. 17-22. Note that Ex. 21 and 22 involve the use of two machines. The result of using machine D and then machine B, as in Ex. 22, results in the original numbers on the tape. In other words, the effect of multiplying a number by 10 can be "undone" by multiplying by 0.1.

Try This: These exercises review and summarize the relationships among units of length, mass, and capacity. The prefix in the name of a unit implies a power of ten. The prefixes emphasized in the first chart are applied in Ex. 1 and 2.

Assessment

Multiply.

1. 10 × 19 190
2. 2.6 × 0.01 0.026
3. 0.05 × 0.1 0.005
4. 1000 × 0.023 23
5. 0.001 × 8200 8.2
6. 100 × 0.006 0.6
7. 0.43 × 10 4.3
8. 0.1 × 4.5 0.45

LESSON OUTCOME

Multiply by 1000, 100, 10, 0.1, 0.01, or 0.001 to express a measurement given in one unit of length in terms of another unit of length

Prerequisite Skills

Express one metre as decimetres, centimetres, and millimetres, and vice versa

Checking Prerequisite Skills

Complete.

1. 1 m = 10 dm
2. 1 dm = 0.1 m
3. 1 m = 100 cm
4. 1 cm = 0.01 m
5. 1 m = 1000 mm
6. 1 mm = 0.001 m

RELATED ACTIVITIES

- Have students complete tables similar to the following.

m	4	4.2	4.27	4.275
cm				
mm				

m	1.5		3.8	
cm				42.5
mm		450		

Changing Measurement Units

Knowing how to multiply by 10, 100, 1000, 0.1, 0.01, or 0.001 can help you change from one measurement unit to another.

1 m = 10 dm	1 dm = 0.1 m
1 m = 100 cm	1 cm = 0.01 m
1 m = 1000 mm	1 mm = 0.001 m

How many centimetres are there in 3.28 m?

1 m = 100 cm
3.28 m = 3.28 × 100 cm
3.28 m = 328 cm

How many metres are there in 25 000 mm?

1 mm = 0.001 m
25 000 mm = 25 000 × 0.001 m
25 000 mm = 25 m

Exercises

Complete.

1. 1 m = ▢ mm 1000
   4.67 m = 4.67 × ▢ mm 1000
   4.67 m = ▢ mm 4670
2. 1 cm = ▢ m 0.01
   62.5 cm = 62.5 × ▢ m 0.01
   62.5 cm = ▢ m 0.625
3. 16 000 mm = ▢ m 16
4. 3.33 m = ▢ cm 333
5. 7 m = ▢ mm 7000
6. 2.75 cm = ▢ mm 27.5
7. 450 cm = ▢ m 4.5
8. 280 mm = ▢ cm 28
9. 2.875 m = ▢ dm 28.75
10. 3.4 dm = ▢ m 0.34
11. 6.3 dm = ▢ cm 63

Change each to millimetres.

12. 0.3 cm 3 mm
13. 1.28 m 1280 mm
14. 75 cm 750 mm
15. 0.33 m 330 mm

Change each to centimetres.

16. 0.7 m 70 cm
17. 7500 mm 750 cm
18. 12.8 m 1280 cm
19. 50 mm 5 cm

Change each to metres.

20. 3500 mm 3.5 m
21. 26 cm 0.26 m
22. 4 mm 0.004 m
23. 1250 cm 12.5 m

Use these facts to help you change

\$1 = 100¢	1¢ = \$0.01

24. 185¢ to dollars. \$1.85
25. \$0.63 to cents. 63¢
26. 250¢ to dollars. \$2.50
27. \$10.80 to cents. 1080¢
28. 4¢ to dollars. \$0.04
29. \$0.30 to cents. 30¢

160

LESSON ACTIVITY

Before Using the Page

- Write a few exercises on the board and complete them with the students to review the work of the previous lesson.

$4.32 \times 100 = 432$ $\quad$ $12.5 \times 10 = 125$
$135 \times 0.01 = 1.35$ $\quad$ $1630 \times 0.1 = 163$
$6.2 \times 1000 = 6200$ $\quad$ $4320 \times 0.001 = 4.32$

Tell the students that multiplications similar to these are useful in changing from one unit of measurement to another. Demonstrate that multiplication by 100 is used to express metres as centimetres, since 1 m is equal to 100 cm. Have students suggest examples and write them on the board.

3 m = 300 cm $\quad$ 4.32 m = 432 cm $\quad$ 5.5 m = 550 cm

Similarly, demonstrate that centimetres can be expressed as metres by multiplying by 0.01, since 1 cm is equal to 0.01 m.

35 cm = 0.35 m $\quad$ 9 cm = 0.09 m $\quad$ 142 cm = 1.42 m

Using the Page

- Draw attention to the six sentences inside the rectangle. Emphasize that knowing these relationships helps in selecting the correct factor. Discuss the two examples provided, paying particular attention to the power of ten used in the multiplication. Work other similar examples on the board as required.

Exercises: The students can use the format shown in Ex. 1 and 2 for the remaining exercises. Some students may be able to perform the first two steps mentally and then write only one sentence. Have students refer to the six sentences at the top of the page as needed. Ex. 24-29 require a similar thought process involving amounts of money.

Assessment

Change to millimetres.

1. 0.4 m 400 mm
2. 34 cm 340 mm
3. 1.45 m 1450 mm

Change to centimetres.

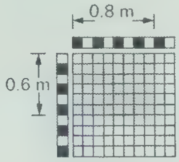
4. 6300 mm 630 cm
5. 0.6 m 60 cm

Change to metres.

6. 9 mm 0.009 m
7. 1250 cm 12.5 m

Multiplying Decimal Tenths, Both Factors Less Than 1.0

The product of two decimals showing tenths is a decimal showing hundredths.

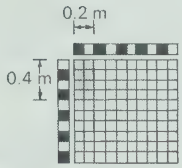


$6 \times 8 = 48$

6 tenths × 8 tenths = 48 hundredths

$0.6 \times 0.8 = 0.48$

A rectangle 0.6 m wide and 0.8 m long covers 0.48 m^2.



$4 \times 2 = 8$

4 tenths × 2 tenths = 8 hundredths

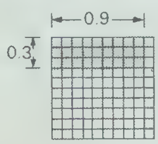
$0.4 \times 0.2 = 0.08$

A rectangle 0.4 m long and 0.2 m wide covers 0.08 m^2.

Working Together

Write a multiplication sentence to match this picture.

1.



0.3 × 0.9 = 0.27

Complete each multiplication. Remember, the product of two decimals showing tenths is a decimal showing hundredths.

2. 3 × 5 = 15
3 tenths × 5 tenths = 15 hundredths
0.3 × 0.5 = 0.15

7	7 tenths	0.7
8	8 tenths	0.8
56	56 hundredths	0.56

3. (above)

4. 0.6 × 0.9 = 0.54

5. 0.4 × 0.7 = 0.28

Exercises

Multiply.

1. 0.3 × 0.2 = 0.06
2. 0.2 × 0.7 = 0.14
3. 0.9 × 0.5 = 0.45
4. 0.3 × 0.3 = 0.09
5. 0.8 × 0.5 = 0.40
6. 0.4 × 0.9 = 0.36
7. 0.6 × 0.4 0.24
8. 0.7 × 0.7 0.49
9. 0.9 × 0.8 0.72
10. 0.1 × 0.7 0.07
11. 0.2 × 0.8 0.16
12. 0.8 × 0.6 0.48
13. 0.6 × 0.5 0.30
14. 0.4 × 0.6 0.24

LESSON OUTCOME

Find the product of two one-place decimals, factors to 0.9

Materials

metre sticks marked in tenths of a metre, models for hundredths or an overhead projector (optional)

Prerequisite Skills

Use multiplication to find the area of a rectangular region in square metres

Checking Prerequisite Skills

Complete the chart by finding the area of each rectangle.

	Length	Width	Area
1.	7 m	4 m	28 m^2
2.	9 m	3 m	27 m^2
3.	8 m	5 m	40 m^2
4.	6 m	2 m	12 m^2

RELATED ACTIVITIES

- Provide students with copies of the diagrams on page T 395 to represent a metre square marked into hundredths of a square metre. Have them shade the diagrams and write multiplication sentences as described in *Before Using the Page*.

LESSON ACTIVITY

Before Using the Page

- Display metre sticks marked in tenths. Have students locate points for such lengths as 0.3 m, 0.4 m, 0.7 m, and 0.9 m.
- In advance of the lesson, draw a square with sides 1 m long on the board and draw grid lines so that the square is divided into hundredths. You may prefer to use an overhead projector so that the image projected on the screen or chalkboard is 1 m^2 in size. Have students observe that the sides of the metre square are marked in tenths of a metre and that the area, 1 m^2, is divided into 100 small squares. Develop that each small square has an area of 0.01 m^2.

  Shade a rectangular region of the metre square in the way shown on page 161. Have students name the length and the width of the region in tenths of a metre and name the number of hundredths of a square metre in the region. Write the corresponding multiplication sentence on the board. Repeat for other rectangular regions.

Using the Page

- The concept developed in *Before Using the Page* is reinforced in the worked example to emphasize that the product of two decimal-tenth factors is a decimal hundredth. Products are also seen as extensions of basic multiplication facts.

Working Together: Use other similar exercises as needed. You may wish to have students sketch diagrams for Ex. 2-5, similar to the one shown in Ex. 1.

Exercises: Observe the students closely as they work to ensure that they do not add the decimal tenths by mistake. Watch to see whether they include the 0 in the tenths' place in the products for Ex. 1, 4, and 10.

Assessment

Multiply.

1. 0.6 × 0.7 = 0.42

2. 0.5 × 0.4 = 0.20

3. 0.9 × 0.9 = 0.81

4. 0.2 × 0.7 0.14

5. 0.8 × 0.1 0.08

UNIVERSITY OF ALBERTA

Pages 162-163

LESSON OUTCOME

Find the product of two one-place decimals, one factor to 0.9; solve related word problems

Materials

models for tenths and hundredths (optional)

Prerequisite Skills

Find the product of two whole numbers, one factor to 9

Checking Prerequisite Skills

Multiply.

1. 34 × 3 = 102 **2.** 16 × 9 = 144 **3.** 123 × 4 = 492

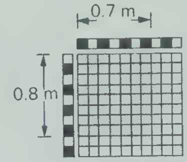
4. 7 × 817 = 5719 **5.** 6 × 355 = 2130

Multiplying Decimal Tenths, One Factor Less Than 1.0

What is the area of a rectangle that is 3.7 m long and 0.8 m wide?

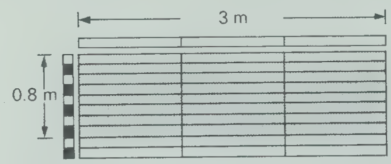
Multiply 0.8 and 3.7.

The product of 0.8 and 0.7 is 0.56.



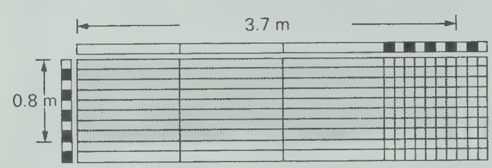
$0.8 \times 0.7 = 0.56$

The product of 0.8 and 3 is 2.4.



$0.8 \times 3 = 2.4$

The product of 0.8 and 3.7 is 2.96.



$0.8 \times 3.7 = 2.96$

The area of the rectangle is 2.96 m^2.

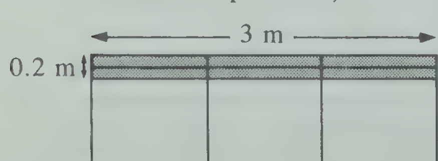
LESSON ACTIVITY

Before Using the Pages

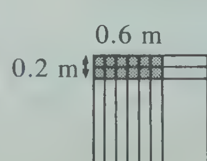
- Have the students complete products for pairs of exercises similar to the following.

3 × 0.2	0.6 × 0.2	2 × 0.4	0.3 × 0.4	8 × 0.7	0.5 × 0.7

Relate each multiplication to a rectangular region, sketching diagrams to illustrate the product. For example, a rectangle 3 m long and 0.2 m wide would have an area of 0.6 m^2. A rectangle 0.6 m by 0.2 m would have an area of 0.12 m^2. (Models of tenths and hundredths may be adapted to illustrate the products.)



$0.2 \times 3 = 0.6$



$0.2 \times 0.6 = 0.12$

Write the following exercises on the board and relate each to the corresponding pair of exercises completed earlier. If possible, lead the students to suggest how the products may be found.

3.6 × 0.2	2.3 × 0.4	8.5 × 0.7

Using the Pages

- The worked example develops the product for 0.8 × 3.7 through three stages using an area model. This can help students to understand that the product of two decimals showing tenths is a decimal showing hundredths. Lead the students through the development on page 162. Have them count the blue parts for each diagram and verify that the product is 2.96. Emphasize that the product shows decimal hundredths. The worked example continues on page 163, illustrating that the product can be found in a way similar to that for whole numbers. Multiplication is carried out from right to left as usual, regrouping where necessary. Draw attention to the portion of the worked example at the bottom

Take another look:

For the product,

$$\begin{array}{r} 3.7 \\ \underline{0.8} \end{array}$$

Use $0.8 \times 0.7 = 0.56$

$$\begin{array}{r} {}^{5} \\ 3.7 \\ \underline{0.8} \\ 6 \end{array}$$

No decimal point is shown in the product yet.

Then, use $0.8 \times 3 = 2.4$

$$\begin{array}{r} {}^{5} \\ 3.7 \\ \underline{0.8} \\ 2.96 \end{array}$$

2 and 4 tenths and 5 tenths more make 2 and 9 tenths.

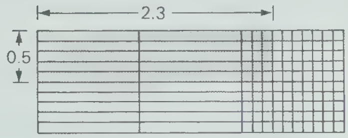
Multiplying $\begin{array}{r} 3.7 \\ \underline{0.8} \end{array}$

is like multiplying $\begin{array}{r} 37 \\ \underline{8} \end{array}$

but you have to put a decimal point in the product when you finish.

Working Together

Write a multiplication sentence to match this picture.



0.5 × 2.3 = 1.15

Complete each multiplication. Remember, the product of two decimals showing tenths is a decimal showing hundredths.

1.

38	38 tenths	3.8
4	4 tenths	0.4
152	152 hundredths	1.52

2.

152	15.2
6	0.6
912	9.12

3. $\begin{array}{r} 5.7 \\ \underline{0.3} \\ 1.71 \end{array}$

4. $\begin{array}{r} 26.3 \\ \underline{0.7} \\ 18.41 \end{array}$

5. 0.5 × 7.9 3.95

6. 0.8 × 36.5 29.20

Exercises

Multiply.

1. $\begin{array}{r} 4.6 \\ \underline{0.3} \\ 1.38 \end{array}$
2. $\begin{array}{r} 3.5 \\ \underline{0.8} \\ 2.80 \end{array}$
3. $\begin{array}{r} 1.8 \\ \underline{0.9} \\ 1.62 \end{array}$
4. $\begin{array}{r} 2.3 \\ \underline{0.4} \\ 0.92 \end{array}$
5. $\begin{array}{r} 17.4 \\ \underline{0.2} \\ 3.48 \end{array}$
6. $\begin{array}{r} 45.6 \\ \underline{0.5} \\ 22.80 \end{array}$
7. $\begin{array}{r} 83.9 \\ \underline{0.3} \\ 25.17 \end{array}$
8. $\begin{array}{r} 27.8 \\ \underline{0.4} \\ 11.12 \end{array}$
9. 0.6 × 5.3 3.18
10. 0.2 × 6.6 1.32
11. 0.5 × 4.7 2.35
12. 0.8 × 7.2 5.76
13. 0.9 × 61.5 55.35
14. 0.7 × 38.9 27.23

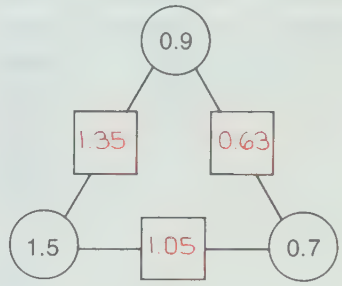
What is the area of a rectangle that is

15. 0.4 m wide and 6.6 m long? 2.64 m²
16. 1.8 m long and 0.7 m wide? 1.26 m²
17. 0.6 m wide and 7.6 m long? 4.56 m²
18. 11.9 m long and 0.9 m wide? 10.71 m²

163

RELATED ACTIVITIES

- For further practice, you may wish to have the students complete some of Ex. 55-66 on page 335.
- Assign further practice in the form of diagrams prepared from copies of page T 390. Write factors in the circles and have students write products in the squares.



- Write one-place decimals less than 1 on the faces of a cube. Write one-place decimals greater than 1 on the faces of another cube. Have students roll the cubes and find the product of the two numbers shown. Suggest that students play the game in pairs and keep a cumulative sum of their scores. After several rounds, the player with the greater sum is the winner.
- Have students complete multiplication exercises similar to the following and observe the patterns in the products.

$\begin{array}{r} 0.7 \\ \underline{\times\ 0.3} \end{array}$ $\begin{array}{r} 0.8 \\ \underline{\times\ 0.3} \end{array}$ $\begin{array}{r} 0.9 \\ \underline{\times\ 0.3} \end{array}$ $\begin{array}{r} 1.0 \\ \underline{\times\ 0.3} \end{array}$

$\begin{array}{r} 1.1 \\ \underline{\times\ 0.3} \end{array}$ $\begin{array}{r} 1.2 \\ \underline{\times\ 0.3} \end{array}$ $\begin{array}{r} 1.3 \\ \underline{\times\ 0.3} \end{array}$ $\begin{array}{r} 1.4 \\ \underline{\times\ 0.3} \end{array}$

of page 163. Emphasize that the decimal point is placed when the multiplication has been completed. Summarize by stating that the product shows hundredths when each of two factors shows tenths.

Working Together: Have students count the blue parts of the picture and write the corresponding multiplication sentence. Then ask them to multiply 0.5 and 2.3 to obtain the product and compare the answers obtained these two ways. Ex. 1 and 2 help students to relate multiplying decimals to multiplying whole numbers. Use other similar exercises as needed before continuing with Ex. 3-6.

Exercises: Ensure that the students write the symbol m² in their answers for Ex. 15-18.

Assessment

Multiply.

1. $\begin{array}{r} 3.4 \\ \underline{0.6} \\ 2.04 \end{array}$
2. $\begin{array}{r} 1.5 \\ \underline{0.9} \\ 1.35 \end{array}$
3. $\begin{array}{r} 14.7 \\ \underline{0.2} \\ 2.94 \end{array}$
4. $\begin{array}{r} 34.2 \\ \underline{0.7} \\ 23.94 \end{array}$

What is the area of a rectangle that is

5. 0.3 m wide and 2.9 m long? 0.87 m²

LESSON OUTCOME

Multiply two one-place decimals; solve related word problems

Prerequisite Skills

Multiply two whole numbers

Checking Prerequisite Skills

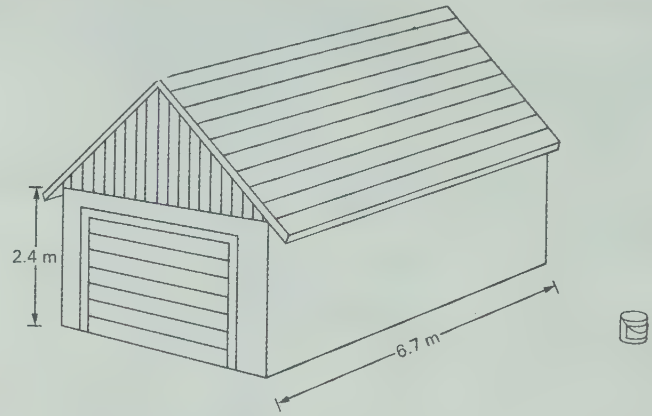
Multiply.

1. $\begin{array}{r} 62 \\ \underline{14} \end{array}$ 868 **2.** $\begin{array}{r} 265 \\ \underline{37} \end{array}$ 9805 **3.** $\begin{array}{r} 180 \\ \underline{294} \end{array}$ 52 920

4. 347 × 419 145 393 **5.** 16 × 1473 23 568

Multiplying Decimal Tenths

The paint can says that there is enough paint inside to cover 15 m². The side of the garage is 6.7 m long and 2.4 m high. Is there enough paint to cover the side of the garage?



Multiply 2.4 and 6.7.

For the product

$$\begin{array}{r} 6.7 \\ \underline{2.4} \end{array}$$

you need to know how to multiply 4 and 67,

$$\begin{array}{r} {}^{2} \\ 6.7 \\ \underline{2.4} \\ 268 \end{array}$$

The decimal point would be here, but there is no need to show it yet.

and how to multiply 2 and 67.

$$\begin{array}{r} {}^{1} \\ 6.7 \\ \underline{2.4} \\ 268 \\ 1340 \end{array}$$

The decimal point would be here, but there is no need to show it yet.

Then add and place the decimal point.

$$\begin{array}{r} 6.7 \\ \underline{2.4} \\ 268 \\ \underline{1340} \\ 16.08 \end{array}$$

The decimal point is here since the product of tenths and tenths is hundredths.

The side of the garage has an area of 16.08 m². There is not enough paint for the side of the garage.

164

LESSON ACTIVITY

Before Using the Pages

- Write several exercises similar to the following on the board. Complete them with the students to review the preceding lessons on multiplication with decimals.

$$\begin{array}{r} 0.7 \\ \underline{\times\ \ 3} \\ 2.1 \end{array} \qquad \begin{array}{r} 2.7 \\ \underline{\times\ \ 3} \\ 8.1 \end{array}$$

$$\begin{array}{r} 0.7 \\ \underline{\times 0.3} \\ 0.21 \end{array} \qquad \begin{array}{r} 2.7 \\ \underline{\times 0.3} \\ 0.81 \end{array}$$

Emphasize that if both factors are decimals showing tenths, the product is a decimal showing hundredths. Point out that until now, one of the two decimal factors in a multiplication has always been less than 1, as in the last two examples above. Write the following exercise on the board, pointing out that each factor is greater than 1 and shows tenths. Ask students how to find the product.

$$\begin{array}{r} 2.7 \\ \underline{\times 6.3} \\ 81 \\ \underline{16\ 20} \\ 17.01 \end{array}$$

If students are not able to suggest the exact product, they may be able to round each factor to the nearest whole number. Then they can multiply to estimate the product (6 × 3 = 18) and determine the location of the decimal point.

Using the Pages

- Have a student read the word problem to introduce the situation. Explain that the solution involves finding the area of the side of the garage and thus multiplication is used. Point out that each factor is a decimal greater than 1 and shows tenths. Ask how many decimal places the product will have.

  Follow the direction of the arrows in leading the students through the solution. Note that the decimal point is placed after adding the partial products. Emphasize the similarity to multiplying whole numbers. Discuss why there would not be enough paint for the side of the garage.

Working Together

Complete each multiplication. Remember, the product of two decimals showing tenths is a decimal showing hundredths.

1.		2.		3.	4.
32	3.2	157	15.7	2.5	36.4
26	2.6	34	34	4.7	15.9
832	8.32	5338	533.8	11.75	578.76

Multiply. Then change the order of the factors to check your work.

5. 5.9 × 3.8 = 22.42 — Check using 3.8 × 5.9
6. 6.8 × 2.3 = 15.64
7. 7.4 × 13.2 97.68
8. 29.5 × 34.6 1020.70

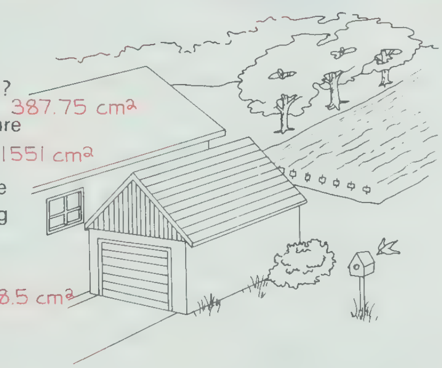
Exercises

Multiply. Then change the order of the factors to check your work.

1. 4.8 × 2.3 = 11.04
2. 4.7 × 8.2 = 38.54
3. 3.7 × 5.5 = 20.35
4. 7.3 × 3.7 = 27.01
5. 6.6 × 8.3 = 54.78
6. 23.4 × 9.6 = 224.64
7. 689 × 2.5 = 1722.5
8. 56.7 × 86.4 = 4898.88
9. 82.5 × 142 = 11715.0
10. 259.2 × 93.6 = 24 261.12
11. 2.9 × 9.5 27.55
12. 55.4 × 7.8 432.12
13. 4.8 × 25.9 124.32
14. 76.5 × 47.4 3626.10

Solve.

15. How many square centimetres of glass are in a pane that is 16.5 cm wide and 23.5 cm long? 387.75 cm²
16. How many square centimetres are in all 4 panes of the window? 1551 cm²
17. The 2 boards on the roof of the birdhouse are each 11.5 cm long and 9.5 cm wide. How many square centimetres of wood are in the roof of the birdhouse? 218.5 cm²
18. The rows in the garden are 0.5 m apart. How far apart are the first and fifth rows? 2.0 m



165

RELATED ACTIVITIES

• For further practice, you may wish to have the students complete some of Ex. 67-80 on page 335.

• Have students use metre sticks marked in tenths of a metre to measure the length and the width of various rectangular objects (to the nearest tenth of a metre). Ask them to multiply to find the approximate area of each object. Display the results in a chart.

Object	Length	Width	Area
door mat	0.9 m	0.5 m	0.45 m²
sidewalk			
table top			
kitchen counter			
piano bench			
rug			
corridor			

Working Together: Ex. 1 and 2 help to show the similarity between multiplying whole numbers and multiplying decimal tenths. Note that the students are to check their work for Ex. 5-8 by changing the order of the factors and multiplying. Ask them to use this method to check the multiplication shown on page 164.

Exercises: Note that the unit of measurement in Ex. 15-17 involves square centimetres. For Ex. 18, suggest to the students that they draw a diagram to represent the situation. They should note that the distance between the first and fifth rows involves four spaces, each 0.5 m wide.

Assessment

Multiply.

1. 2.6 × 1.3 = 3.38
2. 4.7 × 2.9 = 13.63
3. 12.3 × 5.5 = 67.65
4. 22.8 × 4.4 100.32
5. 13.9 × 17.6 244.64

Solve.

6. A garage door is 2.4 m high and 4.5 m wide. How many square metres are there in the area of the door? 10.80

Page 166

OBJECTIVE

Demonstrate competence in multiplying decimals and estimating products

Prerequisite Skills

Round one-place decimals to the nearest whole number

Checking Prerequisite Skills

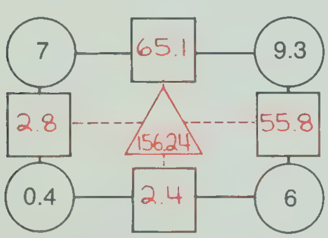
Round to the nearest whole number.

1. 4.1 4 **2.** 3.6 4 **3.** 6.5 7

4. 9.9 10 **5.** 17.4 17 **6.** 32.8 33

RELATED ACTIVITIES

- Provide further practice in the form of diagrams prepared from copies of page T 390. Write factors in the circles and have students write products in the squares. For the example shown below, students may multiply the numbers obtained in opposite squares and show the product in the triangle. If the products are equal, their work is correct. Students may investigate this concept for diagrams for which they write their own factors, using two whole numbers and two decimal tenths.



Practice

Rounding can help you estimate the product of two decimals.

Example:

For the product of 2.8 and 7.3,

round 2.8 → 3
round 7.3 → 7
then multiply → 21

Multiply for the exact product.

2.8
7.3
84
19 60
20.44

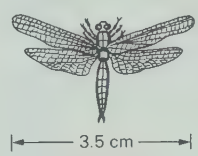
The estimate also helps you remember where to place the decimal point.

Round and multiply to estimate the product. Then find the exact product. Estimates may vary.

1. 8.1 × 5.6 — 45.36 (48)
2. 2.8 × 6.3 — 17.64 (18)
3. 9.2 × 1.4 — 12.88 (9)
4. 7.7 × 4.9 — 37.73 (40)
5. 3.5 × 9.1 — 31.85 (36)
6. 9.8 × 5.3 — 51.94 (50)
7. 71.4 × 8.6 — 614.04 (630)
8. 29.7 × 6.2 — 184.14 (180)
9. 44.9 × 19.5 — 875.55 (900)
10. 31.4 × 28.6 — 898.04 (900)

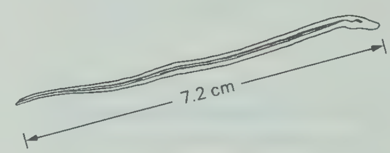
Multiply to answer the questions about these garden creatures.

11. A dragonfly's wingspan can be 1.8 times as wide as this. How wide can it be?



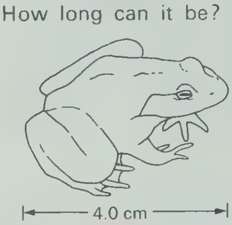
6.30 cm

12. A garter snake can be 12.5 times as long as this. How long can it be?



90.00 cm

13. A frog can be 1.9 times as long as this. How long can it be?



7.60 cm

14. A hummingbird can be 2.2 times as long as this. How long can a hummingbird be?

9.90 cm

LESSON ACTIVITY

Using the Page

- Have the students study the example provided. Then ask several of them to explain the procedure shown. Pay particular attention to placing the decimal point in the exact product, on the basis of the estimate obtained. For example, answers such as 2.044 and 204.4 do not make sense if the estimate of the product is 21.
- Ask the students to write an estimate of the product before finding the exact product for each exercise.

The Floating Decimal Point

When a number is multiplied by 0.1,

	tens	ones	tenths	hundredths		tens	ones	tenths	hundredths
0.1 ×	2	8	6		=		2	8	6

each digit takes a value that is one place less.

On a calculator,

0.1 × 28.6 = 2.86

The decimal point "hops" one place to the left.

The digits stay in the same place in the display.

In many calculator displays, the decimal point can be in any of these positions.

sometimes even here

For a calculator with a floating decimal point, the value for each place in the display depends upon the position of the decimal point.

These calculators have a **floating decimal point**.

What will the display on a calculator with a floating decimal point show for each of these? What will be the place value of the 3 in the display?

1. 10 ⊠ 3 ⊟ 30 3 tens
2. 0.01 ⊠ 3 ⊟ 0.03 3 hundredths
3. 10 ⊠ 10 ⊠ 10 ⊠ 10 ⊠ 3 ⊟ 30 000 3 ten thousands
4. 0.1 ⊠ 0.1 ⊠ 0.1 ⊠ 3 ⊟ 0.003 3 thousandths
5. 10 ⊠ 3 ⊠ 0.1 ⊟ 3 3 ones
6. 0.01 ⊠ 3 ⊠ 10 ⊟ 0.3 3 tenths
7. 3000 ⊠ 0.1 ⊠ 0.1 ⊠ 0.1 ⊠ 0.1 ⊟ 0.3 3 tenths
8. 0.3 ⊠ 100 ⊠ 100 ⊠ 0.1 ⊠ 100 ⊟ 30 000 3 ten thousands
9. 0.01 ⊠ 3.5 ⊠ 1000 ⊠ 0.1 ⊠ 100 ⊠ 0.001 ⊟ 0.35 3 tenths
10. 0.1 ⊠ 3729 ⊠ 0.01 ⊠ 100 ⊠ 0.1 ⊠ 1000 ⊠ 0.1 ⊠ 0.001 ⊟ 3.729 3 ones

Calculator

Page 167

OBJECTIVE

Find the product when one or more factors is a power of ten, using a calculator with a floating decimal point

Materials

calculators (optional)

Vocabulary

floating decimal point

RELATED ACTIVITIES

- Ex. 5 can motivate a discussion about the effect of multiplying a number by 10 and multiplying the product obtained by 0.1. The result is the original number. Assign several exercises to enable students to discover this relationship for 10 and 0.1, for 100 and 0.01, and for 1000 and 0.001. Ask students to make up their own exercises and test them on the calculator. Keycharts can help them in recording exercises and the results.

  4.7 ⊠ 100 ⊠ 0.01 ⊟ 4.7

- The concept of a floating decimal point can be demonstrated on the chalkboard if magnets will adhere to it. Write a five-digit numeral on the board and place a round magnet between two of the digits as a decimal point. Have students change the position of the magnet as you indicate such multipliers as 0.001 and 10.

  12345 ⟶ 12.345

UNIVERSITY OF ALBERTA

LESSON ACTIVITY

Before Using the Page

- On page 158, students examined multiplication when one of two factors is 1000, 100, 10, 0.1, 0.01, or 0.001. In that case, the digits of the other factor move to places with greater values for 10, 100, and 1000, and to lesser values for 0.1, 0.01, and 0.001. In this lesson, students discover that on a calculator, the digits remain in the same position, but the location of the decimal point changes to indicate their new place values.

If calculators are available, review the concept of the lesson on page 158, assign a few multiplications such as 10×1.43 and 0.01×132.6, and let the students discover that the digits remain in the same position as before multiplication, but the decimal point appears in a different position. This will lead them to a discussion of the title of the lesson on page 167.

Using the Page

- These exercises can be completed without a calculator if students compute the multiplications in a left-to-right order.

OBJECTIVE

Write an equation to represent the information in a word problem

Vocabulary
equation

RELATED ACTIVITIES

- The statements expressed by the following equations are false.

$4 + 7 = 20 \qquad 0.5 \times 0.3 = 1.5$

If the symbol = is replaced with >, <, or ≠, the sentences become true statements.

$4 + 7 < 20 \qquad 0.5 \times 0.3 \neq 1.5$

(≠ is read "is not equal to".) Provide students with a list of number sentences and ask them to indicate whether the statements are true or false and to ring those which are equations.

$0.6 \times 2 = 1.2$ T

$3.4 + 1.2 < 5.0$ T

$0.3 \times 0.6 = 1.8$ F

- Although the students have probably completed the word problems on page 49, ask them to write equations for the information in the problems.

Writing Equations

Eva's mother told her their gasoline tank holds 40 L. They bought 22 L to fill the tank. How many litres of gasoline were there in the tank before the "fill-up"?

Eva wrote this equation.

n	+	22	=	40
number of litres in the tank		number of litres added		number of litres in a full tank

An equation is a number sentence that has an equals sign (=). Sometimes all the numbers are shown in the sentence. Sometimes another symbol, like the **n** in Eva's equation, is used for a number.

Write an equation for each of these.
Use **n** to stand for the number that is not given. Equations may vary because relationships may often be expressed in more than one way.

1. The sum of two numbers is 231. One of the numbers is 187.
$187 + n = 231$
2. The product of two numbers is 371. One of the numbers is 7.
$7 \times n = 371$
3. The difference of two lengths is 95.7 cm. The greater of the two lengths is 142.3 cm.
$142.3 - n = 95.7$
4. The difference of two amounts of money is \$4.67. The lesser amount is \$0.86.
$n - \$0.86 = \$4.67$
5. The two numbers 23.7 and 162 are to be multiplied.
$23.7 \times 162 = n$
6. One of 2172 and 2127 is to be subtracted from the other.
$2172 - 2127 = n$

Use **n** and write an equation for each of these. What does the **n** represent in each equation?

7. Eva's mother bought 3 cans of motor oil. They cost \$3.45 in all.
$\$3.45 \div 3 = n$
$n = \$1.15$
8. Eva's mother bought 22 L of gasoline. Each litre cost 22.3¢.
$22 \times 22.3¢ = n$
$n = 490.6¢$ or $\$4.91$
9. Eva's mother paid \$3.45 for the motor oil. Her total bill for the gasoline and motor oil was \$8.36.
$\$8.36 - \$3.45 = n$
$n = \$4.91$

PROBLEM SOLVING

168

LESSON ACTIVITY

Before Using the Page

- Write the expression 13 + 8 on the board and ask what number is named. Write 21 to the right of the expression. Then ask what symbol can be written between the two numerals to show that they are names for the same number.

$13 + 8 \ ?\ 21 \qquad 13 + 8 = 21$

Ask if anyone knows the name given to a number sentence that has the symbol =. Have students write examples of other *equations* on the board.

Using the Page

- Have a student read the word problem at the top of the page to introduce the situation. Ask how the equation n + 22 = 40 differs from the equations represented earlier. Discuss the use of the symbol n to represent the number of litres of gasoline in the tank before 22 L were added to fill the tank to 40 L. Emphasize that n represents the number that is not given.
- The emphasis in these exercises is on the use of a symbol such as n to represent the number that is not given in each problem. Since the relationship among the numbers is given, an equation may be written to show this. Finding the value of n in each equation is not required for Ex. 1-6. However, you may wish to have the students include this step when they have finished the page. Note that different equations are possible for an exercise, depending on a student's interpretation. For example, for Ex. 1, each of the following equations is correct.

$$n + 187 = 231$$
$$231 - n = 187$$
$$187 + n = 231$$
$$231 - 187 = n$$

Checking Up

Multiply.

1. 9.5	2. 9.72	3. 8.363	4. 24.6	5. 7.65	6. 5.038
3	8	2	5	4	7
28.5	77.76	16.726	123.0	30.60	35.266

7. 6 × 8.4 50.4 8. 5 × 2.87 14.35 9. 9 × 15.96 143.64 10. 4 × 4.977 19.908

Round and multiply to estimate each product. Estimates may vary

11. 4.7	12. 6.17	13. 7.495	14. 5.8	15. 9.92	16. 31.09
4	3	6	5	2	8
20	18	42	30	20	240

How much

17. for 5 planters if each costs $4.98? $24.90
18. for 8 plants if each costs $1.49? $11.92
19. for 4 window boxes if each costs $12.95? $51.80

Multiply.

20. 6.1	21. 87.5	22. 186	23. $4.54	24. 279	25. 2.618
37	71	8.5	46	1.9	62
225.7	6212.5	1581.0	$208.84	530.1	162.316

26. 38 × 5.2 197.6 27. 43 × $9.37 $402.91 28. 14.9 × 93 1385.7 29. 58 × 53.46 3100.68

Round and multiply to estimate each product. Estimates may vary

30. 54	31. 3.19	32. 4.905	33. 658	34. 8.38	35. $9.87
6.7	31	26	4.2	294	34
350	90	150	2800	2400	$300

Multiply.

36. 0.01 × 75.2 0.752 37. 12.84 × 10 128.4

Complete.

38. 4800 mm = 4.8 m 39. 35 mm = 3.5 cm

Multiply.

40. 0.6	41. 4.5	42. 5.7	43. 8.2	44. 1.6	45. 24.3
0.4	0.9	8.6	6.5	5.3	2.7
0.24	4.05	49.02	53.30	8.48	65.61

46. 0.8 × 6.7 5.36 47. 7.6 × 7.4 56.24 48. 3.2 × 3.8 12.16 49. 9.5 × 12.9 122.55

Round and multiply to estimate each product. Estimates may vary

50. 6.8	51. 4.4	52. 6.9	53. 3.1	54. 8.3	55. 5.7
8.2	7.5	9.6	5.2	7.9	3.4
56	32	70	15	64	18

169

Page 169

OBJECTIVE

Demonstrate an understanding of the concepts and skills presented in this unit

RELATED ACTIVITIES

• Have the students demonstrate the similarity in multiplying whole numbers and multiplying decimals by completing exercises similar to the following. Note that the basic multiplication fact $2 \times 3 = 6$ is applied in each exercise.

3 tens	3 tenths
× 2	× 2
6 tens	6 tenths
$\boxed{60}$	$\boxed{0.6}$
3	3
× 2 tens	× 2 tenths
6 tens	6 tenths
$\boxed{60}$	$\boxed{0.6}$
3 tens	3 tenths
× 2 tens	× 2 tenths
6 hundreds	6 hundredths
$\boxed{600}$	$\boxed{0.06}$

Skills	Exercises	Related Pages
Multiply a decimal by a one-digit whole number	1-10	T 162-T 165
Round and multiply to estimate a product	11-16	T 166
Multiply a decimal by a two-digit or a three-digit whole number	20-29	T 168-T 169
Round and multiply to estimate a product	30-35 50-55	T 168-T 169 T 180
Multiplying with 0.001, 0.01, 0.1, 10, 100, or 1000 as a factor	36, 37	T 172-T 173
Express a measurement given in one unit of length in terms of another unit of length	38, 39	T 174
Multiply two decimal factors	40 41, 46 42-45, 47-49	T 175 T 176-T 177 T 178-T 179
Solve word problems	17-19	

Comments

Note whether errors are a result of poor recall of basic multiplication facts, difficulty with the algorithm for multiplication, or with the concept of decimals. Diagnostic tests can reveal which basic facts need to be memorized. Copies of pages T 381 and T 391 can be used in preparing and conducting these tests. Lesson suggestions and related activities described for Unit 3 can help students to improve their skill in multiplication with two-digit and three-digit whole numbers. An improvement in that area will carry over to multiplication with decimals, for which emphasis should be placed on knowing where to place the decimal point in the product.

Unit 9 Overview

Geometry

This unit deals with concepts related to various shapes. Lines, line segments, and rays are one-dimensional, polygons are two-dimensional, and solids are three-dimensional figures. Parallel lines and intersecting lines are studied, including lines which are perpendicular to each other. Two rays are seen to combine to form an angle, which may be classified as an acute angle, a right angle, or an obtuse angle. Protractors are used both for measuring and for drawing angles. Line symmetry is examined as it is found in triangles and quadrilaterals. The characteristics of pyramids and prisms are studied and names are assigned to them according to the shapes of the end faces, or bases. The solids with curved surfaces — cylinders, spheres, and cones — are examined briefly. Problems which require two or more steps in their solution are featured in the lesson on problem solving.

Prerequisite Skills

- identify a straight path
- read a scale
- locate points on a scale
- recognize shapes that are alike

Unit Outcomes

- identify, name, and draw lines, line segments, and rays
- identify, name, and draw lines and line segments that are intersecting, parallel, or perpendicular
- identify, name, and draw angles
- trace an angle to determine whether it is congruent to another angle
- use a protractor to measure angles
- classify angles as acute, right, or obtuse
- use a protractor to draw an angle having a given measure
- classify polygons
- identify the angles and sides of polygons
- identify and make shapes having line symmetry
- identify and show lines of symmetry
- classify triangles as equilateral, isosceles, or scalene, according to the number of sides of equal length and/or the number of lines of symmetry
- classify quadrilaterals according to parallel sides and lines of symmetry
- classify pyramids and prisms by the shapes of their bases
- identify the number of faces, edges, and vertices of solid shapes
- identify the solid for a given pattern
- draw a pattern for a given solid
- identify cylinders, spheres, and cones
- solve problems involving two or more steps

Background

At this level, students acquire most of their geometric concepts through manipulation and observation. In some instances they can make comparisons visually and draw conclusions, while in others they are obliged to use tracings and cutouts. Certain basic concepts underlie all aspects of geometry, namely, space, plane, point, line, line segment, ray, and angle. Some of these concepts are undefined and basic assumptions are made concerning them.

The term *point* is undefined. On the abstract level, a geometric point is a particular location in space. A point is immovable and has no length, width, or thickness. The closest approximation to a point is the smallest dot on paper or the point of a needle. All of the other geometric terms are derived in terms of points. These are discussed here as a background for the teacher, but students are not expected to describe them in this manner.

Space is considered to be the set of all points.

A *plane* is the set of all points on a flat surface whose length and width extend endlessly.

A *line* is thought of as a set of points forming a straight path that continues without end in opposite directions.

A *line segment* is part of a line. It has two end points and includes all the points that lie between them.

A *ray* is part of a line. It has one end point and continues without end in one direction.

An *angle* is formed by two rays having a common end point which is called a *vertex*.

The preceding terms which deal primarily with plane figures, called *polygons*, are useful in defining features of solid figures having plane surfaces.

A *face* of a solid is part of a plane.

An *edge* of a solid is the line segment formed where two faces of the solid meet.

A *vertex* of a solid is the common end point of three or more edges.

Polygons are closed figures composed of line segments. It should be pointed out that although polygons are often represented by cutouts of paper or wood, it is the shape, not the surface, made by the line segments that is the polygon. Models made of drinking straws and pipe cleaners approximate the concepts of polygons more closely than cardboard cutouts. The least number of line segments, or *sides*, of a polygon is three. A *triangle* has three sides. A *quadrilateral* has four sides. A quadrilateral with one pair of opposite sides parallel is a *trapezoid*. A quadrilateral with both pairs of opposite sides parallel is a *parallelogram*. A parallelogram with all four sides equal in length is a *rhombus*. A parallelogram with four right angles is a *rectangle*. A rectangle with all four sides equal in length is a *square*. A *pentagon* has five sides; a *hexagon* has six sides; and an *octagon* has eight sides. A polygon is said to be a *regular polygon* if the sides are of equal length and the angles are of equal measure.

If two figures have the same size and shape, they are said to be *congruent*. In terms of their points, they may be matched one to one. Figures which have *line symmetry* can be divided into two congruent parts, but one of the parts must be flipped over to match the other part.

Much of the beauty in nature, in designs, and in manufactured articles comes from symmetrical forms which can often be seen in them. Line symmetry is found in the shape of a butterfly with outstretched wings, of a gull in flight, or of a snowflake. A shape which has line symmetry can be thought of as being divided into two identical parts by a line, known as the *line of symmetry*. Polygons are frequently classified according to the number of equal sides, the number of parallel sides, or the number of equal angles that they have. Some polygons have lines of symmetry and these may be used to classify them. A

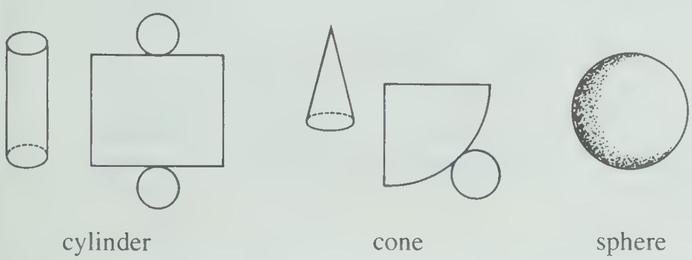
triangle with three lines of symmetry is an *equilateral* triangle; a triangle with one line of symmetry is an *isosceles* triangle; and a triangle with no line of symmetry is a *scalene* triangle. Lines of symmetry are also found in some quadrilaterals. A square has four lines of symmetry, a rectangle has two, and a rhombus also has two.

Some three-dimensional figures, or *solids*, consist of polygons sharing sides. Again, it should be emphasized that it is the shape, not the enclosed space, to which the term *solid* refers. In a solid, two faces meet to form an edge, and three or more edges meet to form a vertex.

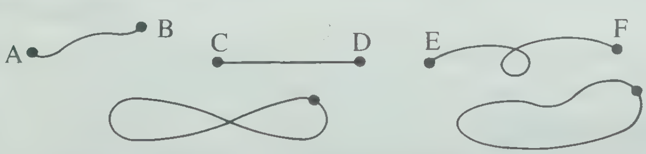
The least number of faces of a solid is four, and this shape is called a *triangular pyramid* (see page 340). Other kinds of pyramids, such as *square pyramid*, *rectangular pyramid*, and *hexagonal pyramid*, derive their names from the shapes of their bases, but in every case the other faces are triangles.

Another type of solid is the *prism* which has two congruent parallel faces and three or more other faces which are rectangles or parallelograms (see page 340). It is the shapes of the bases that give the descriptive names to various kinds of prisms, such as *triangular prism*, *rectangular prism*, and *pentagonal prism*. A prism with each of its six faces a square is called a *cube*.

Solids which have circular faces and surfaces are the *cylinder*, the *cone*, and the *sphere*. A cylinder has two parallel circular bases and a lateral face which is a rolled-up rectangle. A cone has a circular base, and the lateral face comes to a point, or vertex; the lateral face is a portion of another circular shape. A sphere is completely circular in every direction; in other words, all points on a sphere are equidistant from one single point, called its center.



In terms of points, a *curve* is a path between two points. The curve AB is a *simple curve*; the curve CD is the shortest curve between points C and D and is a *line segment*; and the curve EF is not a simple curve because it crosses over itself. A *closed curve* is a curve that returns to its starting point, and a *simple closed curve* does so without crossing over itself.



In this unit the students work with problems for which solutions require two or more steps. In solving a two-step problem it is necessary to determine what operation must be performed first and with which numbers, and then how that result may be used with another number. Complex problems which involve more than two steps require this procedure to be repeated. To solve problems, students require abilities to read with comprehension and to visualize the situations. They need to "feel" what is happening in the situation so that they can sort out the numbers and set them up in appropriate mathematical relationships.

Teaching Strategies

If the class has been grouped by ability for instruction in the preceding units, the students may be regrouped or taken as a single group since the concepts and skills for this unit are not dependent on number.

The activities require, in some instances, special materials such as tracing paper, semitransparent plexiglass mirrors, cardboard strips and paper fasteners, and models of plane and solid figures. If the supply of some items is limited, grouping may be required so that every student is able to have direct experience with the materials. This procedure would provide time for remedial work and practice in operations and for enrichment activities for students who must wait their turn.

In connection with the study of angles it is important to lead the students to realize that the size of an angle is not affected by the lengths of the rays. A protractor does not measure rays; it measures the amount of rotation from one ray to another having the same vertex.

In connection with triangles, students should also be led to see that the total length of any two sides of a triangle is always greater than the length of the third side.

The teacher may wish to follow the *Checking Up* lesson on page 192 with an oral evaluation of a small group of students at one time, using models for the items named or illustrated.

Materials

straight edge for each student, metre stick

tracing paper, a pronged paper fastener and two narrow strips of cardboard shaped like arrows for each student

a protractor for each student

a transparent protractor and an overhead projector, acetate sheets (optional)

scissors, blank sheets of paper for each student; semitransparent plexiglass mirrors; large sheet of paper, magazines, catalogs

a centimetre ruler for each student, copies of the triangles on page T 383

models of prisms and pyramids named on page 188; paper model of a triangular prism prepared from the pattern on page T 386; copies of patterns from pages T 386-T 388 for each student

objects that suggest cylinders, spheres, and cones

Vocabulary

line	rhombus	quadrilateral
line segment	rectangle	pentagon
ray	square	hexagon
point	kite	octagon
end points	angle (∠)	solid
intersecting	vertex	base, face
intersection	congruent angles	pyramid
parallel	protractor	prism
perpendicular	center	triangular pyramid
simple closed curve	base line	square pyramid
line symmetry	degrees	rectangular pyramid
line of symmetry	acute angle	triangular prism
Mira	right angle	rectangular prism
equilateral	obtuse angle	cube
isosceles	straight angle	edge
scalene	polygon	cylinder
trapezoid	vertices	sphere
parallelogram	triangle	cone

LESSON OUTCOME

Identify, name, and draw lines, line segments, and rays

Materials

straight edge for each student

Vocabulary

line, line segment, ray, point, end points

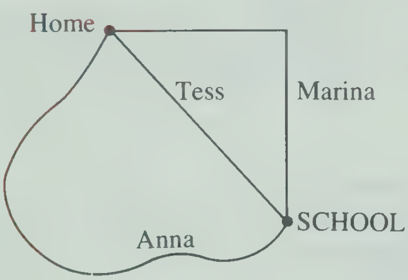
Prerequisite Skills

Identify a straight path

Checking Prerequisite Skills

Which sister walked a straight path home from school?

1. Tess



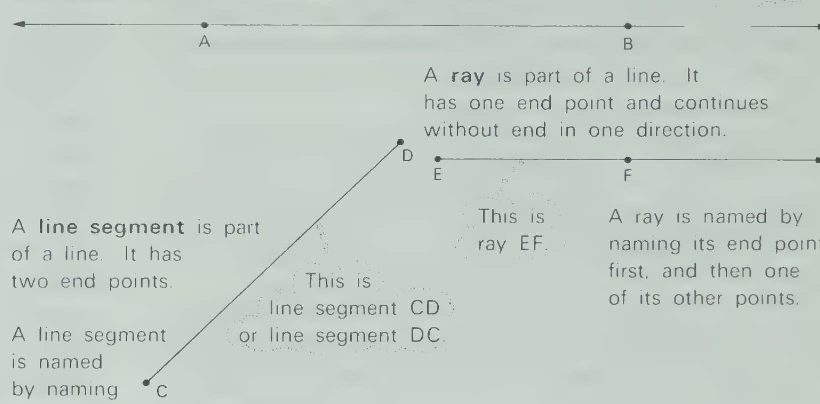
9 GEOMETRY

Lines, Line Segments, and Rays

A **line** is straight. It continues without end in opposite directions.

A line is named by naming two of its points in either order.

This is line AB or line BA.



A **ray** is part of a line. It has one end point and continues without end in one direction.

This is ray EF.

A ray is named by naming its end point first, and then one of its other points.

A **line segment** is part of a line. It has two end points.

This is line segment CD or line segment DC.

A line segment is named by naming its end points in either order.

Working Together

Study the first three rows of this chart. Then complete the rest of the chart.

Sometimes these symbols are used.

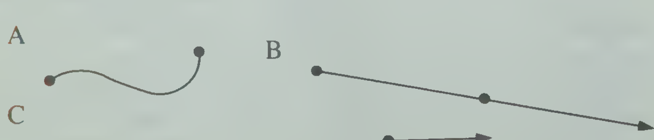
	See	Say or write	
1.	A B	line segment AB or line segment BA	$\overline{AB}$ or $\overline{BA}$
2.	K L	ray KL	$\overrightarrow{KL}$
3.	K L	ray LK	$\overrightarrow{LK}$
4.	C M	line CM or line ? MC	$\overleftrightarrow{CM}$ or $\overleftrightarrow{MC}$
5.	R S ?	line segment RS or ? line segment SR	$\overline{RS}$ or $\overline{SR}$
6.	H G	ray GH ?	$\overrightarrow{GH}$
7.	Z ? Y	line ZY or ? line YZ	$\overleftrightarrow{ZY}$ or $\overleftrightarrow{YZ}$

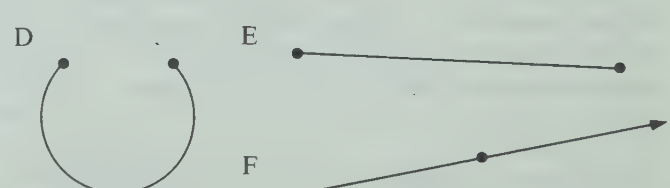
170

LESSON ACTIVITY

Before Using the Pages

- Draw the following diagrams on the board. Have students describe how the diagrams are alike and how they are different. For example, some paths are straight and some are curved; some have arrowheads and others do not. Ask what the dots represent and, if possible, elicit the word *point* from the students. Have a student identify the diagram of a straight path that continues without end in opposite directions (C). Ask what parts of the diagram imply "without end". Then have students identify diagrams that show straight paths that continue in just one direction without end (B and F). Ask how diagram E is different from diagrams B and C.

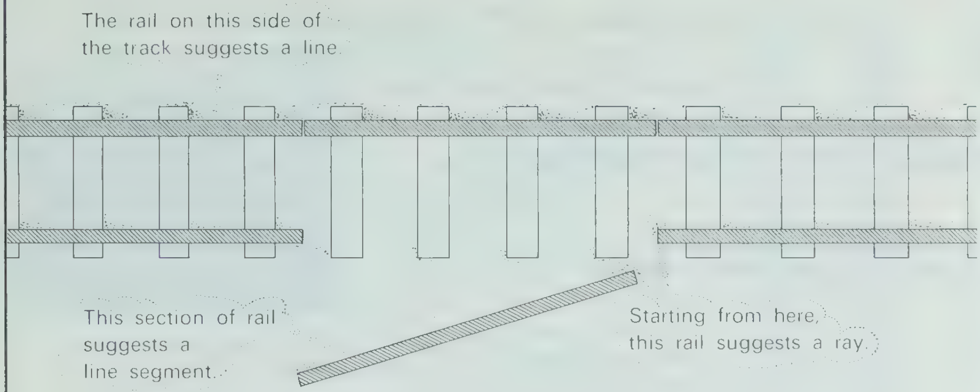




Using the Pages

- The examples at the top of page 170 introduce the terms *line*, *ray*, and *line segment*, as well as the use of capital letters to name points and figures. Have different students read the statements aloud. Pay particular attention to the fact that lines and line segments may be named two ways for two given points because order is not significant. However, in naming a ray, order is significant. For example, ray EF is different from ray FE.

  Have students write capital letters for the points indicated on diagrams B, C, E, and F on the board. Have other students use the letters to state the names of the figures.

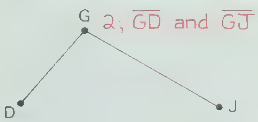


Exercises

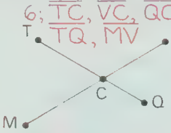
For line segments, the letters may be given in either order.

How many line segments are there? Name them.

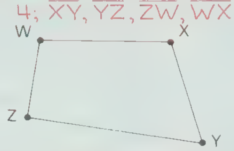
1\. 2; $\overline{GD}$ and $\overline{GJ}$



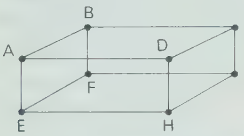
2\. 6; $\overline{TC}$, $\overline{VC}$, $\overline{QC}$, $\overline{MC}$, $\overline{TQ}$, $\overline{MV}$



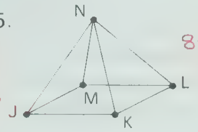
3\. 4; $\overline{XY}$, $\overline{YZ}$, $\overline{ZW}$, $\overline{WX}$



4\. 12; $\overline{AE}$, $\overline{BF}$, $\overline{AB}$, $\overline{EF}$, $\overline{BC}$, $\overline{FG}$, $\overline{EH}$, $\overline{AD}$, $\overline{DC}$, $\overline{HG}$, $\overline{DH}$, $\overline{CG}$

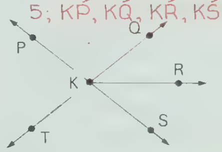


5\. 8; $\overline{NJ}$, $\overline{NM}$, $\overline{NK}$, $\overline{NL}$, $\overline{ML}$, $\overline{LK}$, $\overline{KJ}$, $\overline{JM}$



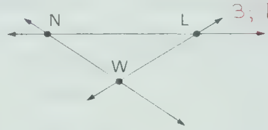
How many rays are there? Name them.

6\. 5; $\overrightarrow{KP}$, $\overrightarrow{KQ}$, $\overrightarrow{KR}$, $\overrightarrow{KS}$, $\overrightarrow{KT}$



How many lines are there? Name them.

7\. 3; $\overleftrightarrow{NL}$, $\overleftrightarrow{WL}$, $\overleftrightarrow{NW}$

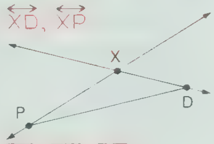


8\. Name two lines, two rays, and two line segments shown in this picture.

$\overleftrightarrow{XD}$, $\overleftrightarrow{XP}$

$\overrightarrow{XD}$, $\overrightarrow{XP}$, $\overrightarrow{PX}$, $\overrightarrow{DX}$

$\overline{PD}$, $\overline{XD}$, $\overline{XP}$



Draw and label these.

9\. line TS   T S

10\. ray EF   E F

11\. line segment YU   Y U

RELATED ACTIVITIES

- Have students contribute suggestions for items in a class chart similar to the following.

These suggest		
lines	rays	line segments
telephone cables	beams of light from the sun	hydro poles

- Have students consider in turn each letter of the alphabet to determine which capital letters can be formed using line segments. Have them print the letters and write the number of line segments they drew to form each letter.

A	E	F
3	4	3

Draw the students' attention to the illustration at the top of page 171. Discuss the suggestions provided and have students give other similar examples for objects in the classroom. For instance, a pencil may suggest a line segment.

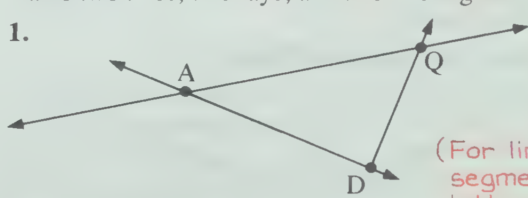
Working Together: Ex. 1-3 are examples that should be discussed with the students before they complete the chart. Have them use their straight edges to draw diagrams for Ex. 5 and 7. Use other similar exercises as required. The chart to the right of Ex. 1-7 shows the symbols that are sometimes used as a short way of naming these figures. Their use is optional at this time because the students will not encounter the symbols in the exercises.

Exercises: Answers may vary for Ex. 8. Remind the students to use their straight edges for Ex. 9-11.

Assessment

Name two lines, two rays, and two line segments in this picture.

1\.



$\overleftrightarrow{AD}$, $\overleftrightarrow{AQ}$

$\overrightarrow{AQ}$, $\overrightarrow{AD}$, $\overrightarrow{DQ}$

$\overline{AQ}$, $\overline{AD}$, $\overline{DQ}$

(For lines and line segments, the letters may be given in either order.)

Draw and label these.

2\. line JK   J K

3\. ray TV   T V

4\. line segment RL   R L

Pages 172-173

LESSON OUTCOME

Identify, name, and draw lines and line segments that are intersecting, parallel, or perpendicular

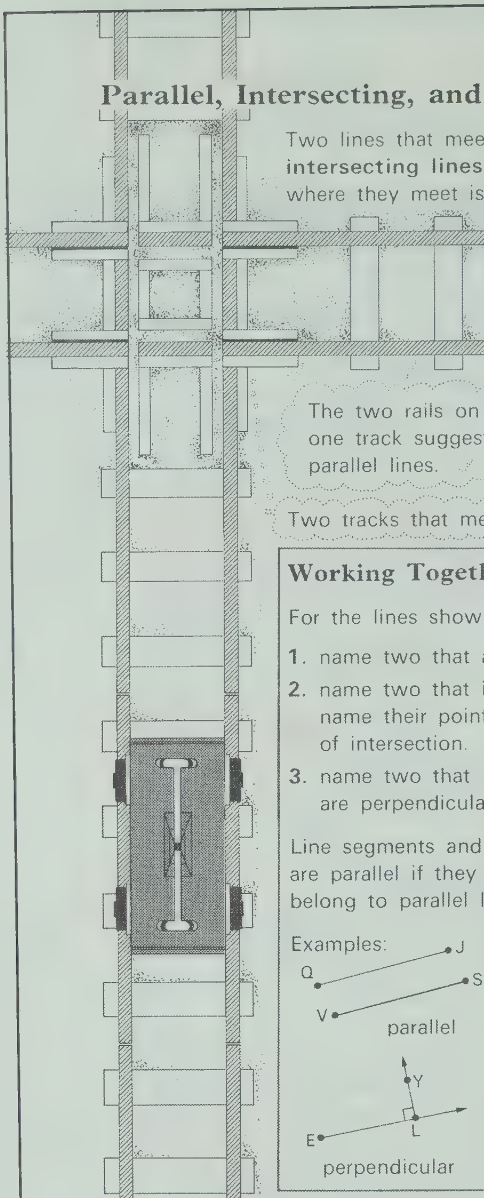
Materials

metre stick, straight edge for each student

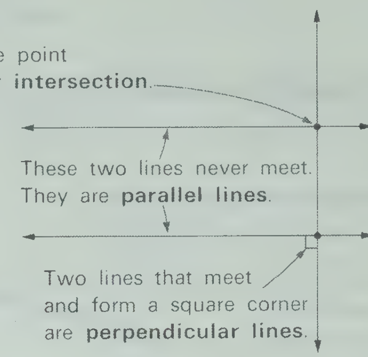
Vocabulary

intersecting, intersection, parallel, perpendicular

Parallel, Intersecting, and Perpendicular Lines



Two lines that meet are **intersecting lines**. The point where they meet is their **intersection**.



These two lines never meet. They are **parallel lines**.

Two lines that meet and form a square corner are **perpendicular lines**.

The two rails on one track suggest parallel lines.

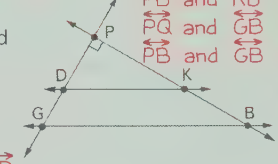
Two tracks that meet suggest intersecting lines.

Working Together

For the lines shown,

1. name two that are parallel. $\overleftrightarrow{DK}$ and $\overleftrightarrow{GB}$
2. name two that intersect and name their point of intersection.
3. name two that are perpendicular. $\overleftrightarrow{PG}$ and $\overleftrightarrow{PB}$

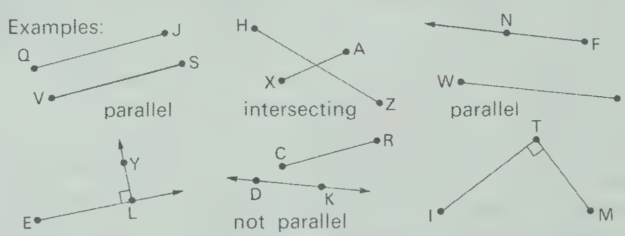
2. $\overleftrightarrow{PG}$ and $\overleftrightarrow{PB}$ at P or
$\overleftrightarrow{PG}$ and $\overleftrightarrow{DK}$ at D or
$\overleftrightarrow{PB}$ and $\overleftrightarrow{KD}$ at K or
$\overleftrightarrow{PQ}$ and $\overleftrightarrow{GB}$ at G or
$\overleftrightarrow{PB}$ and $\overleftrightarrow{GB}$ at B



Line segments and rays are parallel if they belong to parallel lines.

They are perpendicular if they form square corners.

Examples:

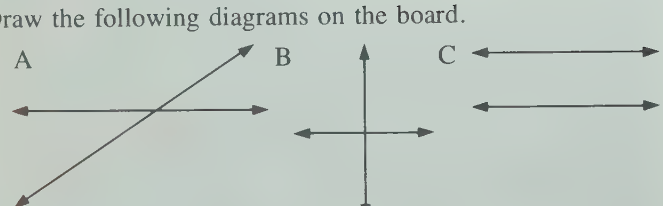


172

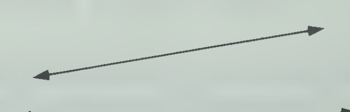
LESSON ACTIVITY

Before Using the Pages

- Draw the following diagrams on the board.



Ask for the number of lines in each diagram and have students describe how the lines are situated. They may say that the lines cross each other in A and B but not in C. Have them describe the way the lines cross in B; for example, they form a square corner. Review the significance of the arrowheads in the diagrams. Ask whether the two lines in C would ever cross each other (if, for instance, the diagram were larger). Place a metre stick on the board and draw two lines that do not intersect; that is, move the chalk along the upper edge and then along the lower edge while the metre stick is held in one place. Ask whether the two lines will ever meet. Then draw the following diagram and repeat the question.



Using the Pages

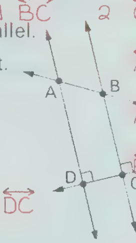
- The examples at the top of page 172 introduce the terms *intersecting*, *intersection*, *parallel*, and *perpendicular*. Have students read the statements aloud, assisting them with pronunciation as required. Point out the symbol that marks the square corner indicating perpendicular lines. Have the students trace parallel lines and perpendicular lines with their fingers in the illustration of the railway tracks on page 172. Then draw their attention to the photograph at the top of page 173 and ask whether the rails

Exercises

For lines and line segments, the letters may be given in either order

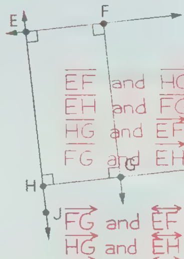
For the lines shown, $\overleftrightarrow{AD}$ and $\overleftrightarrow{BC}$

1. name two that are parallel.
2. name two that intersect. Name their point of intersection.
   2 $\overleftrightarrow{CB}$ and $\overleftrightarrow{AB}$ at B; $\overleftrightarrow{AD}$ and $\overleftrightarrow{AB}$ at A; $\overleftrightarrow{AD}$ and $\overleftrightarrow{DC}$ at D; $\overleftrightarrow{BC}$ and $\overleftrightarrow{DC}$ at C
3. name two that are perpendicular.
   $\overleftrightarrow{AD}$ and $\overleftrightarrow{DC}$; $\overleftrightarrow{BC}$ and $\overleftrightarrow{DC}$



For this diagram,

4. name two intersecting line segments and their point of intersection.
5. name a line segment and a ray that are parallel.
   $\overline{EF}$ and $\overrightarrow{HG}$; $\overline{EH}$ and $\overrightarrow{FG}$; $\overline{HG}$ and $\overrightarrow{EF}$ or $\overrightarrow{FE}$; $\overline{FG}$ and $\overrightarrow{EH}$ or $\overrightarrow{HE}$
6. name a ray and a line that are perpendicular. Also, name their point of intersection.
   $\overrightarrow{FG}$ and $\overleftrightarrow{EF}$ at F; $\overrightarrow{HG}$ and $\overleftrightarrow{EH}$ at H; $\overrightarrow{EF}$ and $\overleftrightarrow{EH}$ at E



4 Possible answers are $\overline{EF}$ and $\overline{EH}$ at E, $\overline{HG}$ and $\overline{GF}$ at G, $\overline{EH}$ and $\overline{HG}$ at H, $\overline{EF}$ and $\overline{FG}$ at F.

Draw

Answers will vary. Examples are given below

7. two intersecting rays.
8. two line segments that are parallel.
9. two line segments that are not parallel and do not intersect.
10. a line segment perpendicular to a ray.
11. a line segment parallel to a line.
12. line AB perpendicular to ray CD.
13. ray RE and ray RG.
14. line segment PQ parallel to ray VW.
15. line segment KL perpendicular to line ST.

Look around. Make a chart like this.

16.

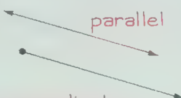
Things parallel	Things perpendicular
fence posts	two edges of the corner of a picture

Answers will vary

Which of these words can you use for each picture?

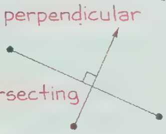
parallel
perpendicular
intersecting

4. parallel



5. perpendicular

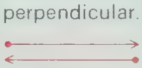
intersecting



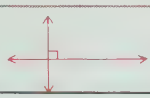
Draw

6. two parallel rays.
7. two lines that are perpendicular.

6.

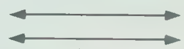


7.



RELATED ACTIVITIES

• Have students help to prepare a large chart showing geometric figures and terms introduced in this unit. It will assist them in remembering new concepts. As new concepts are introduced they may be added to this chart or continued in a new chart.

Name	Diagram
line	
ray	
line segment	
parallel lines	
perpendicular lines	
intersecting lines	

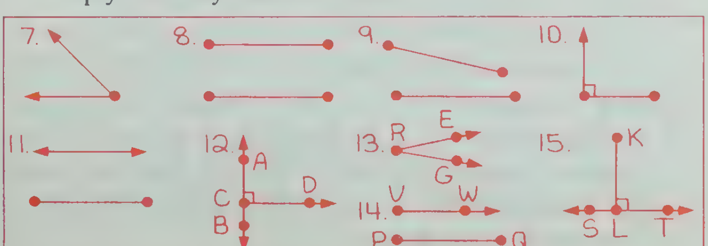
• Have students identify and draw the mathematical symbol for each of the following clues.

Clue	Symbol	Meaning
a line segment parallel to and the same length as another segment	=	equals
a line segment and two points, one on either side of the line segment	÷	divided by
two intersecting line segments that are perpendicular	+	add
one line segment	−	subtract
two line segments that share one end point	< (>)	is less than (is greater than)

of the railway track appear to be parallel or whether they are parallel.

Working Together: Have the students write their answers on the board for Ex. 1-3 and then discuss them. Discuss the examples below Ex. 3. For each example, have a student name the figures and describe them. For instance, ray FN is parallel to line segment WU (line segment UW). Then continue in a similar manner for Ex. 4 and 5. For Ex. 6 and 7, remind the students to use a straight edge. Also, have them use capital letters to name points on their figures.

Exercises: Note that the names given for the two rays in Ex. 13 imply that they share the vertex R.

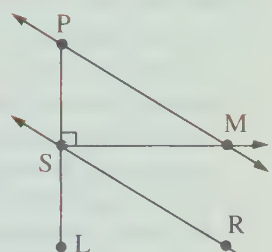


Assessment

For lines and line segments, the letters may be given in either order.

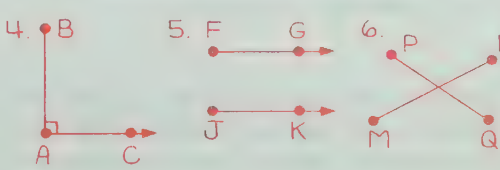
For the diagram at the right, $\overleftrightarrow{PM}$ and $\overleftrightarrow{SR}$

1. name two lines that are parallel.
2. name a line segment and a ray that are perpendicular. $\overline{PS}$ and $\overrightarrow{SM}$
3. name a line and a line segment that intersect. Name their point of intersection. $\overleftrightarrow{SR}$ and $\overline{PL}$, S; $\overleftrightarrow{PM}$ and $\overline{PL}$, P



Draw

4. line segment AB perpendicular to ray AC.
5. ray FG parallel to ray JK.
6. two intersecting line segments, MN and PQ.



Pages 174-175

LESSON OUTCOME

Identify, name, and draw angles; trace an angle to determine whether it is congruent to another angle

Materials

tracing paper and a straight edge for each student, a pronged paper fastener and two narrow strips of cardboard shaped like arrows for each student

Vocabulary

angle (∠), vertex, congruent angles

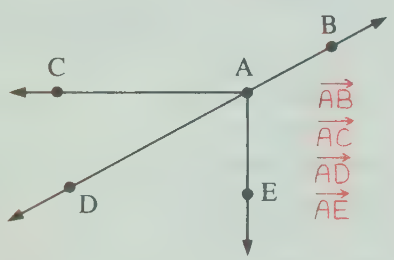
Prerequisite Skills

Identify, name, and draw rays

Checking Prerequisite Skills

How many rays are there? Name them. 4

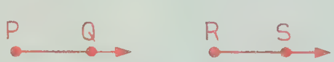
1.



$\overrightarrow{AB}$
$\overrightarrow{AC}$
$\overrightarrow{AD}$
$\overrightarrow{AE}$

Draw and label these.

2. ray PQ **3.** ray RS

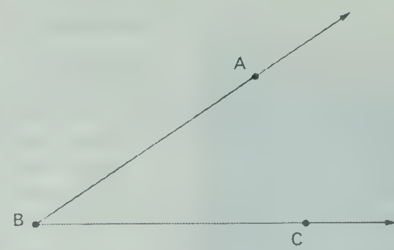


Angles

Two rays that have the same end point form an **angle**.

The common end point is the **vertex** of the angle.

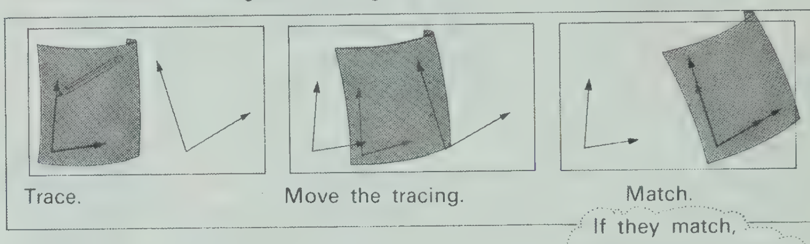
B is the vertex of this angle.



An angle is named by naming the vertex and one other point on each ray.

The vertex letter is always the middle letter in the angle name. The angle above is angle ABC or angle CBA. A shorter way to show the name is ∠ABC or ∠CBA.

Two angles that are the same size are **congruent angles**. You can use tracing paper to test whether two angles are congruent.



Trace. Move the tracing. Match.

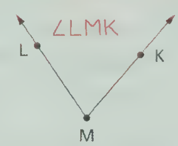
If they match, they are congruent.

Working Together

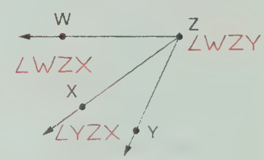
Name each angle. Names will vary; the vertex is always named in the middle.

1.

2.



3.

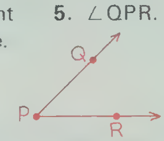


Use tracing paper.

4. Test whether ∠DGH is congruent to ∠KML in the exercises above.
They are not congruent.

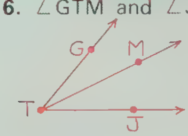
Draw

5. ∠QPR.



Answers will vary

6. ∠GTM and ∠JTM.



174

LESSON ACTIVITY

Before Using the Pages

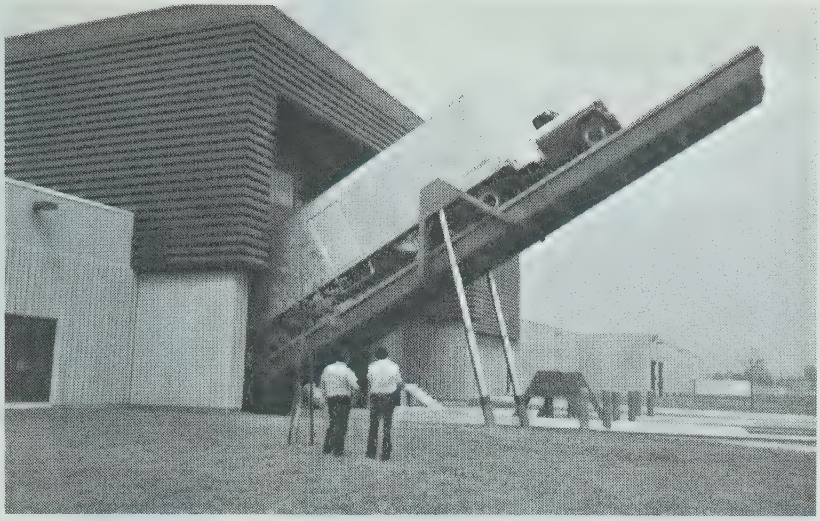
- Give each student a pronged paper fastener and two narrow strips of cardboard, each cut to a point at one end and having a hole in the other end. Tell the students that each strip represents a ray and the hole represents the end point of the ray. Have them use the fastener to join the two "rays" so that they share end points. The strips may now be moved to represent angles of different sizes. Have the students use the strips to match angles formed by objects in the classroom. Many will be angles that form a square corner, but other angles can be found. Some examples are angles formed by the blades of scissors, by two fingers on one hand, and by the hands of a dial clock.

Using the Pages

- The term *angle* probably was mentioned during the preliminary activity. Have a student read the statement at the top of page 174 to formalize the concept. Note, also, the term *vertex* and have students point to the vertex of their cardboard angle models. Have the students use their fingers to trace ∠ ABC in the example, starting first at point A and then again at point C. This will help them to understand why the letter at the vertex is always the middle letter in the name of an angle. Point out the symbol ∠ for "angle" and caution the students to take care in drawing the symbol so that it does not appear to be the symbol < for "is less than". The lower line segment of the symbol for "angle" should be horizontal.

  Introduce the concept of congruent angles and the method of checking for congruent angles by using tracing paper. Also, have each student work with a partner so that one student uses the angle model to form an angle and the other makes a congruent angle using her/his angle model.

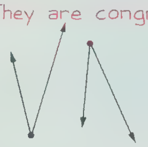
Working Together: Note that Ex. 3 shows three angles. Provide the students with tracing paper for Ex. 4. Have them use a straight edge for Ex. 5 and 6. Point out that the angles named in Ex. 6 share letters, and thus they would share points in the diagram.



Exercises

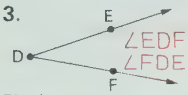
Use tracing paper. They are congruent

1. Test whether these two angles are congruent.

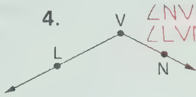


2. Test whether ∠ABC on page 174 and the angle formed by the ground and the ramp holding the truck are congruent.
They are congruent

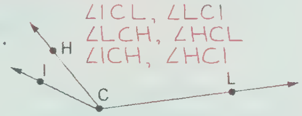
Write two names for each angle shown.

3. 

∠EDF
∠FDE

4. 

∠NVL
∠LVN

5. 

∠ICL, ∠LCI
∠LCH, ∠HCL
∠ICH, ∠HCI

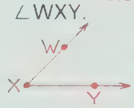
Find

6. two angles that are congruent in Exercises 3, 4, and 5.
∠NVL and ∠HCL

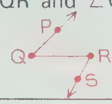
Draw

Answers will vary.

7. ∠WXY.



8. ∠PQR and ∠QRS.



Look around. Make a chart like this.

9.

Examples of angles	Examples of congruent angles
the two hands of a clock	the tips of a ★

Answers will vary.

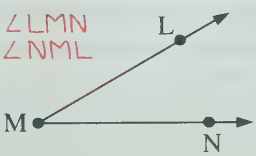
175

Exercises: The students will require tracing paper for Ex. 1, 2, and 6.

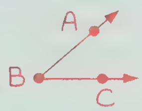
Assessment

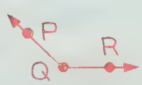
Write two names for this angle.

1. ∠LMN
∠NML



Draw





2. ∠ ABC. 3. ∠ PQR.

Use tracing paper.

4. Test whether ∠ ABC is congruent to ∠ PQR.
Answers will vary.

RELATED ACTIVITIES

- New concepts from this lesson may be added to the chart suggested in *Related Activities* on page T 189.
- Have students use geoboards and/or geopaper (copies of page T 396) to show pairs of line segments that are parallel, perpendicular, and intersecting, and angles that are congruent.
- Many students will enjoy discovering that line segments can form a curve through the process of "curve stitching". A pattern and directions for its use are given on page T 378.

LESSON OUTCOME

Use a protractor to measure angles; classify angles as acute, right, or obtuse

Materials

protractor for each student, angle models prepared in the previous lesson

Vocabulary

protractor, centre, base line, degrees, acute angle, right angle, obtuse angle

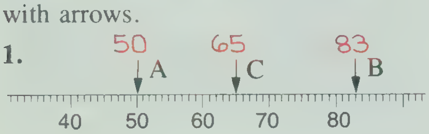
Prerequisite Skills

Read a scale

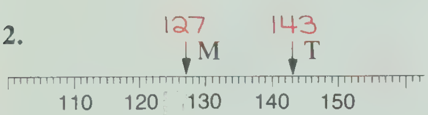
Checking Prerequisite Skills

Write a number for the points marked with arrows.

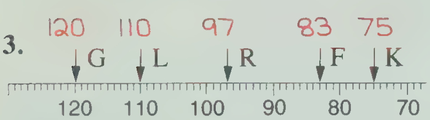
1.



2.



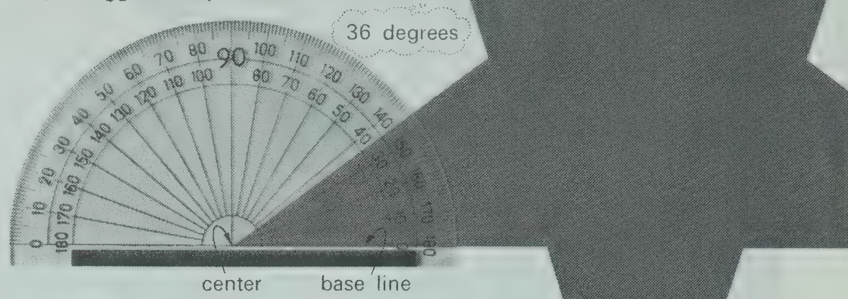
3.



Measuring Angles

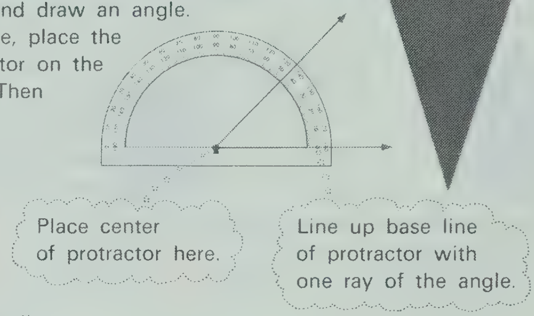
A **protractor** is used for measuring angles. The units used on a protractor are **degrees**.

The angle suggested by the star measures 36°

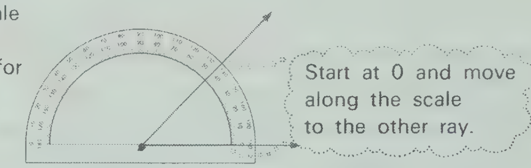


Working Together

Use a straight edge and draw an angle. To measure your angle, place the base line of a protractor on the vertex of the angle. Then line up the base line of the protractor with one ray of the angle.



Start at 0 on the base line and move along the scale to the other ray. Read the number of degrees for the angle measurement.



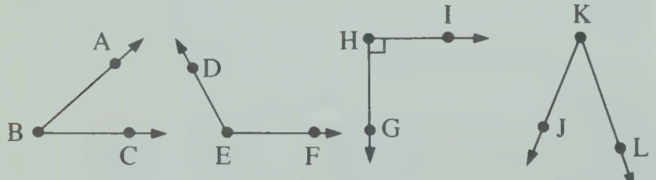
1. What does your protractor show for the measurement of the angle you drew? Answers will vary
2. What does your protractor show for the measurement of the angle shown above? 45°

176

LESSON ACTIVITY

Before Using the Pages

- Draw ∠ ABC on the board so that line segments BA and BC are each at least 50 cm long. Have students come to the board and use their angle models to form an angle congruent to ∠ ABC. Repeat the procedure for the other angles shown below.



Have students suggest which angle is the largest and then use their angle models to check. Develop that the measure of an angle depends on the amount of rotation of one ray from the other. Demonstrate this by matching the two arrows of an angle model with ray BC of ∠ ABC and turning one of the arrows to match ray BA. Have students demonstrate this for the other angles on the board. Emphasize that the length of the strips (or segments of the rays) has no effect on the measure of an angle. Ask which angle suggests a square corner (∠ GHI). Describe each of the other angles in terms of ∠ GHI. For example, ∠ DEF is larger than ∠ GHI. Tell the students that there is a device for measuring the size of an angle.

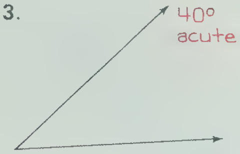
Using the Pages

- Introduce the words *protractor* and *degrees*. Point out the symbol ° for "degrees". These can be contrasted with the words *ruler* and *centimetre* (cm). Have students describe the shape of the protractor and note that there are two scales, each having the range 0° to 180°. The scales may be described as an inner scale and an outer scale.

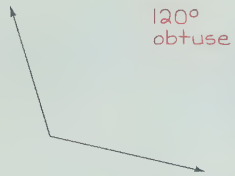
Give each student a protractor and have her/him compare it with the one shown on page 176. Have the students place their protractors on top of the one on the

Measure these angles.

3.



4.



5.

An angle that measures 90 is a **right angle**.

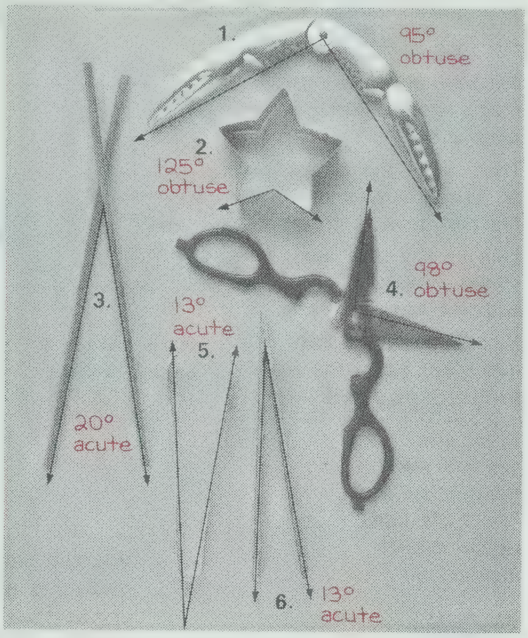
An angle that measures more than 0° but less than 90° is an **acute angle**.

An angle that measures more than 90° but less than 180° is an **obtuse angle**.

6. Are the angles in the above exercises acute, right, or obtuse angles?

Exercises

Measure these angles.



For the exercises above,

7. are the angles acute, right, or obtuse angles?

Complete.

8.

Angle measurement	75	175	90	5	95	112
Kind of angle	acute ?	obtuse ?	right ?	acute ?	obtuse ?	obtuse ?

Look around. Make a chart like this. Answers will vary.

9.

Examples of angles	Measurement of angle	Kind of angle
peak of house	120°	obtuse

RELATED ACTIVITIES

- Have students use protractors to measure angles for various objects in the room. Have them classify the angles as acute, right, or obtuse. The results may be written in a chart.
- Some letters of the alphabet suggest angles, for example, the letter A. Have students draw a large A using a straight edge and measure the angles formed by line segments in the letter.
- Add new concepts from this lesson to the chart described in *Related Activities* on page T 189.
- Have students use geoboards and/or geopaper (copies of pages T396) to show acute angles, right angles, and obtuse angles.
- Have students join dots on geopaper to form different angles. Have them use protractors to measure the angles.
- Have students name the angles shown by the hands on a dial clock for certain hours, for example, 2 o'clock, 3 o'clock, 4 o'clock, and 5 o'clock.
- Discuss with the students through how many degrees the hour hand moves from 12 to 3, from 12 to 6, from 12 to 9, and from 12 to 12 (clockwise only). This would provide enrichment with angles of 180°, 270°, and 360° (one rotation).

UNIVERSITY OF ALBERTA

page to check that the angle measures 36°. Point out that the inner scale is used and for that scale, 0 is located at the right. Emphasize how the center and the base line of the protractor must be placed to measure an angle.

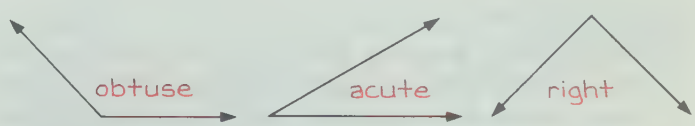
Working Together: Carry out the procedure on the board at the same time as the students work at their desks. It is important that they draw their diagrams large enough for the rays to show under the scale on the protractor. Repeat Ex. 1 for several angles. Then proceed with Ex. 3-5 on page 177.

Introduce the terms *acute angle*, *right angle*, and *obtuse angle*. Have students use these terms to describe the angles on the board from the preliminary activity. Point out that, in many instances, it is possible to tell whether an angle is acute or obtuse without knowing the number of degrees — one needs only to compare the rays with the rays of a right angle.

Exercises: Ex. 1-6 are shown in the photograph. In some instances (Ex. 2 and 4) the rays may not reach the scale on the protractor. If so, have the students hold a straight edge along the ray so that it indicates the number of degrees on the scale. For each measurement, emphasize the need to ask whether the answer makes sense, to help students read the correct scale. If an angle appears to be acute as in Ex. 5, for instance, the measure must be less than 90°.

Assessment

1. Use a straight edge. Draw angles that look like these.



2. Measure each angle. Answers will vary.
3. Tell whether the angles are acute, right, or obtuse angles.

LESSON OUTCOME

Use a protractor to draw an angle having a given measure

Materials

a transparent protractor and an overhead projector or a large protractor; a straight edge and a protractor for each student

Vocabulary

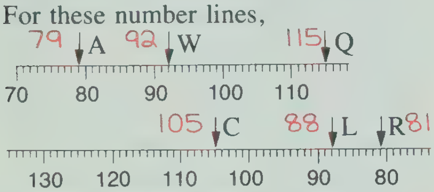
straight angle

Prerequisite Skills

Locate points on a scale

Checking Prerequisite Skills

For these number lines,



1. which letter names the point for 92? W
2. which letter names the point for 105? C
3. which letter names the point for 79? A

RELATED ACTIVITIES

- Have the students classify the angles for Ex. 1-10 as acute, right, or obtuse angles.
- Add the concept of a straight angle to the chart described in *Related Activities* on page T 189.

Drawing Angles

A protractor can be used to draw an angle of a certain size.

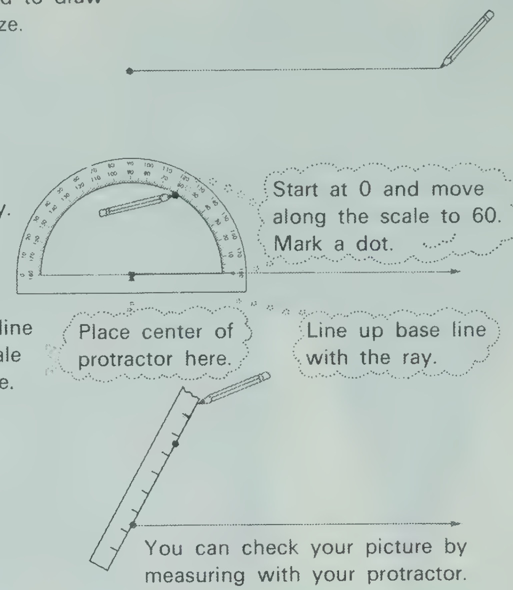
To draw an angle that measures 60°, draw a ray.

Then, place the center of your protractor on the end point of the ray.

Line up the base line of the protractor with the ray.

Start at 0 on the base line and move along the scale to 60. Mark a dot there.

Draw a ray from the end point through the dot. The angle that results should measure 60°.



Exercises

Angles are shown on page T369.

Use your protractor and draw angles that measure

1. 70°.	**2.** 40°.	**3.** 120°.	**4.** 32°.	**5.** 104°.
6. 18°.	**7.** 90°.	**8.** 177°.	**9.** 56°.	**10.** 135°.

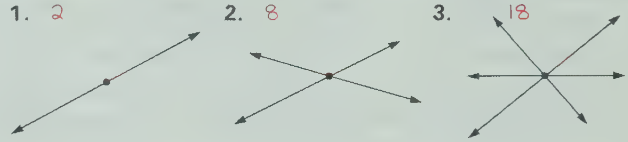
A **straight angle** is an angle that measures 180°.

The rays that form a straight angle also form a line.

try this

How many angles, including straight angles, are in each picture?

1. 2 **2.** 8 **3.** 18



178

LESSON ACTIVITY

Before Using the Page

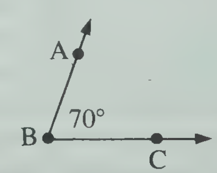
- Use either a transparent protractor with the overhead projector or a large protractor on the chalkboard. Review the use of the protractor to measure an angle for one or two given angles.

Using the Page

- Demonstrate the procedure described in the example, using the overhead projector or the chalkboard. Have students read the statements aloud and, as they do, show the steps to obtain an angle of 60°. Then repeat the steps as the students complete them at the same time. Emphasize positioning the protractor on the ray and reading the appropriate scale. (The appropriate scale is the one for which the mark for 0 coincides with one ray.) Use other examples.

Exercises: You may wish to have the students name each angle and mark its measure as shown.

Ex. 1

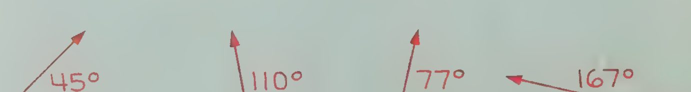


Try This: Have the students copy the diagrams and mark them, if necessary, to find the number of angles.

Assessment

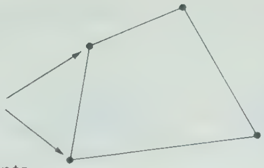
Use your protractor and draw angles that measure

1. 45°. **2.** 110°. **3.** 77°. **4.** 167°.



Polygons

A **polygon** is formed by line segments that share end points like this.

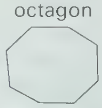


In a polygon, the line segments that meet suggest angles. These are the **angles of the polygon**.

The line segments are the **sides of the polygon**.

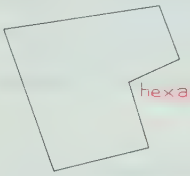
Their end points are the **vertices of the polygon**.

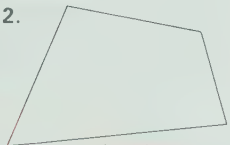
Polygons can have any number of sides.
These are the most common kinds of polygons.

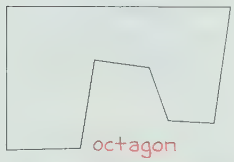
triangle	quadrilateral	pentagon	hexagon	octagon
				
3 sides	4 sides	5 sides	6 sides	8 sides

Exercises

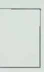
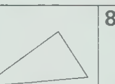
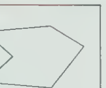
Name the kind of polygon shown.

1.  hexagon

2.  quadrilateral

3.  octagon

Complete.

	4.	5.	6.	7.	8.
					
Kind of polygon	? quadrilateral	? octagon	? pentagon	? triangle	? hexagon
Number of sides	? 4	? 8	? 5	? 3	? 6
Number of vertices	? 4	? 8	? 5	? 3	? 6
Number of angles	? 4	? 8	? 5	? 3	? 6

LESSON OUTCOME

Classify polygons; identify the angles and sides of polygons

Materials

several narrow cardboard strips and pronged paper fasteners

Vocabulary

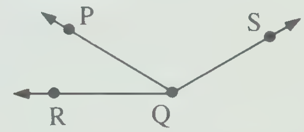
polygon, vertices, triangle, quadrilateral, pentagon, hexagon, octagon

Prerequisite Skills

Identify line segments and angles

Checking Prerequisite Skills

For this diagram,



1. how many line segments are there? 3
2. how many angles are there? 3

RELATED ACTIVITIES

- Have students use geostrips, D-Stix, or straws and pipe cleaners to form polygons. The polygons made of straws and pipe cleaners can be used to make a mobile.
- Prepare a chart to show the names and illustrations of different polygons. It will help students with the spelling and the meaning of the terms if the chart is displayed for several days.

LESSON ACTIVITY

Before Using the Page

- Prepare several narrow cardboard strips with a hole punched in both ends of each strip. Vary the lengths of the strips. Display one and ask what geometric shape it represents (a line segment). Tell the students that the holes represent the end points. Have a student fasten two strips together so that they share one end point. Ask what shape is suggested (an angle). Have a student fasten another strip to the first two so that it shares just one end point with the first strip and just one end point with the second strip. Ask what shape is suggested and ask how many angles and how many sides there are. Ask if 4, 5, or 6 segments could share end points in a similar way. Have students use strips to demonstrate this. Comment on the fact that the only rigid figure is the figure consisting of three line segments.

Using the Page

- Have students read the statements that introduce the terms *polygon*, *sides*, *angles*, and *vertices*. Explain that the prefix "poly" is from the Greek word for "many" and "gon" means "sides". Thus, a polygon describes a many-sided figure. Ask whether a polygon having two sides can be drawn. Have students read the names of the polygons illustrated.

Exercises: Ex. 4-8 will enable the students to discover that a polygon has the same number of sides, angles, and vertices. For example, a pentagon has five sides, five angles, and five vertices.

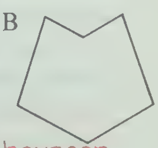
Assessment

For each polygon, name
1. the kind of polygon.
2. the number of sides.
3. the number of vertices.
4. the number of angles.

A

pentagon
5
5
5

B



hexagon
6
6
6

UNIVERSITY OF ALBERTA

Pages 180-181

OBJECTIVE

Demonstrate competence in identifying and naming lines, line segments, rays, angles, and polygons

Materials

tracing paper and a straight edge for each student, a protractor for each student (optional), acetate sheets (optional)

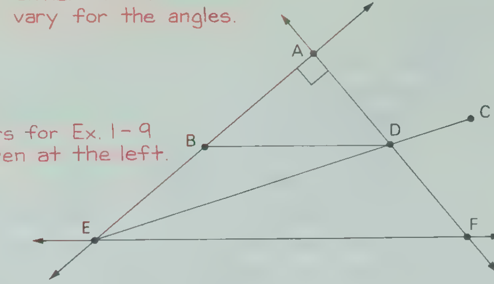
Vocabulary

simple closed curve

1. $\overline{AB}$, $\overline{AE}$, $\overline{BE}$, $\overline{AD}$, $\overline{AF}$, $\overline{DF}$, $\overline{BD}$, $\overline{ED}$, $\overline{EC}$, $\overline{DC}$, $\overline{EF}$
2. $\overrightarrow{AB}$, $\overrightarrow{AE}$, $\overrightarrow{BE}$, $\overrightarrow{EB}$, $\overrightarrow{EA}$, $\overrightarrow{BA}$, $\overrightarrow{AD}$, $\overrightarrow{AF}$, $\overrightarrow{DF}$, $\overrightarrow{FD}$, $\overrightarrow{FA}$, $\overrightarrow{DA}$
3. $\overleftrightarrow{AE}$ ($\overleftrightarrow{AB}$, $\overleftrightarrow{BE}$), $\overleftrightarrow{AF}$ ($\overleftrightarrow{AD}$, $\overleftrightarrow{DF}$), $\overleftrightarrow{EF}$
4. $\angle ABD$, $\angle BDA$, $\angle DAB$, $\angle DBE$, $\angle BED$, $\angle BEF$, $\angle DEF$, $\angle BDE$, $\angle BDF$, $\angle EDF$, $\angle AFE$, $\angle ADC$, $\angle ADE$, $\angle CDF$, $\angle CDB$
5. $\angle EAF$
6. $\angle ABD$, $\angle BED$, $\angle BEF$, $\angle DEF$, $\angle ADB$, $\angle BDE$, $\angle AFE$, $\angle CDF$, $\angle EDA$
7. $\angle DBE$, $\angle FDE$, $\angle FDB$, $\angle CDA$
8. $\overline{BD}$ and $\overline{EF}$
9. $\overleftrightarrow{BA}$ and $\overleftrightarrow{DA}$, $\overrightarrow{BA}$ and $\overrightarrow{DA}$, $\overline{BA}$ and $\overline{DA}$, $\overleftrightarrow{EA}$ and $\overleftrightarrow{FA}$, $\overrightarrow{EA}$ and $\overrightarrow{FA}$, $\overline{EA}$ and $\overline{FA}$

Practice For line segments and lines, the letters may be given in either order. The names may vary for the angles.

For the picture,



1. name three line segments.
2. name three rays.
3. name three lines.
4. name three angles.
5. name one right angle.
6. name one acute angle.
7. name one obtuse angle.
8. name a pair of parallel lines, rays, or line segments.
9. name a pair of perpendicular lines, rays, or line segments.
10. name a pair of lines, rays, or line segments that intersect. Name their point of intersection.
11. name a pair of congruent angles.

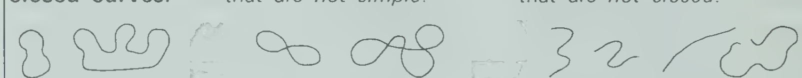
Answers for Ex. 1-9 are given at the left.

Answers for Ex. 10 and 11 are given below.

Which of these cannot be drawn?

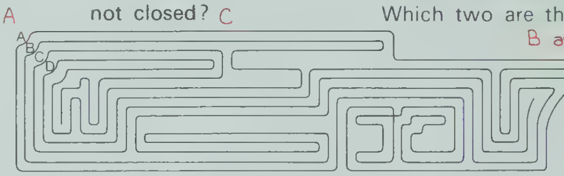
12. two parallel lines that intersect cannot
13. two perpendicular lines that form an angle of 90 can
14. two rays that are not parallel but do not intersect can
15. two angles that match by tracing but have different measurements cannot
16. an acute right angle cannot
17. line PQ parallel to ray QT cannot

These are **simple closed curves.**	These are closed curves that are *not simple.*	These are simple curves that are *not closed.*



1. Which curve below is not simple? A
2. Which curve below is not closed? C
3. Two of A, B, C, and D name the same curve. Which two are they? B and D

try this



LESSON ACTIVITY

Before Using the Pages

- Conduct a brief review of some of the concepts from this unit. Have students use a straight edge on the board to draw and label a line, a line segment, a ray, an angle, two parallel lines, two perpendicular lines, two congruent angles, an acute angle, an obtuse angle, and a right angle.

Using the Pages

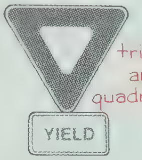
- Draw the students' attention to pentagon ABCDE on page 181. Have them read the other names shown for the pentagon and ask for several other ways to name the same pentagon. Emphasize that the vertices may be named in cyclic order; that is, either clockwise or counterclockwise.

  Some students may need to use a protractor for Ex. 5-7. Provide the students with tracing paper for Ex. 11. An oral discussion of students' answers for Ex. 12-17 would be beneficial.

Try This: Discuss the difference between a closed curve and a curve that is not closed. Point out that a closed curve is not simple if it intersects itself. Provide the students with tracing paper or sheets of clear acetate to trace each of the curves indicated by A, B, C, and D.

10. $\overleftrightarrow{AE}$ and $\overleftrightarrow{AF}$ at A, $\overleftrightarrow{AE}$ and $\overleftrightarrow{EF}$ at E, $\overleftrightarrow{AF}$ and $\overleftrightarrow{EF}$ at F; $\overrightarrow{BA}$ and $\overrightarrow{DA}$ at A, $\overrightarrow{BE}$ and $\overrightarrow{FE}$ at E, $\overrightarrow{EF}$ and $\overrightarrow{AF}$ at F; $\overline{AB}$ and $\overline{AD}$ at A, $\overline{AB}$ and $\overline{BD}$ at B, $\overline{AD}$ and $\overline{BD}$ at D, $\overline{BD}$ and $\overline{DC}$ at D, $\overline{AD}$ and $\overline{DC}$ at D, $\overline{BD}$ and $\overline{DF}$ at D, $\overline{DC}$ and $\overline{DF}$ at D, $\overline{BD}$ and $\overline{DE}$ at D, $\overline{DF}$ and $\overline{EF}$ at F, $\overline{DE}$ and $\overline{EF}$ at E, $\overline{BE}$ and $\overline{DE}$ at E, $\overline{BE}$ and $\overline{BD}$ at B
11. $\angle ABD$ and $\angle AEF$, $\angle ADB$ and $\angle AFE$, $\angle ADE$ and $\angle CDF$, $\angle ADC$ and $\angle EDF$, $\angle BDE$ and $\angle DEF$

Name the kinds of polygons suggested by these traffic signs.

18. 

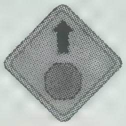
triangle and quadrilateral

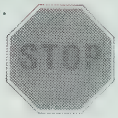
19. 
quadrilateral and triangle

20. 
quadrilateral

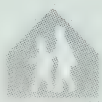
21. 

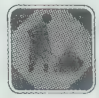
quadrilateral

22. 
quadrilateral and octagon

23. 

octagon

24. 
pentagon

25. 
quadrilateral and octagon

26. 

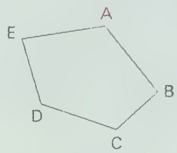
quadrilateral

27. 
quadrilateral

28. 
quadrilateral

29. 
quadrilateral

Polygons can be named by telling the kind of polygon and then naming the vertices in order.

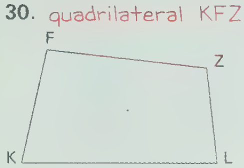


This could be called pentagon ABCDE.

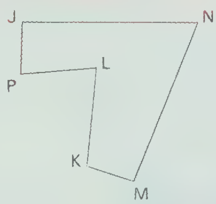
or pentagon EDCBA,
or pentagon CDEAB,
or. . .

Name each polygon.

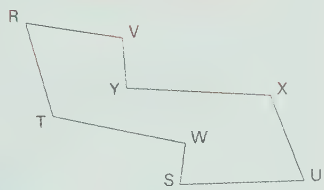
30. quadrilateral KFZL



31. hexagon MKLPJN



32. octagon VYXUSWTR



Look around. Make a chart like this. Answers will vary

33.

Examples of polygons	Kind of polygon
home plate in baseball	pentagon

181

RELATED ACTIVITIES

- Have students identify perpendicular and parallel line segments suggested in the shapes for Ex. 18-29.
- Compile the ideas of the students for Ex. 33 into one large chart for display.
- Some students may be interested in preparing curves similar to those in the *Try This* feature for other students to explore.
- The activity "Multipatterns" described on page T 379 relates the operations of multiplication and addition to geometry. It will be an enjoyable experience for many students.

UNIVERSITY OF ALBERTA

Pages 182-183

LESSON OUTCOME

Identify and make shapes having line symmetry; identify and show lines of symmetry

Materials

tracing paper, scissors, and a straight edge for each student; blank sheets of paper for each student; semitransparent plexiglass mirrors; large sheet of paper; magazines and catalogs

Vocabulary

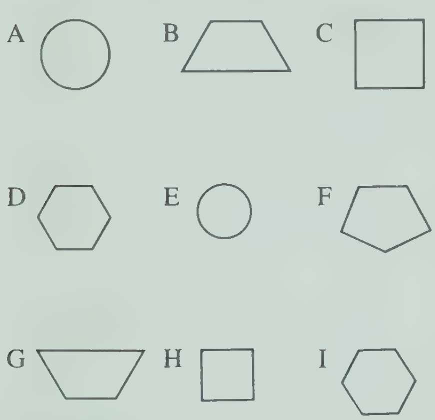
line symmetry, line of symmetry, Mira

Prerequisite Skills

Recognize shapes that are alike

Checking Prerequisite Skills

Name pairs of shapes that are alike.

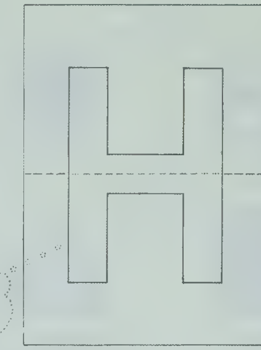


A and E
B and G
C and H
D and I

Line Symmetry

A shape that has **line symmetry** has two matching parts.

If the shape is on paper, one part can be folded onto the other part and the two parts will match.



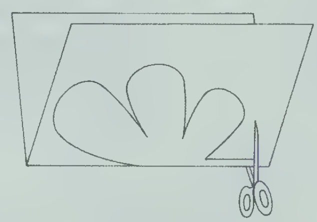
The shape could be cut out of the paper before folding if that would make it easier.

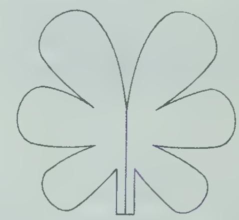
This shape has two lines of symmetry. Can you find the other one?

The line that separates a shape with line symmetry into two matching parts is a **line of symmetry**.

When a fold shows that a shape has line symmetry, the fold shows the line of symmetry.

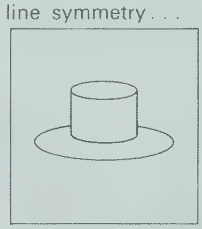
A shape with line symmetry can be made by cutting a folded piece of paper.



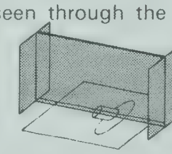


A Mira™ can help you check for line symmetry.

This shape has line symmetry . . .



. . . if the reflection of the part on this side matches the part on the other side as seen through the Mira.



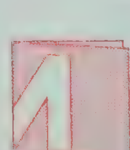
1. Find pictures that suggest line symmetry. Check for line symmetry using the Mira.

182

LESSON ACTIVITY

Before Using the Pages

- Fold a large sheet of paper in half and draw the following shape at the folded edge. Ask the students which letter of the alphabet would be obtained by cutting around the fold and along the lines of the shape. Demonstrate by cutting out the shape. Have students explain how they knew it would be the letter M. Ask if this procedure would work for every letter of the alphabet. Have students give reasons for their answers.



Using the Pages

- The example at the top of page 182 introduces the concept of line symmetry for shapes that have two matching parts.

Have students read the statements and suggest where the second line of symmetry would be located for the letter H. Also, ask whether they think it is easier to cut out the letter H before folding or after folding the sheet of paper. Ask if the letter M (cut out during the preliminary activity) has a second line of symmetry.

On a sheet of paper or on the chalkboard, draw a simple shape having two identical parts that do not match by folding. An example is shown below. Ask if the shape has a line of symmetry and have students give reasons for their answers.

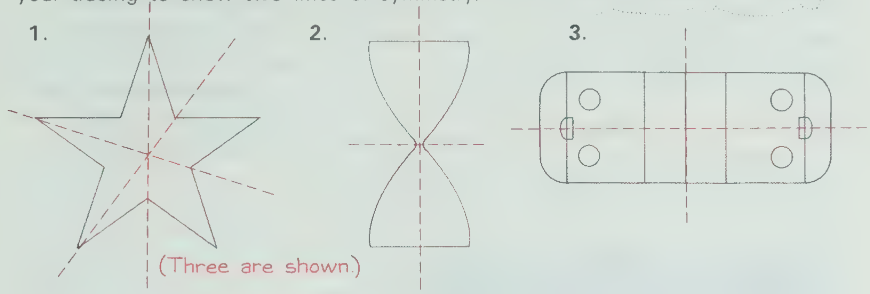
Exercises: Provide the students with tracing paper and scissors for Ex. 1-3. Have them use a straight edge to trace line segments and to draw line segments along the fold lines to

Exercises

Trace each shape. Then make two folds in your tracing to show two lines of symmetry.

You can cut out your tracings if that would make it easier to check.

1. 2. 3.



Use pieces of paper folded once.

4. Cut out shapes that have line symmetry. Mark the lines of symmetry. Answers will vary.

5. Draw pictures on folded pieces of paper as shown on page 182. Then cut out the shapes. Answers will vary.

Use pieces of paper folded twice.

6. Cut out a shape. Mark two lines of symmetry.

7. Draw a picture on a folded piece of paper. Then cut out the shape.

Find out how

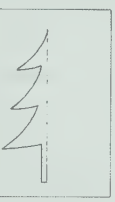
*8. to make three folds and cut out a snowflake.

Look around.

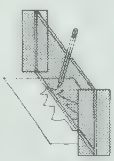
9. Make drawings, or a list, of shapes you see with line symmetry. Show lines of symmetry in the drawings you make. Answers will vary

A Mira can help you draw shapes that have line symmetry.

To complete a shape having line symmetry . . .



. . . draw the other part as suggested by the reflection.



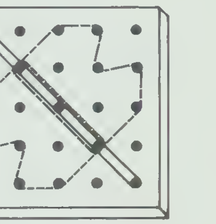
2. Use the Mira to help you draw two shapes having line symmetry.

try this

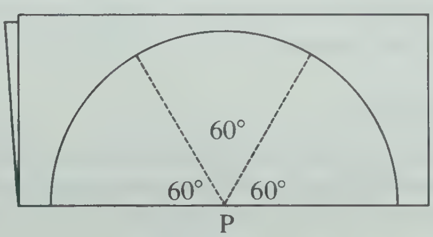
183

RELATED ACTIVITIES

- Display shapes cut by the students for Ex. 4-8 on page 183. Organize the display so that shapes with the same number of lines of symmetry are grouped together. The snowflakes from Ex. 8 would make a particularly attractive display if they are pasted on colored construction paper first.
- Have students use a semitransparent plexiglass mirror to investigate the line symmetry of the diagrams on pages 182 and 183.
- Have students find which letters of the alphabet and which of the digits 0 to 9 have line symmetry.
- Have students work in pairs using geoboards. A rubber band is placed to represent a line of symmetry. Using a rubber band of a different color, a student shows part of a shape on one side of the line. The other student uses another rubber band to complete the shape. The shapes may be copied onto geopaper (copies of page T 396).



show lines of symmetry. Emphasize that the shapes for Ex. 4 and 5 are cut around the fold line. For Ex. 6 and 7, the paper is folded twice by folding once in half and then in half again. Shapes are cut to include both folds. For Ex. 8, the most accurate results can be obtained by using a protractor. The paper is folded once in half. Point P is marked on the fold as the vertex of a straight angle (180°). By using a protractor, points can be marked for rays to form three angles of 60° each. The rays indicate the fold lines for the second and third folds. Designs cut around each fold will produce a snowflake with three lines of symmetry. (Filter paper is circular and is ideal for this activity.)



Try This: A semitransparent plexiglass mirror similar to the one illustrated on page 182 is desirable for a study of line symmetry. As a mirror, it shows the reflection of an object. Being semitransparent, it enables students to match the reflection in testing for line symmetry (Ex. 1) or to trace the reflection in drawing a symmetric shape (Ex. 2). Provide the students with magazines and catalogs to search for pictures that suggest line symmetry.

Assessment Answers will vary for Ex. 1 and 2.

1. Use a piece of paper folded once. Cut out a shape that has one line of symmetry. Mark the line of symmetry.
2. Use a piece of paper folded twice. Cut out a shape. Mark two lines of symmetry.

UNIVERSITY OF ALBERTA

LESSON OUTCOME

Classify triangles as equilateral, isosceles, or scalene, according to the number of sides of equal length and/or the number of lines of symmetry

Materials

plain paper, tracing paper, scissors, and a centimetre ruler for each student; copies of the triangles on page T 383 or a protractor for each student

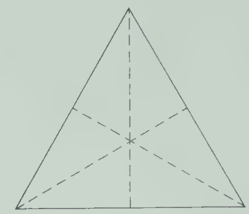
Vocabulary

equilateral, isosceles, scalene

Triangles

A triangle can have all three sides the same length.

Such a triangle has three lines of symmetry.



Trace this triangle and check that

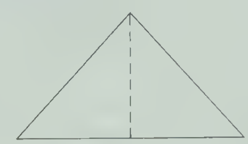
a. all three sides are the same length, and

b. there are three lines of symmetry.

A triangle with three sides the same length or with three lines of symmetry is an **equilateral triangle**.

A triangle can have just two sides the same length.

Such a triangle has just one line of symmetry.



Trace this triangle and check that

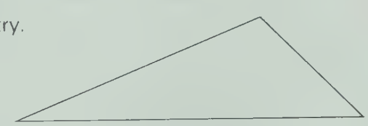
a. just two sides are the same length, and

b. there is just one line of symmetry.

A triangle with two sides the same length or with just one line of symmetry is an **isosceles triangle**.

A triangle can have all three sides with different lengths.

Such a triangle has no lines of symmetry.



Trace this triangle and check that

a. all three sides have different lengths, and

b. there are no lines of symmetry.

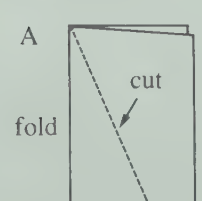
A triangle with all three sides different lengths or with no lines of symmetry is a **scalene triangle**.

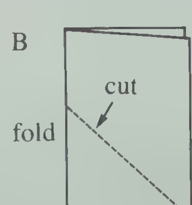
184

LESSON ACTIVITY

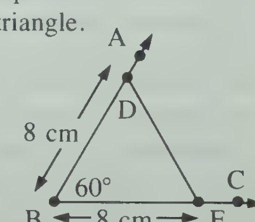
Before Using the Pages

- Have the students fold a sheet of paper in half and cut it as indicated below (A). When they have unfolded the cut portion, have them identify the shape and measure the three sides of the shape. They will discover that two of the sides have the same length. Have them draw a line along the fold to show the line of symmetry. Have them repeat the activity to see if the triangular shape will have two sides of equal length for a slightly different cut (B). It will be easier if they use their rulers to draw the line segments along which they will cut.





- To investigate triangles with three sides of equal length, give each student a copy of one or both triangles on page T 383. Have the students cut out the triangles, measure the sides, and fold each triangle to discover three lines of symmetry. As an alternative, you may prefer to have the students draw their own triangles by using a centimetre ruler and a protractor and following the steps indicated below. They may then cut out and fold the triangle.
  (1) Draw an angle of 60°.
  (2) Label the angle, ABC.
  (3) Mark line segments BD and BE, each 8 cm.
  (4) Join points D and E and measure segment DE.



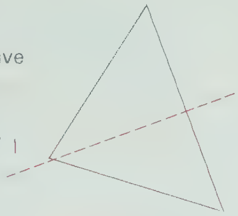
Using the Pages

- Provide the students with tracing paper and let them work independently through the examples on page 184. Discuss their results and help them with pronunciation of the words *equilateral*, *isosceles*, and *scalene*.

Working Together

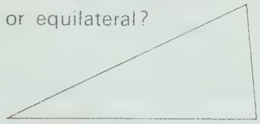
Use tracing paper.

1. How many sides have the same length? 2
2. How many lines of symmetry are there? 1
3. Is this triangle scalene, isosceles, or equilateral?



isosceles

4. Is this triangle scalene, isosceles, or equilateral?

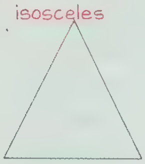


scalene

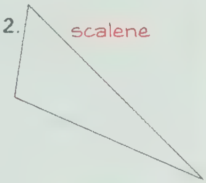
Exercises

Use tracing paper. Are these triangles scalene, isosceles, or equilateral?

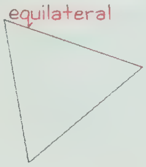
1. isosceles



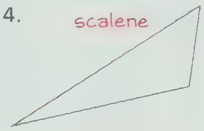
2. scalene



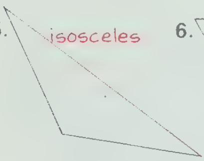
3. equilateral



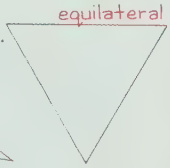
4. scalene

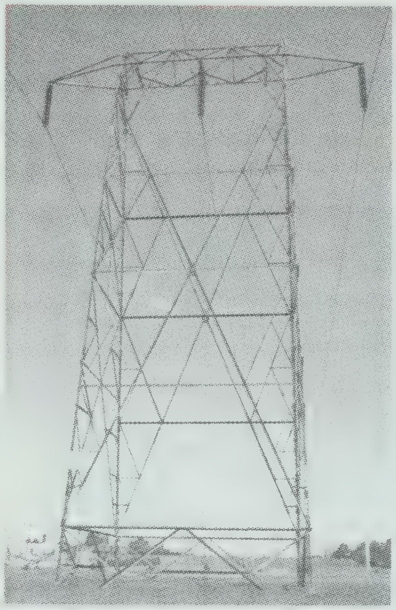


5. isosceles



6. equilateral





Look around, or look at this picture, and make a chart.

7.

Examples of triangles	Kind of triangle
top part of the letter A	isosceles

Answers will vary.

185

RELATED ACTIVITIES

- Have students use a semitransparent plexiglass mirror on the diagrams on pages 184 and 185 to test for line symmetry.
- Have students show various triangles on geoboards and copy the triangles onto geopaper (copies of page T 396). Have them measure the sides and classify the triangles as scalene, isosceles, or equilateral. A semitransparent plexiglass mirror may be used to check for line symmetry.
- Have the students construct triangular shapes using materials such as geostrips, D-Stix, straws and pipe cleaners, or toothpicks and plasticine. Challenge them to find three strips (straws) that will not form a triangle.

Working Together: Have the students tell what they think the results will be for Ex. 1 and Ex. 4 before they measure or trace.

Exercises: Before the students begin, discuss the photograph. Have them identify the object shown, its use, and, if possible, suggest why there are so many triangular shapes in the construction. (Remind them that in a previous lesson they discovered that a triangular shape is a rigid shape.) Give the students tracing paper for Ex. 1-6.

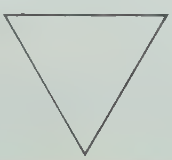
Assessment

Use tracing paper.
Are these triangles scalene, isosceles, or equilateral?

1.

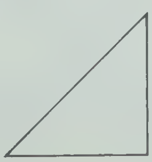
scalene

2.



equilateral

3.



isosceles

UNIVERSITY OF ALBERTA

Pages 186 - 187

LESSON OUTCOME

Classify quadrilaterals according to parallel sides and lines of symmetry

Materials

tracing paper

Vocabulary

trapezoid, parallelogram, rhombus, rectangle, square, kite

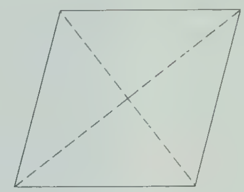
Quadrilaterals

A **trapezoid** has at least one pair of parallel sides.

A **parallelogram** has two pairs of parallel sides

A parallelogram having lines of symmetry like this is a **rhombus**.

Trace the shape and check the lines of symmetry.

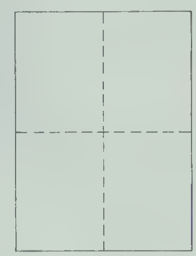


A rhombus has its four sides the same length.

Check the lengths of the four sides.

A parallelogram having lines of symmetry like this is a **rectangle**.

Trace and check the lines of symmetry.

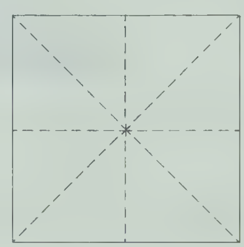


A rectangle has two pairs of sides the same length.

The angles of a rectangle suggest right angles.

A parallelogram having lines of symmetry like this is a **square**.

Trace and check the lines of symmetry.



A square is both a rhombus and a rectangle.

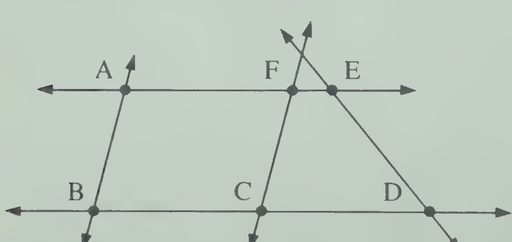
So a square has all the properties of both a rhombus and a rectangle.

186

LESSON ACTIVITY

Before Using the Pages

- Draw the following diagram on the board. Have students name as many pairs of parallel lines as possible. Have them trace line segments in colored chalk to show quadrilaterals. Write the names of the quadrilaterals on the board (ABCF, FCDE, ABDE). For each quadrilateral, have students name sides that are parallel. For instance, in quadrilateral FCDE, side FE is parallel to side CD. Point out situations, such as the example just named, for which line segments are parallel but of unequal length.



Using the Pages

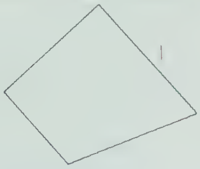
- Provide the students with tracing paper to trace all the shapes shown on page 186. They should fold the tracings of the two trapezoids and the parallelogram to discover that there are no lines of symmetry. Discuss their results when they have finished. Have the students read the names of the shapes: *trapezoid*, *parallelogram*, *rhombus*, *rectangle*, *square*. Help them with the pronunciation as needed. Emphasize that a square has all the properties of both a rhombus and a rectangle and is the only quadrilateral with four lines of symmetry.

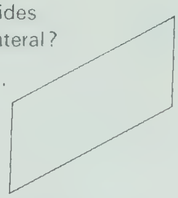
Working Together: The concepts in Ex. 4-6 are important for understanding how the symmetry of a rhombus and a rectangle differ, and thus, how a square differs from each of those.

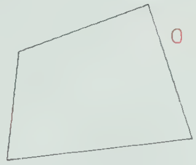
Exercises: Some students may not need tracing paper to answer Ex. 1 and 2. Have students share their results for Ex. 3.

Working Together

How many pairs of parallel sides appear to be in each quadrilateral?

1.  1

2.  2

3.  0

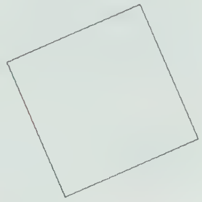
A rhombus and a rectangle each have two lines of symmetry.

4. How are their lines of symmetry different? In a rhombus they go through the vertices and in a rectangle they go through the midpoints of the sides.

Use tracing paper.

5. How many lines of symmetry are there? 4

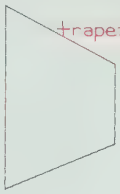
6. Is this a rhombus, a rectangle, or a square? square



Exercises

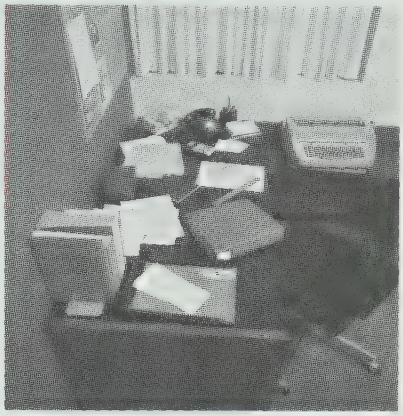
What kind of quadrilateral is each of these? Use tracing paper if needed.

1.  rhombus

2.  trapezoid

A **kite** is a quadrilateral with one line of symmetry.

*3. Draw a kite that has no sides parallel.



Look around, or look at this picture, and make a chart.

4.

Examples of quadrilaterals	Kind of quadrilateral
cover of binder	rectangle

Answers will vary.

187

RELATED ACTIVITIES

- For enrichment, have students draw diagrams for the various kinds of quadrilaterals presented in the lesson and use a protractor to measure the angles of each quadrilateral.
- Have students show examples of the different kinds of quadrilaterals on a geoboard. The shapes may be copied onto geopaper (copies of page T 396).
- You may wish to introduce the concept of an isosceles trapezoid, that is, a trapezoid for which the non-parallel sides are equal in length. Have students draw a large isosceles triangle on geopaper or cut out one as described in *Before Using the Pages* on page T 200. If the triangle is cut into two parts along a line segment parallel to the base, one part will be a smaller isosceles triangle and the other, an isosceles trapezoid having one line of symmetry. Isosceles trapezoids may also be obtained by cutting a regular hexagon and a regular octagon appropriately.
- Have students use a semitransparent plexiglass mirror to check the line symmetry of the figures on pages 186 and 187.

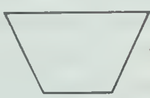
UNIVERSITY OF ALBERTA

Assessment

What kind of quadrilateral is each of these?

1.  square

2.  parallelogram

3.  trapezoid

4.  rhombus

5.  rectangle

Pages 188-189

LESSON OUTCOME

Classify pyramids and prisms by the shapes of their bases; identify the number of faces, edges, and vertices of solid shapes; identify the solid from a given pattern; draw a pattern for a given solid

Materials

models of prisms and pyramids named on page 188; paper model of a triangular prism prepared from the pattern on page T386; copies of patterns from pages T386-T388 for each student

Vocabulary

solid, base, face, pyramid, prism, triangular pyramid, square pyramid, rectangular pyramid, triangular prism, rectangular prism, cube, edge

Prerequisite Skills

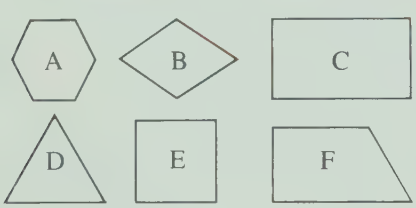
Identify polygons

Checking Prerequisite Skills

For the diagrams below, tell which shape is

1. a square. E **2.** a rectangle. C or E

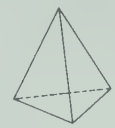
3. a triangle. D **4.** a hexagon. A



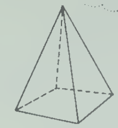
Pyramids and Prisms

A pyramid has one face as its base. Its other faces meet in a point.

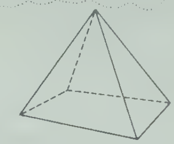
A pyramid is named by using the name of its base.



The base has the shape of a triangle. A pyramid like this is a **triangular pyramid**.



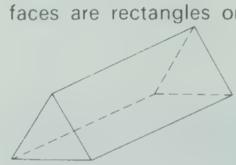
The base has the shape of a square. A pyramid like this is a **square pyramid**.



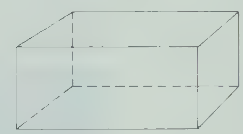
The base has the shape of a rectangle. A pyramid like this is a **rectangular pyramid**.

In a prism, the two faces on the ends can have the shape of any polygon. The other faces are rectangles or parallelograms.

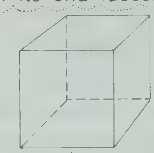
A prism is named by using the name of its end faces.



The end face has the shape of a triangle. A prism like this is a **triangular prism**.

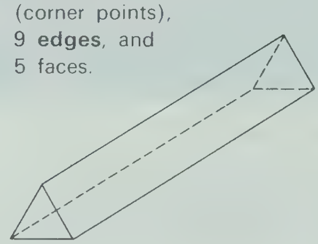


The end face has the shape of a rectangle. A prism like this is a **rectangular prism**.

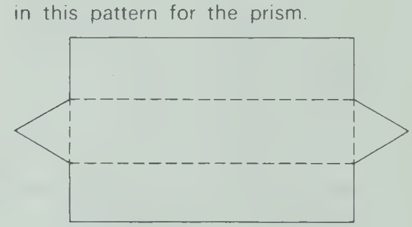


If all the faces have the shape of a square, the prism is a special one called a **cube**.

This triangular prism has 6 **vertices** (corner points), 9 **edges**, and 5 faces.



The 5 faces are seen better in this pattern for the prism.



The pattern could be cut out of paper, folded, and taped to form the prism.

LESSON ACTIVITY

Before Using the Pages

- Display a set of solids for several days in advance of the lesson. Provide opportunities for the students to examine them and observe their likenesses and differences. The terms *base*, *face*, *edge*, *vertex*, and *vertices* may be introduced informally. Have the students trace around the faces of one or more of the solids and identify the shape of each face.

Using the Pages

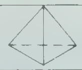
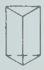
- Have students match each of the six diagrams with the corresponding model from the solid shapes displayed in the classroom. Group the six models into those that are pyramids and those that are prisms, noting that the lateral faces of each pyramid are triangular and meet in a point. Introduce the names *triangular pyramid*, *square pyramid*, and *rectangular pyramid*, explaining that each name is derived from the shape of the base. Stand the triangular prism on one of its triangular faces and turn the prism to display the three rectangular faces. Then stand the prism on its other triangular face to emphasize that end faces have the same shape. Repeat this for the other prisms. Introduce the names *triangular prism*, *rectangular prism*, and *cube*. Have students suggest objects whose shapes are similar to these solids. A tent, for example, can suggest a triangular prism. Introduce the terms *vertices* and *edges*.

  Display a paper model of a triangular prism prepared from the pattern on page T386. Unfold the model to reveal the pattern for the prism and have the students compare it with the one shown at the bottom of page 188.

Exercises: Have the models of the solid shapes available for students who need them to help complete the chart for Ex. 1-6. Provide the students with copies of patterns from pages T386-T388 and have them prepare models prior to the work of Ex. 7-10.

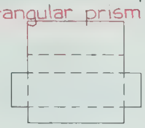
Exercises

Complete.

		Name of solid	Number of vertices	Number of edges	Number of faces	Shapes of the faces
1.		? triangular pyramid	? 4	? 6	? 4	4 triangles
2.		? rectangular prism	? 8	? 12	? 6	? 6 rectangles
3.		? rectangular pyramid	? 5	? 8	? 5	? 1 rectangle 4 triangles
4.		? triangular prism	? 6	? 9	? 5	? 3 rectangles 2 triangles
5.	? 	cube	8?	12?	6?	? 6 squares
6.	? 	square pyramid	? 5	? 8	? 5	? 1 square 4 triangles

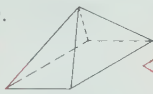
Name the solid that can be made from each pattern.

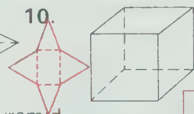
rectangular prism

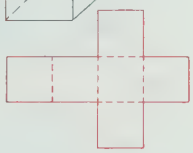
7. 

8. triangular pyramid 

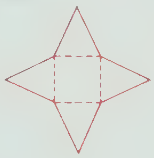
Draw a pattern for each solid.

9. 

10. 



11. a square pyramid



Look around. Make a chart like this.

12.	Object	Kind of solid it suggests
	cereal box	rectangular prism

Answers will vary

Divide. Study the example if needed.

1. $4\overline{)34}$ 8 R2
2. $4\overline{)97}$ 24 R1
3. $3\overline{)59}$ 19 R2
4. $8\overline{)381}$ 47 R5
5. $7\overline{)532}$ 76
6. $2\overline{)\$915}$ $457.50
7. $9\overline{)3417}$ 379 R6
8. $2\overline{)6550}$ 3275
9. $3\overline{)\$50.52}$ $16.84
10. $6\overline{)40\,734}$ 6 789
11. $9\overline{)88\,888}$ 9 876 R4
12. $5\overline{)\$268.55}$ $53.71

Example:

$$\begin{array}{r} 86 \text{ R3} \\ 7\overline{)605} \\ \underline{56} \\ 45 \\ \underline{42} \\ 3 \end{array}$$

KEEPING SHARP

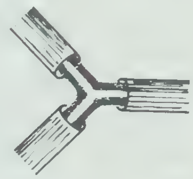
189

RELATED ACTIVITIES

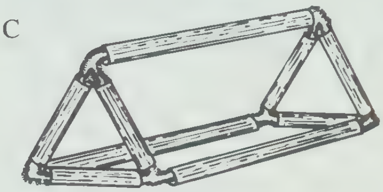
- Have students collect objects, boxes, and so on, that resemble the shapes of the different solids.
- Print the names of the solids on pieces of paper and tape each name to the appropriate solid, for the set that is on display (see *Before Using the Pages*).
- Have the children construct the "skeleton" for a solid of their choice. Have them use drinking straws for edges and pipe cleaners to join the straws. Three straws may be joined at a vertex by bending one pipe cleaner in the following way.

A

B



C

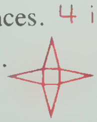


You may wish to extend this concept for pyramids having bases shaped like pentagons or hexagons, and for prisms with those polygons as end faces.

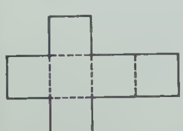
Keeping Sharp: These exercises provide an opportunity to review and practice skills in division in preparation for extending the concept in the next unit. Further practice is provided at the end of this unit.

Assessment

For this solid.

1. give the name of the solid. square pyramid
2. give the number of edges. 8
3. give the number of vertices. 5
4. give the number of faces. 5
5. describe the faces. 4 isosceles triangles and one square
6. draw a pattern. 

For this pattern,



7. name the solid that can be made. cube

Page 190

LESSON OUTCOME

Identify cylinders, spheres, and cones

Materials

objects that suggest cylinders, spheres, and cones; solids that have no curved surfaces (prisms and pyramids)

Vocabulary

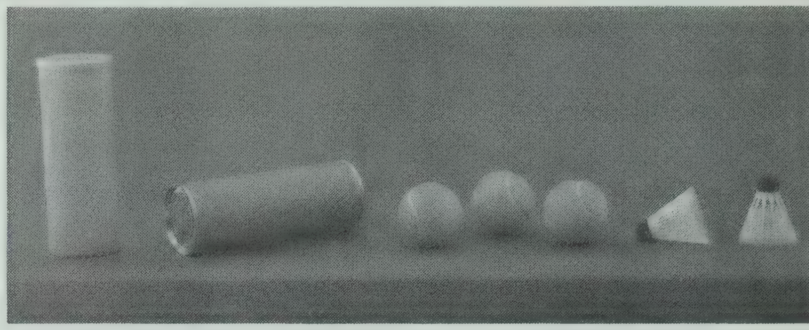
cylinder, sphere, cone

RELATED ACTIVITIES

- Have students collect objects that resemble cylinders, spheres, and cones.
- Have students play "What Is It?" by providing clues such as "The shape suggests a cylinder. It is used to prepare pie crust." Have other students guess the object suggested by the clues.

Cylinders, Spheres, and Cones

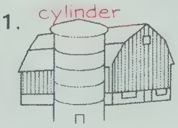
These objects have curved parts.
Which solids do the objects suggest?

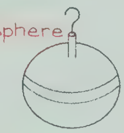


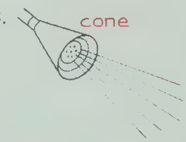
Cylinders Spheres Cones

Exercises

Does the shape suggest a cylinder, a sphere, or a cone?

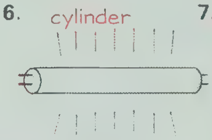
1. cylinder


2. sphere


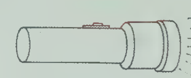
3. cone


4. cylinder


5. cone


6. cylinder


7. sphere


8. cylinder


9. soap bubble sphere
10. sewer pipe cylinder
11. volcano cone
12. skateboard wheel cylinder
13. grapefruit sphere
14. icicle cone

Look around. Make a chart like this.

15.

Object with curved part	Kind of solid it suggests
funnel	cone

Answers will vary.

190

LESSON ACTIVITY

Before Using the Page

- Display objects that suggest cylinders, cones, and spheres, for example, a can, a core from a roll of paper towels, a conical paper hat, and a ball. Display one or two prisms and pyramids in another group. Ask how the shapes in one group differ from the shapes in the other group.

Using the Page

- Introduce the terms *cylinder*, *sphere*, and *cone*. Have students use one of these terms to describe each of the curved objects from the preliminary activity.

Exercises: You may wish to have students identify the objects illustrated in Ex. 1-8 before they begin.

Assessment

Does the shape suggest a cylinder, a sphere, or a cone?

1. scoop of ice cream sphere **2.** rolling pin cylinder **3.** marble sphere
4. drinking straw cylinder **5.** sharpened end of a pencil cone

Solving Problems in Two or More Steps

An average is often found in two steps.

For the average cost of heat each month during the winter,

add

$$\begin{array}{r} \$\ 76.21 \\ 90.07 \\ 95.85 \\ 84.83 \\ 78.36 \\ \underline{72.50} \\ \$497.82 \end{array}$$

then divide.

$$\begin{array}{r} \$\ 82.97 \\ 6\overline{)\$497.82} \\ \underline{48} \\ 17 \\ \underline{12} \\ 5\ 8 \\ \underline{5\ 4} \\ 42 \\ \underline{42} \\ 0 \end{array}$$

The average cost of heat each month was $82.97.

Two or more steps are needed for each of these. Solve each. Show the steps you take.

The steps may vary
Solutions are shown below

1. 8 boys shared $19.60 equally. 3 boys each kept $1.25 of his share and gave the rest to the fund drive. How much was given to the fund drive?
2. Each jar holds 75 plain olives and 38 stuffed olives. How many olives are there in 24 jars?
3. 3 golfers tied for first place in the tournament. They shared the first prize ($1000), the second prize ($750), and the third prize ($275) equally. How much did each golfer win?
4. Each of the 5 boys brought 13 books from home. Each of the 4 girls brought 22 books from home. The boys and girls then shared the books equally. How many books did each get?
5. Each of the 12 old pens holds 68 sheep. 22 sheep were moved from each pen to new pens holding 8 each. The rest were put into new pens holding 6 each. How many new pens are there?
6. 18 boxes have 144 nails each. 23 boxes have 75 nails each. How many nails are there?

PROBLEM SOLVING

191

Page 191

OBJECTIVE

Solve problems involving two or more steps

RELATED ACTIVITIES

- Find examples of other problems for which there are two or more steps in the solution. Have students describe the steps that would be needed to solve each problem. There is no need to show each solution. The numbers may be omitted from the problems.
- Have the students use grocery ads from a newspaper. Have them calculate the total cost of several different items, with more than one of some of the items; for example, 2 cans of soup, 3 loaves of bread, and a bag of potatoes.

1. $8\overline{)\$19.60}$ = $2.45 $\quad$ $2.45 − 1.25 = $1.20 $\quad$ $1.20 × 3 = $3.60

$3.60 was given to the fund drive.

2. 75 + 38 = 113 $\quad$ 113 × 24 = 2712

There are 2712 olives in 24 jars.

3. $1000 + 750 + 275 = $2025 $\quad$ $3\overline{)\$2025}$ = $675

Each golfer won $675.

LESSON ACTIVITY

Using the Page

- Review the fact that if an amount is thought of as being shared equally, the result is an average. (This concept appears in the example on page 93.) Have students tell the number of months that were considered winter months and ask which months these were. Have them check the addition for the sum $497.82 and lead them through the steps of the division to review the procedure. Summarize that two steps were required to complete the solution: addition first and then division.
- Have a student read the directions that precede Ex. 1. Emphasize that there are at least two steps and sometimes more than two steps for the solutions. Have a student read Ex. 1 aloud. Discuss what steps are required and write on the board such key words as *divide*, *subtract*, and *multiply* (or *add*).

When the students have completed the exercises, have several of them show solutions on the board and explain their work. Compare different methods for solving the same problem.

4. 13 × 5 = 65 $\quad$ 22 × 4 = 88 $\quad$ 65 + 88 = 153 $\quad$ $9\overline{)153}$ = 17

Each got 17 books.

5. 68 × 12 = 816 $\quad$ 22 × 12 = 264 $\quad$ 816 − 264 = 552 $\quad$ $8\overline{)264}$ = 33 $\quad$ $6\overline{)552}$ = 92 $\quad$ 33 + 92 = 125

There are 125 new pens.

6. 144 × 18 = 2592 $\quad$ 75 × 23 = 1725 $\quad$ 2592 + 1725 = 4317

There are 4317 nails.

Page 192

OBJECTIVE

Demonstrate an understanding of the concepts and skills presented in this unit

Materials

straight edge, protractor, and tracing paper for each student

RELATED ACTIVITIES

- For enrichment, have students use a protractor to draw an angle whose measure is 80°, for example. Then have them place a semitransparent plexiglass mirror on the vertex and between the two rays. Have them find the position of the mirror for which the reflection of one ray matches the other ray and draw a ray to indicate that position. Have them measure the two angles formed. A similar activity can be carried out by folding the paper so that the two rays of an angle match.
- Have students use plasticine or modeling clay to construct models of prisms, pyramids, cylinders, cones, and spheres. With supervision, some of the models may be sliced into two parts to observe the resulting shapes.

Ex. 5

$\overrightarrow{WX}$ and $\overline{XZ}$, $\overrightarrow{WX}$ and $\overleftrightarrow{XZ}$
$\overrightarrow{WX}$ and $\overrightarrow{XZ}$, $\overline{WX}$ and $\overline{XZ}$
$\overline{WX}$ and $\overleftrightarrow{XZ}$, $\overline{WX}$ and $\overrightarrow{XZ}$
$\overline{WY}$ and $\overline{WX}$, $\overline{WY}$ and $\overrightarrow{WX}$

Checking Up

For lines and line segments the letters may be given in either order.
The names may vary for the angles.

For the picture, name

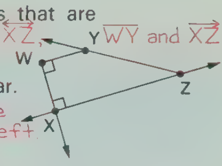
1. a line. $\overleftrightarrow{ML}$
2. a ray. $\overrightarrow{ML}$ or $\overrightarrow{LM}$
3. a line segment. $\overline{ML}$

For the picture, name lines, rays, or line segments that are

$\overline{WY}$ and $\overline{XZ}$, $\overline{WY}$ and $\overleftrightarrow{XZ}$, $\overline{WY}$ and $\overrightarrow{XZ}$

4. parallel.
5. perpendicular.

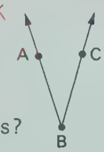
Answers are given at the left.

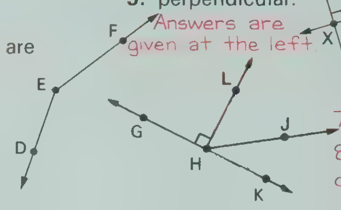


For the angles shown, which are

$\angle ABC$, $\angle LHJ$, $\angle JHK$

6. acute angles?
7. obtuse angles?
8. right angles?
9. congruent angles?





7. $\angle FED$, $\angle GHJ$
8. $\angle GHL$, $\angle LHK$
9. $\angle ABC$ and $\angle JHK$
$\angle GHL$ and $\angle LHK$

Use a protractor and measure

10. angle ABC. 35°
11. angle DEF. 147°

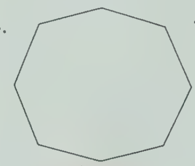
Use a protractor and draw angles that measure

12. 65°.
13. 130°.

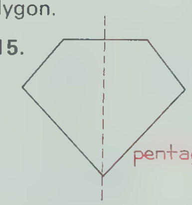
Angles are shown below.

Name the kind of polygon.

14. octagon

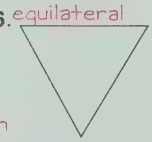


15. pentagon

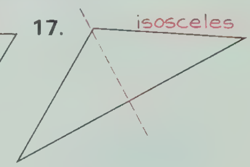


Name the kind of triangle.

16. equilateral

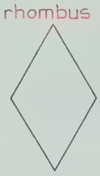


17. isosceles

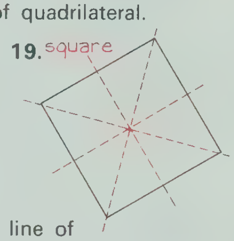


Name the kind of quadrilateral.

18. rhombus



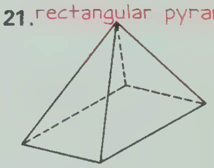
19. square



Name the kind of solid.

20. cylinder

21. rectangular pyramid



Trace and find a line of symmetry for the polygon in

22. Exercise 15.
23. Exercise 17.
24. Exercise 19.

For the solid in Exercise 21,

25. how many vertices are there? 5
26. how many edges are there? 8
27. how many faces are there? 5
28. what kinds of polygons are the faces? 1 rectangle, 4 triangles

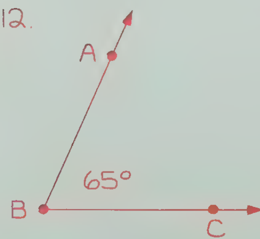
192

Skills	Exercises	Related Pages
Identify and name lines, line segments, and rays	1-3	T 186-T 187
Identify lines, line segments, or rays that are parallel or perpendicular	4, 5	T 188-T 189
Classify angles as acute, right, or obtuse	6-8	T 192-T 193
Identify congruent angles	9	T 190-T 191
Measure angles	10, 11	T 192-T 193
Draw an angle having a given measure	12, 13	T 194
Classify polygons	14, 15	T 195
Classify triangles	16, 17	T 200-T 201
Classify quadrilaterals	18, 19	T 202-T 203
Identify solids	20, 21	T 204-T 206
Show lines of symmetry	22-24	T 198-T 203
Identify faces, edges, and vertices	25-28	T 204-T 205

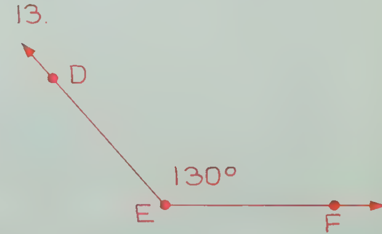
Comments

Before the students begin, point out which diagrams relate to Ex. 1-3, which to Ex. 4 and 5, and which to Ex. 6-9. Remind them to use tracing paper for Ex. 9 and Ex. 22-24.

12.



13.



Checking Skills

Divide.

1. 6)40   6 R4
2. 3)11   3 R2
3. 7)60   8 R4
4. 8)93   11 R5
5. 3)84   28
6. 2)54   27
7. 4)333   83 R1
8. 6)498   83
9. 9)608   67 R5
10. 2)189   94 R1
11. 8)316   39 R4
12. 7)170   24 R2
13. 4)864   216
14. 5)823   164 R3
15. 3)525   175
16. 6)2816   469 R2
17. 9)3730   414 R4
18. 8)7031   878 R7
19. 5)4444   888 R4
20. 4)9839   2459 R3
21. 2)3726   1863
22. 5)47 978   9 595 R3
23. 6)29 593   4 932 R1
24. 9)50 328   5 592
25. 7)18 098   2 585 R3
26. 2)$158 $79
27. 8)$4704 $588
28. 4)$5.80 $1.45
29. 7)$32.83 $4.69
30. 36 ÷ 5   7 R1
31. 50 ÷ 8   6 R2
32. 61 ÷ 9   6 R7
33. 70 ÷ 4   17 R2
34. 192 ÷ 3   64
35. 348 ÷ 9   38 R6
36. 447 ÷ 7   63 R6
37. 419 ÷ 5   83 R4
38. 730 ÷ 2   365
39. 890 ÷ 3   296 R2
40. 4917 ÷ 8   614 R5
41. 2441 ÷ 7   348 R5
42. 1730 ÷ 9   192 R2
43. 2571 ÷ 4   642 R3
44. 9112 ÷ 5   1822 R2
45. 9568 ÷ 3   3189 R1
46. 45 113 ÷ 6   7518 R5
47. 32 704 ÷ 4   8176
48. 42 238 ÷ 8   5279 R6
49. 17 491 ÷ 2   8745 R1
50. 78 987 ÷ 3   26 329
51. 46 666 ÷ 6   7777 R4
52. $335 ÷ 5 $67
53. $7803 ÷ 9 $867
54. $8.89 ÷ 7 $1.27
55. $16.50 ÷ 6 $2.75

Use division to find an average.

1. 13 715 people visited the museum in 5 d. 2743
2. Reggie cut the 675 cm of thread into 9 pieces. 75 cm
3. Penny earned $224 in 8 weeks. $28
4. The scale showed 216 kg when the 6 children stood on it at the same time. 36 kg
5. The temperature at noon each day was 28 C, 21 C, 18 C, 20 C, 26 C, 24 C, and 24 C. 23°C
6. The heights of the girls were 137 cm, 139 cm, 147 cm, and 141 cm. 141 cm
7. The light bulbs lasted for 975 h, 828 h, 1048 h, 963 h, 775 h, and 1009 h. 933 h
8. The containers held 250 mL, 400 mL, 295 mL, 350 mL, 324 mL, 375 mL, 350 mL, and 400 mL. 343 mL
9. The children had $1.08, $1.39, $1.48, $0.75, and $2.25. $1.39

Solve.

10. Rea's parents want to drive about the same distance each day for 3 d on a trip of 2055 km. About how far should they drive each day? 685 km
11. The grocery store chain had 48 384 balloons to give away in its 7 stores. How many balloons should be sent to each store so the stores get the same number of balloons? 6912

193

Page 193

OBJECTIVE

Demonstrate competence in division skills; solve related word problems

RELATED ACTIVITIES

- Have students make up their own addition and subtraction word problems. The problems can be written on index cards and the solutions can be shown on index cards of a different color. Have the students exchange problem cards.

To help students with ideas for writing word problems, provide them with such things as maps that have charts showing distances in kilometres, containers marked in millilitres, and magazines and catalogs with prices of objects shown.

LESSON ACTIVITY

Using the Page

- These exercises review skills in dividing with regrouping and in solving related word problems, including the concept of average. There is a gradual increase in difficulty from Ex. 1-29 and this is repeated in Ex. 30-55. Division with amounts of money is included in Ex. 26-29 and Ex. 52-55. If you wish to assign a number of exercises each day for several days, the column of twelve exercises from Ex. 1-28 along with Ex. 2, 5, 8, and word problems 1, 6, and 10 can constitute a day's assignment. The rest of the exercises can be assigned in a similar way. The results will help you to determine areas in which students may need review or reteaching. Adapt the lesson and activities on the appropriate pages.

UNIVERSITY OF ALBERTA

Unit 10 Overview

Division

The operation of division is extended in this unit to two-digit divisors and to dividends with up to six digits. Place values are emphasized in dealing with digits in dividends and in quotients. Divisors are rounded to the nearest ten to obtain trial divisors, and if trial quotients are incorrect, they are adjusted either upward or downward. Estimation of quotients is presented using rounded divisors and dividends. Word problems are included at every stage of development to relate the operation to practical applications. The lesson on the use of the calculator presents several series of computations which have surprising results. These computations may also be completed without using calculators. The use of equations in solving problems is continued from the earlier introduction in Unit 8.

Prerequisite Skills

- find the quotient and the remainder, divisors and quotients to 9
- multiply by a multiple of ten from 10 to 90
- round whole numbers
- write the family of related facts for addition and subtraction, and for multiplication and division

Unit Outcomes

- divide by a one-digit number, quotients without zeros, dividends with up to five digits
- divide by a one-digit number, zero in one or more places in the quotient, dividends with up to five digits
- divide by a multiple of ten from 10 to 90, dividends with up to six digits, one-digit quotients or quotients that are multiples of ten from 10 to 90, multiples of one hundred from 100 to 900, or multiples of one thousand from 1000 to 9000, remainders zero
- divide by a multiple of ten from 10 to 90, dividends with three to six digits
- divide by a two-digit number when the trial estimates for the digits in the quotient are correct, divisors with the ones' digit to 4, dividends with three to six digits
- divide by a two-digit number when the trial estimates for the digits in the quotient are correct, divisors rounded to the nearest ten, dividends with up to six digits
- divide by a two-digit number when trial estimates for digits in the quotient may be incorrect, dividends with up to six digits
- round the divisor and the dividend and divide to estimate the quotient, and then compare the estimate of the quotient with the exact quotient
- solve word problems involving division
- compute quickly with and without a calculator
- write an equation for information given in a word problem; solve the problem by writing and solving a related equation

Background

In this unit, the shortened standard algorithm is introduced. The quotient is written on one line and the product of the digit in the quotient and the divisor is written in the appropriate place-value position without a zero, or zeros, in the ones', tens', hundreds', or thousands' places. It is important to emphasize place values when the shortened form is used. For instance, in the example shown, the first possible division involves 39 tens and, therefore, the digit 5 of the quotient is written in the tens' place. For the same reason, the product 35 means 35 tens, and if the 5 of the 35 is aligned with the tens' place, it is not necessary to write 350. The remainder 4 represents 4 tens and, combined with 3 ones from the dividend, the next step involves division of 43 ones.

$$\begin{array}{r} 56 \\ 7\overline{)393} \\ \underline{35}\downarrow \\ 43 \\ \underline{42} \\ 1 \end{array}$$

The importance of place value is even greater when zeros occur in quotients. Without concern for place value, students often omit zeros as they hurriedly consider the next digit(s) of the dividend (A); or, sometimes they insert an unnecessary step of multiplying by zero (B).

A
$$\begin{array}{r} 24 \\ 6\overline{)1224} \\ \underline{12}\downarrow\downarrow \\ 024 \\ \underline{24} \\ 0 \end{array}$$

B
$$\begin{array}{r} 204 \\ 6\overline{)1224} \\ \underline{12}\downarrow \\ 02 \\ \underline{00}\downarrow \\ 24 \\ \underline{24} \\ 0 \end{array}$$

C
$$\begin{array}{r} 20 \\ 6\overline{)1224} \\ \underline{12}\downarrow \\ 02 \end{array}$$

D
$$\begin{array}{r} 204 \\ 6\overline{)1224} \\ \underline{12}\downarrow\downarrow \\ 024 \\ \underline{24} \\ 0 \end{array}$$

Careful attention to place value can overcome both of these difficulties. For instance, in the same example, the first division of 12 hundreds is performed with a 2 in the hundreds' place of the quotient and with a remainder of 0 (C). The tens' digit of the dividend is written beside the 0, as indicated by the arrow; since 2 (tens) cannot be divided by 6, a 0 is written in the tens' place of the quotient. This is the step which is most frequently omitted. Since no division is possible with tens, there is no multiplication step and the operation proceeds directly to division of ones with the 4 ones of the dividend combined with the 2 tens to make 24 ones. The digit 4 of the quotient is written in the ones' place (D).

Because basic facts for division and multiplication are limited to one-digit factors, dividing by a two-digit divisor presents new difficulties. It is necessary to round a two-digit divisor to the nearest ten, so that basic facts may be considered using one digit in the quotient and only the tens' digit of the divisor. For example, in the case of 1376 ÷ 43 (E), rounding the divisor 43 to 40 brings the 4 into use as a factor. The digit 4 and the number 13 are examined to discover what the digit should be in the quotient. Since $4 \times 3 = 12$, and $40 \times 3 = 120$, the first digit (3) of the quotient is in the tens' place, and $43 \times 3 = 129$ is used in the algorithm. The next step involves 4 (from 40) and 8 (from 86) from which the digit 2 is chosen for the quotient. Sometimes the estimated digits for the quotient are correct as in the preceding example, but frequently they are too great and must be decreased (F), or they are too small and must be increased (G).

E
$$\begin{array}{r} 32 \\ 43\overline{)1376} \\ \underline{129}\downarrow \\ 86 \\ \underline{86} \\ 0 \end{array}$$

F
$$\begin{array}{r} \square \\ 34\overline{)2665} \end{array}$$
Round 34 to 3(0).
$3 \times 8 = 24$
$30 \times 8 = 240$
Try 34×8 (tens).

$$\begin{array}{r} 8 \\ 34\overline{)2665} \\ \underline{272} \end{array}$$
Decrease 8 to 7.

$$\begin{array}{r} 7\square \\ 34\overline{)2665} \\ \underline{238}\downarrow \\ 285 \end{array}$$

G

$36\overline{)1553}$ with quotient □	$36\overline{)1553}$ with quotient 3	$36\overline{)1553}$ with quotient 4□
Round 36 to 4(0).	108	144
4 × 3 = 12	47	113
40 × 3 = 120	Increase 3 to 4.	
Try 36 × 3 (tens).		

Because such steps can occur for each digit in the quotient for a division, it is obvious that long division is a complex process involving a certain amount of trial and error as well as competent use of the basic facts and operations in multiplication and subtraction.

In Unit 8 the *Problem Solving* lesson introduced the use of equations to structure mathematical relationships. This approach is continued in this unit by actually solving the equations. Sometimes the unknown may be found directly as in 365 ÷ 5 = n and 3600 + 1495 = n. However, equations must often be rewritten to establish workable relationships. In some cases, only the known elements need to be rearranged; for example, 4750 − n = 2436 can be rewritten as 4750 − 2436 = n, and 3450 ÷ n = 75 as 3450 ÷ 75 = n. In other cases, inverse operations are required. For example, to solve 3765 + n = 7200, the unknown addend is found by using the subtraction 7200 − 3765 = n; to solve 48 × n = 1776, the unknown factor is found by using division in 1776 ÷ 48 = n.

Two-step solutions are encountered in one of the sets of exercises. If the same kind of operation is used in both steps, the order of considering the numbers is not critical. For example, if a boy delivers 46 papers each day for 2 weeks (6 issues each week), the number of papers delivered in that time may be found in two ways, since both steps use multiplication.

2 × 6 = 12	6 × 46 = 276
12 × 46 = 552	2 × 276 = 552
He delivered 552 papers.	He delivered 552 papers.

On the other hand, where more than one kind of operation is involved, it is necessary to determine which numbers and which operation to use first. If a girl who earns money by baby-sitting is paid $1.00 an hour for 3 hours, and is given a tip of 50¢, there are two different operations, multiplication and addition, and the correct numbers and operations must be selected. In this case, multiplication is performed first using 3 and 100, and then 50 is added.

3 × 100 = 300
300 + 50 = 350
She received $3.50.

Therefore, careful reading is required for word problems, not only to comprehend and visualize the situations, but, in complex situations, to determine which numbers and operations to use and the order of performing the operations.

There are several ways to check the accuracy of work in division. Estimating the quotient before performing the actual division is a quick way of detecting some types of errors. Multiplication of the quotient and the divisor and the addition of any remainder to that product is a method which uses the inverse operations.

Teaching Strategies

For the first two lessons of the unit, models of thousands, hundreds, tens, and ones may be used to demonstrate the regrouping which frequently occurs in dividing numbers by one-digit divisors. It should be pointed out that the use of such models relates to partitive division, that is, the sharing or partitioning of a number into equal groups. For example, in 2184 ÷ 4, the dividend is first considered as 2 thousands 1 hundred 8 tens 4 ones, and then as 21 hundreds 8 tens 4 ones.

	5 hundreds	4 tens	6 ones
4)	21 hundreds	8 tens	4 ones
	20 hundreds		
	1 hundred		
		18 tens	
		16 tens	
		2 tens	
			24 ones
			24 ones
			0

After dividing the 21 hundreds into 4 equal groups of 5 hundreds, there is 1 hundred left to regroup with 8 tens as 18 tens; after dividing the 18 tens into 4 equal groups of 4 tens, there are 2 tens left to regroup with 4 ones as 24 ones, and these are divided into 4 equal groups of 6 ones. Each of the 4 equal groups contains 5 hundreds 4 tens 6 ones, and therefore, 2184 ÷ 4 = 546.

In the directives for most of the exercises, students are asked to check their work and this should not be overlooked. The expression "practice makes perfect" loses its value if students practice incorrect methods. It is important that any errors be detected at once so that reteaching or review may be planned without delay.

Accuracy in the work with division in this unit is dependent on complete mastery of basic multiplication facts, competence in multiplication of one-digit and two-digit numbers by one-digit numbers, and in subtraction of three-digit numbers. For instance, in the example shown, the basic facts 7 × 4, 7 × 2, 3 × 4, and 3 × 2 are used, multiplication with regrouping is used twice, and subtraction with regrouping is also used twice. It is important, then, to include drill of these facts and skills on a regular basis.

$$\begin{array}{r} 73 \\ 24\overline{)1761} \\ \underline{168} \\ 81 \\ \underline{72} \\ 9 \end{array}$$

Place values should be emphasized at all times in the work with division. Some students may be helped by telling in advance the name of the place for the first digit of the quotient. For example, in 1761 ÷ 24, it is obvious that neither 1 thousand, nor 17 hundreds can be used, but 176 tens can; thus, the first digit of the quotient will be in the tens' place. From this decision it is also apparent that there will be two digits in the quotient. Students may be asked to begin exercises by writing the place-value name for the first digit in the quotient and the number of digits in the quotient for each exercise. For example, for Ex. 9 on page 205, the students would write "hundreds, 3".

Materials

models for thousands, hundreds, tens, and ones
tracing paper (optional)
graph paper for each student
circular objects for tracing (optional)
calculators (optional)

Vocabulary

vendor	pavilion
tonne, t	turnstile

Pages 194-195

LESSON OUTCOME

Divide by a one-digit number, quotients without zeros, dividends with up to five digits

Vocabulary

vendor

Prerequisite Skills

Find the quotient and the remainder, divisors and quotients to 9

Checking Prerequisite Skills

Find the quotient and the remainder.

1. $3\overline{)25}$ 8 R1 **2.** $5\overline{)49}$ 9 R4

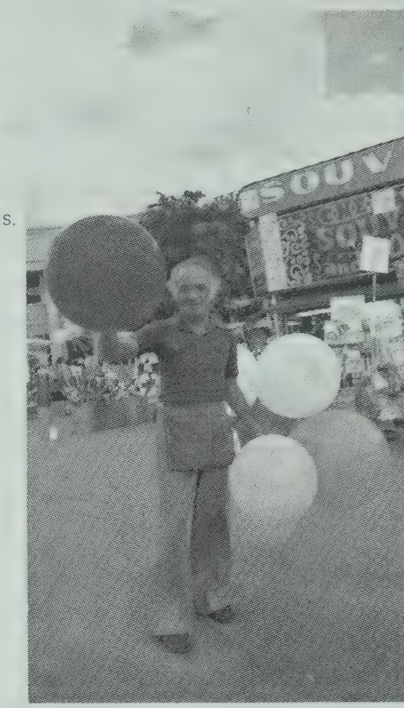
3. $8\overline{)66}$ 8 R2 **4.** $7\overline{)30}$ 4 R2

5. $2\overline{)15}$ 7 R1 **6.** $9\overline{)32}$ 3 R5

10 DIVISION

Dividing by a One-Digit Number

1590 balloons were divided equally among 6 balloon vendors. How many balloons did each vendor receive?



Divide 1590 by 6. $6\overline{)1590}$

1590 shows 1 thousand. Since 1 is less than 6, think of 1 thousand 5 hundreds as 15 hundreds. Then divide the 15 hundreds.

6 × 2 = 12
6 × 3 = 18 . . . too great!

Use 6 × 2 hundreds = 12 hundreds.

Write
$$\begin{array}{r} 2 \\ 6\overline{)1590} \\ 12 \\ \hline 3 \end{array}$$

Think of the 3 hundreds 9 tens that remain as 39 tens.

$$\begin{array}{r} 2 \\ 6\overline{)1590} \\ 12\downarrow \\ \hline 39 \end{array}$$

Then divide the 39 tens.

6 × 6 = 36
6 × 7 = 42 . . . too great!

Use 6 × 6 tens = 36 tens.

Write
$$\begin{array}{r} 26 \\ 6\overline{)1590} \\ 12 \\ \hline 39 \\ 36 \\ \hline 3 \end{array}$$

Think of the 3 tens 0 ones that remain as 30 ones.

$$\begin{array}{r} 26 \\ 6\overline{)1590} \\ 12 \\ \hline 39 \\ 36\downarrow \\ \hline 30 \end{array}$$

Then divide the 30 ones.

6 × 5 = 30

Write
$$\begin{array}{r} 265 \\ 6\overline{)1590} \\ 12 \\ \hline 39 \\ 36 \\ \hline 30 \\ 30 \\ \hline 0 \end{array}$$

Each vendor received 265 balloons.

194

LESSON ACTIVITY

Before Using the Pages

- Review place value in numerals with up to five digits using examples similar to the following.

  1. $\boxed{4}$ 1 — 4 tens; $\boxed{4\ 1}$ — 41 ones
  2. $\boxed{5}$ 8 4 — 5 hundreds; $\boxed{5\ 8}$ 4 — 58 tens
  3. $\boxed{1}$ 3 2 6 — 1 thousand; $\boxed{1\ 3}$ 2 6 — 13 hundreds
  4. $\boxed{2}$ 4 6 1 3 — 2 ten thousands; $\boxed{2\ 4}$ 6 1 3 — 24 thousands

- Have the students recall that sharing a number of objects equally is associated with the operation of division. Review that division is carried out place by place from left to right. For example, for $2\overline{)486}$, the hundreds are divided first, then the tens, and then the ones. Without performing division for the following, have students tell what the place value would be for the first digit of the quotient. For example, in $4\overline{)183}$, 1 is less than 4, and 1 hundred 8 tens is thought of as 18 tens. Thus, the first digit of the quotient will be in the tens' place.

$6\overline{)732}$ $4\overline{)183}$ $4\overline{)8513}$ $9\overline{)4726}$ $3\overline{)15\ 462}$

Using the Pages

- Have a student read the word problem at the top of page 194. Discuss the meaning of the word *vendor*. Tell the students that the man in the photograph is a balloon vendor and ask for another way of expressing the same idea.

  Lead the students through the worked example, asking questions about the steps shown in red, to review the process of division. For instance, ask why the first digit of the quotient is in the hundreds' place rather than the thousands' place, how the numbers 12 and 3 are obtained, what place value 12 and 3 represent (hundreds), and so on. Emphasize that division is carried out place by place from left to right, regrouping as needed. Point out that the remainder in this division is zero.

Working Together

Complete.

1. 4)276 — quotient 6▢ (9)
   24
   36
   ▢36 (36)
   ▢ 0 (0)

2. 7)6892 — quotient 9▢▢ R▢ (84, 4)
   63
   5▢ (9)
   ▢56 (56)
   ▢32 (32)
   ▢28 (28)
   ▢ 4 (4)

Divide.

3. 5)385 — 77
4. 9)2799 — 311
5. 53 678 ÷ 6 — 8946 R2
6. $51 448 ÷ 8 — $6431

Exercises

Divide.

1. 5)215	43	2. 3)276	92
3. 4)330	82 R2	4. 6)566	94 R2
5. 2)6935	3467 R1	6. 7)4994	713 R3
7. 8)2788	348 R4	8. 5)3211	642 R1
9. 2)72 895	36 447 R1	10. 3)28 447	9 482 R1
11. 9)68 609	7 623 R2	12. 8)51 296	6 412
13. 4)21 094	5 273 R2	14. 7)36 283	5 183 R2
15. 9)3939	437 R6	16. 5)32 411	6 482 R1
17. 6)$10 338	$ 1 723	18. 8)$9016	$1 127
19. 28 069 ÷ 9	3118 R7	20. 42 264 ÷ 8	5283
21. 12 350 ÷ 3	4116 R2	22. 3646 ÷ 4	911 R2
23. $30 702 ÷ 7	$4386	24. $2052 ÷ 6	$342

Copy and complete each of these.

*25. 6)25 78▢ — quotient 4 ▢98 (2; 8)

*26. 8)▢7▢7▢ — quotient 2 14▢ (7; 1, 1, 6)

*27. ▢)55 ▢▢▢ — quotient 6 118 (9; 062)

*28. 7)▢▢ ▢▢▢ — quotient 3 221 (22 547)

Study these division sentences.

5 ÷ 5 = 1	5 ÷ 1 = 5
6 ÷ 6 = 1	6 ÷ 1 = 6
7 ÷ 7 = 1	7 ÷ 1 = 7
0 ÷ 5 = 0	3 ÷ 4 = 0 R3
0 ÷ 6 = 0	2 ÷ 4 = 0 R2
0 ÷ 7 = 0	1 ÷ 4 = 0 R1

Divide.

1. 2)2 — 1	2. 1)9 — 9	3. 5)3 — 0 R3
4. 4)0 — 0	5. 8)2 — 0 R2	6. 4)4 — 1
7. 1)3 — 3	8. 6)4 — 0 R4	9. 3)0 — 0
10. 8)0 — 0	11. 3)2 — 0 R2	12. 8)8 — 1
13. 9)8 — 0 R8	14. 7)3 — 0 R3	15. 9)0 — 0
16. 2)1 — 0 R1	17. 1)0 — 0	18. 1)4 — 4
19. 8)5 — 0 R5	20. 3)3 — 1	21. 1)1 — 1
22. 9)9 — 1	23. 2)0 — 0	24. 9)7 — 0 R7
25. 1)8 — 8	26. 6)3 — 0 R3	
27. 7)5 — 0 R5	28. 1)2 — 2	
29. 3)1 — 0 R1	30. 8)4 — 0 R4	

try this

195

RELATED ACTIVITIES

- Challenge some of the students to find solutions for Ex. 25-28 so that remainders other than zero are also obtained. For instance, for Ex. 25, although the quotient will be the same as before (4298), a dividend of 25 789 would result in a remainder of 1. Similarly, for Ex. 26, the ones' digit of the dividend may be 6, 7, 8, or 9, to give remainders of 0, 1, 2, and 3, respectively. Two examples for Ex. 27 are shown below.

```
   6 118 R1          6 118 R8
9)55 063          9)55 070
  54                54
   1 0               1 0
     9                 9
     16                17
      9                 9
      73                80
      72                72
       1                 8
```

- Have students complete sequences of exercises similar to the following and note patterns in remainders for a given divisor.

```
 2 R2    2 R1    2 (R0)
3)8     3)7     3)6
 1 R2    1 R1    1 (R0)
3)5     3)4     3)3
 0 R2    0 R1    0 (R0)
3)2     3)1     3)0

 6 R8    6 R7        5 (R0)
9)62    9)61  ...  9)45
```

Ask what operation can be used to check division and have a few students demonstrate this on the board for the division on page 194.

Working Together: The partially completed divisions of Ex. 1 and 2 enable the students to study the steps shown and complete the divisions. It would be desirable for this to be an oral discussion. For Ex. 3-6, you may wish to have students predict the place value of the first digit of the quotient, and thus tell the number of digits for each quotient.

Exercises: Have the students note that there is only one solution for each of Ex. 25-28 because the divisions have a remainder of zero. Although these may be solved by thinking of division, suggest that there is another way to find the unknown numbers. Do not tell the students how to proceed, but watch for those who multiply the divisor and the quotient.

Try This: These exercises help to prepare the students for the concept in the following lesson. At that time they will encounter zeros in the quotients, for example, in the tens' place in 4)432 (108). In this feature, exercises such as 4)4 and 4)3 are presented. Have the students use multiplication to check each division. Some examples are shown below.

1. 2)2 — 1; ×|1 over 2|2   11. 3)2 — 0 R2; ×|0 over 3|0 0 + 2 = 2

Assessment

Divide.

1. 3)516 — 172
2. 7)8065 — 1152 R1
3. 5)3389 — 677 R4
4. 8)12 909 — 1 613 R5

Pages 196 - 197

LESSON OUTCOME

Divide by a one-digit number, zero in one or more places in the quotient, dividends with up to five digits

Materials

models for thousands, hundreds, tens, and ones

Prerequisite Skills

Divide by a one-digit number, quotients without zeros, dividends with up to five digits

Checking Prerequisite Skills

Divide.

1. $3\overline{)642}$ 214
2. $5\overline{)276}$ 55 R1
3. $2\overline{)5236}$ 2618
4. $7\overline{)9876}$ 1410 R6
5. $4\overline{)6364}$ 1591
6. $9\overline{)12\ 096}$ 1 344

Zeros in the Quotient

The roller coaster ride costs 3 tickets. In 2 h, 2724 tickets were collected for the roller coaster. How many rides were taken on the roller coaster in 2 h?

Divide 2724 by 3. $3\overline{)2724}$

2724 shows 2 thousands. Since 2 is less than 3, think of 2 thousands 7 hundreds as 27 hundreds.
Then divide the 27 hundreds.

$3 \times 9 = 27$

Use 3×9 hundreds = 27 hundreds.

Write
$$\begin{array}{r} 9 \\ 3\overline{)2724} \\ 27 \\ \hline 0 \end{array}$$

There are 0 hundreds left, but there are still 2 tens to divide.

$$\begin{array}{r} 9 \\ 3\overline{)2724} \\ 27\downarrow \\ \hline 02 \end{array}$$

Divide the 2 tens.

Since 2 is less than 3, write 0 in the tens place.

Write
$$\begin{array}{r} 90 \\ 3\overline{)2724} \\ 27 \\ \hline 02 \end{array}$$

Think of the 2 tens 4 ones that remain as 24 ones.

$$\begin{array}{r} 90 \\ 3\overline{)2724} \\ 27\ \downarrow \\ \hline 024 \end{array}$$

Then divide the 24 ones.

$3 \times 8 = 24$

Write
$$\begin{array}{r} 908 \\ 3\overline{)2724} \\ 27 \\ \hline 024 \\ 24 \\ \hline 0 \end{array}$$

The zero is needed in the quotient to show that 2724 divided by 3 is 908 and not 98.

908 rides were taken on the roller coaster in 2 h.

196

LESSON ACTIVITY

Before Using the Pages

- Write the exercises $3\overline{)148}$, $3\overline{)815}$, and $3\overline{)612}$ on the board. For each exercise, have students tell the place value of the first digit of the quotient, explain how this is known, tell the number of digits there will be in the quotient, and explain how this is determined without carrying out the division. For example, for $3\overline{)612}$, because 6 is greater than 3, the first digit of the quotient will be in the hundreds' place. To complete the numeral for the quotient, a tens' digit and a ones' digit will be required. Thus, the quotient will have three digits.

  Assign the division $3\overline{)612}$, reminding the students that it was agreed that the quotient would have three digits. When they have arrived at a quotient, have them use multiplication to check their work. For students whose quotients are incorrect, use models to find the quotient, emphasizing that in each of the three equal groups, there are 0 tens. The quotient is shown as 2 hundreds 0 tens 4 ones. Have a student write the standard numeral on the board. Discuss the importance of writing 0 in the tens' place. Check the quotient 204 by using multiplication.

Using the Pages

- The worked example shows and explains the steps in a division for which there are 0 tens in the quotient. Have a student read the word problem at the top of page 196. Point out that the thousands' digit of the dividend is 2 and the divisor is 3. Ask what the place value will be of the first digit of the quotient and how many digits there will be in the quotient. Lead the students through the example, emphasizing the place-by-place aspect of division. Dividing 27 hundreds gives 9 hundreds in the quotient, dividing 2 tens gives 0 tens in the quotient, and dividing 24 ones gives 8 ones in the quotient.

  Have some students use multiplication to check the division and have others use models.

Working Together: Ask questions about each of Ex. 1-3 and complete these on the board with the students. Ask, for

Working Together

Give the next digit
for each quotient.

1. 6)420   70
   42
   00

2. 2)609   30
   6
   00

3. 3)12 027   4 00
   12
   0 02

Complete.

4. 5)1015   2 03
   10
   01 5
   15
   0

5. 8)53 600   6 7 00
   48
   5 6
   5 6
   00

6. 7)63 214   9 030 R 4
   63
   021
   21
   04

Divide.

7. 6)6335   1055 R5
8. 9)$63 009 $ 7 001
9. $120.80 ÷ 4 $30.20

Exercises

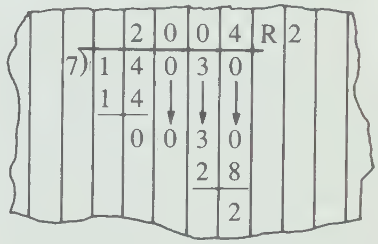
Divide.

1. 2)180   90
2. 5)3350   670
3. 4)242   60 R2
4. 6)37 383   6 230 R3
5. 3)27 272   9 090 R2
6. 9)7254   806
7. 7)14 030   2 004 R2
8. 8)48 244   6 030 R4
9. 7)37 122   5 303 R1
10. 6)3000   500
11. 4)30 803   7 700 R3
12. 9)72 050   8 005 R5
13. 6)48 000   8 000
14. 8)24 322   3 040 R2
15. 9519 ÷ 5   1903 R4
16. 2720 ÷ 3   906 R2
17. 18 199 ÷ 2   9099 R1
18. 6086 ÷ 8   760 R6
19. 54 005 ÷ 9   6000 R5
20. 43 804 ÷ 6   7300 R4
21. 2802 ÷ 7   400 R2
22. 10 382 ÷ 5   2076 R2
23. $6356 ÷ 7 $908
24. $18 603 ÷ 3 $6201

197

RELATED ACTIVITIES

- For further practice, you may wish to have the students complete Ex. 33-36 on page 336.
- Have students use multiplication (and addition for the remainders) and/or models to check a few of the division exercises on page 197.
- Have students write division exercises for other students to complete.
- Using lined paper turned sideways is effective in helping students write division exercises with careful attention to place value. Using arrows as shown in the worked example on page 194 is also helpful. For the example below, the subtraction 14 − 14 gives a difference of 0 (thousands). The first arrow is drawn to indicate hundreds and then 0 is written in the hundreds' place of the quotient. The rest of the division proceeds in a similar manner.



example, why the first digit of the quotient is in the tens' place for Ex. 1 and what is divided after 42 tens. Some students may work independently on Ex. 4-6 while you work with the other students at the board. Have them answer questions similar to the following for Ex. 4.

"Can we divide the thousands?"
"How many hundreds are divided?"
"How many hundreds are in the quotient?"
"How many hundreds are left?"
"What is divided after the hundreds?"
"Can the tens be divided?"
"How many tens are in the quotient?"
"What is divided after the tens?"

If necessary, use models to demonstrate the steps in a division. Note that there will be a decimal point in the quotient for Ex. 9.

Exercises: Observe the students as they work and provide assistance as required.

Assessment

Divide.

1. 2)140   70
2. 5)518   103 R3
3. 3)1524   508
4. 9)27 848   3 094 R2

OBJECTIVE

Demonstrate competence in division; solve related word problems

Materials

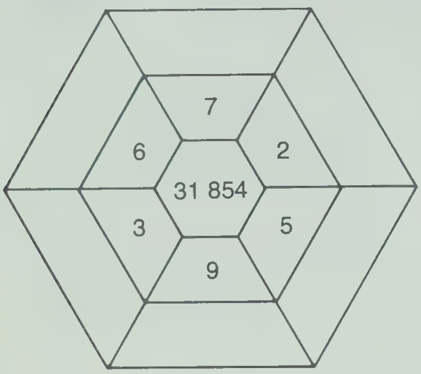
tracing paper (optional)

RELATED ACTIVITIES

- Rapid drills, both oral and written, to practice basic multiplication facts can help to improve performance in division. Include exercises that involve one missing factor.

  4 × 9 = ____ 7 × ____ = 56

- Prepare diagrams for students to practice division, using copies of page T 390.



9)31 854

Practice

Divide. Then check the six most difficult exercises.

Remember you can multiply the divisor and the quotient, then add the remainder to check your work.

1. 7)602 86	2. 4)360 90	3. 8)567 70 R7	4. 5)$245 $ 49
5. 6)2723 453 R5	6. 9)6318 702	7. 6)2404 400 R4	8. 8)$5680 $ 710
9. 2)1002 501	10. 8)1460 182 R4	11. 3)1507 502 R1	12. 4)$3372 $ 843
13. 4)5032 1258	14. 7)33 810 4 830	15. 5)21 029 4 205 R4	16. 6)$24 174 $ 4 029
17. 8)38 404 4 800 R4	18. 3)21 005 7 001 R2	19. 9)63 454 7 050 R4	20. 8)$32 000 $ 4 000
21. 434 ÷ 9 48 R2	22. 494 ÷ 7 70 R4	23. 1529 ÷ 3 509 R2	24. $21.50 ÷ 5 $4.30
25. 2000 ÷ 4 500	26. 4784 ÷ 8 598	27. 42 002 ÷ 6 7000 R2	28. $98.74 ÷ 2 $49.37
29. 12 178 ÷ 3 4059 R1	30. 33 333 ÷ 6 5555 R3	31. 26 454 ÷ 5 5290 R4	32. $320.36 ÷ 4 $80.09

Solve.

33. Sandy and her six friends earned $39.90 at the fair. If they share the money equally, how much will each have? $5.70

34. Jim and his seven friends have 102 tickets altogether. How many rides that cost 2 tickets each can they take? 6. There will be 6 tickets left.

35. The Wild Mouse costs 4 tickets. Each of 29 students has 27 tickets. How many rides on the Wild Mouse can they take? 195. They will have 3 tickets left.

36. Each of 23 students has 19 tickets. How many rides costing 3 tickets will their tickets buy? 145. They will have 2 tickets left.

Help Martha find the path to the Ferris wheel by following the exercises with even remainders.

	4)20 015 5 003 R3	9)54 809 6 089 R8	6)1222 203 R4	7)4517 645 R2
3)18 032 6 010 R2	6)545 90 R5	3)12 008 4 002 R2	8)1779 222 R3	5)3042 608 R2
9)9566 1062 R8	5)15 023 3 004 R3	7)629 89 R6	6)25 813 4 302 R1	8)24 038 3 004 R6
4)2242 560 R2	7)2869 409 R6	3)776 258 R2	4)16 037 4 009 R1	9)4171 463 R4
2)4937 2468 R1	5)6878 1375 R3	8)3279 409 R7	6)4324 720 R4	Ferris wheel

198

LESSON ACTIVITY

Using the Page

- Have a student select one of the exercises from Ex. 1-32 and write it on the board. Have other students help to complete the exercise, explaining the steps of the procedure. Draw particular attention to zeros in the quotient and the need for them. Use multiplication to check the division. Repeat as needed for other exercises to review the work of the two previous lessons.

  Have the students study Ex. 33-36. Ask how many persons are sharing the money in Ex. 33. Ask how many steps are needed to solve Ex. 36.

  If you wish, give each student a sheet of tracing paper on which to mark the path to the Ferris wheel after they have completed the necessary division exercises. You may need to review the concept of even numbers and odd numbers.

Dividing Evenly by a Multiple of 10

Byron and his brother are arranging 1200 chairs in rows for a concert. They are placing 30 chairs in each row. How many rows will there be?

Divide 1200 by 30.

1200 shows 1 thousand.
1200 shows 12 hundreds.

Since 12 is less than 30, think of 12 hundreds 0 tens as 120 tens. Then divide the 120 tens.

30 × 4 = 120

Use 30 × 4 tens = 120 tens.

Write

```
    4
30)1200
   120
     0
```

There are 0 tens 0 ones left to divide.

```
    4
30)1200
   120↓
    00
```

30 × 0 = 0

Write

```
    40
30)1200
   120
    00
```

There will be 40 rows.

Working Together

Divide.

1. 30)60 2
2. 80)4000 50
3. 30 000 ÷ 60 500
4. 720 000 ÷ 90 8000

Exercises

Divide.

1. 30)90 3
2. 50)3500 70
3. 20)600 30
4. 40)160 000 4 000
5. 80)32 000 400
6. 10)40 000 4 000
7. 90)810 9
8. 70)7000 100
9. 4800 ÷ 60 80
10. 300 000 ÷ 50 6000
11. 2700 ÷ 30 90
12. 10 000 ÷ 20 500
13. 36 000 ÷ 40 900
14. 6300 ÷ 90 70
15. 420 ÷ 70 6
16. 560 000 ÷ 80 7000

199

Page 199

LESSON OUTCOME

Divide by a multiple of ten from 10 to 90, dividends with up to six digits, one-digit quotients or quotients that are multiples of ten from 10 to 90, multiples of one hundred from 100 to 900, or multiples of one thousand from 1000 to 9000, remainders zero

Prerequisite Skills

Multiply by a multiple of ten from 10 to 90; divide by a one-digit number

Checking Prerequisite Skills

Multiply.

1. 30 × 4 120
2. 60 × 8 480
3. 50 × 9 450
4. 20 × 8 160

Divide.

5. 9)2700 300
6. 8)32 000 4 000

RELATED ACTIVITIES

- Completing pairs of exercises similar to the following can help students gain an insight into the concept of division. Emphasize how to determine the place value of the first digit in each quotient.

1.	6)30	60)300
2.	6)300	60)3000
3.	6)3000	60)30 000
4.	6)30 000	60)300 000

LESSON ACTIVITY

Before Using the Page

- Review place value in numerals with up to six digits using examples similar to the following.

$\boxed{1}$200	1 thousand	$\boxed{1}$60 000	1 hundred thousand
$\boxed{12}$00	12 hundreds	$\boxed{16}$0 000	16 ten thousands
$\boxed{120}$0	120 tens	$\boxed{160}$ 000	160 thousands

- Have students suggest missing numbers to make true sentences for examples similar to the following.

40 × 9 = _____	70 × 6 = _____
40 × _____ = 3 600	70 × _____ = 4 200
40 × _____ = 36 000	70 × _____ = 42 000

- Have students tell the place value of the first digit in each quotient for the following exercises. Ask how many digits there will be in each quotient and how the divisors in the last two exercises differ from those in the first two.

3)4200 6)4200 30)4200 60)4200

Using the Page

- Have a student read the word problem at the top of page 199. Ask questions about the solution. For example, ask why 1 thousand or 12 hundreds cannot be divided first. Emphasize the need to show 0 ones in the quotient to indicate that the ones were divided after the tens. Use multiplication to check the division.

Working Together: Ex. 1-4 lead gradually from two-digit dividends to six-digit dividends. Use other similar exercises as required.

Exercises: Ensure that the students write the first digit of a quotient in the correct place-value position to help reveal the number of other digits (zeros) there will be in the quotient.

Assessment

Divide.

1. 50)5000 100
2. 20)400 20
3. 70)49 000 700
4. 40)16 000 400

Pages 200-201

LESSON OUTCOME

Divide by a multiple of ten from 10 to 90, dividends with three to six digits; solve related word problems

Materials

graph paper for each student

Prerequisite Skills

Divide by a one-digit number; divide by a multiple of ten from 10 to 90, quotients are multiples of 10, 100, or 1000

Checking Prerequisite Skills

Divide.

68 R1 1. 2)137	99 2. 6)594
1020 R3 3. 8)8163	7 502 4. 5)37 510
40 5. 60)2400	50 6. 30)1500
700 7. 80)56 000	8 000 8. 90)720 000

Dividing by a Multiple of 10

Shin took 1300 photographs of the fair. He took 20 photographs with each roll of film. How many rolls of film did he use?

Divide 1300 by 20.

1300 shows 1 thousand 3 hundreds or 13 hundreds. Since 13 is less than 20, think of 13 hundreds 0 tens as 130 tens. Then divide the 130 tens.

20 × 6 = 120
20 × 7 = 140. . . too great!

Use 20 × 6 tens = 120 tens.

```
          6
Write 20)1300
         120
          10
```

Think of the 10 tens 0 ones that remain as 100 ones.

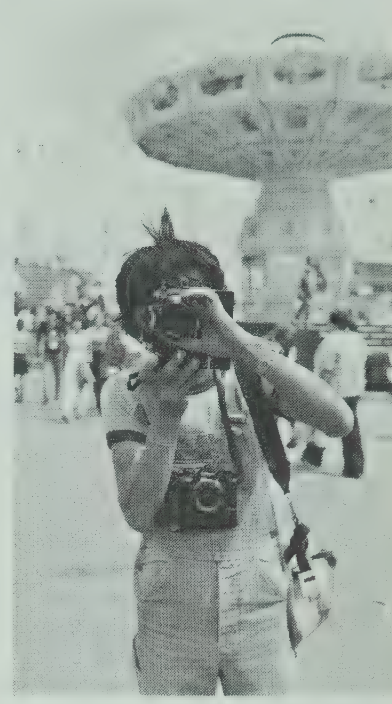
```
       6
   20)1300
      120↓
       100
```

Then divide the 100 ones.

20 × 5 = 100

```
          65
Write 20)1300
         120
          100
          100
            0
```

Shin used 65 rolls of film.



Working Together

Complete.

```
       3   10
       9▒ R▒
1. 70)6520
      630
       ▒220
       ▒210
       ▒ 10
```

Divide. Then check.

2. 50)280 — 5 R30 3. 30)1920 — 64

4. 31 347 ÷ 60 — 522 R27 5. 84 040 ÷ 80 — 1050 R40

200

LESSON ACTIVITY

Before Using the Pages

- Write the following exercises on the board. Have students tell the place value of the first digit in the quotient and the number of digits in the quotient for each exercise. Tell students to show a square for each digit there will be in a quotient, as indicated in the first exercise below. Ask students to explain their answers.

  □□□
  2)1 8 0 0 20)1800 20)1860

  Have the students complete the divisions. Ask a few students to show their work on the board. Discuss similarities and differences among divisors, dividends, quotients, and the steps required to arrive at the quotients.

Using the Pages

- The worked example shows and explains the steps in the division 20)1300. Have a student read the word problem at the top of page 200. Direct the students' attention to the photograph and ask whether anyone can identify the fair (Canadian National Exhibition). You may wish to have the students turn to other pages of this unit to see some of the photographs Shin took at the fair (pages 194, 202, and 208).

  Lead the students through the steps of the solution, paying particular attention to the statements that explain why the division starts with 130 tens and not with 1 thousand or 13 hundreds. Then have the students complete the same division on their own and check their solution with the worked example.

  Have the students turn to page 199 and recall that all the exercises on that page had divisors that were multiples of ten. Ask whether the same is true of the exercises on page 201. Have the students recall that all the quotients for the exercises on page 199 ended in one (or more) zeros. In the worked example on page 200, point out that the quotient ends in 5. This will establish how these exercises differ from those in the preceding lesson.

Exercises

Divide as shown on each box of film. Match each result to the photograph topics in this chart. Then name the topic of the photographs.

Example: $80\overline{)7212}$ = 90 R12   concerts
720
12

Result	Topics of the photographs
8 R17	Agriculture
90 R12	Concerts
800 R17	Contests
9227 R14	Exhibits
877 R14	Fireworks
9287 R14	Games
903 R14	Parades
8007 R15	Rides

1. $20\overline{)18\,074}$ — 903 R14 Parades
2. $30\overline{)257}$ — 8 R17 Agriculture
3. $70\overline{)61\,404}$ — 877 R14 Fireworks
4. $50\overline{)464\,364}$ — 9 287 R14 Games
5. $40\overline{)35\,094}$ — 877 R14 Fireworks
6. $20\overline{)160\,155}$ — 8 007 R15 Rides
7. $60\overline{)5412}$ — 90 R12 Concerts
8. $80\overline{)70\,174}$ — 877 R14 Fireworks
9. $90\overline{)81\,284}$ — 903 R14 Parades
10. $80\overline{)64\,017}$ — 800 R17 Contests
11. $90\overline{)737}$ — 8 R17 Agriculture
12. $30\overline{)278\,624}$ — 9 287 R14 Games
13. $60\overline{)553\,634}$ — 9 227 R14 Exhibits
14. $90\overline{)720\,645}$ — 8 007 R15 Rides
15. $20\overline{)177}$ — 8 R17 Agriculture
16. $50\overline{)400\,365}$ — 8 007 R15 Rides
17. $40\overline{)320\,295}$ — 8 007 R15 Rides
18. $70\overline{)650\,104}$ — 9 287 R14 Parades
19. $60\overline{)48\,017}$ — 800 R17 Contests
20. $70\overline{)560\,505}$ — 8 007 R15 Rides

21. Use a tally chart to show the number of rolls of film for each topic.
A tally chart is shown at the right.

Solve.

22. Draw a graph to show the number of photographs of each topic if each film contains 20 photographs.
A graph is shown below.

23. Shin took 2000 photographs with 50 rolls of film. He took the same number of photographs with each roll of film. How many photographs did he take with each roll of film? 40

24. Shin took 1200 photographs. He took 30 photographs with each roll of film. How many rolls of film did he use? 40

201

RELATED ACTIVITIES

• For further practice, you may wish to have the students complete Ex. 37-44 on page 336.

• Prepare tables for students to practice division, using copies of page T 390. Division by 10 is especially interesting because of the relationship between the dividend and the quotient.

÷	10	20	90
68 432			

÷	10
476	47 R6
3 929	
16 052	
423 168	

21.

Topic of the photographs	Number of rolls of film
Agriculture	III
Concerts	I
Contests	II
Exhibits	I
Fireworks	III
Games	II
Parades	III
Rides	~~IIII~~

Working Together: Note that the students are to use multiplication to check their divisions for Ex. 2-5. In these exercises, remind the students to pay particular attention to the position of the first digit of a quotient because this enables them to determine the number of digits in the quotient.

Exercises: Have students read the topics named in the chart at the top of page 201 and the directions to the left of the chart. Relate the quotient 90 R12 in the example to the topic, concerts, in the chart. Point out that Ex. 1-20 are shown on shapes that suggest boxes of film. Tell the students that the quotients obtained can be related to the topics in the chart to reveal the kinds of photographs taken on each film.

After the students have had an opportunity to correct any errors in Ex. 1-20, assign Ex. 21 and 22. Provide them with graph paper and review the need for careful consideration of title, headings, and scale for the graph.

Assessment

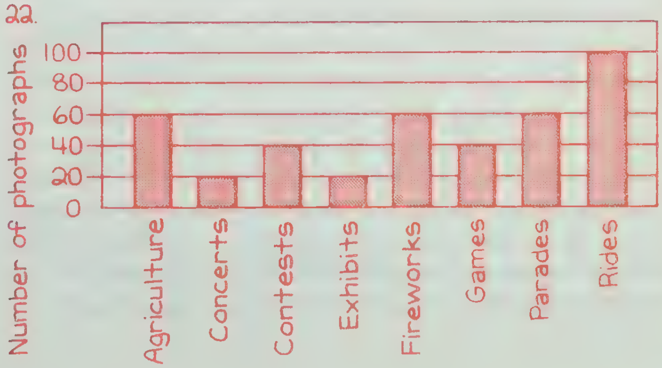
Divide.

1. $20\overline{)348}$ 17 R8
2. $60\overline{)12\,345}$ 205 R45
3. $40\overline{)100\,898}$ 2 522 R18
4. $90\overline{)810\,437}$ 9 004 R77

Solve.

5. Shin took 1400 photographs with 70 identical rolls of film. How many photographs did he take with each roll of film? 20

22.



LESSON OUTCOME

Divide by a two-digit number when the trial estimates for the digits in the quotient are correct, divisors with the ones' digit to 4, dividends with three to six digits; solve related word problems

Vocabulary

tonne, t, pavilion

Prerequisite Skills

Divide by a one-digit number; divide by a multiple of ten from 10 to 90

Checking Prerequisite Skills

Divide.

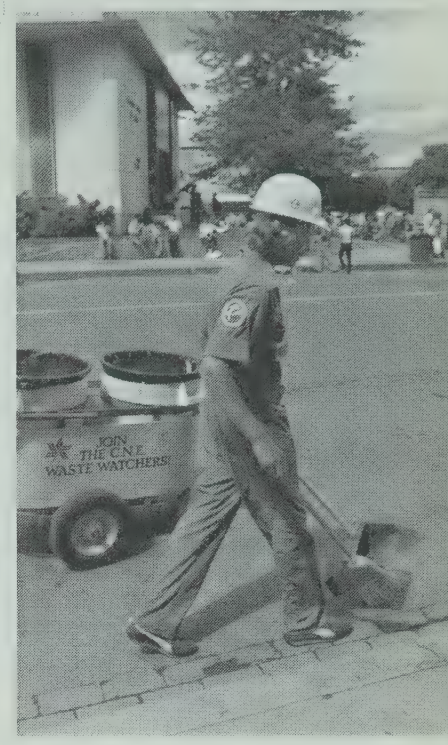
1. $9\overline{)428}$ 47 R5
2. $4\overline{)1216}$ 304
3. $7\overline{)13\ 205}$ 1 886 R3
4. $3\overline{)165\ 165}$ 55 055
5. $60\overline{)2100}$ 35
6. $20\overline{)3375}$ 168 R15
7. $50\overline{)45\ 440}$ 908 R40
8. $80\overline{)58\ 465}$ 730 R65

Dividing by a Two-Digit Number

In August, 1178 t of garbage were collected at a fairgrounds. What was the average mass of garbage collected each day?

The symbol t stands for **tonne**.

1 t = 1000 kg



Divide 1178 by 31.

For $31\overline{)1178}$, think of $30\overline{)1178}$.

1178 shows 1 thousand 1 hundred or 11 hundreds. Since 11 is less than 30, think of 11 hundreds 7 tens as 117 tens. Then divide the 117 tens.

30 × 3 = 90
30 × 4 = 120 . . . too great!

Use 31 × 3 tens = 93 tens.

Write

$$\begin{array}{r} 3 \\ 31\overline{)1178} \\ \underline{93} \\ 24 \end{array}$$

Think of the 24 tens 8 ones that remain as 248 ones.

$$\begin{array}{r} 3 \\ 31\overline{)1178} \\ \underline{93\downarrow} \\ 248 \end{array}$$

Then divide the 248 ones.

30 × 8 = 240
30 × 9 = 270 . . . too great!

Use 31 × 8 = 248.

Write

$$\begin{array}{r} 38 \\ 31\overline{)1178} \\ \underline{93} \\ 248 \\ \underline{248} \\ 0 \end{array}$$

The average mass of garbage collected each day was 38 t.

202

LESSON ACTIVITY

Before Using the Pages

- Write the following exercises on the board. Discuss the place value of the first digit of the quotient and the number of digits in the quotient for each exercise.

  $2\overline{)1495}$ $\qquad$ $20\overline{)1495}$

  Have the students complete the divisions and ask two students to show and explain their work on the board. Write the exercise $21\overline{)1495}$ on the board and have the students note that it is very similar to the exercise $20\overline{)1495}$. Suggest that the quotients must be close in value because the dividends are equal and the divisors differ by one. Ask whether the first digit of the quotient for $21\overline{)1495}$ will be in the thousands' place, the hundreds' place, or the tens' place. If the quotient is similar to that of $20\overline{)1495}$ it will be in the tens' place. Also, for $21\overline{)1495}$, 1 is less than 21, and 14 is less than 21, thus, the division begins with 149 (tens) ÷ 21. Ask the students what the tens' digit of the quotient will probably be for $21\overline{)1495}$. They will likely suggest 7 because it was the tens' digit of the quotient for $20\overline{)1495}$. Have the students copy the exercise and try to complete it. Discuss the results obtained. Summarize that the division $20\overline{)1495}$ was completed first because the divisor, 20, being a multiple of ten was helpful in dividing by 21, which is not a multiple of ten.

Using the Pages

- Begin with a brief discussion of the photographs, having students explain the importance of keeping the fairgrounds (or any area of the environment) clean and tidy. You may wish to discuss the caption "waste watchers" seen in the photograph and compare it with the expression "waist watchers".

  Have a student read the word problem at the top of page 202, noting the symbol t for *tonne* and the fact that 1 t is equal to 1000 kg. Point out that although the required

Working Together

For each, give the divisor you could think of to help you find the quotient.

1. 12)4838 10
2. 71)7526 70

Complete.

3. 51)3586   7□ R□   0   16
   357
   16

4. 43)2666   6□   2
   258
   □86
   □86
   □ 0

Divide.
Then check.

5. 64)327 5 R7
6. 81)20 272 250 R22
7. 93)30 699 330 R9
8. 32)$67 456 $ 2 108

Exercises

Divide.

1. 41)328 8
2. 62)411 6 R39
3. 54)873 16 R9
4. 83)625 7 R44
5. 51)4325 84 R41
6. 21)2526 120 R6
7. 94)7683 81 R69
8. 63)3194 50 R44
9. 43)39 184 911 R11
10. 84)37 168 442 R40
11. 92)40 020 435
12. 52)42 082 809 R14
13. 11)66 027 6 002 R5
14. 72)108 577 1 508 R1
15. 81)248 730 3 070 R60
16. 93)306 999 3 301 R6
17. 19 152 ÷ 42 456
18. 68 125 ÷ 74 920 R45
19. 13 488 ÷ 12 1124
20. $22 387 ÷ 61 $367
21. $11 077 ÷ 53 $209
22. $59 942 ÷ 82 $731
23. $762 680 ÷ 23 $33 160
24. $296 143 ÷ 31 $9553
25. $101 244 ÷ 44 $2301

Solve.

26. 492 kg of garbage were collected from a dozen pavilions. What was the average amount of garbage collected from each pavilion? 41 kg
27. There are 11 rides on 14 652 m^2 of fairground. What is the average amount of space for each ride? 1332 m^2
28. The same number of lightbulbs was used for each of 22 pavilions. 4510 lightbulbs were used altogether. How many lightbulbs were used for each pavilion? 205
29. 132 kg of nails were used to build 33 stands at a fair. What was the average mass of nails used to build each stand? 4 kg

203

RELATED ACTIVITIES

- Have students find the missing digits in divisions similar to the following.

```
      1 [3][2] R5
12) 1 5 [8] 9
    [1][2]
      [3] 8
      [3][6]
        [2][9]
        [2][4]
            5
```

```
          [4] 0 8 R [1][7]
34) [1][3] 8 8 9
     1 3 6
       [2][8] 9
       [2][7][2]
           1 [7]
```

- Write several division exercises on the board or prepare a work sheet. Have the students draw squares to indicate the number of digits in the quotient and their place values, as shown in the examples below.

```
      □ □ □ □
44) 8 0 1 5 6

        □ □ □ □
72) 1 0 3 2 1 9
```

division is 31)1178, it is helpful to think of 30)1178 for finding the first digit of the quotient. Write the exercise 30)1178 on the board. Underline digits of the dividend to show the following three steps for finding the number of digits in the quotient.

30)<u>1</u> 1 7 8   (1 < 30)   30)<u>1 1</u> 7 8   (11 < 30)   30)<u>1 1 7</u> 8   □□

Ask what the tens' digit of the quotient will be and compare the answer with the development on page 202. Continue with a discussion of the worked example, emphasizing that in dividing 248 ones, the divisor is again thought of as 30 to obtain 8 as the ones' digit for 31)1178.

Working Together: Ex. 1 and 2 deal with choosing the best multiple of ten as a trial divisor. For all exercises on this page, the multiples of ten can be obtained by rounding the divisors down to the nearest ten. Ex. 3 and 4 provide partially completed divisions to familiarize students with the procedure. Discuss which multiple of ten is thought of in the divisions for Ex. 3-8.

Exercises: Some students may find it helpful to write exercises showing a multiple of ten as the divisor first.

```
      [8]          8
40)3 2 8     41)3 2 8
                3 2 8
                    0
```

Assessment

Divide.

1. 21)365 17 R8
2. 44)398 9 R2
3. 83)49 966 602
4. 62)114 095 1 840 R15

Solve.

5. Last year, 387 972 people visited the fair in 12 days. Find the average number of people who visited the fair each day. 32 331

Pages 204-205

LESSON OUTCOME

Divide by a two-digit number when the trial estimates for the digits in the quotient are correct, divisors rounded to the nearest ten, dividends with up to six digits

Prerequisite Skills

Divide by a multiple of ten from 10 to 90; divide by a two-digit number when the trial estimates for the digits in the quotient are correct, divisors with the ones' digit to 4

Checking Prerequisite Skills

Divide.

1. 30)175 — 5 R25
2. 60)9327 — 155 R27
3. 20)19 482 — 974 R2
4. 70)70 140 — 1002
5. 51)1734 — 34
6. 32)25 646 — 801 R14
7. 84)52 249 — 622 R1
8. 63)291 586 — 4 628 R22

Rounding the Divisor

Cecil and his friends collected 3104 pop bottles at the fair and placed them in cases of 48. How many full cases of bottles did they collect? How many bottles for another case did they collect?

Divide 3104 by 48.

For 48)3104, think of 50)3104.

48 rounded to the nearest ten is 50.

3104 shows 3 thousands 1 hundred or 31 hundreds. Since 31 is less than 50, think of 31 hundreds 0 tens as 310 tens. Then divide the 310 tens.

$50 \times 6 = 300$
$50 \times 7 = 350$. . . too great!

Use 48×6 tens = 288 tens.

```
          6
Write 48)3104
         288
          22
```

Think of the 22 tens 4 ones that remain as 224 ones.

```
      6
  48)3104
     288↓
      224
```

Then divide the 224 ones.

$50 \times 4 = 200$
$50 \times 5 = 250$

Use $48 \times 4 = 192$.

```
          64 R32
Write 48)3104
         288
          224
          192
           32
```

They collected 64 full cases of bottles and 32 bottles for another case.

204

LESSON ACTIVITY

Before Using the Pages

- Write an exercise such as 42)1369 on the board. Complete it with the students to review the procedure presented in the previous lesson. Point out that 40 is the multiple of ten that is thought of as the divisor, to help obtain digits of the quotient. Ask if 40 is likely the best multiple of ten to help with the division 48)3104. Have students give reasons for their answers.

Using the Pages

- The worked example introduces the procedure of rounding a divisor to the nearest ten to obtain a trial divisor. Have a student read the word problem to introduce the situation. Lead the students through the solution, noting that the procedure is the same as in the previous lesson, but the multiple of ten is obtained by rounding the divisor to the nearest ten, in this case, from 48 up to 50. Review that 50 as a divisor helps to determine each digit of the quotient, not just the first digit.

Working Together: Ex. 1-3 review rounding numbers. Have students recall the rule for rounding numbers, particularly for examples similar to Ex. 3. Ex. 4 and 5 help students to apply the procedure in finding digits of the quotient after the first digit. It is important to discuss the solutions for Ex. 4-9.

Exercises: Before the students begin the exercises, ensure that they understand what is required in Ex. 23 and 24. (A remainder in one exercise does not form part of the divisor in the next exercise.) The solutions for Ex. 25 and 26 require more than one step. For example, in Ex. 26, one solution is as follows. The price of one can was 30¢ and \$385.20 was received in all. It will be necessary to think of both amounts as cents and complete the division 30)38 520 to find the number of cans sold. This quotient, in turn, divided by 36 will determine the number of cartons of cans and the number of extra cans sold.

Working Together

Round to the nearest ten.

1. 63 60 2. 78 80 3. 35 40

Complete.

4. $94\overline{)6120}$ — quotient 6▒ R▒ (5, 10)
   564
   48▒ 0
   ▒ 470
   ▒ 10

5. $29\overline{)10\,336}$ — quotient 3▒▒ R▒ (56, 12)
   8 7
   1 63
   ▒ 145
   ▒ 186
   ▒ 174
   ▒ 12

Divide.

6. $76\overline{)574}$ 7 R42
7. $31\overline{)2859}$ 92 R7
8. 418 141 ÷ 52 8041 R9
9. $1068.90 ÷ 21 $50.90

Exercises

Divide.

1. $22\overline{)792}$ 36
2. $87\overline{)663}$ 7 R54
3. $64\overline{)539}$ 8 R27
4. $35\overline{)870}$ 24 R30
5. $11\overline{)1786}$ 162 R4
6. $37\overline{)2053}$ 55 R18
7. $45\overline{)1660}$ 36 R40
8. $88\overline{)3970}$ 45 R10
9. $61\overline{)44\,103}$ 723
10. $19\overline{)84\,600}$ 4 452 R12
11. $33\overline{)76\,109}$ 2 306 R11
12. $47\overline{)41\,162}$ 875 R37
13. $23\overline{)138\,098}$ 6 004 R6
14. $46\overline{)362\,320}$ 7 876 R24
15. $78\overline{)332\,110}$ 4 257 R64
16. $92\overline{)277\,092}$ 3 011 R80
17. 49 210 ÷ 28 1757 R14
18. 56 928 ÷ 85 669 R63
19. 34 172 ÷ 69 495 R17
20. $260.15 ÷ 43 $6.05
21. $6480.72 ÷ 72 $90.01
22. $1435.90 ÷ 83 $17.30

Use the quotient in each block as the missing divisor in the next.

23.	2132 ÷ 52 41	2378 ÷ ▒ (41) 58	5182 ÷ ▒ (58) 89 R20	36 144 ÷ ▒ (89) 406 R10
24.	4874 ÷ 77 63 R23	3591 ÷ ▒ (63) 57	1834 ÷ ▒ (57) 32 R10	129 988 ÷ ▒ (32) 4062 R4

Solve.

25. At the fair, Marvin sold 2246 tickets. Each book of tickets has 48 tickets. How many whole books of tickets did he sell? 46 How many tickets from another book did he sell? 38

*26. Each carton has 36 cans of juice. Gina sold cans of juice for 30¢ each. She received $385.20 for the cans of juice. How many whole cartons of cans of juice did she sell? 35 How many cans from another carton did she sell? 24

205

RELATED ACTIVITIES

- You may wish to demonstrate the following procedure for checking division, known as "casting out nines". The procedure is shown below for the division given in the worked example on page 204.

$$\begin{array}{r} 64 \text{ R}32 \\ 48\overline{)3104} \end{array}$$

Step 1. For the divisor, 48, $4 + 8 = 12$, $1 + 2 = 3$.

Step 2. For the quotient, 64, $6 + 4 = 10$, $1 + 0 = 1$.

Step 3. For the remainder, 32, $3 + 2 = 5$.

Step 4. For the dividend, 3104, $3 + 1 + 0 + 4 = 8$.

Step 5. For the numbers obtained in the previous four steps, multiply the divisor and the quotient and add the remainder. The result should be the same as the number obtained for the dividend.

$$(3 \times 1) + 5 = 8$$

The same procedure is shown below for Ex. 1 on page 205.

$$\begin{array}{r} 36 \\ 22\overline{)792} \\ \underline{66} \\ 132 \\ \underline{132} \\ 0 \end{array}$$

divisor: $2 + 2 = 4$
quotient: $3 + 6 = 9$
remainder: 0
dividend: $7 + 9 + 2 = 18$, $1 + 8 = 9$

$$(4 \times 9) + 0 = 36$$
$$3 + 6 = 9$$

Assessment

Divide.

1. $27\overline{)638}$ 23 R17
2. $42\overline{)1320}$ 31 R18
3. $65\overline{)14\,101}$ 216 R61
4. $79\overline{)100\,305}$ 1 269 R54

Solve.

5. For the fair, 3100 tickets were sold. Each book held 45 tickets. How many whole books of tickets were sold? 68 How many tickets from another book were sold? 40

LESSON OUTCOME

Divide by a two-digit number when trial estimates for digits in the quotient may be incorrect, dividends with up to six digits

Prerequisite Skills

Divide by a two-digit number when the trial estimates for the digits in the quotient are correct

Checking Prerequisite Skills

Divide.

1. $78\overline{)8169}$ 104 R57
2. $41\overline{)16\ 845}$ 410 R35
3. $89\overline{)63\ 869}$ 717 R56
4. $33\overline{)716\ 168}$ 21 702 R2

Dividing by a Two-Digit Number

Divide 6558 by 84.

For $84\overline{)6558}$, think of $80\overline{)6558}$.

84 rounded to the nearest ten is 80.

For $80\overline{)6558}$, think of 6558 as 655 tens 8 ones. Then divide the 655 tens.

$80 \times 8 = 640$
$80 \times 9 = 720$ . . . too great!

Try using 84×8 tens.

84×8 tens = 672 tens

$$\begin{array}{r} 8 \\ 84\overline{)6558} \\ 672 \end{array}$$

cannot subtract 672 from 655

Use 84×7 tens instead.

84×7 tens = 588 tens

$$\begin{array}{r} 7 \\ 84\overline{)6558} \\ \underline{588} \\ 67 \end{array}$$

Then complete the division.

$$\begin{array}{r} 78 \text{ R6} \\ 84\overline{)6558}\phantom{\text{ R6}} \\ \underline{588}\phantom{\text{ R6}} \\ 678\phantom{\text{ R6}} \\ \underline{672}\phantom{\text{ R6}} \\ 6\phantom{\text{ R6}} \end{array}$$

$6558 \div 84 = 78$ R6

Divide 5493 by 66.

For $66\overline{)5493}$, think of $70\overline{)5493}$.

66 rounded to the nearest ten is 70.

For $70\overline{)5493}$, think of 5493 as 549 tens 3 ones. Then divide the 549 tens.

$70 \times 7 = 490$
$70 \times 8 = 560$ . . . too great!

Try using 66×7 tens.

66×7 tens = 462 tens

$$\begin{array}{r} 7 \\ 66\overline{)5493} \\ \underline{462} \\ 87 \end{array}$$

greater than the divisor

Use 66×8 tens instead.

66×8 tens = 528 tens

$$\begin{array}{r} 8 \\ 66\overline{)5493} \\ \underline{528} \\ 21 \end{array}$$

Then complete the division.

$$\begin{array}{r} 83 \text{ R15} \\ 66\overline{)5493}\phantom{\text{ R15}} \\ \underline{528}\phantom{\text{ R15}} \\ 213\phantom{\text{ R15}} \\ \underline{198}\phantom{\text{ R15}} \\ 15\phantom{\text{ R15}} \end{array}$$

$5493 \div 66 = 83$ R15

206

LESSON ACTIVITY

Before Using the Pages

- Write the following exercises on the board and ask how to obtain a multiple of ten as a divisor to help in the divisions. Have the students complete the divisions. Then ask two students to show and explain their work on the board.

$23\overline{)691}$ $\quad$ $39\overline{)5260}$

Assign the division $42\overline{)2438}$ and let the students discover the subtraction step for which the subtrahend is greater than the minuend. Ask for suggestions to overcome the difficulty. Have students demonstrate their ideas on the board. (A trial divisor of 40 suggests 6 tens in the quotient since $40 \times 6 = 240$; but $42 \times 6 = 252$.) Emphasize that the situation is not the result of an error in computation. Explain that it arises sometimes as the result of using a rounded divisor and a simple adjustment of the digit in the quotient corrects the situation.

$$\begin{array}{r} 6 \\ 40\overline{)2438} \\ \underline{240} \end{array} \longrightarrow \begin{array}{r} 6 \\ 42\overline{)2438} \\ \underline{252} \end{array} \longrightarrow \begin{array}{r} 58 \text{ R2} \\ 42\overline{)2438}\phantom{\text{ R2}} \\ \underline{210}\phantom{\text{ R2}} \\ 338\phantom{\text{ R2}} \\ \underline{336}\phantom{\text{ R2}} \\ 2\phantom{\text{ R2}} \end{array}$$

Using the Pages

- The worked examples present two divisions to illustrate the situations that may arise as a result of rounding the divisor to obtain digits in the quotient. First, for $84\overline{)6558}$, it is necessary to decrease the tens' digit of the quotient from 8 to 7 in order to carry out the subtraction step. Secondly, for $66\overline{)5493}$, it is necessary to increase the tens' digit of the quotient from 7 to 8 to prevent the difference in the subtraction step from being greater than the divisor. Discuss each example in turn, asking questions such as the following.

Working Together

Complete.

1\. 7 62 / 6■ R■■
64)4350
384
510
■■448
■■ 62

2\. 07 19 / 2■■ R■■
47)9748
94
34■ 8
■■329
■ 19

Divide. Then check.

3\. 26)886 34 R2

4\. 12)3165 263 R9

5\. $73 416 ÷ 92 $798

6\. $2114.70 ÷ 35 $60.42

Exercises

Divide. Check the six most difficult exercises.

1\. 52)153 2 R49
2\. 29)265 9 R4
3\. 73)948 12 R72
4\. 86)710 8 R22
5\. 34)6574 193 R12
6\. 23)6002 260 R22
7\. 19)7986 420 R6
8\. 67)5772 86 R10
9\. 27)67 518 2 500 R18
10\. 75)46 229 616 R29
11\. 42)74 660 1 777 R26
12\. 18)37 200 2 066 R12
13\. 37)106 250 2 871 R23
14\. 53)206 324 3 892 R48
15\. 74)412 984 5 580 R64
16\. 81)321 123 3 964 R39
17\. $58 575 ÷ 55 $1065
18\. $94 458 ÷ 21 $4498
19\. $256 496 ÷ 92 $2788
20\. $371.68 ÷ 46 $8.08
21\. $487.50 ÷ 65 $7.50
22\. $3336.08 ÷ 44 $75.82

Use division to solve each of these. Tell what you would do with the remainder to get the most reasonable answer.

1\. Sandy and Chris arranged 425 exhibits on tables. Each table holds 16 exhibits. How many tables did they need? 27
Increase the quotient by 1.

2\. They packed 107 prizes in boxes. Each box holds 8 prizes. How many boxes did they fill? 13
Ignore the remainder.

3\. Sandy and Chris shared $125 equally for their work at the fair. How much did each receive? $62.50
Extend the dividend to show cents.

4\. Sandy and Chris were to share 53 tickets for rides. How many tickets would each receive? 26
Ignore the remainder.

PROBLEM SOLVING

RELATED ACTIVITIES

- For further practice, you may wish to have the students complete selected exercises from Ex. 45-60 on page 336.
- Have students complete the divisions below and write the remainders in the corresponding squares of a smaller diagram. If the divisions are correct, the remainders give a Magic Square. (The sums of the remainders in the columns, rows, and diagonals are the same.)

A 27)3026 B 41)12 963
C 48)2550 D 92)29 081
E 57)2627 F 73)22 850
G 16)676 H 65)1953
I 84)24 620

A 2	B 7	C
D	E	F
G	H	I

"Why is seventy the best trial divisor for 66)5493?"
"Why is seven tens not the correct tens' digit?"
"What can be done to correct this?"
"How does the trial divisor help to find the ones' digit?"
In these two examples it is only the tens' digit of the quotient that requires correction. Tell the students that sometimes more than one digit of a quotient must be corrected.

Working Together: Discuss the steps that are shown in the partially completed divisions of Ex. 1 and 2. Ask what the trial divisor is, what is divided first, how the first digit of the quotient is obtained by using the trial divisor, how it is known that the digit is correct, and so on. Use similar questions to help the students complete the divisions. For Ex. 3-6, the students are to use multiplication to check their work.

Exercises: The students are to choose the six exercises that gave them the most difficulty and use multiplication to check the divisions. Remind them that for some exercises they will need to show the symbol $, and for others, a decimal point will be necessary.

Problem Solving: These exercises enable students to consider remainders in division as they relate to situations in real life. At times a remainder requires that the quotient be increased by 1 (Ex. 1); at other times it may be disregarded or some other solution sought (Ex. 4).

Assessment

Divide.

1\. 27)590 21 R23
2\. 43)802 18 R28
3\. 65)4125 63 R30
4\. 88)17 869 203 R5

LESSON OUTCOME

Round the divisor and the dividend and divide to estimate the quotient, and then compare the estimate of the quotient with the exact quotient

Prerequisite Skills

Round whole numbers; divide by a two-digit number, dividends with up to six digits

Checking Prerequisite Skills

Round

1. 3964 to the nearest hundred. 4000
2. 61 325 to the nearest thousand. 61 000
3. 387 156 to the nearest ten thousand. 390 000

Divide.

4. $34\overline{)669}$ 19 R23
5. $67\overline{)7301}$ 108 R65
6. $73\overline{)21\ 390}$ 293 R1
7. $46\overline{)575\ 649}$ 12 514 R5

Estimating the Quotient

A tally chart that was used one day showed that a game was played 1729 times and 91 prizes were won. About how many times was the game played for each prize won?

Rounding the divisor and the dividend can help you to estimate the quotient.

For $91\overline{)1729}$,

round

$90\overline{)1700}$

For $90\overline{)1700}$, think

$9\overline{)17}$ 1 or 2? . . . 2 is closer!

Use 2 in the quotient.

the rounded divisor

$9 \times 2 = 18$
$90 \times 2 = 180$
$90 \times 20 = 1800$

1800 is close to 1700!

20 is an estimate for $1729 \div 91$.

The game was played about 20 times for each prize that was won.

For the exact quotient, divide in the usual way.

$$\begin{array}{r} 19 \\ 91\overline{)1729} \\ \underline{91} \\ 819 \\ \underline{819} \\ 0 \end{array}$$

Working Together

Give the rounded divisor and the rounded dividend.

1. $64\overline{)8249}$ 60 8000
2. $75\overline{)6385}$ 80 6400

Give the first digit in the quotient.

3. $47\overline{)32\ 987}$ 6 (from rounding)

Give the number of digits in the quotient.

4. $64\overline{)5276}$ 2

Round and divide to estimate the quotient. Then divide to find the quotient. Estimates will vary

5. $11\overline{)497}$ 45 R2 (50)
6. $56\overline{)3847}$ 68 R39 (60)
7. $7264 \div 34$ 213 R22 (240)
8. \$476 204 ÷ 68 \$7003 (\$7000)

208

LESSON ACTIVITY

Before Using the Pages

- Write a few exercises on the board to review the work of the previous lesson. Have students write solutions on the board and explain their work.

  $12\overline{)317}$ $23\overline{)2915}$ $85\overline{)174\ 456}$

  Tell the students that it is helpful at times to estimate quotients. Ask what kind of numbers are useful in making estimates, rather than using the exact numbers. Lead them to suggest the use of rounded numbers.

Using the Pages

- Begin with a brief discussion of the photograph. Have students describe the game shown and tell of their own experiences in a similar game at a fair. Then have a student read the word problem to introduce the situation. Emphasize the word "about" in the problem, since this implies estimating the answer.

Have students explain the rounding of the divisor and the dividend. Draw attention to the fact that estimating begins by thinking of the division $9\overline{)17}$ first and this leads to use of the rounded number 90 in 90×2. It would be helpful to write the following sequence on the board to emphasize that there is no need to go beyond $90 \times 20 = 1800$. It also helps to illustrate a pattern that might be helpful in other exercises.

$$\begin{array}{rcl} 90 \times 2 &=& 180 \\ 90 \times 20 &=& 1\ 800 \longleftarrow 90\overline{)1800}^{\,20} \\ 90 \times 200 &=& 18\ 000 \\ 90 \times 2000 &=& 180\ 000 \end{array}$$

Note that the exact quotient is 19, which is quite close to the estimate, 20.

Working Together: Ex. 1 and 2 deal with rounding divisors and dividends. It is obvious that divisors are rounded to the nearest ten, but the choice of place value for rounding the dividends varies. Discuss this aspect with the students to help them make their decisions. For Ex. 1 they may choose

Exercises

Estimate each quotient. Then divide to find the quotient. Estimates will vary.

1. $22\overline{)473}$ 21 R11 (20)
2. $43\overline{)346}$ 8 R2 (8)
3. $88\overline{)7063}$ 80 R23 (80)
4. $49\overline{)3509}$ 71 R30 (70)
5. $13\overline{)1486}$ 114 R4 (100)
6. $72\overline{)6445}$ 89 R37 (90)
7. $59\overline{)58\ 723}$ 995 R18 (1000)
8. $41\overline{)47\ 380}$ 1 155 R25 (1200)
9. $27\overline{)170\ 052}$ 6 298 R6 (6000)
10. $93\overline{)840\ 260}$ 9 035 R5 (9000)
11. $38\overline{)80\ 397}$ 2 115 R27 (2000)
12. $62\overline{)74\ 021}$ 1 193 R55 (1000)
13. 13 760 ÷ 18 764 R8 (700)
14. 462 462 ÷ 79 5853 R75 (6000)
15. 327 277 ÷ 54 6060 R37 (6000)
16. \$650 349 ÷ 81 \$8029 (\$8000)
17. \$95 552 ÷ 32 \$2986 (\$3000)
18. \$326 300 ÷ 65 \$5020 (\$5000)

For each exercise, estimate the quotient. Then choose three of the numbers shown on the chart that could be the quotient. Find each quotient, and score one point for each reasonable estimate. Estimates will vary.

19. $61\overline{)43\ 676}$ 716 (700)
20. $28\overline{)229\ 740}$ 8 205 (8000)
21. $51\overline{)216\ 036}$ 4 236 (4000)
22. $82\overline{)176\ 218}$ 2 149 (2000)
23. $37\overline{)2701}$ 73 (70)
24. $19\overline{)836}$ 44 (40)
25. $69\overline{)55\ 407}$ 803 (800)
26. $91\overline{)39\ 403}$ 433 (400)
27. $76\overline{)6232}$ 82 (80)
28. $42\overline{)294\ 756}$ 7 018 (7000)

4280 21	7018 28	803 25	728 19
808 25	44 24	438 26	4236 21
71 23	1083	8208 20	72 23
716 19	435 26	7241 28	83 27
224	4207 21	203	2048 22
81 27	8205 20	7156 28	802 25
433 26	216	73 23	1438
724 19	43 24	8210 20	2149 22
42 24	82 27	9893	2108 22

Solve by estimating. Answers may vary.

29. During an 11 d fair, \$1144 was collected at a game. About what was the average amount collected each day at the game?
Estimate: \$100

30. A game was played 5336 times during the 58 h of the fair. About what was the average number of times the game was played each hour?
Estimate: 90

209

RELATED ACTIVITIES

• Challenge some of the students with exercises similar to the following. Tell them that the divisor and the quotient are equal. Have them find the number and complete the division.

1. $\boxed{2}\,5\overline{)6\ 2\ 5}$ quotient $\boxed{2}\boxed{5}$
2. $1\boxed{9}\overline{)3\ 6\ 1}$ quotient $\boxed{1}\boxed{9}$
3. $\boxed{7}\boxed{2}\overline{)5\ 1\ 8\ 4}$ quotient $\boxed{7}\boxed{2}$
4. $\boxed{5}\boxed{3}\overline{)2\ 8\ 0\ 9}$ quotient $\boxed{5}\boxed{3}$

to round to the nearest hundred or to the nearest thousand. For Ex. 2, it is preferable to round to the nearest hundred. Ex. 3 concentrates on finding the first digit of the quotient. To do this, the divisor and the dividend are rounded first to give, for example, $50\overline{)33\ 000}$. This leads to thinking of the division $5\overline{)33}$ and a first digit of 6. In Ex. 4, the steps of the previous exercises are repeated and the skill of determining the place value of the first digit is considered. This, in turn, makes it possible to name the number of digits in the quotient, and thus give an estimate of the quotient. This sequence is shown below for Ex. 7.

$$34\overline{)7264} \longrightarrow 30\overline{)7300} \longrightarrow 3\overline{)7}^{\,2} \longrightarrow 30\overline{)7\ 3\ 0\ 0}^{\,2\,\square\square}$$

$$\begin{aligned} 3 \times 2 &= 6 \\ 30 \times 2 &= 60 \\ 30 \times 20 &= 600 \\ 30 \times 200 &= 6000 \end{aligned}$$

The estimate of the quotient is 200.

With practice, students can carry out much of the procedure mentally. Note that the estimate 200 for Ex. 7 can be derived from the step that shows $30\overline{)73\ 0\ 0}^{\,2\,\square\square}$.

Exercises: It might be best to work Ex. 19 with the students to ensure that they understand the use of the chart.

$$61\overline{)43\ 676} \longrightarrow 60\overline{)44\ 0\ 0\ 0}^{\,7\,\square\square} \longrightarrow \text{Estimate: } 700$$

There are three possible numbers in the chart for the quotient (728, 716, 724). By division, the exact quotient is 716. A student would score one point if 716 is one of the three numbers selected as a possibility. Ensure that the students write their estimates before finding the exact quotients. Note that Ex. 29 and 30 require only estimating the averages.

Assessment

Estimates may vary.

Estimate each quotient. Then find the exact quotient.

1. $28\overline{)627}$ 22 R11 (20)
2. $75\overline{)5894}$ 78 R44 (70)
3. $42\overline{)93\ 185}$ 2 218 R29 (2000)
4. $49\overline{)687\ 173}$ 14 023 R46 (14 000)

OBJECTIVE

Demonstrate competence in division; solve related word problems

Vocabulary
turnstile

Practice

First, estimate the quotient without doing any work on paper. Then divide and compare the quotient with your estimate. Estimates will vary.

1. 20)4800 240 (240)
2. 69)3657 53 (50)
3. 51)5559 109 (110)
4. 24)69 432 2893 (3000)
5. 62)28 560 460 R40 (500)
6. 31)309 999 9999 R30 (10 000)
7. 10)67 300 6730 (6700)
8. 86)657 980 7650 R80 (7000)
9. 55)387 865 7052 R5 (7000)
10. 16)144 116 9007 R4 (9000)
11. 74)46 472 628 (600)
12. 29)26 229 904 R13 (900)
13. \$70 680 ÷ 76 \$930 (\$900)
14. \$65 065 ÷ 13 \$5005 (\$5000)
15. \$2596 ÷ 44 \$59 (\$60)
16. \$3013.50 ÷ 35 \$86.10 (\$90.00)
17. \$321.08 ÷ 92 \$3.49 (\$3.00)
18. \$5426.33 ÷ 67 \$80.99 (\$80.00)

Complete three exercises in each box. Write the result for the fourth exercise by using the pattern.

19. 30)8100 270	20. 40)11 600 290	21. 70)21 700 310	22. 90)29 700 330
23. 47)37 088 789 R5	24. 56)38 594 689 R10	25. 72)42 423 589 R15	26. 48)23 492 489 R20
27. 78)78 118 1001 R40	28. 85)170 200 2002 R30	29. 94)282 302 3003 R20	30. 84)336 346 4004 R10

31. Make up a fifth exercise to fit each of the patterns above. Answers will vary.

Study these division sentences.

60 ÷ 10 = 6	400 ÷ 100 = 4	5 000 ÷ 1000 = 5
720 ÷ 10 = 72	1 100 ÷ 100 = 11	33 000 ÷ 1000 = 33
6340 ÷ 10 = 634	82 300 ÷ 100 = 823	601 000 ÷ 1000 = 601
9100 ÷ 10 = 910	60 000 ÷ 100 = 600	400 000 ÷ 1000 = 400

Give a rule that helps you find the quotient when

1. the divisor is 10. Move the digits in the dividend one place to the right.
2. the divisor is 100. two places to the right.
3. the divisor is 1000. three places to the right.

Make up some division exercises to test your rules.
Answers will vary.

try this

210

LESSON ACTIVITY

Using the Pages

- Complete one of Ex. 1-18 on the board with the students to review the steps in estimating a quotient and in finding the exact quotient. For Ex. 19-30, ensure that the students understand what is required. Note that most of the problems on page 211 require more than one step to solve.

Try This: On page 51, the *Try This* feature shows multiplication sentences in which one of the two factors is ten, one hundred, or one thousand. This concept is extended on page 131 to include both factors. Now, similar sentences are encountered for the operation of division, and students can discover how the quotient is related to the dividend when the divisor is 10 (100, 1000) and the dividend is a multiple of 10 (100, 1000). Although three-digit divisors are involved in exercises such as 33 000 ÷ 1000 = 33, the numbers are not difficult to work with and thinking of multiplication to perform the division, or to check it, is useful.

Solve.

32. There were 34 people on each roller coaster ride on a day when the turnstile showed 6222. How many times was the roller coaster used that day? 183

33. There were 73 rides on the Ferris wheel on a day when the turnstile showed 2336. If the same number of people rode each time, how many people were on each ride? 32

34. For each Wild Mouse ride, there were 4 people in each of the 7 cars. The turnstile showed 4368. How many times was the Wild Mouse ride used that day? 156

35. The 6 people in Hilda's family spent $104.16 altogether in 2 d at the fair. What was the average amount each person spent each day? $8.68

36. During the first week of the fair, 93 307 tickets were sold for a ride. The next week, 101 223 tickets were sold for that ride. The fair was open each day for 2 weeks. What was the average number of tickets sold each day for that ride? 13 895

37. This year, $118 860 was collected in 12 d for the rides. Last year, the average amount collected each day for the rides was $9236. How much more was the average amount collected each day for the rides this year than last year? $669

38. Julia's family has 22 films with 12 photographs each. They want to take the same number of photographs on each of their 11 d at the fair. How many should they take each day? 24

39. Each roll of dimes contains 50 dimes. 863 dimes from the rolls were used at a ride. How many whole rolls of dimes were used? 17 How many dimes from another roll were used? 13

RELATED ACTIVITIES

- For further practice, you may wish to have the students complete Ex. 61-68 on page 336.
- Students may play the following game in groups of three using a set of numeral cards for the numbers from 0 to 9. Each player draws a diagram to indicate a two-digit divisor and a four-digit dividend (A). One player draws six numeral cards in turn and reads the numbers aloud. As each number is read, the players write it in any place value of the divisor or the dividend to form a division exercise (B). Once a number is written its position may not be changed. After the players complete their own divisions, points are awarded as follows: ''highest'' quotient, 3; ''middle'' quotient, 2; ''lowest'' quotient, 1. After ten rounds the player with the most points is the winner.

A $\square\square\overline{)\square\square\square\square}$

B $\boxed{4}\boxed{1}\overline{)\boxed{3}\boxed{9}\boxed{0}\boxed{8}}$

Page 212

OBJECTIVE

Demonstrate competence in division

Materials

circular objects for tracing (optional)

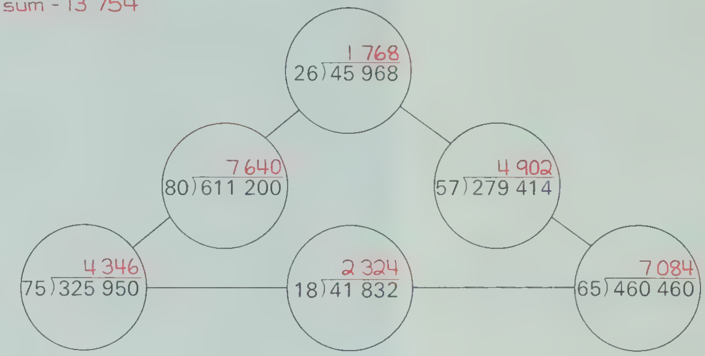
RELATED ACTIVITIES

- For further practice, you may wish to have the students complete Ex. 69-76 on page 336.
- For a small group of students, write a division exercise on the board. Have one student divide to find the first digit of the quotient. Have a second student complete the multiplication step and a third complete the subtraction step. Continue in a similar manner to complete the division.

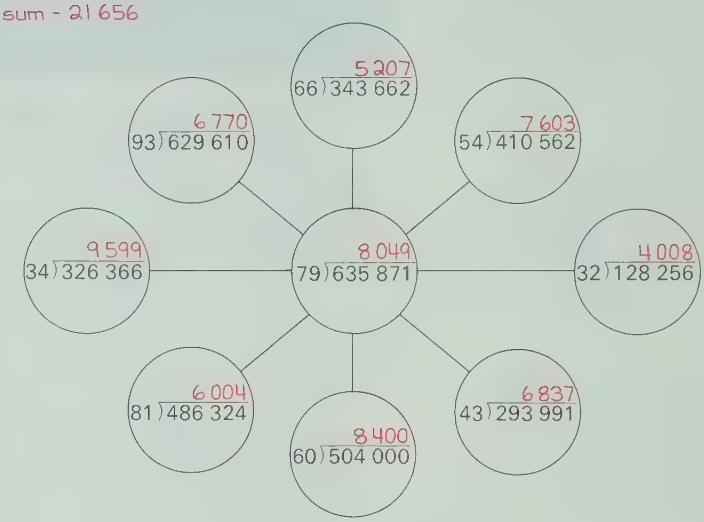
Practice

Copy these diagrams and divide as shown.
Then add the quotients along each line segment.
If your quotients are correct,
the sums in line will be equal.

1. sum - 13 754



2. sum - 21 656



212

LESSON ACTIVITY

Using the Page

- Have a student read the instructions at the top of the page. Then ask another student to tell in her/his own words what is required. You may wish to have the students trace around circular objects in drawing their diagrams. Lids from small containers such as those for 35 mm film are useful, as are coins and plastic templates. Note that only the quotients are written inside the circles after the exercises have been completed on another part of the page.

Computing Quickly

When you can compute in your head, you can find the result more quickly than you can when you use a calculator.

Choose a number from 1 to 10.	5
Add 4.	9
Multiply by 8.	72
Subtract 12.	60
Divide by 4.	15
Subtract 1.	14
Divide by 2.	7
Subtract the number you started with.	2
Your result is 2.	Yes!?!

Try each of these three times. What is special about your three results?

Each result is the original number.

1. Use a number from 1 to 10. Add 5. Multiply by 3. Add 6. Divide by 3. Subtract 7.

The result is the original number

2. Use a number from 1 to 10. Multiply by 20. Subtract 8. Divide by 4. Add 2. Divide by 5.

3. Think of a number from 10 to 20. Add 16. Multiply by 20. Subtract 340. Divide by 10. Add 2. Divide by twice the number you started with.

The result is always 1

When the numbers become greater, you may need paper and pencil or a calculator to help you.

4. Use this year. Double it. Add 5. Multiply by 50. Add your age. Add 365. Subtract 615.

Try this with some other years too.

The result is a combination of the year and the age

5. Use a number from 10 to 100. Add 57. Multiply by 57. Subtract 2166. Divide by 19. Subtract 57. Divide by 3.

The result is the original number.

Calculator

213

Page 213

OBJECTIVE

Compute quickly with and without a calculator

Materials

calculators (optional)

RELATED ACTIVITIES

- Some students may be able to write exercises similar to those on the page. The exercises should be tested by using several examples before they are given to other students to try.

LESSON ACTIVITY

Using the Page

- This lesson is one that can be assigned with a minimum of discussion so that the students may make their own discoveries. You may wish to have one student play the part of the magician and the other students compute mentally the result of each step of the worked example. For each of the exercises, the result does not change even though a different number is used to begin the exercise. Calculators may be used when computation is too difficult to perform mentally.

OBJECTIVE

Write an equation for information given in a word problem; solve the problem by writing and solving a related equation

Prerequisite Skills

Write the family of related facts for addition and subtraction, and for multiplication and division

Checking Prerequisite Skills

Complete the sentences to show a family of addition and subtraction facts.

1. 3 + 9 = 12
12 − 9 = 3
9 + 3 = 12
12 − 3 = 9

Complete the sentences to show a family of multiplication and division facts.

2. 9 × 5 = 45
45 ÷ 9 = 5
5 × 9 = 45
45 ÷ 5 = 9

RELATED ACTIVITIES

- Have students play the following game. Say, "I am thinking of a number. When I subtract 5 the result is 14." The first student to suggest the correct equation gives similar information for the next equation.

UNIVERSITY OF ALBERTA

Solving Equations

The band and 9 other persons formed a marching unit in the parade. There were 65 persons in the unit. How many were in the band?

To find the value of n in n + 9 = 65, subtract 9 from 65.

This equation tells the story.

$$n + 9 = 65$$

number in the band / number of other marchers / number in the unit

This related equation helps solve the problem.

$$65 - 9 = n$$

This completes the solution.

$$65 - 9 = 56$$

There were 56 persons in the band.

Here are examples of related number sentences.

Addition and Subtraction		Multiplication and Division	
56 + 9 = 65	9 + 56 = 65	8 × 7 = 56	7 × 8 = 56
65 − 9 = 56	65 − 56 = 9	56 ÷ 8 = 7	56 ÷ 7 = 8

Copy each equation. Write a related equation. Then find the solution. Equations will vary.

1. n − 4 = 7 7 + 4 = n 11
2. 8 + n = 20 20 − 8 = n 12
3. n ÷ 8 = 6 6 × 8 = n 48
4. n × 7 = 56 56 ÷ 7 = n 8
5. n + 13 = 32 32 − 13 = n 19
6. n ÷ 3 = 9 9 × 3 = n 27
7. 9 × n = 81 81 ÷ 9 = n 9
8. n − 10 = 34 34 + 10 = n 44

Write an equation for each of these. Then write a related equation. Find the solutions. Equations will vary.

9. The band members marched in 8 rows with the same number in each row. There were 56 band members. How many were there in each row? 7
8 × n = 56
56 ÷ 8 = n

10. For the parade, all marching units were divided among 4 parade sections. That placed 7 units in each section. How many marching units were there? 28
n ÷ 4 = 7
7 × 4 = n

11. From all the floats in the parade, 5 were chosen for prizes. There were 9 other floats. How many floats were in the parade? 14
n − 5 = 9
9 + 5 = n

PROBLEM SOLVING

214

LESSON ACTIVITY

Before Using the Page

- You may wish to have the students turn to page 168 to review the introduction to equations and the use of a symbol such as n for a number that is not given.

Using the Page

- The worked example demonstrates the use of an equation to represent information stated in a word problem and the use of a related sentence to solve the equation. Have a student read the problem at the top of the page. Ask how the equation tells the given story in a simpler way. Since the equation expresses addition, point out that the related sentence involving subtraction is helpful in solving the problem.

Discuss the examples of related sentences shown below the worked example. Then discuss Ex. 1 with the students. Ask what operation the related sentence would show and then ask what the sentence is. Any of the following are acceptable: n = 4 + 7; n = 7 + 4; 4 + 7 = n; 7 + 4 = n.

For Ex. 9-11, remind the students that concluding statements are required. They may follow the solution provided in the worked example.

Checking Up

Divide.

1. 6)378 **63**	2. 9)8323 **924 R7**	3. 8)5672 **709**	4. 7)35 566 **5 080 R6**
5. 10)980 **98**	6. 30)2700 **90**	7. 60)45 180 **753**	8. 90)403 302 **4 481 R12**
9. 22)699 **31 R17**	10. 31)2198 **70 R28**	11. 49)16 908 **345 R3**	12. 83)602 005 **7 253 R6**
13. 54)403 **7 R25**	14. 76)8289 **109 R5**	15. 93)45 546 **489 R69**	16. 67)549 809 **8 206 R7**
17. 72)650 **9 R2**	18. 65)5855 **90 R5**	19. 40)24 340 **608 R20**	20. 88)792 529 **9 006 R1**

21. 22 580 ÷ 35 **645 R5** 22. 178 709 ÷ 28 **6382 R13** 23. 59 790 ÷ 61 **980 R10**
24. 505 170 ÷ 73 **6920 R10** 25. 70 995 ÷ 87 **816 R3** 26. 40 482 ÷ 52 **778 R26**
27. $210 800 ÷ 68 **$3100** 28. $361 196 ÷ 44 **$8209** 29. $765 000 ÷ 85 **$9000**
30. $355.68 ÷ 57 **$6.24** 31. $718.41 ÷ 77 **$9.33** 32. $7399.56 ÷ 92 **$80.43**
33. $1027.25 ÷ 25 **$41.09** 34. $2440.64 ÷ 58 **$42.08** 35. $3957.11 ÷ 79 **$50.09**

Solve.

36. In the parking lot at the fair, there are 708 cars in 12 equal rows. How many cars are in each row? **59**

37. 4 girls had $15.40 to share equally at the fair. How much would each receive? **$3.85**

38. During the fair, a photographer took 6000 photographs. Each film has 30 photographs. How many films were used? **200**

39. 10 881 souvenirs were sold at a stand during a 9 d fair. What was the average number of souvenirs sold each day? **1209**

40. After the fair, Anita wanted to pack the remaining 3327 souvenirs in boxes. Each box held 36 souvenirs. How many whole boxes of souvenirs would she pack? How many souvenirs **92** would she pack in another box? **15**

41. Jack is arranging 918 exhibits on 27 tables at the fair. He wants to place the same number of exhibits on each table. How many exhibits should he place on each table? **34**

42. There are 91 prizes for a contest. 3458 people entered the contest. On the average, how many people entered the contest for each prize? **38**

43. After the fair, Adam packed the remaining 126 prizes. He placed 18 prizes in each box. How many boxes did he use? **7**

215

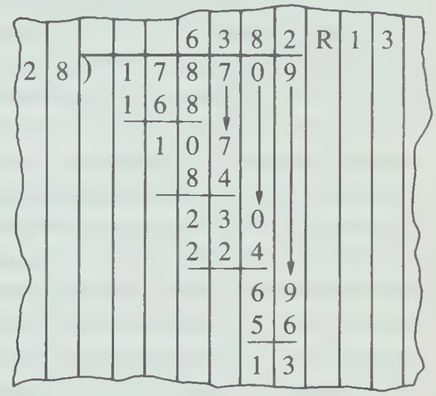
Page 215

OBJECTIVE

Demonstrate an understanding of the concepts and skills presented in this unit

RELATED ACTIVITIES

- The use of lined paper turned sideways as described in *Related Activities* on page T215 for working with one-digit divisors can be adapted to help those students having difficulty with two-digit divisors.



Skills	Exercises	Related Pages
Divide by a one-digit number	1, 2	T212-T213
Divide with zeros in the quotient	3, 4	T214-T215
Divide evenly by a multiple of ten	6	T217
Divide by a multiple of ten	5, 7, 8, 19	T218-T219
Divide by a two-digit number	9, 10	T220-T221
Divide by a two-digit number by rounding the divisor to the nearest ten to obtain a trial divisor	11-18, 20-35	T222-T225
Solve division problems	36-43	

Comments

Note carefully where errors occur in the students' work. For example, do errors occur more frequently in multiplication, in subtraction, in finding a trial divisor, or do the students have difficulty remembering the sequence of steps. Division by a one-digit divisor must be mastered before division by a two-digit number can be dealt with. Choose activities from lesson suggestions and *Related Activities* in this unit or in Unit 5 to review or reteach concepts as required.

Unit 11 Overview

Measurement

The topic of volume is introduced by counting cubes in structures of various shapes, as well as in rectangular prisms. Multiplication of the number of centimetre cubes in one layer by the number of layers is presented as a method for finding the volume of a rectangular prism. The relationships between a volume of 1000 cm^3 and a capacity of 1 L, and between 1 cm^3 and 1 mL are established. Then the method for finding volume in cubic centimetres is used to determine in terms of millilitres and litres the capacities of containers which are in the shape of rectangular prisms. Conversions from millilitres to litres and vice versa are made by using the factors 0.001 and 1000 as multipliers. Mass, in terms of grams and kilograms, is first studied in connection with familiar objects. The relationships between 1 cm^3, 1 mL, and 1 g of water, and between 1 kg and 1 L of water are developed. Two ways of recording time to the second are studied and numeric dating is presented as an efficient method for recording dates. Patterns inherent in various geometric relationships are explored in the lesson on problem solving. Two *Try This* features encourage students to experiment with rectangular prisms made of cubes. Two *Keeping Sharp* features provide exercises to maintain skills in addition, subtraction, and multiplication of whole numbers and decimals. These skills are also assessed at the end of the unit in sets of exercises and related word problems.

Prerequisite Skills

- use multiplication to find the number of objects in a rectangular array
- multiply a number to 999 by a multiple of ten from 10 to 90
- multiply a two-digit number by a two-digit number
- multiply a decimal and a whole number by 1000 or by 0.001
- write numerals for times shown on a 12-hour clock

Unit Outcomes

- count the cubes in a shape
- find volume by counting centimetre cubes
- multiply the number of centimetre cubes in one layer of a rectangular prism by the number of layers to find the volume in cubic centimetres
- associate a volume of 1000 cm^3 with a capacity of 1 L
- express cubic centimetres as litres and vice versa, for a whole number of litres
- find capacity in litres by finding volume in cubic centimetres, volumes are a multiple of 1000 cm^3
- estimate capacity to the nearest 500 mL
- express litres as millilitres and vice versa; solve related word problems
- associate a volume of 1 cm^3 with a capacity of 1 mL
- find capacity in millilitres by finding volume in cubic centimetres
- express an amount given in one unit in terms of another unit for millilitres, litres, and cubic centimetres
- estimate mass to the nearest 500 g
- express grams as kilograms and vice versa; solve related word problems
- associate the mass of 1 L of water with 1 kg; associate the mass of 1 mL of water with 1 g
- relate the volume of a container, the amount of water needed to fill it, and the mass of the water
- write numerals for times to the second for a 24-hour clock
- add and subtract times
- write and interpret dates using numeric dating (year, month, day)
- solve problems by searching for patterns and following the patterns

Background

The origin, structure, and convenience of the metric system are discussed in the Overview for Unit 7. It is recommended that reference be made to those comments in relation to volume, capacity, and mass at this time. It should be pointed out that the concepts of volume and capacity are virtually the same. Briefly, volume refers to how much space an object occupies, and capacity to how much space is inside a container. For instance, an empty carton takes up a certain amount of space (volume), but it can hold almost exactly the same amount of material (capacity). A cube, or a container, with edges 1 cm long occupies a space, or has a capacity, of one cubic centimetre (1 cm^3). Thus, both volume and capacity may be measured in cubic centimetres. However, since the capacities of irregularly-shaped containers cannot be calculated easily from their linear measurements, a unit called the *litre* is used. One litre (1 L) has the same capacity as one thousand cubic centimetres (1000 cm^3), and one *millilitre* (1 mL) has the same capacity as one cubic centimetre (1 cm^3).

The metric units of measurement are further interrelated by one litre of water having a mass of one *kilogram* (1 kg), or one thousand *grams* (1000 g). Thus, 1 cm^3 of water has a mass of 1 g. Mass refers to the quantity of matter in an object and may be measured by balancing an object and standard units of mass on balance scales. Mass is not the same as weight. Weight is the measure of the force required to lift an object and it can vary because it is affected by the force of gravity. Mass cannot vary. The chart below shows some of the relationships between measures of volume, capacity, and the mass of water.

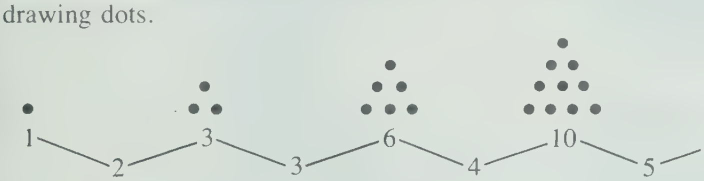
Volume	1 cm^3	10 cm^3	100 cm^3	1000 cm^3
Capacity	1 mL	10 mL	100 mL	1000 mL
	0.001 L	0.01 L	0.1 L	1 L
Mass of water	1 g	10 g	100 g	1000 g
	0.001 kg	0.01 kg	0.1 kg	1 kg

Time is not measured in metric units, but the *second* (s) is recognized as an SI unit. The other units of time — hour (h), minute (min), and day (d) — although non-metric, are accepted universally. There seems to be no international symbol for week, month, or year, although a, for *annum*, is sometimes used for year. Time on a 24-hour basis may be indicated by using two digits for each hour, minute, and second. Spaces may separate the pairs of digits, or colons may be placed between them. Thus, 18:45:50 indicates 18 h, 45 min, and 50 s. Since each day begins and ends at midnight, that time may be, for example, either Tuesday at 24:00:00 or Wednesday at 00:00:00. On a 24-hour basis, hours after noon are greater than 12 (13 to 24). It should be noted that if the number for the hours, the minutes, or the seconds is less than 10, a zero is written to the left of the single digit so that each time unit is represented by two digits. For instance, 08:05:03 indicates 8 h, 5 min, and 3 s after

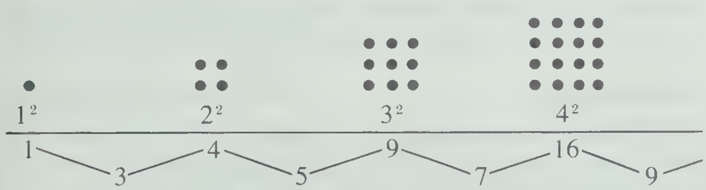
midnight. Because time uses a non-metric system of relationships, conversions and regrouping in operations cannot be done in terms of ten, but rather in terms of 24 and 60 (1 d = 24 h, 1 h = 60 min, 1 min = 60 s).

Numeric dating, that is, writing dates without the names of months in words, is becoming standard practice. The year, the month, and the day, in descending order of size, are indicated with eight digits: four for the year, two for the month, and two for the day. Again, if the number for the month or the day is less than ten, a zero is written to the left of the single digit. Therefore, July 1, 1867, the day of Confederation for Canada, is indicated by 1867 07 01. In numeric dating, spaces are left between the pairs of digits. If the century is obvious, the first two digits of the date may be omitted, as in 69 07 20 (July 20, 1969), the date when man first landed on the moon.

There is a fascination in numbers, not only in the relationships between operations and the basic facts used in them, but also in patterns which frequently emerge in sequences of numbers. Some of these are examined in the *Problem Solving* lesson on page 233. Among them is the pattern found in a series of *triangular numbers*. Triangular numbers are those that can be represented by dots arranged in triangular form as shown. The fascinating feature is the pattern of increase from one number to the next: 1 to 3 (2), 3 to 6 (3), 6 to 10 (4). The increase is always one more than the previous one, and by this pattern the series of triangular numbers may be extended arithmetically without drawing dots.



Another pattern of differences may be seen between *square numbers*. Square numbers are, as the name implies, those for which dots may be arranged in squares. The pattern of differences this time is the series of odd numbers and it may be extended to name other square numbers. It is interesting to note that any square number may be formed by adding two consecutive triangular numbers; for example, 4 = 1 + 3, 9 = 3 + 6, 16 = 6 + 10.



Teaching Strategies

It is important that all students have direct experiences in measuring volume, capacity, and mass. This is sometimes impossible due to lack of sufficient materials, space, and time. It is recommended that instructional groups be formed and that a schedule be arranged for them to engage in many of the preliminary activities suggested in *Before Using the Pages*. Activity cards are suggested to guide students with a minimum of teacher direction. While some students are engaged in the activities, others may be working at the exercises from the *Keeping Sharp* features on pages 223 and 227, and from *Checking Skills* on pages 236 and 237. The actual lessons in the textbook may be completed as outlined, either with one group at a time or after all the groups have had the preliminary experiences.

Converting millilitres to litres and grams to kilograms involves multiplication by 0.001, and the opposite conversions involve multiplication by 1000. It may be valuable to refer to the work on pages 158 and 159, where the products of such multiplications were written directly. For example, the conversions between millilitres and litres on page 223 and between kilograms and grams on page 227 should be completed by merely moving digits in relation to the decimal points, and by adding zeros if necessary.

If a demonstration 24-hour clock is not available, extra tabs for 13 to 24 may be affixed to a 12-hour clock face. On these pages, no regrouping is involved in the exercises which require the use of addition and subtraction; this feature should be observed if other exercises are assigned for practice. The regrouping of seconds to minutes to hours, and vice versa, is in terms of 60, and up to this time the students have regrouped only by tens.

The results of the *Checking Up* at the end of the unit should be examined carefully to determine whether the concepts and skills have been developed adequately. According to the analysis, it may be necessary to reteach or to review some topics with specific students and to assign follow-up practice. The same procedure should be followed with the exercises and problems on pages 236 and 237. Students should be encouraged to assume some responsibility for overcoming any apparent weaknesses by referring to lessons in the book where the skills are presented and developed.

Materials

centimetre cubes
centimetre rulers (optional)
rectangular prism
various containers that hold 1 L as described in *Before Using the Pages* on page T 240
material for pouring, such as cereal, sand, water, rice; funnel
a variety of containers that hold less than 1 L, about 1 L, and more than 1 L
measuring cups marked in litres and millilitres
container having the shape of a cube with edges 10 cm long; 3-by-3 section of centimetre graph paper cut from copies of page T 397, scissors, and tape for each student
one-millilitre measuring spoon
balance scales, masses from 1 g to 3 kg, objects for measuring mass (pencils, books, corks, scissors, coins, nails of different sizes)
masses from 1 g to 1 kg, water, containers such as plastic bottles and paper cups, waterproof container having the shape of a cube with edges 10 cm long
demonstration 24-hour clock with movable hands

Vocabulary

volume	hour hand
cubic centimetre, cm^3	minute hand
litre, L	second hand
capacity	words for times to the second
gram, g	diagonal of a polygon
kilogram, kg	triangular number
a.m., p.m.	numeric dating
24-hour clock	marathon race

Page 216

LESSON OUTCOME

Count the cubes in a shape

Materials

centimetre cubes

RELATED ACTIVITIES

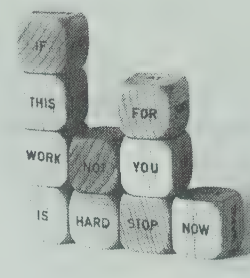
- Have students build shapes using a certain number of cubes, for example, 12 cubes. Display the different shapes obtained. Emphasize that each shape occupies the same amount of space.

11 MEASUREMENT

Counting Cubes

Marina used word blocks to form a sentence.

How many blocks did she use?
Count them.



10 blocks were used.

She had these blocks left over.
How many are there? Count them.

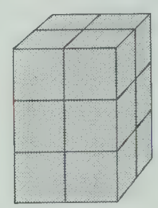
Think of how many there are in each layer.

11 blocks were left over.

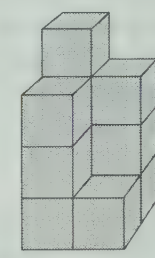
Exercises

Count the cubes in each shape.

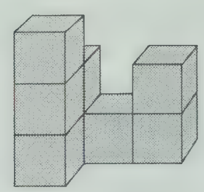
1. 12



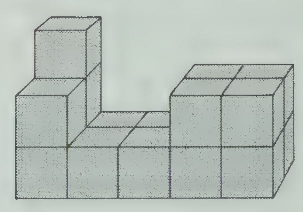
2. 11



3. 8



4. 17



216

LESSON ACTIVITY

Before Using the Page

- Have the students work in small groups building shapes with centimetre cubes. Ask them to exchange their shapes and count the cubes that make up each shape. Discuss that some of the cubes may be hidden when the shape is viewed from a particular position. Ask for ways of counting the hidden cubes without turning the shapes to view them from a different side.

Using the Page

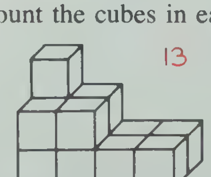
- Have students read the sentence that Marina formed and count the blocks used. Note that the blocks are the same size and shape, and that their shape suggests a cube. Discuss that some of the blocks are hidden from view in the arrangement of blocks left over. Point out the suggestion in the ''thought cloud'' for helping to count the blocks.

Exercises: If necessary, provide students with centimetre cubes for building the shapes, in order to check their answers.

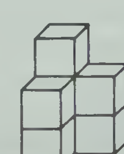
Assessment

Count the cubes in each shape.

1. 13



2. 7

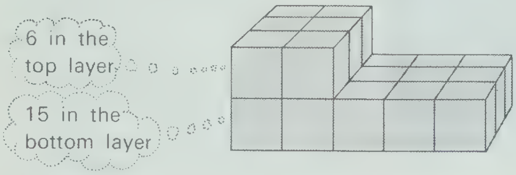


UNIVERSITY OF ALBERTA

Volume in Cubic Centimetres

Each edge of this cube is 1 cm long.
The volume of the cube is 1 cm^3.

You can find the volume of larger solids by counting centimetre cubes.



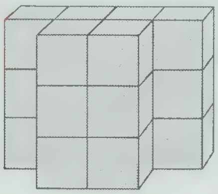
There are 21 centimetre cubes.

The volume of this solid is 21 cm^3.

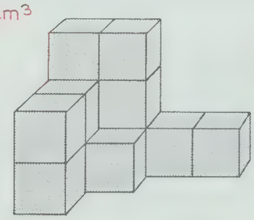
Exercises

Find the volume in cubic centimetres.

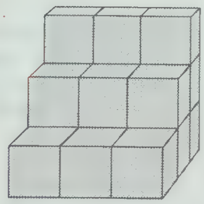
1. 18 cm^3



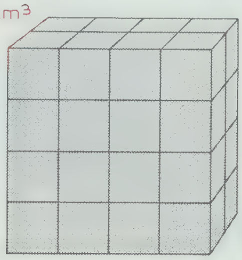
2. 13 cm^3



3. 18 cm^3



4. 32 cm^3



217

Page 217

LESSON OUTCOME

Find volume by counting centimetre cubes

Materials

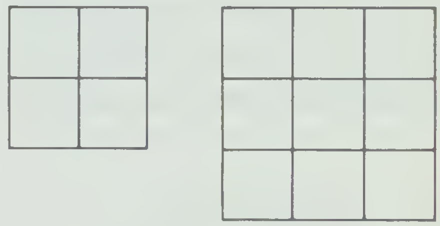
centimetre cubes, centimetre rulers (optional)

Vocabulary

volume, cubic centimetre, cm^3

RELATED ACTIVITIES

- Have students find how many centimetre cubes are needed to build a larger cube. For example, ask whether a cube can be built using 4, 8, or 9 centimetre cubes. Discuss their results. You may wish to provide patterns as shown below for the bases of the cubes. The students may build a cube on each base and then count the centimetre cubes.



LESSON ACTIVITY

Before Using the Page

- Review that all the edges of a cube have the same length. Have students suggest the length of each edge of the cubes used to build shapes in the previous lesson. If necessary, have them measure the edges using a centimetre ruler. Ask what name can be given to a cube having edges 1 cm long.

Using the Page

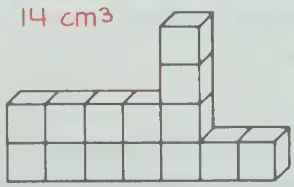
- Have a student read the two statements at the top of the page. Draw attention to the symbol cm^3 for *cubic centimetre*, explaining that the numeral 3 represents the word ''cubic''. (A cube is a rectangular prism with each of its six faces a square.) Associate the word *volume* with the amount of space a shape occupies and thus with the number of centimetre cubes used to build the shape. Note that counting the centimetre cubes in each layer helps to find that there are 21 in all, although some of the cubes are hidden.

Exercises: It would be beneficial to have several students explain their procedure for finding the volume of each shape. For instance, in Ex. 4, some students may have used multiplication by thinking of 4 layers of 8 for $4 \times 8 = 32$.

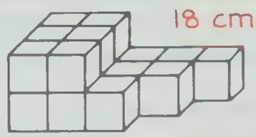
Assessment

Find the volume in cubic centimetres.

1. 14 cm^3



2. 18 cm^3



Pages 218-219

LESSON OUTCOME

Multiply the number of centimetre cubes in one layer of a rectangular prism by the number of layers to find the volume in cubic centimetres

Materials

centimetre cubes, rectangular prism

Prerequisite Skills

Use multiplication to find the number of objects in a rectangular array

Checking Prerequisite Skills

For this diagram,

● ● ● ● ● ● ● ● ●

● ● ● ● ● ● ● ● ●

● ● ● ● ● ● ● ● ●

1. how many rows are there? 3
2. how many ●'s are in each row? 9
3. how many ●'s are there in all? 27

Tell the number of ●'s there would be

4. for 7 rows with 6 ●'s in each row. 42
5. for 12 rows with 8 ●'s in each row. 96

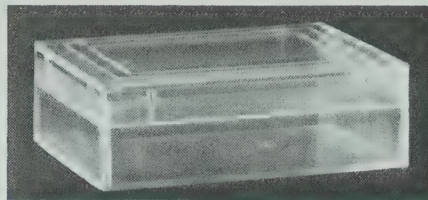
UNIVERSITY OF ALBERTA

Volume of a Rectangular Prism

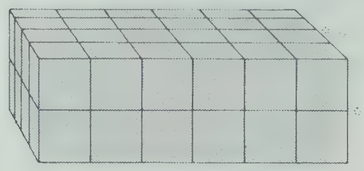
Multiplication can help to find the volume of a rectangular prism.

How much space does this box take?

Find the volume.



The box and this stack of centimetre cubes take the same amount of space.



6 along this edge

4 along this edge

2 along this edge

For the bottom layer,

$6 \times 4 = 24$

there are 24 cubes.

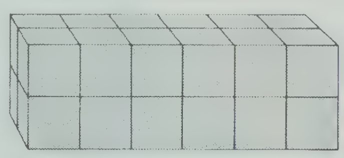
There are 2 layers

$2 \times 24 = 48$

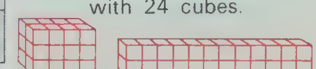
or 48 cubes in all.

There are 48 centimetre cubes in this stack.
The volume of the box is 48 cm^3.

Tess made this rectangular prism with 24 cubes.



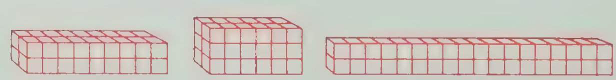
1. Show two other rectangular prisms that can be made with 24 cubes.



try this

2. Show three rectangular prisms that can be made with 36 cubes.

218



LESSON ACTIVITY

Before Using the Pages

- Display a rectangular prism and ask what name is given to the solid. If students have forgotten its name, refer them to page 188. Have the students work in small groups using centimetre cubes to build shapes that are rectangular prisms. Have them exchange shapes, find the volume, and explain the method used to find it.

Using the Pages

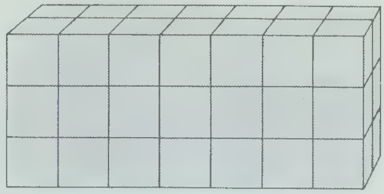
- Have a student name the shape of the plastic container in the photograph. Review that in finding the amount of space an object occupies, it is necessary to find its volume. Explain that since this implies counting centimetre cubes, it is necessary to make a copy of the shape by stacking centimetre cubes. Have the students use centimetre cubes to copy the stack of cubes shown. Ask for a quick way to find the number of cubes in the stack. Emphasize that multiplication is useful because each layer of cubes in a rectangular prism has the same number of cubes. Have a student read the concluding statements.

Working Together: Ex. 1-3 develop the sequence of steps to arrive at the number of centimetre cubes in the bottom layer of a rectangular prism, using multiplication. Note that the terms "one bottom edge" and "the other bottom edge" refer to the length and the width. Ex. 4 and 5 complete the sequence for finding the volume. Have the students follow the same steps to complete Ex. 6 and other similar exercises as required.

Exercises: Diagrams are provided for Ex. 1 and 2. Some students may need to use centimetre cubes to check their answers for Ex. 3-7, but encourage them to complete the exercises without using cubes. Ask what is special about the prism for Ex. 7. (It is a cube.)

Try This: Some students may be able to think of prisms for these exercises by thinking only of multiplication. Have centimetre cubes available for those who need them. The results may be shown either with cubes or a diagram. You

Working Together

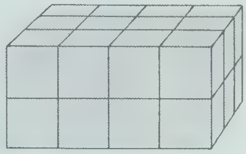


For this solid, how many centimetre cubes are

1. along one bottom edge? 7
2. along the other bottom edge? 2
3. in the bottom layer? Give 14 the multiplication sentence. $7 \times 2 = 14$

For the same solid,

4. how many layers are there? 3
5. give the volume in cubic centimetres. Give the 42 cm³ multiplication sentence. $14 \times 3 = 42$



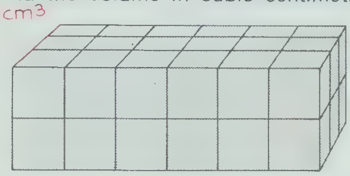
For this solid,

6. use multiplication to find the volume in cubic centimetres. 24 cm³

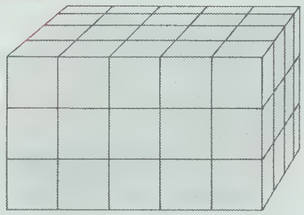
Exercises

For each solid, use multiplication to find the volume in cubic centimetres.

1. 36 cm³



2. 60 cm³



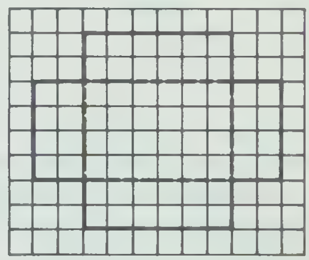
Complete this chart.

	Number of centimetre cubes along one bottom edge	along the other bottom edge	in the bottom layer	Number of layers	Volume in cubic centimetres
3.	6	2	? 12	4	? 48 cm³
4.	7	6	? 42	5	? 210 cm³
5.	9	7	? 63	8	? 504 cm³
6.	9	9	? 81	7	? 567 cm³
7.	10	10	? 100	10	? 1000 cm³

219

RELATED ACTIVITIES

- Explain what is meant by the surface area of a solid. Have students find the surface area in square centimetres for different rectangular prisms that they build with centimetre cubes. Also, have them find the surface area of the prisms suggested for Ex. 1 and 2 of the *Try This* feature. They may be surprised to learn that rectangular prisms may have the same volume but different surface areas.
- Provide students with small containers such as jewelry boxes. Have them fill the boxes with centimetre cubes to find their volume. Before filling the boxes, have the students estimate the number of cubes that would be used for each box.
- Use copies of page T 397 to prepare patterns similar to the one shown below. Have students cut around the patterns. Tell them to fold each pattern along the dotted lines and tape edges to form a box. However, before this is done, tell them they are to write the number of centimetre cubes they think it would take to fill the box. They may use centimetre cubes afterward, if necessary, to check their estimates.



may wish to summarize the possibilities suggested by the students in a chart similar to the one shown for Ex. 3-7.

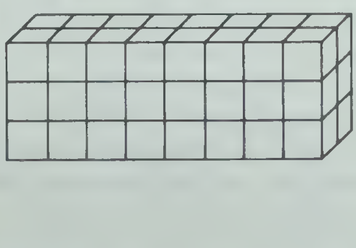
Assessment

For a rectangular prism, there are 5 centimetre cubes along one bottom edge and 4 along the other bottom edge.

1. How many centimetre cubes are in the bottom layer? 20
2. If there are 6 layers, what is the volume of the rectangular prism? 120 cm3

Use multiplication to find the volume in cubic centimetres.

3. 48 cm3



LESSON OUTCOME

Associate a volume of 1000 cm^3 with a capacity of 1 L; express cubic centimetres as litres and vice versa, for a whole number of litres; find capacity in litres by finding volume in cubic centimetres, volumes are a multiple of 1000 cm^3

Materials

various containers that hold 1 L as described in *Before Using the Pages*; material for pouring, such as cereal, sand; funnel; centimetre cubes

Vocabulary

litre, L

Prerequisite Skills

Multiply a number to 999 by a multiple of ten from 10 to 90; multiply a two-digit number by a two-digit number; find the volume of a rectangular prism in cubic centimetres

Checking Prerequisite Skills

Multiply.

1. 800 × 50 40 000 **2.** 450 × 80 36 000

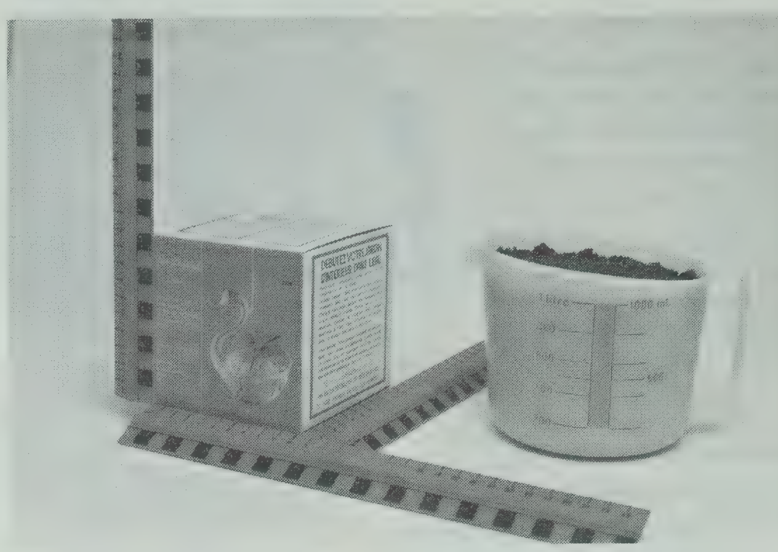
3. 25 × 25 625 **4.** 45 × 15 × 25 16 875

For a rectangular prism, there are 7 centimetre cubes along one bottom edge, 4 centimetre cubes along the other bottom edge, and 5 layers.

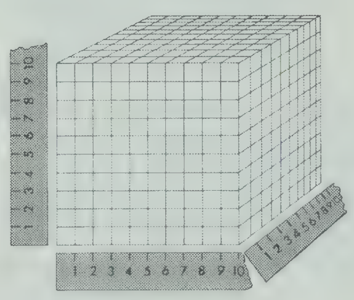
5. What is the volume of the prism? 140 cm3

Cubic Centimetres and Litres

1 L of soil will fill a box this size.



1000 centimetre cubes would fill a box the same size as the one above.



For the bottom layer, $10 \times 10 = 100$ there are 100 cubes.

For 10 layers, $10 \times 100 = 1000$ there are 1000 cubes.

1000 cm^3 will fill the box.

1 L and 1000 cm^3 take the same amount of space.

220

LESSON ACTIVITY

Before Using the Pages

- Review the litre as a unit of capacity. Have students help to fill containers using a measuring cup that holds 1 L and is labeled ''one litre''. The containers should have a capacity of 1 L, for example, a plastic ice cream container. There should also be an unmarked container the shape of a cube having edges 10 cm long. Such cubes may be purchased or one may be prepared easily by obtaining a large milk carton that is 10 cm long and 10 cm wide, and cutting it at a height of 10 cm. Through this activity the students can see that containers having different shapes can hold 1 L, particularly the container having the shape of a cube.
- Remove all the containers from the previous activity except the one having the shape of a cube. Have a student identify the shape. Ask how to find its volume. Some students may suggest using centimetre cubes to fill the container or to copy the shape. Ask how many centimetre cubes they think would be needed. Have students match centimetre cubes with the length, width, and height of the container to help them answer the question. You may prefer to have them match copies of the diagrams on page T 395 with each face of the container.

Using the Pages

- Read the introductory statement and ask what the shape of the box is and what its dimensions are. Then draw attention to the diagram below the photograph, pointing out that it has the same shape and size as the box. Lead the students through the procedure of finding the volume. Return to a discussion of the photograph and associate filling the box with 1 L of soil with the similar activity suggested in *Before Using the Pages*. Emphasize that 1 L and 1000 cm^3 take the same amount of space. Ask how many litres take the same amount of space as 3000 cm^3 and how many cubic centimetres take the same space as 9 L.

Working Together: Ex. 1 and 2 deal with the relationship between litres and cubic centimetres. Ex. 3-5 review the

Working Together

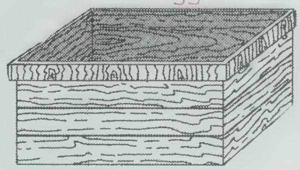
Complete.

1. 12 L take the same space as ? cm³. 12 000

2. 35 000 cm³ take the same space as ? L. 35

This planter could be filled with centimetre cubes so there would be

40 cubes along one bottom edge,
30 cubes along the other bottom edge,
and 20 layers of cubes



3. For the bottom layer, $40 \times 30 =$? 1200 there are ? cm³. 1200

4. For 20 layers, $20 \times 1200 =$? 24 000 there are ? cm³. 24 000

5. 24 000 cm³ = ? L 24 This planter can hold ? L of soil. 24

Exercises

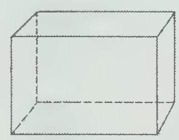
Complete these charts.

	cm³	L
1.	8 000	? 8
2.	13 ?000	13
3.	19 ?000	19
4.	21 000	? 21
5.	35 000	? 35

		6.	7.	8.
Centimetre cubes in bottom layer		400	500	350
Number of layers		20	24	20
Amount of space	cm³	8000 ?	12 ?000	7000 ?
	L	8 ?	12 ?	7 ?

		9.	10.	11.	12.
Centimetre cubes along one bottom edge		30	40	25	55
Centimetre cubes along the other bottom edge		30	10	32	35
Centimetre cubes in bottom layer		? 900	? 400	? 800	? 1925
Number of layers		20	15	25	40
Amount of space	cm³	18 ?000	6?000	20 ?000	77 ?000
	L	18 ?	6 ?	20 ?	77 ?

Think of rectangular prisms



to help you with these, if needed.

221

RELATED ACTIVITIES

- Have students cut copies of the diagrams on page T 395 and paste six diagrams onto cardboard. Have them cut around the diagrams on the cardboard to obtain six faces for a cube and tape the faces so that the cube shows 100 cm² on each face. Some students may be able to make their cubes by folding a pattern similar to the one on page T 386. To do this, have them tape copies of page T 397 together to form a large sheet of graph paper and outline the pattern for a cube having edges 10 cm long. It may then be pasted to cardboard, cut out, and folded.
- Have the students build shapes by stacking cubes and name the amount of space taken in cubic centimetres and in litres.

sequence developed in the previous lesson for using multiplication to find the volume of a rectangular prism. Since larger numbers are involved here, it may be advisable to review the steps for smaller numbers first.

Exercises: Before the students begin, have them study the three charts. Have students explain in their own words what is required for each chart.

Assessment

Complete.

1. 7000 cm³ take the same space as __7__ L.
2. 22 L take the same space as __22 000__ cm³.

3.

Centimetre cubes in bottom layer		450
Number of layers		20
Amount of space	cm³	9000
	L	9

4.

Centimetre cubes along one bottom edge		25
Centimetre cubes along the other bottom edge		22
Centimetre cubes in bottom layer		550
Number of layers		20
Amount of space	cm³	11 000
	L	11

Pages 222-223

LESSON OUTCOME

Estimate capacity to the nearest 500 mL; express litres as millilitres and vice versa; solve related word problems

Materials

a variety of containers that hold less than 1 L, about 1 L, and more than 1 L; material for pouring, such as sand, water, rice; a funnel; measuring cups marked in litres and millilitres

Vocabulary

capacity

Prerequisite Skills

Multiply a decimal and a whole number by 1000 or by 0.001

Checking Prerequisite Skills

Multiply.

1. 1.5 × 1000 1500 **2.** 3.25 × 1000 3250

3. 450 × 0.001 0.450 or 0.45 **4.** 1450 × 0.001 1.450 or 1.45

Capacity in Litres and Millilitres

Each spray uses about 1 mL (millilitre) from these bottles.

The small bottle holds about 250 mL. The large bottle holds about 500 mL when half full. The large bottle holds about 1 L when full.

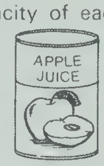
The large bottle can be used for about 1000 sprays.

1 L = 1000 mL 1 mL = 0.001 L

Working Together

Choose the best estimate for the capacity of each.

1.



1 L

1 mL	500 mL	1 L

2.

500 mL

1 mL	500 mL	1 L

Complete.

3. 1 L = ___ mL 1000
 2.5 L = 2.5 × ___ mL 1000
 2.5 L = ___ mL 2500
4. 8.25 L = ___ mL 8250
5. 0.47 L = ___ mL 470
6. 1 mL = ___ L 0.001
 750 mL = 750 × ___ L 0.001
 750 mL = ___ L 0.750 or 0.75
7. 1250 mL = ___ L 1.250 or 1.25
8. 820 mL = ___ L 0.820 or 0.82

222

LESSON ACTIVITY

Before Using the Pages

- Prepare an area of the classroom for activities involving pouring and measuring. A sink or water table with water would be suitable, or a table with such materials as rice, wheat, or sand.

  Several days before the lesson, ask the students to bring various empty cans and plastic containers to school. Then provide opportunities for them to measure the capacity of containers using measuring cups marked in litres and millilitres. Ask them to find containers having capacities of 250 mL, 500 mL, 750 mL, 1 L, 1.5 L, 2 L, and 2.5 L, for example. Ask them to find how many millilitres are the same amount as 1 L, and how many litres are the same as 1500 mL.

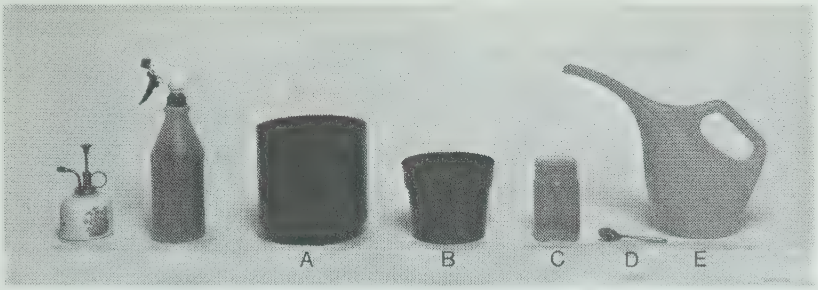
Using the Pages

- Have a student read the title at the top of page 222. Associate the word *capacity* with the amount each bottle holds. Have students read the statements above and below the photograph. Ask how many millilitres the large bottle holds when full. Emphasize the relationship 1000 mL = 1 L. Develop that one spray is about one-thousandth of a litre (1 mL = 0.001 L) since the large bottle holds 1 L and since it can be used for about one thousand sprays. It may be helpful to develop a chart on the board to help students understand the relationship.

L	0.001	0.010	0.100	0.500	1	1.110
mL	1	10	100	500	1000	1110

Working Together: If students are not familiar with the containers suggested in Ex. 1 and 2, substitute others of similar capacity. Ex. 3-5 deal with expressing litres as millilitres. This involves multiplication with 1000 as a factor. For Ex. 6-8, multiplication with 0.001 as a factor is involved in the reverse procedure of expressing millilitres as litres. The multiplications may be performed mentally rather than in writing.

UNIVERSITY OF ALBERTA



Exercises

For the objects shown above,

1. choose the best estimate for the capacity of each. →

D 1 mL	C 500 mL	B 1.5 L
E 2 L		A 3 L

Complete this chart. Remember to use: 1 L = 1000 mL, 1 mL = 0.001 L

2.

mL	2000	? 7000	? 3500	6400	? 2250	? 750	? 800	300
L	? 2	7	3.5	6.4 ?	2.25	0.75	0.8	? 0.3

Mr. Kelly raises African violets. Each month he has to add 8 mL of plant food to each of the planters he uses. How many millilitres of plant food does he need

3. for one planter in one year? 96
4. for 25 planters in one year? 2400

If a new bottle contains 500 mL of plant food,

5. how many years would the bottle last for one planter? about 5 years
6. how many bottles would he need in one year for 25 planters? 5

Add.

1. 387 + 406 = 793
2. 2958 + 1627 = 4585
3. 884 + 2496 3380
4. 3857 + 7649 11 506

Subtract.

5. 364 − 291 = 73
6. 4438 − 2852 = 1586
7. 2323 − 674 1649
8. 7000 − 1796 5204

Multiply.

9. 9768 × 6 = 58 608
10. 6897 × 9 = 62 073
11. 7986 × 78 = 622 908

KEEPING SHARP

223

RELATED ACTIVITIES

• Have students collect containers (boxes, cans, cartons, jars, and so on) for which the capacity is clearly marked in millilitres or litres. These may be used for different activities as follows.

1. Prepare a display using the labels of the containers to show items that are sold by capacity in millilitres and in litres.
2. Have students express an amount shown in litres as millilitres and vice versa.
3. Have students arrange a set of containers in order of capacity from least to greatest.
4. Write word problems about some of the containers and have students solve the problems by using one or more of the operations addition, subtraction, multiplication, and division.

Exercises: To help the students with Ex. 1, the photograph on page 223 includes the two bottles which were discussed in the example on page 222. Review that the small bottle holds about 250 mL and the large bottle holds about 1 L. Then they may consider which of the remaining containers might hold more than 250 mL but less than 1 L to help to identify the container having a capacity of 500 mL, which container might hold as much as the 1 L bottle and the 500 mL container together, and so on.

Keeping Sharp: These exercises help to maintain skills in adding, subtracting, and multiplying whole numbers. Some students may complete these exercises while others are involved in the measuring activities of *Before Using the Pages*.

Assessment

Complete the chart.

1.

mL	3000	4500	1800	900	3250
L	3	4.5	1.8	0.9	3.25

Choose the best estimate for

1 mL
500 mL
1 L

2. the amount of water needed to boil one egg. 500 mL
3. the amount of lemonade for four people to drink. 1 L

Solve.

4. Mr. Kelly's geranium needs 250 mL of water each week. How many litres of water does it need in one year? 13

Pages 224-225

LESSON OUTCOME

Associate a volume of 1 cm^3 with 1 mL; find capacity in millilitres by finding volume in cubic centimetres; express an amount given in one unit in terms of another unit for millilitres, litres, and cubic centimetres

Materials

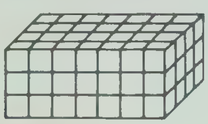
container having the shape of a cube with edges 10 cm long; 3-by-3 section of centimetre graph paper cut from copies of page T 397, scissors, and tape for each student; one-millilitre measuring spoon

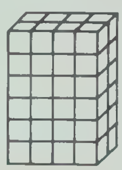
Prerequisite Skills

Use multiplication to find the volume of a rectangular prism in cubic centimetres; express litres as millilitres and vice versa

Checking Prerequisite Skills

Use multiplication to find the volume in cubic centimetres.

1. 
84 cm3

2. 
48 cm3

Complete.

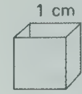
3. 8 L = $\square$ mL 8000

4. 700 mL = $\square$ L 0.7

5. 1360 mL = $\square$ L 1.36

Cubic Centimetres and Millilitres

How many millilitres of water will fill a container whose volume is 1 cm^3?

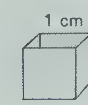


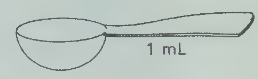
1 L and 1000 cm^3 take the same amount of space.
1 L and 1000 mL take the same amount of space.

1000 cm^3 and 1000 mL take the same amount of space.

That means 1 cm^3 and 1 mL also take the same amount of space.

1 mL of water will fill a container whose volume is 1 cm^3.





Working Together

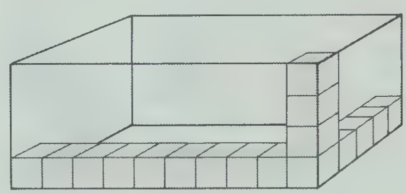
Complete.

1. 370 mL take the same space as $\square$ cm^3. 370

2. 650 cm^3 take the same space as $\square$ mL. 650

	3.	4.	5.	6.	7.	8.
cm^3	1	1000	1200	? 2700	400	? 750
mL	1	? 1000	? 1200	2700	? 400	? 750
L	0.001	? 1	? 1.2	? 2.7	? 0.4	0.750

The picture shows how this rectangular prism could be filled with centimetre cubes.



9. How many centimetre cubes would be in one layer? 50

10. How many layers would there be? 4

11. How many cubic centimetres would fill the prism? 200

12. How many millilitres would fill the prism? 200

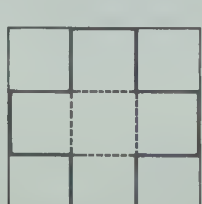
224

LESSON ACTIVITY

Before Using the Pages

- Display the container having the shape of a cube with edges 10 cm long, as suggested in *Before Using the Pages* on page T 240. Have the students recall that 1000 centimetre cubes would fill it and 1 L of water would fill it. Review the relationship 1 L = 1000 mL and ask how many millilitres take the same space as 1000 cm^3.

  Give each student a 3-by-3 section of centimetre graph paper cut from copies of page T 397. Have the students cut out a pattern as shown below and fold and tape it to make a container.



Have them identify the shape of the container and the length of each edge. Review the term *one cubic centimetre*. Ask how many millilitres of water they think would fill the container.

Using the Pages

- Read the worked example with the students to develop that 1 cm^3 and 1 mL take the same amount of space. If possible, demonstrate with a one-millilitre measuring spoon and a container 1 cm by 1 cm by 1 cm that 1 mL of water will fill the container.

Working Together: Ex. 1 and 2 deal with expressing cubic centimetres as millilitres and vice versa. Most students will have no difficulty since the numerals are the same for each. Discuss Ex. 3 with the students and have a few of them explain how to obtain numerals to complete the chart for Ex. 4-8. The diagram for Ex. 9-11 shows only those cubes that match the length, width, and height of the prism. Ask why it is not necessary to show all the centimetre cubes for the prism.

UNIVERSITY OF ALBERTA

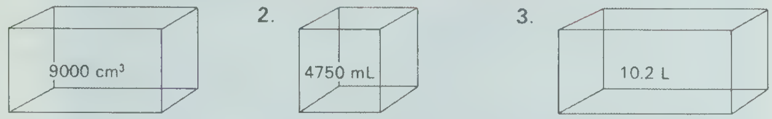
Exercises

For each tank, complete this sentence:

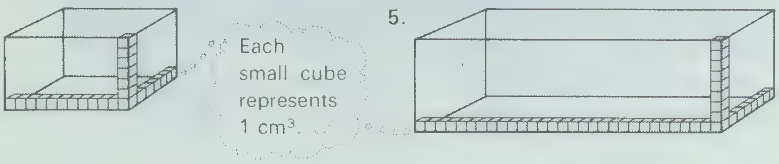
Answers are given below.

The tank will hold ___ cm³, ___ mL, or ___ L of water.

1. 2. 3.



4. 5.



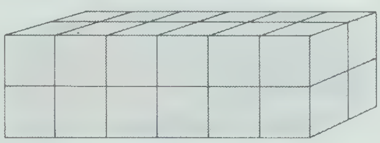
Complete this chart.

	6.	7.	8.	9.	10.	11.	12.
cm³	1000	1600	2400	3750	5240	300	250
mL	1000	1600	2400	3750	5240	300	250
L	1	1.6	2.4	3.75	5.24	0.3	0.25

Write this sentence using "L".
The motorcycle has a 0.5 L engine.
13. The motorcycle has a 500 cm³ engine.

Write this sentence using "cm³".
The automobile has a 3600 cm³ engine
14. The automobile has a 3.6 L engine.

Tess made a rectangular prism with 24 cubes. She glued the cubes together and then painted each side of the prism blue.



After being painted, how many of the cubes have blue

1. on 3 faces only? 8
2. on 2 faces only? 16
3. on 1 face only? 0
4. on 0 faces? 0

Answer the same questions for Possible answers are given on page T369

5. other prisms that use 24 cubes.
6. a prism that uses 36 cubes.
7. a prism that uses 27 cubes.

PROBLEM SOLVING

225

RELATED ACTIVITIES

- If there are unused fish tanks available in the school, exercises similar to Ex. 1-5 may be assigned. For example, have students measure the capacity in millilitres of a fish tank and then write the amount of water it will hold in litres and in cubic centimetres. This corresponds to Ex. 2. Activities that correspond to Ex. 4 and 5 will involve matching the length, width, and height of the fish tank with centimetre cubes.

Exercises: The steps suggested by Ex. 9-12 of *Working Together* show the procedure for completing Ex. 4 and 5.

Problem Solving: Provide the students with cubes for building the prisms suggested in the exercises. They should sketch a diagram of each prism that they build and show their answers beside the diagram.

Assessment

Complete the chart.

	cm³	mL	L
1.	3000	3000	3
2.	1520	1520	1.52
3.	1400	1400	1.4
4.	700	700	0.7
5.	3650	3650	3.65
6.	750	750	0.75

1. The tank will hold 9000 cm³, 9000 mL, or 9 L of water.
2. The tank will hold 4750 cm³, 4750 mL, or 4.75 L of water.
3. The tank will hold 10 200 cm³, 10 200 mL, or 10.2 L of water.
4. The tank will hold 588 cm³, 588 mL, or 0.588 L of water.
5. The tank will hold 2160 cm³, 2160 mL, or 2.16 L of water.

Pages 226-227

LESSON OUTCOME

Estimate mass to the nearest 500 g; express grams as kilograms and vice versa; solve related word problems

Materials

balance scales, masses from 1 g to 3 kg, objects for measuring mass (pencils, books, corks, scissors, coins, nails of different sizes)

Vocabulary

gram, g, kilogram, kg

Prerequisite Skills

Multiply a decimal and a whole number by 1000 or by 0.001

Checking Prerequisite Skills

Multiply.

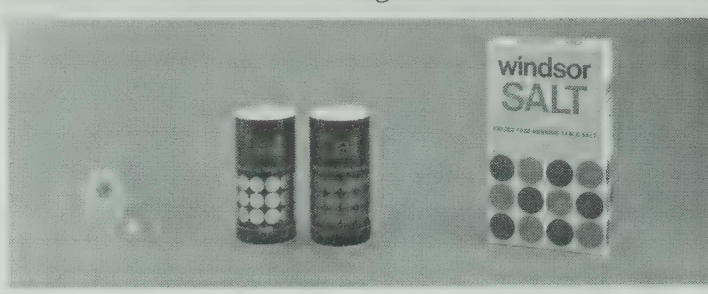
1. 2.5 × 1000 2500

2. 7.45 × 1000 7450

3. 350 × 0.001 0.350 or 0.35

4. 520 × 0.001 0.520 or 0.52

Mass in Grams and Kilograms



The mass of the salt in a packet of salt is about 1 g (gram).

The mass of the salt in these two shakers is about 500 g.

The mass of the salt in this carton is about 1 kg.

The carton can be used for about 1000 packets of salt.

$$1 \text{ kg} = 1000 \text{ g} \qquad 1 \text{ g} = 0.001 \text{ kg}$$

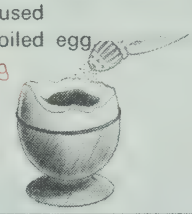
Working Together

Choose the best estimate for the mass of each.

1. The salt in the 4 shakers. 1 kg

1 g	500 g	1 kg

2. The salt used on the boiled egg. 1g



1 g	500 g	1 kg

Complete.

3. 1 kg = ▒ g 1000
   3.5 kg = 3.5 × ▒ g 1000
   3.5 kg = ▒ g 3500
4. 6.45 kg = ▒ g 6450
5. 0.28 kg = ▒ g 280
6. 1 g = ▒ kg 0.001
   750 g = 750 × ▒ kg 0.001
   750 g = ▒ kg 0.750 or 0.75
7. 2540 g = ▒ kg 2.540 or 2.54
8. 320 g = ▒ kg 0.320 or 0.32

226

LESSON ACTIVITY

Before Using the Pages

- Prepare an area of the classroom for activities in measuring mass. Have the students use balance scales to measure various objects having masses less than 1 kg. Have them find objects having masses of about 1 g, 250 g, 500 g, 750 g, and 1 kg. Ask questions such as "How many nails are needed to balance five hundred grams?", "How many grams are the same as one kilogram?", and "How many kilograms are the same as fifteen hundred grams?"

Using the Pages

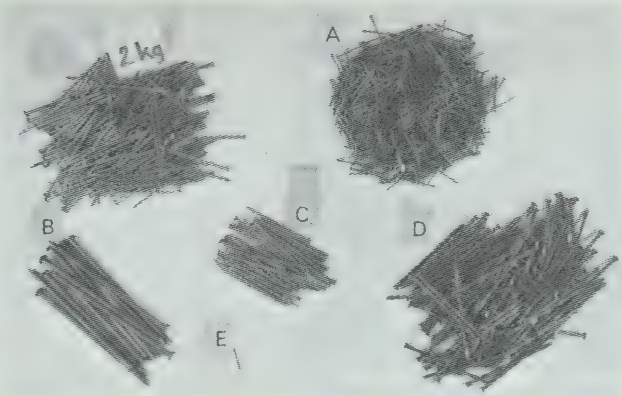
- Have a student read the title at the top of page 226. Associate the word *mass* with finding how heavy an object is. Ask whether a mass of 1 g is heavier than or lighter than a mass of 1 kg. Have students read the statements below the photograph. Ask what the mass of the salt would be in each of the two shakers (250 g) and ask how many shakers of salt would be needed to fill the carton. Summarize by stating these two relationships: 1 kg = 1000 g, and 1 g = 0.001 kg. A chart similar to the following may be developed on the board to help students understand the relationship between grams and kilograms.

kg	0.001	0.010	0.100	0.500	1	1.510
g	1	10	100	500	1000	1510

Working Together: Have students explain their answers for Ex. 1 and 2. For Ex. 3-5, kilograms are expressed as grams by thinking of multiplication with 1000. For Ex. 6-8, thinking of multiplication with 0.001 helps in expressing grams as kilograms. The multiplication is usually performed mentally rather than in writing.

Exercises: Ex. 1 refers to the piles of nails in the photograph. The solution to Ex. 3 involves the use of division by a two-digit number. Ex. 4 may be solved using either multiplication or division and the relationship 2 kg = 2000 g.

Keeping Sharp: These exercises help to maintain skills in addition, subtraction, and multiplication with decimals.



Exercises

For the objects shown above,

1. choose the best estimate of the mass of each. →

E	C	B
1 g	500 g	1 kg
A 1.5 kg	D 3 kg	

Complete this chart. Remember to use: 1 kg = 1000 g, 1 g = 0.001 kg

2.								
g	2000	? 3000	? 4700	1500	? 3620	? 470	50?	720
kg	? 2	3	4.7	? 1.5	3.62	0.47	0.05	? 0.72

Solve.

3. The mass of a loaf of bread is 675 g. There are 27 slices. About how many grams are there in one slice? 25

4. 45 g of peanut butter are used for each sandwich. Is there enough peanut butter in a 2 kg jar for 35 sandwiches? Yes; 35 sandwiches require 1575 kg of peanut butter

KEEPING SHARP

Add.

1. 75.8 + 61.2 = 137.0
2. 2.69 + 2.88 = 5.57
3. 3.876 + 4.927 8.803
4. \$9.21 + \$0.95 \$10.16

Subtract.

5. 64.1 − 34.3 = 29.8
6. 7.44 − 5.56 = 1.88
7. 8.013 − 7.537 0.476
8. \$6.52 − \$4.68 \$1.84

Multiply.

9. 61.3 × 8 = 490.4
10. 7.9 × 46 = 363.4
11. 73 × \$2.74 \$200.02

227

Some students may complete these while others are involved in the measuring activities suggested in *Before Using the Pages*.

Assessment

Choose the best estimate for the mass of

1 g
500 g
1 kg

1. a pair of shoes. 500 g
2. a candle for a birthday cake. 1 g

Complete the chart.

3.					
g	4000	5500	1200	40	1200
kg	4	5.5	1.2	0.04	1.20

Solve.

4. The mass of one egg is 62.5 g. Is the mass of a dozen eggs more than or less than 1 kg? less than

RELATED ACTIVITIES

- Adapt the activity described in *Related Activities* on page T 243 for containers and objects for which the mass is clearly marked in grams or kilograms.
- Have students carry out activities similar to the following which combine tasks in measurement and calculation.

1. Measure the mass of a new box of chalk. Measure the mass of the empty box. Without measuring, find the mass of one stick of chalk.
2. Measure the mass of one mathematics book. Use your answer to find the mass of 24 of these books.
3. Measure the mass of one unopened can of soup. Use your answer to find the mass of 18 cans of this soup.
4. Read the mass marked on a package of dry soup mix. Find the mass of the empty package without opening it.
5. Measure the mass of one penny. Find the mass of the pennies worth one dollar.
6. Measure the mass of one nickel. Find the value of a bag of nickels having a mass of about 4600 g.

Pages 228-229

LESSON OUTCOME

Associate the mass of 1 L of water with 1 kg; associate the mass of 1 mL of water with 1 g; relate the volume of a container, the amount of water needed to fill it, and the mass of the water

Materials

balance scales, masses from 1 g to 1 kg, water, containers such as plastic bottles and paper cups, waterproof container having the shape of a cube with edges 10 cm long

Prerequisite Skills

Express grams as kilograms and kilograms as grams; express millilitres as litres and litres as millilitres

Checking Prerequisite Skills

Complete the charts.

1.

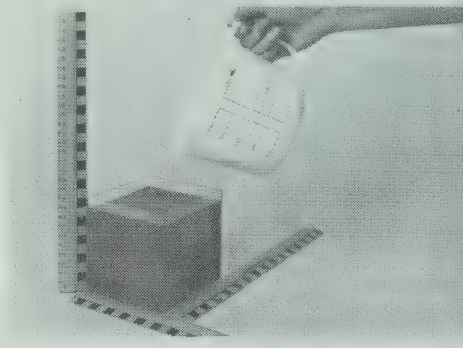
mL	1000	450	6250	800
L	1	0.45	6.25	0.8

2.

g	1000	1200	320	5700
kg	1	1.2	0.32	5.7

The Mass of Water

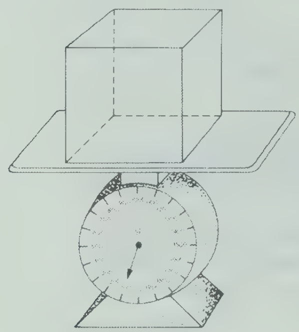
Larry knows that 1 L of water will fill this container.

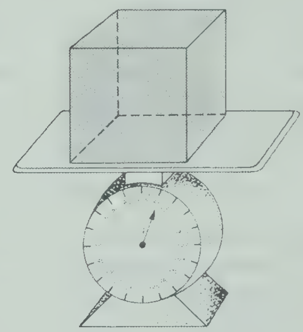


He used a scale he found in the kitchen to get an idea of how heavy 1 L of water is.

For the empty container the scale showed 105 g.

For the full container the scale showed 1105 g.





The number of grams of water is the difference of 1105 and 105.

1105 − 105 = 1000

The mass of 1 L of water is 1000 g.

or 1 kg

The mass of 1000 mL of water is 1000 g.

The mass of 1 mL of water is 1 g.

228

LESSON ACTIVITY

Before Using the Pages

- Review that we measure its mass when we want to find how heavy an object is. Have students name various objects for which they have measured the masses during the work of this unit. Then ask how it might be possible to measure the mass of a quantity of liquid such as water. They will likely suggest pouring the liquid into a container and then measuring the mass. Demonstrate this and, if no one suggests it, lead the students to realize that the mass of the container must also be taken into consideration. Students will likely recognize that subtraction is involved if you write statements similar to the following on the board.

mass of container and water	275 g
mass of empty container	50 g

- Display a waterproof container having the shape of a cube with edges 10 cm long. (One may be prepared from a large milk carton as described in *Before Using the Pages* on page T 240.) Ask how much water will fill the container and how the mass of the water can be found. Have students help to demonstrate the procedure.

Using the Pages

- The example on page 228 shows the procedure suggested in the preliminary activities. Discuss the steps shown to arrive at a mass of 1000 g (or 1 kg) for 1 L of water, emphasizing the need for subtraction. Then have students explain why the mass of 1 mL of water must be 1 g. Although it may be difficult to demonstrate, some students may wish to try. This can lead to a discussion of the need for precision and more suitable scales.

Working Together: Ex. 1 involves multiples of one thousand millilitres (grams) and Ex. 2 deals with a whole number of litres (kilograms). Use other similar exercises before dealing with such numbers as those in Ex. 3 and 4. Complete Ex. 5 with the students, noting that they may find the numbers for an exercise in an order that seems preferable to them. For example, in Ex. 6, some students may write the number of kilograms first.

Working Together

Complete.

1. The mass of 5000 mL of water is ___ g. 5000
2. The mass of 4 L of water is ___ kg. 4
3. The mass of ___ mL of water is 810 g. 810
4. The mass of ___ L of water is 1.6 kg. 1.6

	5.	6.	7.	8.	9.	10.
L	? 5	4	? 0.25	? 0.81	? 1.6	? 3.16
mL	5000	? 4000	250	? 810	? 1600	? 3160
g	? 5000	? 4000	? 250	810	? 1600	3160
kg	? 5	? 4	? 0.25	? 0.81	1.6	? 3.16

Exercises

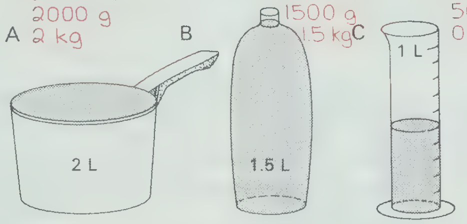
For each container, give the amount of water

1. in millilitres.
2. in grams.
3. in kilograms.

A 2000 mL 2000 g 2 kg

B 1500 mL 1500 g 1.5 kg

C 500 mL 500 g 0.5 kg



Complete.

4. The mass of 640 mL of water is ___ g. 640
5. The mass of 4.3 L of water is ___ kg. 4.3
6. The mass of ___ mL of water is 970 g. 970
7. The mass of ___ L of water is 1.85 kg. 1.85

		8.	9.	10.	11.	12.
Volume of container	cm^3	4000	3250	? 14 700	? 1650	? 250
Amount of water needed to fill it	mL	4000	? 3250	? 14 700	? 1650	? 250
	L	? 4	? 3.25	14.7	? 1.65	? 0.25
Mass of the water	kg	? 4	? 3.25	? 14.7	? 1.65	0.25
	g	? 4000	? 3250	? 14 700	1650	? 250

229

Exercises: Note that the students must interpret the scale in diagram C for Ex. 1-3. For Ex. 8-12, it may be necessary to review that 1000 cm^3 and 1000 mL (or 1 L) take the same amount of space (see page 224).

Assessment

Complete.

1. The mass of 540 mL of water is 540 g.
2. The mass of 320 mL of water is 320 g.
3. The mass of 6.5 L of water is 6.5 kg.
4. The mass of 2.45 L of water is 2.45 kg.

		5.	6.
Volume of container	cm^3	1650	750
Amount of water needed to fill it	mL	1650	750
	L	1.65	0.75
Mass of the water	kg	1.65	0.75
	g	1650	750

RELATED ACTIVITIES

- Provide opportunities for the students to fill various containers with water and measure the amounts of water in millilitres or litres. Then have them write the mass of each amount of water in grams or kilograms.
- Have the students compare the mass of one litre of water with one litre of sand, salt, rice, or other suitable substance. Have them compare the mass of one litre of water with one litre of water in which an amount of sugar or salt has been dissolved.

Pages 230-231

LESSON OUTCOME

Write numerals for times to the second for a 24-hour clock; add and subtract times

Materials

demonstration 24-hour clock with movable hands

Vocabulary

a.m., p.m., 24-hour clock, hour hand, minute hand, second hand, words for times to the second, marathon race

Prerequisite Skills

Write numerals for times shown on a 12-hour clock

Checking Prerequisite Skills

Write the time shown.

1.

8:20

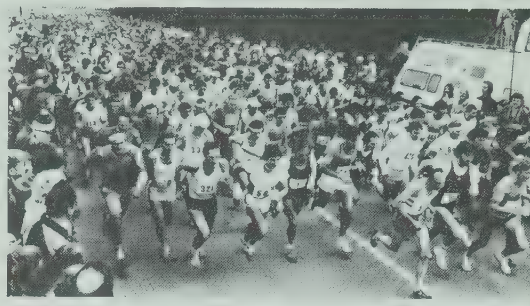
2.

1:37

The 24-Hour Clock

The 24-hour clock shows when the marathon race began.

ten-thirty in the morning



The winning runner took 2 h, 17 min, and 43 s. At what time did he cross the finish line?

Add. 10:30:00
02:17:43
12:47:43

47 min and 43 s after noon

Working Together

This 24-hour clock shows when the last runner finished in the afternoon.

1. Look at the hour hand. What hour has just passed? 13

2. Look at the minute hand. What minute has just passed? 56
3. Look at the second hand. How many seconds are shown? 17
4. Write the time. Show hours, then minutes, then seconds. 13:56:17
5. How long did it take the last runner to finish? 3 h, 26 min, 17 s

Is the time shown below before noon or after noon? How can you tell?

6. 19:35:16 after noon
7. 04:42:18 before noon
(Hours after noon are greater than 12.)

What time is it when the clock hands are

8. like this before noon? 04:05:10
9. like this after noon? 16:05:10

230

LESSON ACTIVITY

Before Using the Pages

- Write the following numerals for times on the board and ask how their meanings differ.

  2:15 a.m. 2:15 p.m.

  Ask for another way to indicate a time after noon without using the symbol p.m. If necessary, draw attention to the numerals on the 24-hour demonstration clock. Write the numeral 14:15 on the board and review that the numerals 13 to 23 (or 24) can be used to identify hours after noon since there are 24 hours in the day. Review that a time such as 14:15 is read "fourteen fifteen hours" or simply "fourteen fifteen". Note that for a 24-hour clock, two digits are always used to write the hour, as in 02:15.

- Show times on the demonstration clock, name an event for each time, and have the students write the numerals for the times. For example, set the hour hand at 6, the minute hand at 12, and say, "It is time for supper." The students should write the numeral 18:00. Use other examples such as 04:30, 11:55, and 21:10.

- Write the numeral 14:15:42 on the board and ask how such a numeral for time might be interpreted. Lead the students to suggest that the 42 represents seconds.

Using the Pages

- The photograph at the top of page 230 shows the start of a marathon race. Explain that a marathon is a long distance race which originally covered about 42 km. Ask what time is shown on the dial clock for the start of the race. Then read the statement above the illustration on page 231 to learn how long it took the winning runner to complete the race. Note the symbols h, min, and s for hours, minutes, and seconds. Draw attention to the statements below the photograph on page 230 and ask what operation will be required to answer the question.

  Have students interpret the numerals for the times shown. Ask how the vertical arrangement of the numerals helps in performing the addition. Point out that the numeral

The winning runner crossed the finish line 2 h, 17 min, and 43 s later.

Exercises

Write the time shown on the clock

1. when it is time for dinner. 17:35:10

2. when it is time for school. 08:50:25

Choose the better time for

3. delivering the morning paper.

   06:10:45 or 18:10:45 06:10:45

4. eating an after-school snack.

   03:50:30 or 15:50:30 15:50:30

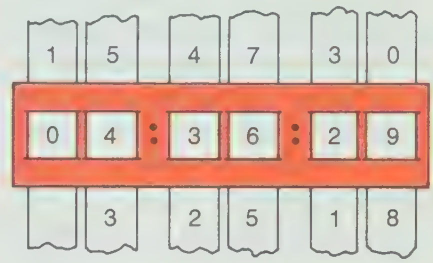
Complete this chart.

	Runner	Starts	Finishes	Time taken
5.	Ed	10:30:12	12:57:14	2 ? h 27 ? min 2 ? s
6.	Nancy	10:30:12	13:36:21	3 h 6 min 9 s
7.	Ivan	10:30:36	13:45:40	3 h 15 min 4 s
8.	Dino	10:31:00	14:50:41	4 ? h 19 ? min 41 ? s
9.	Anna	10:31:24	13:40:53	3 ? h 9 ? min 29 ? s

231

RELATED ACTIVITIES

- Use the demonstration clock to practice reading and showing times with small groups of students. The clock should have a second hand.
- At different times throughout the day, have students write numerals for times shown on the classroom clock.
- Students may make a simple form of digital clock to use for showing times to the second according to a 24-hour clock. Strips of stiff paper showing the necessary numerals are threaded into six slits in a rectangular sheet of cardboard. The ends of each strip are joined to form a movable loop.



12:47:43 is read "twelve forty-seven forty-three". Emphasize that hours are named first, then minutes, and then seconds, noting that numerals for times to the second always show six digits.

Working Together: Ex. 1-4 show the steps for writing times to the second. Note that the directions specify "in the afternoon". Have the students follow similar steps to complete Ex. 8 and 9. The solution to Ex. 5 is found by subtraction. Remind the students to align the numerals vertically.

Exercises: The suggestions given in Ex. 1-4 determine whether times are before noon or after noon. You may need to discuss aspects of the chart for Ex. 5-9 to ensure that the students understand what is required.

Assessment

Write the time shown on the clock

1. when it is time for breakfast. 07:20:45

Choose the better time for

2. eating dinner.

   05:45:20 or 17:30:15

Solve.

3. Dirk started the race at 12:40:05. He finished 1 h, 17 min, and 46 s later. At what time did he finish the race? 13:57:51

Page 232

LESSON OUTCOME

Write and interpret dates using numeric dating (year, month, day)

Vocabulary
numeric dating

RELATED ACTIVITIES

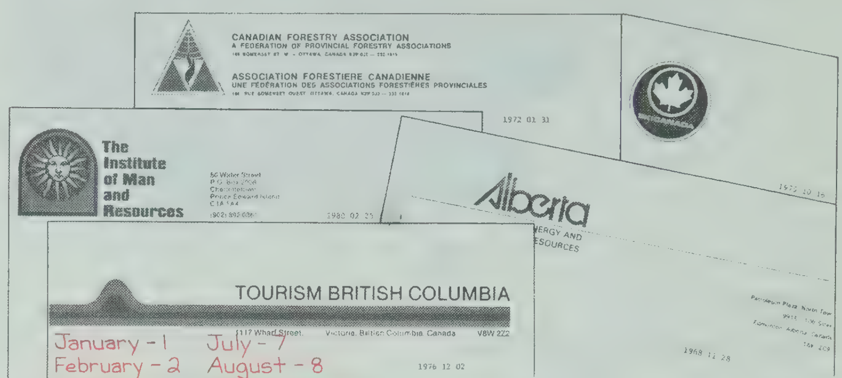
- Have a different student write the date with and without numeric dating each morning.
- Have students find and bring to school examples of numeric dating in magazines, catalogs, newspapers, and packaged items. Display the examples and have students interpret them.
- Ask students to find the pattern for the following sequence of letters and use it to continue the sequence for four more letters.

O T T F F S S E N ________
(one, two, three, . . .)

Numeric Dating

The SKICANADA letter was written on October 16, 1975.

The first four digits show the year, the next two show one of the 12 months, and the last two show the day of that month.



Working Together

Tell

1. which number names each month.

January - 1, February - 2, March - 3, April - 4, May - 5, June - 6, July - 7, August - 8, September - 9, October - 10, November - 11, December - 12

For the letter from The Institute of Man and Resources,

2. what year is shown? 1980
3. what month is shown? February
4. what day of the month is shown? 25

Give the month, day, and year.

5. 1973 01 08 January 8, 1973
6. 1976 08 01 August 1, 1976

Give the date of your birth

7. in two ways. Answers will vary

Exercises

Give the month, day, and year

1. for the Canadian Forestry letter. January 31, 1972
2. for the Alberta letter. November 28, 1968
3. for the British Columbia letter. December 2, 1976

Using 8 digits, what date would you put on a letter

4. on May 5, 1982? 1982 05 05
5. on August 8, 1908? 1908 08 08
6. on July 11, 1867? 1867 07 11
7. on the day after Dec. 31, 1999? 2000 01 01

Write the months of the year in the order shown. Tell why they are listed in this order.

*8. 4, 8, 12, 2, 1, 7, 6, 3, 5, 11, 10, 9

April, August, December, February, January, July, June, March, May, November, October, September

The months have been put in alphabetical order.

232

LESSON ACTIVITY

Using the Page

- Begin with a brief discussion of the illustration. Have students read the different letterheads shown. Draw attention to the dates indicated on the letters and ask how the numerals are interpreted. Point out the statement in the "thought cloud", noting that there are eight digits in each date. Emphasize that the left-to-right sequence names the year, then the month, then the day of the month. Have a student read the title of the lesson. Discuss that simple *numeric dating* involves the use of eight digits as described.

Working Together: Ex. 1 reviews the sequence of the months in a year and the number associated with each month. Ex. 2-6 deal with the left-to-right sequence of interpreting numeric dating. Ex. 7 examines the ability to name a date with and without numeric dating.

Exercises: Ex. 8 is starred because its solution depends on recognizing the pattern. (The names of the months are given in alphabetical order.)

Assessment

Give the date for today

1. in two ways. Answers will vary.

Give the month, day, and year.

2. 1980 02 29 February 29, 1980

Using 8 digits, what date would you put on a letter

3. on September 7, 1988? 1988 09 07

Finding Patterns

Draw the first four pictures for each chart. Count and record the results. Then complete the chart by using the number pattern.

1. Cutting pizza

Number of cuts	1	2	3	4	9	12
Number of pieces	2	4	6 ?	8 ?	18 ?	24 ?

2. Diagonals from one vertex of a polygon

Number of sides	3	4	5	6	8	11
Number of diagonals	0	1 ?	2 ?	3 ?	5 ?	8 ?

3. Diagonals in a polygon

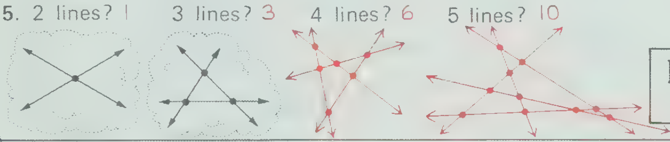
Number of sides	3	4	5	6	7	12	20
Number of diagonals	0	2	5 ?	9 ?	14 ?	54 ?	170 ?

4. Triangular numbers

Number of rows	1	2	3	4	5	8	13
Number of dots	1	3	6 ?	10 ?	15 ?	36 ?	91 ?

How many points of intersection can there be for

5. 2 lines? 1 3 lines? 3 4 lines? 6 5 lines? 10



PROBLEM SOLVING

233

Page 233

OBJECTIVE

Solve problems by searching for patterns and following the patterns

Vocabulary

diagonal of a polygon, triangular number

RELATED ACTIVITIES

- Have students complete Ex. 1 so that the cuts do not pass through the center of the pizza. However, tell them to cut the pizza to obtain the greatest number of pieces possible with each new cut. The pieces will not be the same size. The following pattern is obtained.

Number of cuts	1	2	3	4	5
Number of pieces	2	4	7	11	16

2 3 4 5

- Have students investigate square numbers using a chart as shown below.

	Number of rows	Number of dots
•	1	1
•• ••	2	4
••• ••• •••	3	
	4	
	5	
	8	
	13	

LESSON ACTIVITY

Using the Page

- It would be desirable to complete Ex. 1 with the students to ensure that they understand the procedure. Emphasize that the cuts are to pass through the center of the pizza. A very different pattern emerges if the cuts do not pass through the center (see *Related Activities*).

For Ex. 2 and 3, review that a diagonal of a polygon is a line segment having two non-adjacent vertices of the polygon as its end points.

For Ex. 4, have students suggest why the numbers in the row for "Number of dots" are described as *triangular numbers*.

For all the exercises, emphasize that finding a pattern can be a very helpful way of solving each problem because the pattern can be continued. Once the patterns have been discovered, students can carry out further investigations of the numbers in the patterns. For example, the use of subtraction for the pattern in Ex. 4 leads to other patterns that can be explored.

Number of dots	1	3	6	10	15

2 3 4 ?

1 ? ?

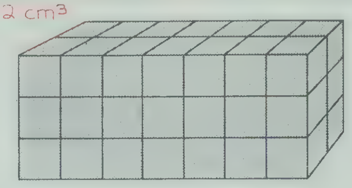
OBJECTIVE

Demonstrate an understanding of the concepts and skills presented in this unit

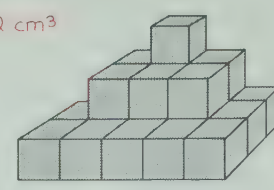
Checking Up

Each small cube represents a cubic centimetre.
Give each volume in cubic centimetres.

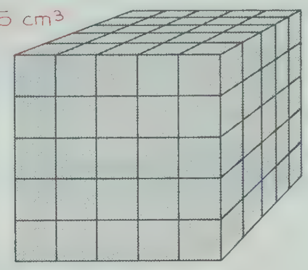
1. 42 cm³



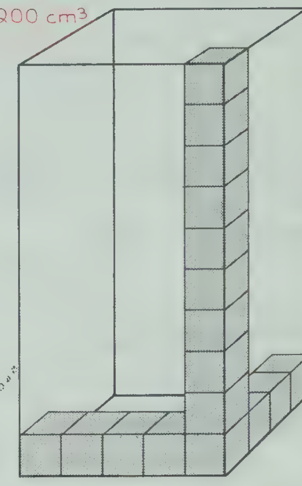
2. 22 cm³



3. 125 cm³



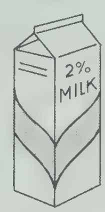
4. 200 cm³



Find how many centimetre cubes will fill this rectangular prism.

Choose the best estimate for

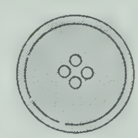
5. the capacity.



1 L

1 mL	500 mL	1 L

6. the mass.



1 g

1 g	500 g	1 kg

234

Skills	Exercises	Related Pages
Find volume in cubic centimetres	1-4	T 236-T 239
Estimate capacity	5	T 242-T 243
Estimate mass	6	T 246-T 247
Express millilitres as litres and litres as millilitres	7-9	T 242-T 243
Express grams as kilograms and kilograms as grams	10-12	T 246-T 247
Relate millilitres and cubic centimetres	13, 14	T 244-T 245
Relate litres and cubic centimetres	15, 16	T 240-T 241 T 244-T 245
Relate cubic centimetres, grams (kilograms), and millilitres (litres) for amounts of water	17-24	T 248-T 249
Tell time on a 24-hour clock	25, 26	T 250-T 251
Add and subtract times	27-30	T 250-T 251
Write and interpret dates using numeric dating (year, month, day)	31-34	T 252

Comments

An understanding of different kinds of measurement and the relationships among different units is best promoted through many activities in measuring objects. Choose from those suggested in *Related Activities* and *Before Using the Pages* for lessons in this unit.

UNIVERSITY OF ALBERTA

Complete.

7. 4000 mL = L 4
8. 6.7 L = mL 6700
9. 450 mL = L 0.45
10. 3000 g = kg 3
11. 1400 g = kg 1.4
12. 0.35 kg = g 350
13. 480 cm^3 take the same space as mL. 480
14. 675 mL take the same space as cm^3. 675
15. 12 000 cm^3 take the same space as L. 12
16. 8.45 L take the same space as cm^3. 8450
17. The mass of 250 mL of water is g. 250
18. The mass of 3.2 L of water is kg. 3.2
19. The mass of L 0.8 of water is 0.8 kg.
20. The mass of mL 2500 of water is 2500 g.

21. The fish tank holds 35 000 cm^3.
35 000 mL of water will fill the tank.
22. L of water will fill the tank. 35
23. The tank can hold kg of water. 35
24. The tank can hold g of water. 35 000

For this 24-hour clock,

25. write the time shown if it is before noon. 07:25:18
26. write the time shown if it is after noon. 19:25:18

27. Write the time it would be 3 h 14 min 6 s earlier in the afternoon. 16:11:12
28. Write the time it would be 3 h 14 min 6 s later in the morning. 10:39:24

If the time shown is before noon, write the time for

29. 6 h 20 min 9 s earlier. 01:05:09
30. 6 h 20 min 9 s later. 13:45:27

Complete this chart to show each date two ways.

31.	32.	33.	34.
Nov. 6, 1959	April 8, ?1931	Dec 11, ?1850	June 6, 2066
1959 11?06	1931 04 08	1850 12 11	2066 06?06

235

RELATED ACTIVITIES

• Have students use copies of page T 397 to prepare patterns for containers having the shape of a rectangular prism with a given volume. For example, ask them to make and cut out a pattern for a box having a volume of 24 cm^3. Discuss the different possibilities and let them experiment with cubes before drawing the pattern. When there is sufficient confidence with small containers, assign volumes of 400 cm^3 and 6000 cm^3, for example. On each container, have students show the volume in cubic centimetres, the amount of water the container would hold in millilitres or litres, and the mass of the water it would hold in grams or kilograms.

Pages 236-237

OBJECTIVE

Demonstrate competence in addition, subtraction, and multiplication skills; solve related word problems

Checking Skills

Add.

1.	425 343	768
2.	26.4 70.2	96.6
3.	$2.51 3.37	$5.88
4.	364 174	538
5.	21.3 19.5	40.8
6.	$12.62 13.95	$26.57
7.	343 389	732
8.	4.64 2.36	7.00
9.	$28.73 49.51	$78.24
10.	967 584	1551
11.	29.7 88.3	118.0
12.	$16.58 9.72	$26.30
13.	9685 7286	16 971
14.	24.559 7.549	32.108
15.	124 2037 69	2230
16.	195.6 240.9 38.2	474.7
17.	10.52 0.06 12.98 1.49	25.05
18.	$403.19 4.98 68.73 135.27	$612.17

19. 325 + 61 386
20. 6.33 + 1.26 7.59
21. 764 + 2165 2929
22. 41.3 + 4.7 46.0
23. 5738 + 2846 8584
24. 2.694 + 2.531 5.225
25. 1271 + 6968 8239
26. 28.87 + 15.79 44.66
27. 40 558 + 49 894 90 452
28. 52.794 + 7.368 60.162
29. $126.48 + $84.93 $211.41
30. 405 + 3839 + 63 4307
31. 64.8 + 9.3 + 126.9 201.0
32. $78.49 + $104.98 + $28.89 $212.36
33. 16.97 + 1.24 + 0.55 + 29.76 48.52

Subtract.

1.	734 321	413
2.	95.6 42.4	53.2
3.	$6.44 3.41	$3.03
4.	850 24	826
5.	34.7 5.0	29.7
6.	$11.75 5.13	$6.62
7.	525 138	387
8.	17.37 9.65	7.72
9.	$24.93 15.37	$9.56
10.	7317 5428	1889
11.	625.0 358.3	266.7
12.	$58.14 29.39	$28.75
13.	2704 1237	1467
14.	460.3 171.5	288.8
15.	$25.02 6.73	$18.29
16.	3006 1687	1319
17.	500.2 72.5	427.7
18.	$73.00 16.28	$56.72
19.	67 323 8 394	58 929
20.	8362.3 1394.9	6967.4
21.	42.60 3.71	38.89
22.	$514.95 356.96	$157.99
23.	50 030 1 641	48 389
24.	42.600 3.715	38.885

25. 858 − 215 643
26. 8.37 − 6.34 2.03
27. 3918 − 2381 1537
28. $6.67 − $4.02 $2.65
29. 6315 − 5872 443
30. 1.358 − 0.684 0.674
31. 7833 − 3894 3939
32. $93.11 − $7.19 $85.92
33. 600 − 409 191
34. 700.1 − 248.7 451.4
35. $62.00 − $13.56 $48.44
36. 50 000 − 35 676 14 324
37. 42.000 − 19.123 22.877
38. $60 000 − $43 295 $16 705

236

LESSON ACTIVITY

Using the Pages

- These exercises provide practice in addition, subtraction, and multiplication with whole numbers and with decimals. The exercises are grouped according to operation and, generally, the difficulty increases in working sequentially through one group. Note that there are two groups suggested for each of addition and subtraction: one for vertical arrangement of numbers and another for horizontal arrangement.

  The word problems on page 237 are not grouped according to operation. Several problems involve more than one step and the use of more than one operation in the solution.

  The exercises may be assigned one section at a time to allow students to concentrate on one operation. Alternatively, you may assign a few exercises from each section to be completed together.

Multiply.

1. 36 2 72	**2.** 8.3 6 49.8	**3.** 27¢ 4 108¢ or $1.08
4. 746 3 2238	**5.** 39.7 9 357.3	**6.** $8.55 7 $59.85
7. 4567 6 27 402	**8.** 3.782 8 30.256	**9.** $19.93 5 $99.65
10. 62 73 4526	**11.** 0.37 41 15.17	**12.** 46¢ 59 2714¢ or $27.14
13. 323 68 21 964	**14.** 13.4 95 1273.0	**15.** $6.81 39 $265.59
16. 7454 27 201 258	**17.** 9.18 24 220.32	**18.** $22.53 75 $1689.75
19. 535 834 446 190	**20.** 84.9 192 16 300.8	**21.** $8.84 679 $6002.36
22. 3735 715 2 670 525	**23.** 2.673 926 2475.198	**24.** $19.72 483 $9524.76
25. 18 545 8 148 360	**26.** 266.79 5 1333.95	
27. 42.835 6 257.010	**28.** $137.97 9 $1241.73	
29. 0.9 0.2 0.18	**30.** 0.4 0.6 0.24	**31.** 0.2 0.4 0.08
32. 6.5 0.6 3.90	**33.** 1.7 0.3 0.51	**34.** 9.4 0.8 7.52
35. 5.8 4.7 27.26	**36.** 8.6 2.5 21.50	**37.** 8.1 8.3 67.23

Solve.

1. A record album costs $5.95. A "single" costs $1.29. How much would both cost? $7.24
2. A cassette tape recorder costs $39.98. Batteries cost $4.87. A microphone costs $4.95. Tape cassettes cost $10.76. An earphone costs 98¢. How much does the complete set cost? $61.54
3. A color television set costs $289.39. A black and white set costs $99.95. How much more does the color set cost? $189.44
4. How much change is there from a twenty-dollar bill when $9.98 is paid for a record album? $10.02
5. Each cassette has 45 min of recording time. How many minutes are on 8 cassettes? 360
6. Each reel of recording tape costs $5.49. How much would 6 reels cost? $32.94
7. A cassette has 86.56 m of tape. How many metres would there be on 12 cassettes? How much more than 1 km is this? 1038.72 38.72 m
8. Each tape cassette costs $2.69. How much would 24 cassettes cost? $64.56
9. Each tape cassette costs $2.69. For 3 cassettes, how much change would there be from a ten-dollar bill? $1.93
10. 1675 record albums that sold for $5.95 each were put on sale for $4.79 each. How much less would they sell for in all? $1943.00

237

RELATED ACTIVITIES

- Choose from related activities and games in preceding units for review and enrichment. Extra practice and review is possible by having students complete one or more of the following.

1. Use addition to check the answers for several subtraction exercises.
2. Write the word name for each addend in a selection of addition exercises.
3. Use an abacus to illustrate sums and differences for whole numbers and for decimals.
4. Estimate answers by rounding numbers.
5. Write a word problem to match the addition (subtraction, multiplication) indicated in an exercise.

Unit 12 Overview

Dividing Decimals

The division of decimals by one-digit and two-digit whole numbers proceeds through a series of lessons, first with no regrouping and later with regrouping. The same procedures are used as for division of whole numbers, and place values are emphasized throughout the presentations. The major new feature in division of decimals is the possibility of continuing the process by writing zeros in decimal places to the right of the ones' place. This principle is also applied to division of whole numbers which are less than the divisors. The lesson on the use of the calculator emphasizes the need for using estimation as a check to detect any errors which may arise through pressing the wrong keys. The lesson on developing problem-solving skills involves the use of logical thinking to solve everyday problems.

Prerequisite Skills

- divide by a one-digit number, dividends with up to five digits, remainder zero
- rename mixed numbers and decimals representing two place values in terms of the lesser value
- round a two-digit number to the nearest ten

Unit Outcomes

- Divide a one-place decimal to 9.9 by a one-digit number, no regrouping, quotients greater than 1
- divide a one-place decimal by a one-digit number, regrouping, quotients greater than 1
- divide a decimal with two or three decimal places by a one-digit number, regrouping, dividends with up to five digits, quotients greater than 1
- divide a decimal with up to three decimal places by a one-digit number, regrouping, quotients less than 1
- divide a whole number or a decimal by a one-digit number, using zeros in the dividend, quotients terminating by the third decimal place
- divide a decimal or a whole number by a two-digit number, using zeros in the dividend, quotients terminating by the third decimal place
- solve word problems involving division with decimals
- recognize incorrect results for operations performed with a calculator
- solve a problem through a process of logical thinking

Background

Division of decimals involves the same general understanding and skill as division of whole numbers because of the decimal nature of our numeration system. Each place value of a dividend is considered in succession from left to right.

Just as there are many numerals for naming the same number, a numeral may be interpreted in more than one way. For example, the numeral 375 may be thought of as 3 hundreds 7 tens 5 ones, or 37 tens 5 ones, or 375 ones. Similarly, decimals may be interpreted in several ways, for example, 2.67 may be considered as 2 ones 6 tenths 7 hundredths, or 26 tenths 7 hundredths, or 267 hundredths. In the case of decimals, numerals for the same number may be extended by writing zeros in places to the right of the given digits. For example, if 2.67 is written as 2.670, it can be interpreted as 2670 thousandths. Thus, any whole number or decimal may be extended by writing a zero or zeros to the right of the last digit in the decimal part.

It is customary to extend dividends by adding zeros for two reasons. It is often possible that a division with a remainder greater than zero at a particular place value may become a division with a remainder of zero if the process is continued for one or more places. For instance, in 3.4 ÷ 8 the quotient to tenths is 0.4, but there is a remainder of 2 tenths. If the dividend is extended to hundredths, there is a remainder of 4 hundredths. However, if the dividend is extended further to thousandths, there is a remainder of zero and the quotient is exactly 0.425.

$$\begin{array}{r} 0.4 \\ 8\overline{)3.4} \\ \underline{3\ 2} \\ 2 \end{array} \qquad \begin{array}{r} 0.42 \\ 8\overline{)3.40} \\ \underline{3\ 2} \\ 20 \\ \underline{16} \\ 4 \end{array} \qquad \begin{array}{r} 0.425 \\ 8\overline{)3.400} \\ \underline{3\ 2} \\ 20 \\ \underline{16} \\ 40 \\ \underline{40} \\ 0 \end{array}$$

The other situation in which it is customary to extend dividends is when the divisor is greater than the dividend, particularly when both are whole numbers. For example, the division 6 ÷ 8 may be completed if the dividend 6 is extended first to 6.0 (60 tenths) and then to 6.00 (600 hundredths). In this case the quotient terminates in the second decimal place. Sometimes it is impossible to get a terminating quotient, that is, to eliminate a remainder greater than zero, so, according to the circumstances, the division is continued to an acceptable number of places in the quotient. It is usual to do this to one more place so that the quotient may be rounded. This stage is not reached in *Starting Points in Mathematics 5*, but students may encounter examples of this type if they continue the activity of calculating "batting averages" presented on page 247. For instance, in the first example shown below, the quotient for 3 ÷ 8 terminates in the third decimal place. In the second example given, 3 ÷ 7, the quotient does not terminate (0.4285714. . .), but rounded to three decimal places the average is 0.429. In the third example, 4 ÷ 9, the quotient does not terminate (0.4444. . .), but rounded to three decimal places the average is 0.444.

$$\begin{array}{r} 0.75 \\ 8\overline{)6.00} \\ \underline{5\ 6} \\ 40 \\ \underline{40} \\ 0 \end{array}$$

Hits	Times at bat	Average
3	8	0.375
3	7	0.429
4	9	0.444

Unit pricing of items in supermarkets is another application of dividing with decimals, and customers are now giving more attention to such calculations in their shopping. Some shoppers also make other calculations on their own to determine the best buys. One of the *Problem Solving* features and a set of exercises give students experiences of this type. Often such calculations require the use of only one-digit divisors and it is an advantage if they can be performed without the use of paper and pencil. In this connection, estimation is often sufficient to make comparisons. For instance, if the price for 2 bottles of ginger ale is 69¢, the cost of one bottle is about 35¢ (70 ÷ 2); whereas if the price for 3 bottles is 98¢, the cost of one bottle is about 33¢ (99 ÷ 3).

$$\begin{array}{r} 34.5 \\ 2\overline{)69.0} \\ \underline{6} \\ 09 \\ \underline{8} \\ 1\,0 \\ \underline{1\,0} \\ 0 \end{array} \qquad \begin{array}{r} 32.66 \\ 3\overline{)98.00} \\ \underline{9} \\ 08 \\ \underline{6} \\ 2\,0 \\ \underline{1\,8} \\ 20 \\ \underline{18} \\ 2 \end{array}$$

With the advent of inexpensive calculators into everyday life, many people have been lulled into an expectation of complete accuracy. Calculators themselves are accurate, but it is relatively easy for the wrong keys to be pressed, and since the calculators are so fast, errors are often undetected. Because of the possibility of undetected errors, students should be encouraged to estimate results. Estimation is one of the easiest and fastest ways to check accuracy. This can be done quickly by using rounded numbers and basic facts. For example, the cost of 80 m of wire at 37.5¢ for one metre can be estimated by using a rounded price of 40¢ and the basic fact $8 \times 4 = 32$ extended to $80 \times 40 = 3200$; this product, converted to \$32.00, can be rounded to give an estimate of about \$30.00.

The lesson on problem solving involves only small numbers and computational skills are not important. Logical thinking is the key and this is dependent first upon careful reading to establish what kinds of relationships exist between the numbers and what unusual factors are involved in specific situations. This reading is then followed by unique types of reasoning or strategies. This same problem-solving skill is needed whenever one encounters situations in which one is inclined to think "How shall I go about it?" Each situation is apt to be different and clear thinking is the basic requirement for solving such problems.

Teaching Strategies

The partitive, or sharing, concept of division is used in the lesson presentations because it is suited to the step-by-step division of successive place values from left to right. This approach can also be demonstrated effectively with models showing ones, tenths, hundredths, and thousandths. Because the last-named value is not easily represented, it is suggested that models be used rather fully with the other values to establish the various division steps meaningfully, particularly the renaming of remainders with the next values. If a good understanding has been developed and the students are able to carry out the steps meaningfully as far as the second decimal place (hundredths), they should have no difficulty in extending the operation to the third decimal place.

In some of the presentations on the chalkboard, it may be helpful to write the numerals in place-value charts or columns as shown. Place values in the dividend and in the quotient should be emphasized at all times. In the second step, for instance, the partial dividend is 17 tenths (1 one and 7 tenths), in the third step, 10 hundredths (1 tenth 0 hundredths), and in the final step, 24 thousandths (2 hundredths 4 thousandths).

	T	O	t	h	th
		3	4	2	6
4)	1	3 .	7	0	4
	1	2			
		1	7		
		1	6		
			1	0	
				8	
				2	4
				2	4
					0

Before the *Practice* lesson on page 244, it is suggested that there be a quick review of the concepts and methods related to perimeter and area. Attention is directed particularly to the lessons in Unit 7 on pages 134 and 135 and pages 140 and 141.

It may be advisable to check students' abilities to divide whole numbers by two-digit divisors before the lesson on page 250 where the division of decimals by two-digit divisors is presented. In Unit 10 the procedures are outlined in the lessons on pages 202-207. The important skills to watch are the correct rounding of the two-digit divisor to the nearest ten and any necessary adjustments of estimated digits for the quotients. Both of these skills are also required in the division of decimals.

Some students may be helped to accept the annexing of zeros to dividends by completing sets of equivalent numbers, as shown. These can then be used to complete the division examples given. Similar sets of equivalent numbers may be made for the dividends in Ex. 1-24 on page 247.

Number	Hundredths	Thousandths
6.4	6.40	6.400
36		
2.7		
0.45		

$5\overline{)6.4}$ $\quad 24\overline{)36}$ $\quad 4\overline{)2.7}$ $\quad 6\overline{)0.45}$

Students may need help in understanding the generalization in the "thought cloud" at the top of page 245; namely, that the quotient is less than 1 when the divisor is greater than the dividend. Prior to this stage of their mathematical understanding they only knew that such a division was not possible for whole numbers. Careful attention, however, needs to be given to reading the numerals, because the number of digits does not always determine the size of a number. When decimals greater than 1 are involved, attention should be directed to the whole-number part of the decimal. In $8\overline{)6.75}$ the dividend has three digits, but the number 6.75 is less than the divisor 8, and therefore the quotient must be less than 1. It may be helpful to provide a set of examples, similar to those given below, and have the students quickly identify those in which the quotients will be less than 1.

$24\overline{)15.6}$ $\quad 3\overline{)0.279}$ $\quad 6\overline{)8.7}$

$4\overline{)3.5}$ $\quad 16\overline{)8}$ $\quad 28\overline{)9.1}$

The lesson on the use of the calculator emphasizes the importance of estimation to detect errors when using calculators. The lesson may achieve the same goal even if calculators are not available, although it will require more time for the students to discover the errors and to determine their causes.

Materials

models for ones, tenths, and hundredths prepared as described on page T 109

calculators (optional)

Vocabulary

optical laboratory

batting average

LESSON OUTCOME

Divide a one-place decimal to 9.9 by a one-digit number, no regrouping, quotients greater than 1; solve related word problems

Materials

models for ones and tenths prepared as described on page T 109

Vocabulary

optical laboratory

Prerequisite Skills

Divide a two-digit number by a one-digit number, no regrouping, remainder zero

Checking Prerequisite Skills

Divide.

1. $3\overline{)96}$ 32
2. $2\overline{)48}$ 24
3. $4\overline{)88}$ 22
4. $2\overline{)26}$ 13

---

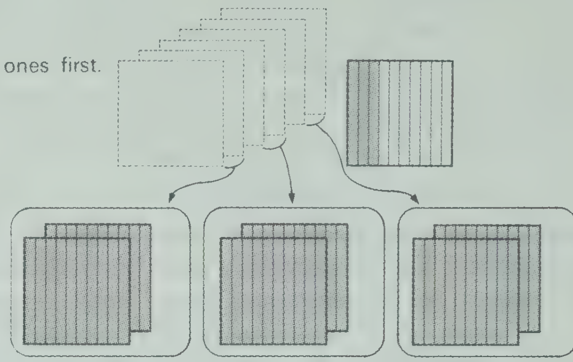
12 DIVIDING DECIMALS

Sharing Ones and Tenths

Divide 6.3 by 3.

For $3\overline{)6.3}$, share the 6 ones first.

Think $3 \times 2 = 6$



ones tenths

Write
$$\begin{array}{r} 2 \\ 3\overline{)6.3} \\ 6 \\ \hline 0 \end{array}$$

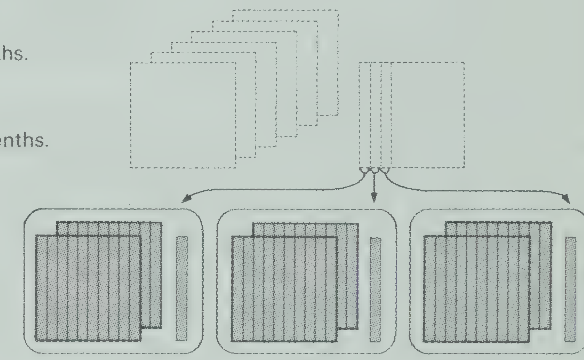
There are 0 ones left, but there are still 3 tenths to share.

$$\begin{array}{r} 2 \\ 3\overline{)6.3} \\ 6\downarrow \\ \hline 0\;3 \end{array}$$

Then share the 3 tenths.

$3 \times 1 = 3$

Use 3×1 tenth = 3 tenths.



Write
$$\begin{array}{r} 2.1 \\ 3\overline{)6.3} \\ 6 \\ \hline 0\;3 \\ 3 \\ \hline 0 \end{array}$$

Place the decimal point in the quotient above the decimal point in the dividend.

When 6.3 is divided by 3, the quotient is 2.1.

238

LESSON ACTIVITY

Before Using the Pages

- Name a decimal such as 8.4 and have a student write the numeral on the board. Have another student name the number of ones and the number of tenths and use models to represent 8.4. Ask how many ones and tenths each person will receive if the models are shared equally between two. Repeat the question for sharing the models equally among four. For each question, have students demonstrate the answer using the models. Repeat the procedure for other similar examples. Then ask what operation is associated with the process of sharing equally. Have students write division sentences for statements encountered earlier. For example, for "8.4 shared equally between 2 gives 4.2 to each", they would write $8.4 \div 2 = 4.2$.

Using the Pages

- The worked example demonstrates that dividing a decimal is similar to dividing a whole number. The division is performed place by place from left to right and the use of basic multiplication facts helps to complete the division.

  Lead the students through the steps of the example, associating the process with the sharing of models as indicated in the diagrams. Ask questions such as
  "What is divided first?"
  "What multiplication fact is helpful in dividing six ones by three?"
  "Are there any ones left over?"
  "What is divided next?"
  "How is this shown in the diagram?"

  Emphasize that the decimal point is placed in the quotient above the decimal point in the dividend. Draw attention to the fact that the decimal point is not shown in the work below the dividend, but place values are aligned as indicated by the arrow.

Working Together: Ex. 1-3 establish that ones are shared first and then the tenths. This corresponds to dividing the dividend place by place from left to right in Ex. 4-6.

Working Together

Complete.

1. Share 8 ones between 2 and each gets ones. 4
2. Share 6 tenths between 2 and each gets tenths. 3
3. For 2)8.6, first share ones, 8 then share tenths. Each will 6 get 4 ones and 3 tenths, or 4.3.

Divide. Remember to place the decimal point in the quotient.

4. 2)8.6 4.3 5. 3)6.9 2.3 6. 4)8.4 2.1

Exercises

Divide.

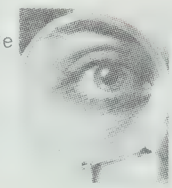
1. 4)4.8 1.2	2. 2)2.8 1.4	3. 3)3.6 1.2
4. 2)8.2 4.1	5. 3)6.6 2.2	6. 2)4.8 2.4
7. 5)5.5 1.1	8. 2)8.8 4.4	9. 3)9.6 3.2
10. 2)6.4 3.2	11. 3)9.3 3.1	12. 2)6.2 3.1
13. 4)8.8 2.2	14. 2)2.6 1.3	15. 2)4.6 2.3
16. 2)2.4 1.2	17. 3)9.9 3.3	18. 2)6.8 3.4

Solve.

19. It took Kelly 6.6 s to run across the room and back. If it took the same time to run each way, how long did it take him to run one way? 3.3 s
20. The temperature went up 3.9 C in 3 h. If it went up the same amount in each hour, how much did it go up in the first hour? 1.3°C

Glass lenses are ground in optical laboratories.

1. The glass is 9.03 mm thick. It must be ground until it is 8.25 mm thick for the lens of a magnifying glass. How much must be ground from the glass? 0.78 mm
2. For a thickness of 8.25 mm, 2.89 mm had to be ground from the glass in all. How thick was the glass before grinding? 11.14 mm
3. The magnifying glass can magnify 3.2 times. How wide will an object that is 1.3 cm wide appear in the magnifying glass? 4.16 cm



4. A magnifying glass costs $8.98. Pam and her 6 friends each want one. Every second Saturday they earn $15 mowing lawns. From the first day they mow, how long will it take them to earn enough for the magnifying glasses? up to the end of 8 weeks

PROBLEM SOLVING

RELATED ACTIVITIES

• Exercises similar to the following can help students relate division of whole numbers and division of decimals.

1. 3)63 3)6.3
2. 2)84 2)8.4

You may wish to have students label columns above each dividend to emphasize place values. For example, Ex. 1 above would be written as shown below.

tens | ones		ones | tenths
3)6 3	and	3)6 . 3

Exercises: Remind the students to show the decimal point in each quotient and to write concluding statements for Ex. 19 and 20.

Problem Solving: These exercises involve decimals and the operations of addition, subtraction, and multiplication. Although Ex. 4 may suggest the use of division, tell the students to solve the problem without using it. This may be done in the following way.

Each of 7 persons wants a magnifying glass costing $8.98.

$$\begin{array}{r} \$8.98 \\ \times \quad 7 \\ \hline \$62.86 \end{array}$$

$62.86 is needed.

Sat.	Sat.	Sat.	Sat.	Sat.	Sat.	Sat.	Sat.	Sat.
$15		$15		$15		$15		$15

On the ninth Saturday (or at the end of eight weeks) they will have enough money.

Assessment

Divide.

1. 7)7.7 1.1 2. 2)6.2 3.1 3. 4)8.4 2.1

Solve.

4. The length of a ribbon is 3.6 m. If Kay cuts it into three pieces of equal length, how long will each piece be? 1.2 m

Pages 240-241

LESSON OUTCOME

Divide a one-place decimal by a one-digit number, regrouping, quotients greater than 1; solve related word problems

Materials

models for ones and tenths

Prerequisite Skills

Divide a three-digit number by a one-digit number, regrouping, remainder zero; divide a one-place decimal by a one-digit number, no regrouping

Checking Prerequisite Skills

Divide.

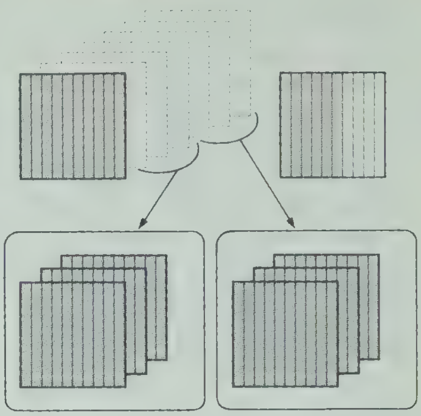
1. $2\overline{)174}$ 87	**2.** $9\overline{)234}$ 26
3. $6\overline{)504}$ 84	**4.** $8\overline{)704}$ 88
5. $2\overline{)8.2}$ 4.1	**6.** $8\overline{)8.8}$ 1.1
7. $3\overline{)6.3}$ 2.1	**8.** $2\overline{)6.6}$ 3.3

Dividing Ones and Tenths with Regrouping

Divide 7.6 by 2.

For $2\overline{)7.6}$, divide the 7 ones first.

2 × 3 = 6
2 × 4 = 8 . . . too great!



Use 2 × 3 = 6. Write

$$\begin{array}{r} 3 \\ 2\overline{)7.6} \\ 6 \\ \hline 1 \end{array}$$

Think of the 1 one 6 tenths that remain as 16 tenths.

$$\begin{array}{r} 3 \\ 2\overline{)7.6} \\ 6 \\ \hline 1\ 6 \end{array}$$

Then divide the 16 tenths.

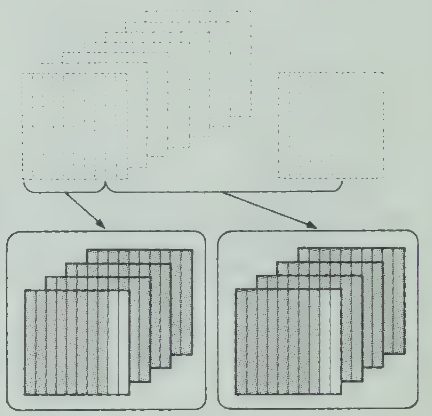
2 × 8 = 16

Use 2 × 8 tenths = 16 tenths.

Place the decimal point in the quotient above the decimal point in the dividend.

Write

$$\begin{array}{r} 3.8 \\ 2\overline{)7.6} \\ 6 \\ \hline 1\ 6 \\ 1\ 6 \\ \hline 0 \end{array}$$



When 7.6 is divided by 2, the quotient is 3.8.

240

LESSON ACTIVITY

Before Using the Pages

- Write sentences similar to the following on the board and have the students complete them. Use models to demonstrate the regrouping as required.

1 one and 2 tenths = _____ tenths
2 ones and 4 tenths = _____ tenths
3 ones and 2 tenths = _____ tenths
5 ones and 6 tenths = _____ tenths

- Display 4 ones and 2 tenths and ask what number is represented. Choose three students to share the models and, if necessary, ask for assistance from other students. Lead them to suggest regrouping 1 one as 10 more tenths. Summarize the steps of sharing ones, regrouping 1 one, and sharing tenths. Ask which step is needed in this case which was not required in the previous lesson.

Using the Pages

- The worked example presents a division for which it is necessary to regroup 1 one as 10 more tenths. The steps are highlighted with blue on the numerals and are related to corresponding steps of sharing and regrouping models. Question the students as you lead them through the example. Emphasize the position of the decimal point in the quotient. Ask what the quotient would be if the decimal point were not shown. Have the students use multiplication to check the division.

Working Together: Ex. 1 and 2 review regrouping ones as tenths. Ex. 3-5 deal with determining the first digit of the quotient by applying a basic multiplication fact. Ex. 6 and 7 help to establish the sequence of steps in division. The students should think through the steps that are shown before they complete the divisions. When they finish Ex. 8-10, ask how Ex. 9 differs from Ex. 8 (there are tens as well as ones in the dividend) and how Ex. 10 differs from Ex. 9 (no regrouping of ones as tenths is required).

Working Together

Complete.

1. 1 one and 2 tenths = 12 tenths
2. 3 ones and 5 tenths = 35 tenths

Give the first multiplication fact you can use to find the quotient.

Example: For 4)9.6, use 4 × 2 = 8.

3. 2)7.2 2 × 3 = 6
4. 5)23.5 5 × 4 = 20
5. 3)12.6 3 × 4 = 12

Complete.

6. 4)9.6 2.4
   8
   1 6
   16
   0

7. 5)23.5 4.7
   20
   35
   35
   0

Divide. Remember to place the decimal point in the quotient.

8. 2)7.2 3.6
9. 4)22.4 5.6
10. 3)12.6 4.2

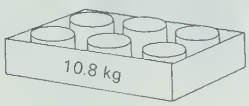
Exercises

Divide.

1. 4)5.2 1.3
2. 3)8.4 2.8
3. 2)9.6 4.8
4. 5)11.5 2.3
5. 6)8.4 1.4
6. 7)16.8 2.4
7. 8)9.6 1.2
8. 9)15.3 1.7
9. 4)16.8 4.2
10. 6)21.6 3.6
11. 2)11.2 5.6
12. 5)6.5 1.3
13. 3)11.1 3.7
14. 7)40.6 5.8
15. 2)8.4 4.2

Solve.

16. How heavy is each can? 1.8 kg



17. How much does each jug hold? 2.8 L

18. What is the average mass of each melon? 5.2 kg

19. What is the average mass of each cabbage? 1.3 kg

241

RELATED ACTIVITIES

• For further practice, you may wish to have the students complete Ex. 1-5 on page 337.

• Assign exercises similar to the following to help students relate division of whole numbers and division of decimals.

1. 2)34 2)3.4
2. 7)112 7)11.2

Again, you may wish to have students label columns above each dividend as suggested in *Related Activities* on page T 261.

Exercises: Remind the students to write concluding statements for Ex. 16-19 and to include the unit of measurement for each.

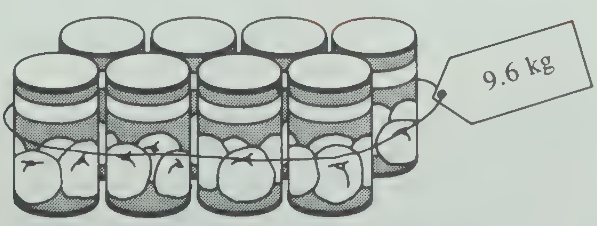
Assessment

Divide.

1. 4)6.4 1.6
2. 8)17.6 2.2
3. 5)32.5 6.5
4. 6)29.4 4.9

Solve.

5. How heavy is each can? 1.2 kg



Pages 242-243

LESSON OUTCOME

Divide a decimal with two or three decimal places by a one-digit number, regrouping, dividends with up to five digits, quotients greater than 1; solve related word problems

Prerequisite Skills

Rename mixed numbers and decimals representing two place values in terms of the lesser value; divide by a one-digit number, dividends with up to five digits, remainder zero; divide a one-place decimal by a one-digit number, regrouping

Checking Prerequisite Skills

Complete.

1. 4 tens 3 ones = 43 ones
2. 2 ones and 6 tenths = 26 tenths
3. 3 tenths 5 hundredths = 35 hundredths
4. 1 hundredth 2 thousandths = 12 thousandths

Divide.

5. $3\overline{)1461}$ 487
6. $7\overline{)21\ 357}$ 3 051
7. $4\overline{)96\ 344}$ 24 086
8. $5\overline{)60\ 850}$ 12 170
9. $9\overline{)21.6}$ 2.4
10. $6\overline{)34.8}$ 5.8

Dividing Hundredths and Thousandths

Holly, Kris, and Ian earned \$8.64 delivering coupons. They shared the money equally. How much did each earn?

Divide 8.64 by 3.

For $3\overline{)8.64}$, divide the 8 ones first.

3 × 2 = 6
3 × 3 = 9 . . . too great!

Use 3 × 2 = 6.

Write
```
   2
3)8.64
  6
  2
```

Think of the 2 ones 6 tenths that remain as 26 tenths.

```
   2
3)8.64
  6↓
  2 6
```

Then divide the 26 tenths.

3 × 8 = 24
3 × 9 = 27 . . . too great!

Use 3 × 8 tenths = 24 tenths.

Write
```
   2.8
3)8.64
  6
  2 6
  2 4
    2
```

Think of the 2 tenths 4 hundredths that remain as 24 hundredths.

```
   2.8
3)8.64
  6
  2 6
  2 4↓
    24
```

Then divide the 24 hundredths.

3 × 8 = 24

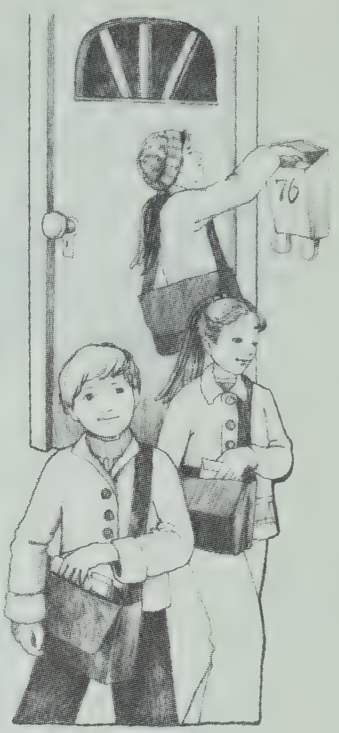
Use 3 × 8 hundredths = 24 hundredths.

Write
```
   2.88
3)8.64
  6
  2 6
  2 4
    24
    24
     0
```

Take another look:
```
   $2.88
3)$8.64
   6
   2 6
   2 4
     24
     24
      0
```

Holly, Kris, and Ian each earned \$2.88.

242



LESSON ACTIVITY

Before Using the Pages

- Write the division $3\overline{)477}$ on the board and have the students complete it. Ask one student to show the work on the board and explain the steps. Write the statements beside the corresponding lines as indicated below, during the explanation of the division.

```
  159
3)477
  3↓
  17
  15↓
   27
   27
    0
```

1. Divide 4 hundreds.
   Use 3 × 1 hundred = 3 hundreds.
2. Divide 17 tens.
   Use 3 × 5 tens = 15 tens.
3. Divide 27 ones.
   Use 3 × 9 ones = 27 ones.

Mark a decimal point between the two 7's of the dividend and ask what number is represented in the dividend as a result of this change. Ask how the quotient will change. The students will likely suggest that a decimal point is required between the 5 and the 9. Indicate that this is acceptable if the steps of the explanation can be altered to justify the quotient for the new dividend. For each underlined word in turn, erase the word and have students write another word to describe the corresponding step of the new division. For example, *hundreds* would be replaced by *tens*. The completed division is shown below.

```
  15.9
3)47.7
  3↓
  17
  15↓
   2 7
   2 7
     0
```

1. Divide 4 tens.
   Use 3 × 1 ten = 3 tens.
2. Divide 17 ones.
   Use 3 × 5 ones = 15 ones.
3. Divide 27 tenths.
   Use 3 × 9 tenths = 27 tenths.

Have the students use multiplication to check the quotient. You may wish to repeat the procedure for $3\overline{)4.77}$.

UNIVERSITY OF ALBERTA

Working Together

Complete.

1. $5\overline{)7.95}$ — quotient 1.59; work: 5, 29, 25, 45, 45, 0

2. $6\overline{)20.10}$ — quotient 3.35; work: 18, 21, 18, 30, 30, 0

3. $4\overline{)5.612}$ — quotient 1.403; work: 4, 16, 16, 012, 12, 0

Divide.

4. $3\overline{)7.62}$ 2.54
5. $5\overline{)9.765}$ 1.953
6. $2\overline{)\$11.56}$ \$ 5.78
7. $4\overline{)4.248}$ 1.062
8. $6\overline{)18.036}$ 3.006

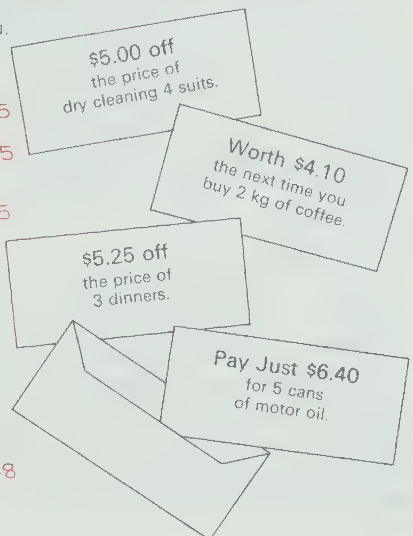
Exercises

Divide.

1. $4\overline{)5.08}$ 1.27
2. $2\overline{)9.38}$ 4.69
3. $3\overline{)11.97}$ 3.99
4. $5\overline{)8.690}$ 1.738
5. $7\overline{)\$7.84}$ \$1.12
6. $8\overline{)8.64}$ 1.08
7. $4\overline{)6.32}$ 1.58
8. $9\overline{)26.01}$ 2.89
9. $3\overline{)5.409}$ 1.803
10. $5\overline{)\$9.35}$ \$1.87
11. $2\overline{)5.70}$ 2.85
12. $7\overline{)8.54}$ 1.22
13. $5\overline{)20.005}$ 4.001
14. $7\overline{)9.996}$ 1.428
15. $8\overline{)\$16.32}$ \$ 2.04
16. $3\overline{)8.85}$ 2.95
17. $3\overline{)7.77}$ 2.59
18. $9\overline{)43.434}$ 4.826
19. $6\overline{)6.012}$ 1.002
20. $5\overline{)\$24.55}$ \$ 4.91

Solve. Use these coupons to help you.

21. How much less does it cost to clean each suit with the coupon? \$1.25
22. How much less does 1 kg of coffee cost with the coupon? \$2.05
23. How much less does each dinner cost with the coupon? \$1.75
24. How much does each can of oil cost with the coupon? \$1.28
25. The students were paid \$8.64 for delivering 6 bundles of coupons. How much did they earn for each bundle? \$1.44
26. The 3 students shared the money equally. How much did *each* student earn for *each* bundle? \$0.48



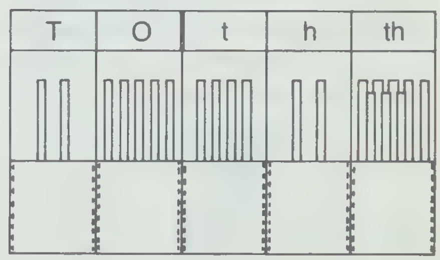
243

RELATED ACTIVITIES

- For further practice, you may wish to have the students complete Ex. 6-20 on page 337.
- Prepare a work sheet with exercises similar to the following.

> Complete the first division. Use your answer to write the quotients for the other divisions.
>
> $3\overline{)42.57}$ $3\overline{)425.7}$
>
> $3\overline{)4.257}$ $3\overline{)42\ 570}$

- Students having difficulty may find it helpful to use a place-value pocket chart to complete division exercises with decimals. A chart similar to the one shown below may be prepared from Bristol board. Cut several cardboard strips for use in representing numbers. The number shown in the chart below is 25.428. To show $3\overline{)25.428}$, the 2 tens' strips are removed and 20 more strips are placed in the ones' pocket. The 25 ones are shared equally in 3 groups. The 1 one remaining is removed and 10 more strips are placed in the tenths' pocket. The sharing and regrouping is continued in a similar manner to complete the division.



Using the Pages

- The worked example demonstrates division of decimal hundredths with two regroupings. Have students help to explain the steps of the division, giving careful attention to the basic multiplication facts that help derive digits of the quotient. Emphasize the position of the decimal point in the dividend and the quotient and review that decimal points are not shown in the work below the dividend. Draw attention to the solution shown with the symbol \$ at the bottom of page 242. This is the form the students will use in exercises such as Ex. 6 of *Working Together*.

Working Together: Have students work at the board while others work at their desks. Pay particular attention to the place value of each digit of the quotient. In Ex. 2, for example, some students may be careless and write $\overset{33.5}{6\overline{)20.10}}$ rather than $\overset{3.35}{6\overline{)20.10}}$. Have them use multiplication to check each division.

Exercises: Some students may need assistance with Ex. 21-26. Ex. 26 is starred for two reasons: some information must be obtained from the previous problem; the first digit of the quotient is zero and it must be shown because it is the only digit to the left of the decimal point. If students have different solutions, have them shown on the board.

Assessment

Divide.

1. $6\overline{)\$7.44}$ \$1.24
2. $3\overline{)75.48}$ 25.16
3. $8\overline{)34.736}$ 4.342
4. $7\overline{)84.063}$ 12.009

Solve.

5. How much does each litre of ice cream cost with the coupon? \$1.09

> Pay just \$3.27 for 3 one-litre tubs of ice cream.

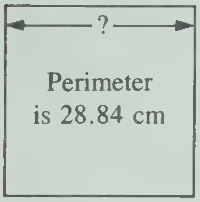
Page 244

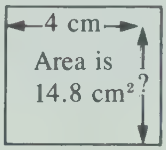
OBJECTIVE

Demonstrate competence in dividing decimals by one-digit numbers

RELATED ACTIVITIES

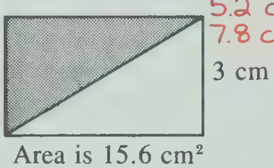
• Exercises similar to Ex. 1-12 may be shown on sheets of paper of the appropriate shape. Students may complete an exercise and exchange papers to obtain a new exercise.



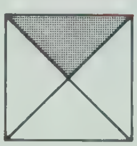


Other exercises similar to the following will provide a challenge for some students.

1. What is the length of the rectangle and the area of the gray triangle? 5.2 cm 7.8 cm²



2. The perimeter of the square is 18.4 cm. What is the area of the gray triangle? 21.16 cm²

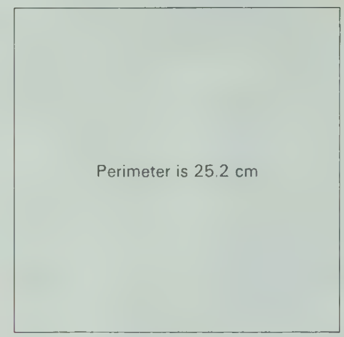


Practice

For each square, divide the perimeter by 4 to find the length of each side.

1.

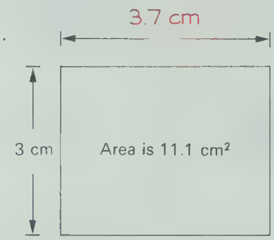
2.



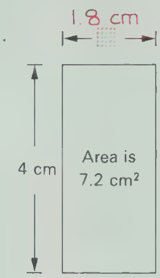
3. The perimeter of a square is 15.88 m. 3.97 m
4. The perimeter of a square is 5.52 km. 1.38 km

For each rectangle, divide the area by the length of one side to find the length of the other side.

5.



6.



	One Side	Area	
7.	2 cm	15.2 cm²	7.6 cm
8.	9 cm	31.5 cm²	3.5 cm
9.	5 m	9.0 m²	1.8 m
10.	8 m	22.08 m²	2.76 m
11.	3 km	7.5 km²	2.5 km
12.	2 km	7.36 km²	3.68 km

Divide. Multiply to check.

13. $4\overline{)8.4}$ 2.1
14. $3\overline{)6.63}$ 2.21
15. $5\overline{)6.5}$ 1.3
16. $4\overline{)4.08}$ 1.02
17. $2\overline{)17.8}$ 8.9
18. $3\overline{)7.272}$ 2.424
19. $7\overline{)12.6}$ 1.8
20. $2\overline{)12.58}$ 6.29
21. $6\overline{)24.174}$ 4.029
22. $5\overline{)5.015}$ 1.003

Divide to find the other number.

	Product of two numbers	One of the numbers	
23.	8.45	5	1.69
24.	20.4	3	6.8
25.	9.576	8	1.197
26.	18.97	7	2.71

244

LESSON ACTIVITY

Before Using the Page

• Briefly review the concepts of perimeter of a square and area of a rectangle. Ask questions similar to the following.
"If the perimeter of a square is 16 cm, what is the length of each side of the square?"
"The area of a rectangle is 12 cm² and the length is 6 cm. What is the width of the rectangle?"
Draw diagrams on the board to help the students.

Using the Page

• Discuss the instructions for each group of exercises to ensure that the students understand what is required.

Quotients Less Than 1

Divide 5.25 by 7.

For 7)5.25, divide the 5 ones.

7 × 0 = 0
7 × 1 = 7 . . . too great!

Use 7 × 0 = 0. Write 7)5.25 with 0 above.

When the divisor → 7)5.25 is greater than the dividend, the quotient is less than 1. The first digit in the quotient is 0.

Think of the 5 ones 2 tenths as 52 tenths. (0 over 7)5.25)

Then divide the 52 tenths. Write 0.7 over 7)5.25; 49; 3

7 × 7 = 49
7 × 8 = 56 . . . too great!

Use 7 × 7 tenths = 49 tenths.

Think of the 3 tenths 5 hundredths that remain as 35 hundredths. (0.7 over 7)5.25; 49; 35)

Then divide the 35 hundredths. Write 0.75 over 7)5.25; 49; 35; 35; 0

7 × 5 = 35

Use 7 × 5 hundredths = 35 hundredths.

When 5.25 is divided by 7, the quotient is 0.75.

Working Together

Give the first digit in each quotient.

1. 3)1.5 — 0
2. 7)3.78 — 0
3. 4)11.2 — 2

Divide.

4. 6)2.4 — 0.4
5. 2)1.616 — 0.808
6. 8)$2.00 — $0.25

Exercises

Use the first quotient to help you write the other quotients.

1. 3)2256 = 752; 3)22.56 = 7.52; 3)2.256 = 0.752
2. 8)1872 = 234; 8)18.72 = 2.34; 8)1.872 = 0.234

Divide.

3. 5)1.25 — 0.25
4. 2)0.76 — 0.38
5. 7)4.9 — 0.7
6. 5)3.0 — 0.6
7. 3)$1.59 — $0.53
8. 8)2.728 — 0.341
9. 6)1.8 — 0.3
10. 4)3.216 — 0.804
11. 8)3.000 — 0.375
12. 9)$4.86 — $0.54

245

LESSON OUTCOME

Divide a decimal with up to three decimal places by a one-digit number, regrouping, quotients less than 1

Materials

models for ones and tenths

Prerequisite Skills

Rename decimals showing ones and tenths as tenths; divide by a one-digit number, remainder zero; divide a decimal by a one-digit number, quotients greater than 1

Checking Prerequisite Skills

Complete.

1. 3 ones 2 tenths = 32 tenths
2. 1.5 = 15 tenths
3. 2.4 = 24 tenths

Divide.

4. 3)126 — 42
5. 6)1596 — 266
6. 8)4904 — 613
7. 7)43.26 — 6.18
8. 5)26.155 — 5.231

RELATED ACTIVITIES

- For further practice, you may wish to have the students complete exercises from Ex. 21-35 on page 337.
- Prepare a work sheet showing several division exercises. Have the students ring the exercises for which the quotients are less than 1.

3)12.9   7)6.44   5)11.215

LESSON ACTIVITY

Before Using the Page

- Have students use models to find the quotient for 3)2.4. Write the quotient above the dividend, emphasizing the need for a zero in the ones' place: 3)2.4 with 0.8 above. Draw attention to the fact that the amount for each group is less than one whole.
- Write the division 3)258 on the board and have the students complete it. Ask one student to complete it on the board. Write the division 3)25.8 on the board and ask what the quotient will be. Write 3)2.58 on the board and repeat the question. Discuss the need for showing 0 as the first digit of the quotient in the last example.

3)258 = 86   3)25.8 = 8.6   3)2.58 = 0.86

Using the Page

- The worked example shows and explains the steps in dividing a decimal when the first digit of the quotient is 0 ones. Have students help to explain the steps. Emphasize that the digits of the quotient must be written in their correct places directly above the digits of the dividend.

Working Together: Have a student explain why a 0 is not written as the first digit of the quotient for Ex. 3. Ask how to tell when a 0 will be needed. (The whole number part of the dividend will be less than the divisor.)

Exercises: For Ex. 1, it is necessary to write only the quotients. However, it is advisable to have the students use multiplication to check one of the two decimal divisions in each of Ex. 1 and 2.

Assessment

Divide.

1. 2)1.8 — 0.9
2. 4)2.12 — 0.53
3. 8)6.408 — 0.801
4. 3)$2.97 — $0.99

Pages 246-247

LESSON OUTCOME

Divide a whole number or a decimal by a one-digit number, using zeros in the dividend, quotients terminating by the third decimal place

Materials

models for ones, tenths, and hundredths

Vocabulary

batting average

Prerequisite Skills

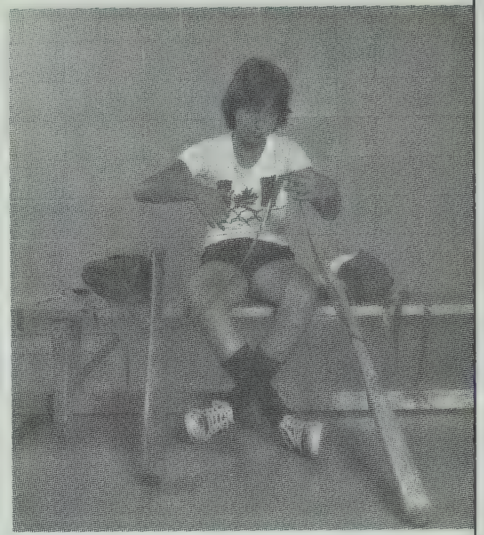
Divide a decimal by a whole number

Checking Prerequisite Skills

Divide.

1. $3\overline{)4.41}$ 1.47
2. $9\overline{)6.39}$ 0.71
3. $6\overline{)1.350}$ 0.225
4. $4\overline{)3.00}$ 0.75

Using More Decimal Places



Lisa wants to cut 2 m of red tape into 5 pieces that are the same length and 3 m of blue tape into 4 pieces that are the same length. How long should each piece be?

To cut 2 m of red tape into 5 pieces, divide 2 by 5.

For $5\overline{)2}$, divide the 2 ones.

$5 \times 0 = 0$ Write $\begin{array}{r} 0 \\ 5\overline{)2} \end{array}$

Think of the 2 ones as 2 ones 0 tenths, or 20 tenths. $\begin{array}{r} 0 \\ 5\overline{)2.0} \end{array}$

Then divide the 20 tenths.

$5 \times 4 = 20$

Use 5 × 4 tenths = 20 tenths. Write $\begin{array}{r} 0.4 \\ 5\overline{)2.0} \\ \underline{2\,0} \\ 0 \end{array}$

Each piece of red tape would be 0.4 m long.

To cut 3 m of blue tape into 4 pieces, divide 3 by 4.

For $4\overline{)3}$, first use 4 × 0 = 0. → Write $\begin{array}{r} 0 \\ 4\overline{)3} \end{array}$ → Next, think of the 3 ones as 30 tenths. $\begin{array}{r} 0 \\ 4\overline{)3.0} \end{array}$

Divide. Use 4 × 7 tenths = 28 tenths. → Think of the 2 tenths as 20 hundredths. → Divide. Use 4 × 5 hundredths = 20 hundredths.

Write $\begin{array}{r} 0.7 \\ 4\overline{)3.0} \\ \underline{2\,8} \\ 2 \end{array}$ $\begin{array}{r} 0.7 \\ 4\overline{)3.00} \\ \underline{2\,8\downarrow} \\ 20 \end{array}$ Write $\begin{array}{r} 0.75 \\ 4\overline{)3.00} \\ \underline{2\,8} \\ 20 \\ \underline{20} \\ 0 \end{array}$

Each piece of blue tape would be 0.75 m long.

246

LESSON ACTIVITY

Before Using the Pages

- Review that 1 whole, 10 tenths, and 100 hundredths all represent the same number, and that 1, 1.0, and 1.00 are different names for that number. Use other examples such as 4.2 and 4.20, and 4.11 and 4.110.
- Write the division $2\overline{)1}$ on the board and have students interpret it in different ways. For example, they may suggest "one divided by two" or "one shared equally between two". Because the division involves two whole numbers for which the divisor is greater than the dividend, they may even suggest that such a division cannot be performed. Display a model of 1 whole marked into 10 tenths and ask a student to cut it so that it may be shared equally between two students. Summarize that thinking of 1 whole as 10 tenths helps to complete the division. Write the following on the board.

$$\begin{array}{r} ? \\ 2\overline{)1} \end{array} \longrightarrow \begin{array}{r} \text{5 tenths} \\ 2\overline{)\text{10 tenths}} \end{array}$$

Follow a similar procedure using a model of 1 whole marked into 100 hundredths to demonstrate $4\overline{)1}$.

$$\begin{array}{r} ? \\ 4\overline{)1} \end{array} \longrightarrow \begin{array}{r} \text{25 hundredths} \\ 4\overline{)\text{100 hundredths}} \end{array}$$

Using the Pages

- The examples demonstrate the steps in division when zeros are required in decimal places of the dividend. Because the process of regrouping to divide is not new, emphasis can be placed on the need to show zeros in decimal places of the dividend. For example, to divide $5\overline{)2}$ we must think of 2 as 20 tenths and a 0 is needed in the tenths' place to show 2.0. Review that 2 and 2.0 are names for the same number, as are 3, 3.0, and 3.00. Summarize that the extra zeros do not change the value of a number but help to complete the division.

Here are other examples that show how zeros in decimal places can help you complete a division.

Divide 1 by 2:	Divide 11 by 4:	Divide 2.6 by 8:
0.5 2)1.0 1 0 0	2.75 4)11.00 8 3 0 2 8 20 20 0	0.325 8)2.600 2 4 20 16 40 40 0

Working Together

Complete.

1. 5)12.0 = 2.4
   10
   2 0
   20
   0
2. 2)1.3 = 0.65
   12
   10
   10
   0

Complete this chart.

3.	5	5.0	5.00	5.000
4.	32	32.0	32.00	32.000
5.		2.6	2.60	2.600
6.			10.73	10.730

Divide.

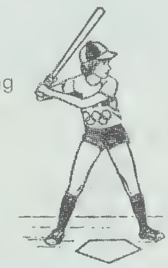
7. 5)7.32 1.464 8. 8)3 0.375 9. 4)$2 $0.50

Exercises

Divide. Use more zeros when needed.

1. 5)3 0.6 2. 4)8 2 3. 6)3 0.5
4. 8)4 0.5 5. 5)1 0.2 6. 4)1 0.25
7. 5)10 2 8. 8)7 0.875 9. 2)3 1.5
10. 5)4 0.8 11. 6)9 1.5 12. 8)2 0.25
13. 4)$7 $1.75 14. 5)$7 $1.40 15. 8)$12 $1.50
16. 3)5.7 1.9 17. 2)5.7 2.85 18. 5)12.3 2.46
19. 4)9.4 2.35 20. 2)0.75 0.375 21. 5)6.25 1.25
22. 4)6.5 1.625 23. 8)10.6 1.325 24. 4)3.1 0.775

A ball player's "batting average" is found by dividing the number of hits by the number of times at bat. The quotient always shows 3 decimal places.



What was Lisa's batting average?

1. In one game, Lisa had 1 hit in 4 times at bat. 0.250
2. After two games, she had 5 hits in 8 times at bat. 0.625
3. For the season, Lisa had 15 hits in 40 times at bat. 0.375

PROBLEM SOLVING

247

In the examples at the top of page 247, blue is used to indicate the digits that were not in the original dividend but were needed to complete each division. Choose one of the exercises and develop it on the board with the students. Discuss that no more zeros are required in the dividend when the subtraction step gives a difference of zero, and thus the division is completed.

Working Together: Ex. 1 and 2 help develop the steps that will be applied in Ex. 7-9. Ex. 3-6 emphasize writing different names for a number through the use of zeros in decimal places. For Ex. 7-9, discuss that one zero is required for Ex. 7, two zeros for Ex. 9, and three for Ex. 8.

Exercises: Remind the students to show decimal points in the quotients and the dividends, particularly for Ex. 1-15. Emphasize the words "when needed" in the instructions.

RELATED ACTIVITIES

- For further practice, you may wish to have the students complete selected exercises from Ex. 36-68 on page 337. Note that the dividends for Ex. 51-68 are whole numbers, whereas the dividends for Ex. 36-50 are decimals.
- If the students are involved in playing softball during physical education classes, they may be interested in finding their batting average for the total number of games played. (It is possible that a division may still have a remainder after dividing thousandths. If this happens, have the students give the quotient to three decimal places without rounding to the nearest thousandth.)
- Prepare a work sheet with exercises similar to the following. Have students complete the first division in each row and use that quotient to write the quotients for the other divisions in the row.

1. 4)500 4)50 4)5
2. 2)150 2)15 2)1.5

- Students may discover and use patterns in completing divisions similar to the following.

1. 4)3 8)6 12)9 16)12
2. 5)2 10)4 15)6 20)8

Problem Solving: Draw attention to the fact that the quotient for a batting average always shows three decimal places. Ensure that the students know which number is the dividend and which is the quotient.

Assessment

Divide. Use more zeros when needed.

1. 5)7 1.4 2. 8)6 0.75 3. 6)8.46 1.41
4. 4)1.1 0.275 5. 5)1.63 0.326 6. 2)5.7 2.85

Pages 248-249

OBJECTIVE

Demonstrate competence in dividing decimals

Practice

Dividing decimals is just like dividing whole numbers.
Study these examples.

Divide 3456 by 8.	Divide 345.6 by 8.	Divide 34.56 by 8.
4 8)34[redacted] 32 2	4 8)34[redacted] 32 2	4 8)34[redacted] 32 2
4 8)345[redacted] 32↓ 25	4 8)345[redacted] 32↓ 25	4 8)34.5[redacted] 32↓ 2 5
43[redacted] 8)345[redacted] 32 25 24 1	43[redacted] 8)345[redacted] 32 25 24 1	4.3[redacted] 8)34.5[redacted] 32 2 5 2 4 1
43 8)3456 32 25 24↓ 16	43 8)345.6 32 25 24↓ 1 6	4.3 8)34.56 32 2 5 2 4↓ 16
432 8)3456 32 25 24 16 16 0	43.2 8)345.6 32 25 24 1 6 1 6 0	4.32 8)34.56 32 2 5 2 4 16 16 0
$3456 \div 8 = 432$	$345.6 \div 8 = 43.2$	$34.56 \div 8 = 4.32$

What do you think the result will be for $3.456 \div 8$?

248

LESSON ACTIVITY

Using the Pages

- The three divisions completed on page 248 differ only in that two of the dividends show a decimal point. These examples demonstrate that the steps in dividing decimals are the same as those for dividing whole numbers. You might assign the division with whole numbers (8)3456) for the students to complete independently and have them check their work with the exercise on page 248. Then have them study the work shown for 8)345.6 and 8)34.56, discussing the similarities and differences. Finally, have them answer the question at the bottom of the page.
- Note that no division is required for Ex. 1 and only the first division must be completed for Ex. 2. The other quotients are written on the basis of the first quotient. For Ex. 18-23, you may need to remind the students how to find the average. Discuss that Ex. 18 concerns the average time taken to run a certain distance, Ex. 22 concerns the average amount of money for the monthly phone bill, and so on.

Use the first quotient to help you write the other quotients.

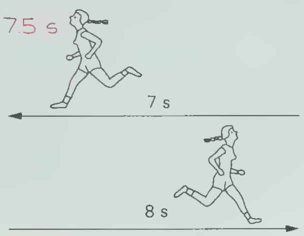
1.	154 3)462	15.4 3)46.2	1.54 3)4.62	0.154 3)0.462	$1.54 3)$4.62
2.	375 8)3000	37.5 8)300	3.75 8)30	0.375 8)3	$ 3.75 8)$30.00

Divide.

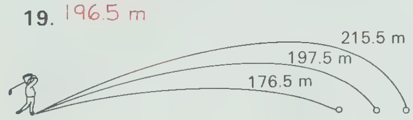
3. 0.8 6)4.8	4. 6.32 2)12.64	5. 1.55 4)6.2	6. 4.6 5)23	7. 3.92 8)31.36
8. 0.75 8)6	9. 0.376 7)2.632	10. 2.372 3)7.116	11. 1.08 6)6.48	12. 0.9 9)8.1
13. 3.25 4)13	14. 0.25 3)0.75	15. 8.572 7)60.004	16. 3.08 2)6.16	17. 3.64 5)18.2

Add, then divide to find the average.

18. 7.5 s

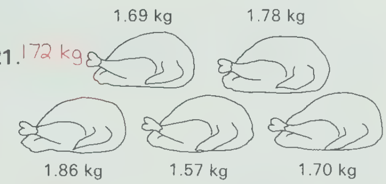


19. 196.5 m



20. 4.75 L

21. 1.72 kg



22. $19.11

PHONE BILL $9.45 | PHONE BILL $18.76 | PHONE BILL $24.20

PHONE BILL $19.75 | PHONE BILL $15.89 | PHONE BILL $26.61

23. 37.0°C

My Body Temperature

Sun.	37.2°C	Mon.	36.5°C
Tues.	36.7°C	Wed.	37.0°C
Thurs.	37.8°C	Fri.	37.3°C
Sat.	36.5°C		

RELATED ACTIVITIES

- For further practice, you may wish to have the students complete selected exercises from Ex. 36-68 on page 337. Note that the dividends for Ex. 51-68 are whole numbers, whereas the dividends for Ex. 36-50 are decimals.
- For practice in multiplying when one of the numbers is a decimal, have students use multiplication to check their quotients for Ex. 3-17.
- Draw attention to advertisements showing items priced at 2 for $1.95, for example, and have students suggest how to find the cost of one item. This will involve writing 0 in the dividend (2)$1.950) and rounding the quotient to the nearest cent.

```
   $0.975
 2)$1.950
   1 8
     15
     14
      10
      10
       0
```

One item would cost $0.98.

Have students find similar advertisements in newspapers and magazines and bring them to school. Use only those examples for which the quotient terminates at the third decimal place.

Pages 250-251

LESSON OUTCOME

Divide a decimal or a whole number by a two-digit number, using zeros in the dividend, quotients terminating by the third decimal place

Prerequisite Skills

Round a two-digit number to the nearest ten; divide a decimal by a one-digit number

Checking Prerequisite Skills

Round to the nearest ten.

1. 27 30 **2.** 32 30

3. 45 50 **4.** 16 20

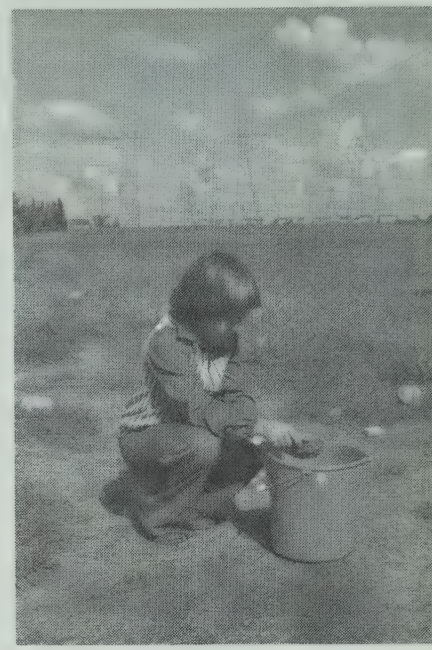
Divide.

5. 2)84.29 42.145 **6.** 7)7.497 1.071

7. 3)6.864 2.288 **8.** 5)3.41 0.682

Dividing by a Two-Digit Number

Anne gathered 21 rocks for her collection. The mass of all the rocks was 35.7 kg. What was the average mass of each rock?



Divide 35.7 by 21.

21 rounded to the nearest ten is 20.

For 21)35.7, think of 20)35.7.

For 20)35.7, think of the 3 tens 5 ones as 35 ones. Then divide the 35 ones.

20 × 1 = 20
20 × 2 = 40 . . . too great!

Use 21 × 1 = 21.

```
         1
Write 21)35.7
         21
         14
```

Think of the 14 ones 7 tenths that remain as 147 tenths.

```
         1
Write 21)35.7
         21↓
         14 7
```

Then divide the 147 tenths.

20 × 7 = 140
20 × 8 = 160 . . . too great!

Use 21 × 7 tenths = 147 tenths.

```
         1 7
Write 21)35.7
         21
         14 7
         14 7
            0
```

Place the decimal point in the quotient above the decimal point in the dividend.

The average mass of each rock was 1.7 kg.

250

LESSON ACTIVITY

Before Using the Pages

- Write two or three division exercises such as 29)10 672, 45)46 620, and 24)84 288 on the board. Review the procedure of rounding the divisor to obtain trial digits for the quotient. Review that sometimes a trial digit may be too small or too great. Have the students complete the divisions. Then write a division exercise such as 29)10.672 and have the students tell what the quotient will be by using one of the quotients found earlier. Use several other examples as shown below.

```
   368        36.8        3.68        0.368
29)10 672  29)1067.2  29)106.72  29)10.672
```

Have students explain how to tell whether the quotient for 24)84.288 and for 45)4.662 will be less than 1 or greater than 1.

Using the Pages

- Have a student read the word problem to introduce the situation. Lead the students through the steps of the solution and draw attention to the statement in the ''thought cloud'' at the bottom of the page. Emphasize that the division process is the same for decimals as for whole numbers. Students who have rock collections may be able to suggest whether the average mass of 1.7 kg for rocks in a collection is heavy or light compared with the rocks in their collection.

Working Together: Draw attention to the statement in the ''thought cloud'' at the top of page 251. It will assist students in completing Ex. 1 and 2. The sub-skills dealt with are: determining the first digit of the quotient (Ex. 1 and 2); and relating division of decimals to division of whole numbers (Ex. 3 and 4). The ability to apply these skills is examined in Ex. 5-8.

Working Together

Give the first digit in each quotient.

1. 43)110.08	2	2. 12)6.48	0

When the divisor → is greater than the dividend, the quotient is less than 1.

Use this division to help you with this one.

14)182 = 13 (14, 42, 42, 0)	→	3. 14)1.82	0.13
15)3120 = 208	→	4. 15)31.2	2.08

Divide.

5. 11)23.87	2.17	6. 38)$78.28	$ 2.06
7. 16)32.4	2.025	8. 25)$85	$ 3.40

Exercises

Divide.

1. 12)28.08	2.34	2. 16)6.24	0.39
3. 21)21.84	1.04	4. 25)$48.50	$ 1.94
5. 22)31.9	1.45	6. 64)56	0.875
7. 72)90	1.25	8. 23)$7.36	$0.32
9. 24)17.4	0.725	10. 17)18.632	1.096
11. 74)103.6	1.4	12. 14)$49	$ 3.50
13. 86)64.5	0.75	14. 43)18.404	0.428
15. 32)12	0.375	16. 57)$144.78	$ 2.54
17. 14)4.270	0.305	18. 28)103.6	3.7

try this

Study these patterns.

7 × 10 = 70	2 × 100 = 200
83 × 10 = 830	32 × 100 = 3 200
502 × 10 = 5020	460 × 100 = 46 000

1. Give a rule for multiplying by 10 and a rule for multiplying by 100. Rules are given below

Study these patterns.

30 ÷ 10 = 3	700 ÷ 100 = 7
400 ÷ 10 = 40	2 800 ÷ 100 = 28
2460 ÷ 10 = 246	90 000 ÷ 100 = 900

2. Give a rule for dividing by 10 and a rule for dividing by 100. Rules are given below

Study these patterns.

7.2 × 10 = 72	3.9 × 100 = 390
3.64 × 10 = 36.4	8.51 × 100 = 851
0.42 × 10 = 4.2	0.235 × 100 = 23.5

3. Do your rules for Exercise 1 still work? yes

Study these patterns.

6 ÷ 10 = 0.6	3 ÷ 100 = 0.03
0.735 ÷ 10 = 0.0735	5.8 ÷ 100 = 0.058
14.2 ÷ 10 = 1.42	279.3 ÷ 100 = 2.793

4. Do your rules for Exercise 2 still work? yes

RELATED ACTIVITIES

- If students are not careful in writing digits of the quotient in their correct places, errors similar to the following can result.

Incorrect	Correct
5)29. = 58	5)29.0 = 5.8

Division exercises completed on lined paper turned sideways can be helpful in overcoming this difficulty (see *Related Activities* on page T215).

- Students can prepare a numeral card for a five-digit number such as 24 036. A clear acetate sleeve showing a decimal point can be made to slide along the card. The sleeve can be moved to show the product or the quotient when multiplying or dividing by 10 or 100. The device can be used to encourage mental computation.

2 4 | 0.3 | 6

240.36 ÷ 10 = 24.036

2 | 4.0 | 3 6

- Students with rock collections may be interested in bringing to school several small rocks from their collections. The total mass of the rocks may be measured and the average mass of each rock may be calculated.

Exercises: Remind the students that in some exercises they will need to use zeros in the dividends, and in others they will need to write a decimal point as well as zeros. Tell the students to use multiplication to check the divisions with which they had difficulty. Exercises such as Ex. 6 may present difficulty because the dividend is less than the divisor, and it will be necessary to use zeros in decimal places of the dividend.

Try This: These exercises review the product of a whole number and 10 or 100, and the quotient when a whole number is divided by 10 or 100. The concepts are then extended to a decimal either as one factor or a dividend. Students can discover that the same rules apply with a decimal factor (dividend) as with a whole number: the digits remain in the same sequence but their place values change (see page 158).

Assessment

Divide.

1. 13)55.042	4.234	2. 19)4.199	0.221
3. 25)85	3.4	4. 36)12.672	0.352

Try This

1. When multiplying by 10, the digits of the other factor move one place to the left and a zero is written on the right.
   When multiplying by 100, the digits of the other factor move two places to the left and two zeros are written on the right.
2. When dividing by 10, the digits of the dividend move one place to the right.
   When dividing by 100, the digits of the dividend move two places to the right.

Pages 252-253

OBJECTIVE

Demonstrate competence in dividing decimals

Practice

Divide.

1. $8\overline{)23.2}$ 2.9
2. $19\overline{)57.95}$ 3.05
3. $5\overline{)4.1}$ 0.82
4. $24\overline{)156}$ 6.5
5. $7\overline{)\$22.47}$ \$3.21
6. $9\overline{)6.993}$ 0.777
7. $8\overline{)29}$ 3.625
8. $35\overline{)57.4}$ 1.64
9. $2\overline{)0.73}$ 0.365
10. $33\overline{)\$22.11}$ \$0.67
11. $32\overline{)4}$ 0.125
12. $58\overline{)178.06}$ 3.07
13. $3\overline{)19.26}$ 6.42
14. $16\overline{)9.2}$ 0.575
15. $8\overline{)\$6}$ \$0.75
16. $4\overline{)8.024}$ 2.006
17. $43\overline{)30.1}$ 0.7
18. $29\overline{)9.57}$ 0.33
19. $6\overline{)3}$ 0.5
20. $52\overline{)\$169}$ \$3.25

Use >, <, or = to make true statements.

Example: $1.23 \div 3$ ● $12.3 \div 3$

$3\overline{)1.23}$ 0.41 $3\overline{)12.3}$ 4.1 Write $1.23 \div 3 < 12.3 \div 3$

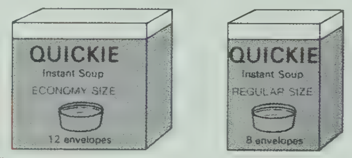
21. $18.25 \div 5$ ● $11.25 \div 3$ <
22. $26.25 \div 7$ ● $33.75 \div 9$ =
23. $21 \div 8$ ● $15 \div 6$ >
24. $2.54 \div 4$ ● $3.75 \div 6$ >
25. $20.35 \div 5$ ● $18.8 \div 4$ <
26. $75.15 \div 9$ ● $58.45 \div 7$ =

These are special division pairs.
Divide. Then multiply the quotients.

27. $2\overline{)5}$ 2.5; $5\overline{)2}$ 0.4 $2.5 \times 0.4 = 1$
28. $7\overline{)28}$ 4; $28\overline{)7}$ 0.25 $4 \times 0.25 = 1$
29. $1\overline{)8}$ 8; $8\overline{)1}$ 0.125 $8 \times 0.125 = 1$

What is the cost of each envelope of soup

1. in the Economy-size package? \$0.18
2. in the Regular-size package? \$0.19
3. in the Economy-size package with this coupon? \$0.14



\$2.16 \$1.52

PROBLEM SOLVING

252

LESSON ACTIVITY

Using the Pages

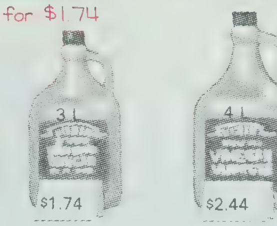
- Discuss the example that precedes Ex. 21 to ensure that the students understand what is required. Review the procedure that would be used to complete Ex. 30-36. Ex. 36 is starred because it is necessary to interpret quotients such as \$0.145. For example, if the cost of 8 pencils is \$1.16, the price of 1 pencil is found from $8\overline{)\$1.16}$, which gives \$0.145 or 14.5¢.

Problem Solving: These exercises encourage students to be aware of the cost for one unit when purchasing items offered in packages of different sizes. The exercises are similar to those on page 253.

Find the better offer in each pair.

30. 5 kg for $9.45

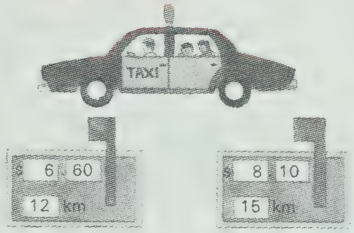
31. 3 L for $1.74



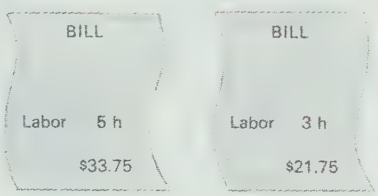
32. 12 for $1.44

33. 3 m for $7.41

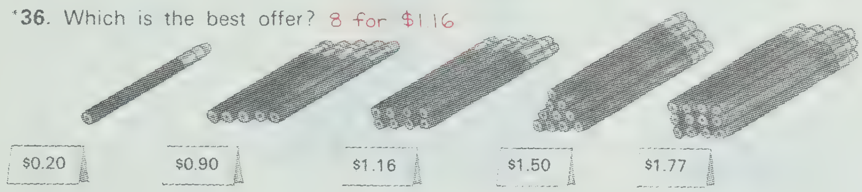
34. 15 km for $8.10



35. 5 h for $33.75



***36.** Which is the best offer? 8 for $1.16



253

RELATED ACTIVITIES

- For further practice, you may wish to have the students complete selected exercises from Ex. 69-90 on page 337. Note that Ex. 81-90 require extra zeros in the dividends, whereas Ex. 69-80 do not.
- Have the students find the cost of each envelope of soup in the regular-size package, using the coupon indicated in the *Problem Solving* feature.
- If the containers collected for measuring mass and capacity in Unit 11 are still available, these may be used now. Have the students use the prices and amounts shown to find the price for one unit of each item. For example, for a container marked $4.60 for 5 L, the division $5\overline{)\$4.60}$ would give the price for one litre.
- Have students complete exercises similar to those on page 253 for items advertised in newspapers.
- Students may play the game ''Shopping Spree'' described on page T 377 and include statements similar to the following for some of the spaces.

  ''Purchase one record album if the price of 3 is $11.97.''
- Assign other exercises similar to Ex. 27-29 for which the product of the quotients is 1. The number of decimal places in a quotient should not exceed three.

Page 254

OBJECTIVE

Recognize incorrect results for operations performed with a calculator

Materials

calculators (optional)

RELATED ACTIVITIES

- Have students write exercises similar to those on page 254 for other students to complete.
- Have students use calculators to complete exercises similar to the following. Use their answers (correct and incorrect) to write exercises similar to those on the page.

1. 42.7 $\boxed{\times}$ 2.9 $\boxed{=}$ ____
2. 33.18 $\boxed{+}$ 67.05 $\boxed{-}$ 80.12 $\boxed{=}$ ____
3. 17.9 $\boxed{\times}$ 8.8 $\boxed{\div}$ 3 $\boxed{=}$ ____

- Have students use a calculator to investigate the relationship between multiplying a number by 0.1 and dividing the same number by 10. Use exercises similar to the following.

1. 42.8 ÷ 10 = _____
   42.8 × 0.1 = _____
2. 3.608 ÷ 10 = _____
   3.608 × 0.1 = _____

Extend the concept to include multiplying by 0.01 and dividing by 100.

3. 8100 ÷ 100 = _____
   8100 × 0.01 = _____
4. 28 500 ÷ 100 = _____
   28 500 × 0.01 = _____

Recognizing Incorrect Results

When she multiplied 274.6 and 1.2, Natalie expected a product close to 274.6.

274.6 × 1.2 should be close to 274.6 × 1, or 274.6.

But her calculator display showed

3295.2

I must have made a mistake!

Did I forget to press the decimal point key for one of the factors?

Can you find a mistake that Natalie might have made to get a product of 3295.2?

____

One display is correct. For the other displays, either an incorrect operation key was pressed or a decimal was entered incorrectly. Find the correct display. Find a mistake that would give the other displays.

1. 482.9 $\boxed{\times}$ 67 $\boxed{=}$ 32354.3

3235413.	32354.3	549.9
4829 $\boxed{\times}$ 67		482.9 $\boxed{+}$ 67

2. 76.49 $\boxed{+}$ 25.98 $\boxed{-}$ 57.75 $\boxed{=}$ 44.72

44.72	160.22	2616.74
	76.49 $\boxed{+}$ 25.98 $\boxed{+}$ 57.75	76.49 $\boxed{+}$ 2598 $\boxed{-}$ 57.75

3. 14.5 $\boxed{\times}$ 9.6 $\boxed{\div}$ 32 $\boxed{=}$ 4.35

107.2	43.5	4.35
14.5 $\boxed{\times}$ 9.6 $\boxed{-}$ 32	145 $\boxed{\times}$ 9.6 $\boxed{\div}$ 32 or 14.5 $\boxed{\times}$ 96 $\boxed{\div}$ 32 or 14.5 $\boxed{\times}$ 9.6 $\boxed{\div}$ 32	

Rounding and estimating can help you find the correct display and the mistakes that gave the other displays.

Calculator

254

LESSON ACTIVITY

Before Using the Page

- Write a few exercises similar to the following on the board. Have students estimate the answer for each without any written work. Encourage quick responses. Have them explain how they arrived at their estimates.

1. 19.8 + 14.5 − 5.2
2. 28.2 × 2.3
3. 102.6 ÷ 21

Using the Page

- The example provided can lead to discussion about the importance of estimating answers in conjunction with using a calculator. Ask what numbers were to be multiplied, how an estimate of the product was obtained, and how it helped to reveal an error in the display. Have students suggest what Natalie's mistake may have been and write an exercise to demonstrate their suggestion. It is probable that the decimal point key was forgotten for one of the two factors.
- Of the three displays shown for each exercise only one is correct, and the correct one can be determined by estimating. For example, for Ex. 3, an estimate of 5 can be found from 15 × 10 ÷ 30 = 150 ÷ 30. The correct display must be 4.35. To determine the source of error for other displays, the computation may have to be performed with the given numbers. For Ex. 3, the display 43.5 is simply an incorrect entry of a decimal. The display 107.2 is obtained if the $\boxed{-}$ key is pressed instead of the $\boxed{\div}$ key.

Logical Thinking

Use the information given and list the students in order from tallest to shortest. Ryan, Laura, Leslie, Greg, Anne

Laura Greg Anne Ryan Leslie

Laura is taller than Anne.	Greg is not the shortest.	Laura is as heavy as Leslie.	Ryan is not as short as Laura.	Leslie is taller than Greg but not as tall as Laura.

When you think you have them in order, turn the page to check your list.

After the pointer stops, the only direction it can move is down the scale. It will not move up the scale until another coin is used.

1. Mr. Koll has just one coin. He wants to find how heavy he and each of his 3 bags is. Tell how he could do this.

Cheryl has a card that is 5 cm long and 3 cm wide.

2. Show how she can use the card to draw a line segment 2 cm long.

PROBLEM SOLVING

Answers are given at the right for Ex 1 and 2

255

Pages 255-256

OBJECTIVE

Solve a problem through a process of logical thinking

1. He can start with the three bags and himself. He can take each bag off one by one. The difference in each case will be the mass.
2. Draw a line segment using the 5 cm side. From one end of the line segment, measure 3 cm using the 3 cm side. The remaining part of the line segment is 2 cm long.

LESSON ACTIVITY

Before Using the Pages

- You might begin with a brief discussion of various games that are available commercially. Have students name those that are familiar to them and tell what they like and dislike about the games. Discuss the difference between games that are based on chance, such as "Snakes and Ladders", and games that are based on logical thinking, such as chess or checkers. Some games such as backgammon and card games combine elements of chance and logic through a choice of moves or plays that may be made. Tell the students that logical thinking is involved in solving problems.

Using the Pages

- Have students read the problem at the top of page 255, but ensure that they do not turn the page until you tell them to do so. Give them a few moments to study and explore the problem. Have them present their ideas on the board for the others to consider. Listing the names of the children beginning with the information in one statement and then including the information in the other statements, one at a time, may help in solving the problem. For example, "Laura is taller than Anne" can be shown as follows.

  Laura
  Anne

  "Ryan is not as short as Laura" can be considered next.

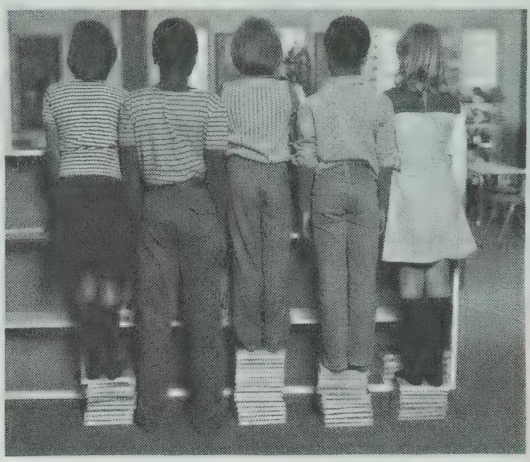
  Ryan
  Laura
  Anne

  When the students have all the names listed, have them turn the page to check their answers. Draw attention to the statement in the "thought cloud" beside the photograph on page 256. Return to the clues below the photograph on page 255 and discuss the ways in which they were or were not useful in solving the problem.

RELATED ACTIVITIES

- Students can be encouraged to play games that rely on logical thinking and discuss the moves made during the game. Chess, checkers, Chinese checkers, regular tick-tack-toe and three-dimensional Tic Tac Toe are suitable.

Does the list you made for the problem on page 255 agree with what you see in this picture?



When you solve problems, always read carefully and be sure you understand the given information.

For Ex. 3 - 10, the chart below shows how many times to trace each line segment. For example, for Ex. 6, trace 7 cm three times to make a line segment 21 cm long. From it cut off 4 cm four times. The remaining part of the line segment will be 5 cm long.

One of these line segments is 7 cm long. The other is 4 cm long. Use pencil, eraser, tracing paper, and an unmarked straight edge to draw a line segment that is

You can do these in any order.

3. 1 cm long.
4. 2 cm long.
5. 3 cm long.
6. 5 cm long.
7. 6 cm long.
8. 8 cm long.
9. 9 cm long.
10. 10 cm long.
11. What other lengths could you draw? There are numerous possibilities.

Read these carefully and be sure you understand the given information.

12. If your doctor gives you 3 pills and says to take one every half hour, how long will they last? 1 h

13. When you go to bed one night at 20:00 and set the alarm to get up at 09:00, how many hours of sleep will you get?

14. A rancher had 17 steers. All but 9 were sold. How many steers were left? 9

PROBLEM SOLVING

13. 1 h with a dial clock
13 h with a digital clock

256

- Assign the exercises and have the students work independently. When they have finished they may share their ideas.

  The answer to Ex. 13 may be one hour for a conventional dial clock. Digital clocks, however, usually have alarms that may be set for a.m. or p.m., and it is possible for the answer to be 13 hours.

Ex.	Line segment	Number of times 7 cm	Number of times 4 cm
3	1 cm	1	2
4	2 cm	2	3
5	3 cm	1	1
6	5 cm	3	4
7	6 cm	2	5
8	8 cm	4	5
9	9 cm	3	3
10	10 cm	2	1

Checking Up

Divide.

1. 2)7.6 3.8
2. 8)51.2 6.4
3. 3)$17.16 $ 5.72
4. 7)3.346 0.478
5. 5)3.5 0.7
6. 9)40.545 4.505
7. 7)$6.02 $0.86
8. 6)15 2.5
9. 4)2.5 0.625
10. 5)7.1 1.42
11. 4)$26 $ 6.50
12. 8)7 0.875

Solve.

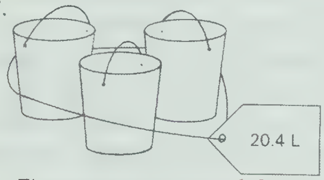
13. Georgette ran 2 laps in 35 s. If each lap took the same time, how long did it take to run 1 lap? 17.5 s

14. Bob bought rolls by the baker's dozen for $2.99. There are 13 in a baker's dozen. How much did Bob pay for 1 roll? $0.23

15. The perimeter of this square is 5.5 km. How long is one side? 1.375 km

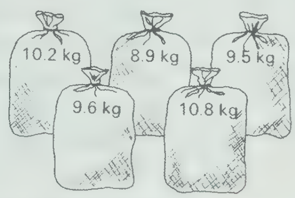
16. 6 m — The area of this rectangle is 57 m². How long is this side? 9.5 m

17.



The average is L. 6.8

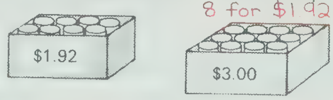
18.



The average is kg. 9.8

19. The product of two numbers is 64.26. One number is 14. What is the other number? 4.59

20. If all the cans are the same size, which carton is the better buy? 8 for $1.92



Divide.

21. 24)73.68 3.07
22. 15)18.6 1.24
23. 32)$217.28 $ 6.79
24. 19)13.87 0.73

257

Page 257

OBJECTIVE

Demonstrate an understanding of the concepts and skills presented in this unit

RELATED ACTIVITIES

- Copy students' exercises that show various errors in the division process. Prepare a work sheet showing the exercises without revealing their source. Have the students find and correct the errors.
- Students may be interested in listing the kinds of errors discovered in the preceding activity. They may prepare a tally sheet to show the frequency of each kind of error and graph the results.

1. multiplication 𝍸 ||
2. quotient digits
3. subtraction
4. regrouping

Skills	Exercises	Related Pages
Divide a decimal by a one-digit number, quotients greater than 1	1-3, 6	T 260-T 265
Divide a decimal by a one-digit number, quotients less than 1	4, 5, 7	T 267
Divide a whole number or a decimal by a one-digit number, using zeros in the dividend, quotients terminating by the third decimal place	8-12	T 268-T 269
Solve division problems	13-20	
Divide a decimal by a two-digit number	21-24	T 272-T 273

Comments

Students who had little difficulty with long division with whole numbers will likely have little difficulty dividing a decimal by a whole number. It may be necessary to help other students improve in areas such as the rapid recall of basic multiplication facts and the rounding of two-digit divisors to obtain trial digits for the quotient. Finding the product of a two-digit number and a one-digit number is also required in the procedure and some students may need more practice writing such products in one line. It is important to detect whether errors are being made in a particular step of the algorithm and to help students with that step, rather than to assign more division exercises to be completed.

Unit 13 Overview

Fractions

In this unit, simple fractions are associated with objects and with sets. The terms *numerator* and *denominator* are presented in the first lesson in both these contexts. Equivalent fractions are noted by means of diagrams and are later obtained by multiplication and division. Cross products are used not only to check for equivalence of fractions but also to find a missing term in a pair of equivalent fractions. Fractions with like denominators are compared by examining their numerators. Fractions with unlike denominators are converted to equivalent fractions having like denominators and then their numerators are compared. Improper fractions are changed to whole numbers or to numbers in mixed form, and vice versa, first through the use of models and later by multiplication and division. Addition and subtraction of fractions and numbers in mixed form with like denominators is presented, with regrouping in some cases. Division and then multiplication are used to find a number for part of a set. Relationships between decimals and fractions are established to develop fuller understanding of fraction concepts and to strengthen skills in adding and subtracting fractions. Students learn to find the decimal equivalent for a fraction by dividing the numerator by the denominator. The skill presented in the *Problem Solving* lesson directs attention to the need to select information and to exclude unneeded information.

Prerequisite Skills

- multiply and divide whole numbers
- simplify an expression involving multiplication and addition
- write decimals
- divide using zero to show more decimal places in the dividend, quotients terminating by the third decimal place
- add and subtract decimals

Unit Outcomes

- write numerals for fractions less than one for part of a whole and part of a set, denominators from 2 to 10 and also 12; draw diagrams to illustrate fractions
- write two or more equivalent fractions for diagrams showing part of a whole or part of a set
- use multiplication or division to find equivalent fractions
- find the missing term in two equivalent fractions
- use cross products to determine whether two fractions are equivalent
- use cross products to find the missing term in two equivalent fractions
- compare fractions with like denominators; compare fractions with unlike denominators by writing equivalent fractions with like denominators
- express a whole number as an improper fraction; express a number in mixed form as an improper fraction
- express an improper fraction as a whole number or as a number in mixed form
- add two proper fractions or two numbers in mixed form with like denominators, regrouping
- subtract proper fractions or numbers in mixed form with like denominators, regrouping
- find the number for part of a set when the number of the set is a multiple of the denominator of the fraction
- express a fraction as a decimal by first writing an equivalent fraction with a denominator of 10 or 100, for halves, fourths, and fifths
- divide the numerator of a fraction by the denominator to express a fraction as a decimal
- compare fractions using their decimal equivalents; add and subtract fractions and their decimal equivalents, and then compare the results
- solve word problems involving fractions
- select the necessary information for solving a problem, and then solve the problem

Background

The word *fraction* comes from the Latin word *frangere* meaning to break; hence a fraction represents part of a whole or a set. The lower numeral in a fraction is called the *denominator* and refers to the number of equal parts in the whole object or the whole set. The denominator gives the name of each part and indicates the size of one part. For instance, in $\frac{2}{5}$ the 5 provides the name "fifth" for each part, which is one of five equal parts. The upper numeral in a fraction is called the *numerator* and represents the number of parts being considered. In the fraction $\frac{2}{5}$, two equal parts, called "fifths", are indicated.

The numerals $\frac{4}{10}$, $\frac{6}{15}$, $\frac{10}{25}$, and $\frac{18}{45}$ all name the fractional number which is $\frac{2}{5}$ in its simplest form. Equivalent fractions are usually obtained by multiplying or dividing the numerator and the denominator by the same number. It is important to realize why this is an acceptable way. It is readily accepted that multiplying or dividing by 1, the *identity element* for these operations, does not alter the value of a number ($8 \times 1 = 8$, $9 \div 1 = 9$). If 1 is expressed as $\frac{2}{2}$, $\frac{3}{3}$, $\frac{4}{4}$, and so on, the value of the number remains unchanged. For example, if $\frac{3}{5}$ is multiplied by 1 in these forms, equivalent fractions are obtained. Similarly, equivalent fractions may be obtained by dividing a fraction by appropriate fractional names for 1. In effect, then, multiplying or dividing the numerator and the denominator of a fraction by the same number is merely an application of the identity element for these two operations.

$$\boxed{\frac{3}{5} \times 1} \quad \frac{3}{5} \times \frac{2}{2} = \frac{6}{10} \qquad \boxed{\frac{16}{24} \div 1} \quad \frac{16}{24} \div \frac{2}{2} = \frac{8}{12}$$

The same application of the identity element underlies the use of cross products to test for equivalence of fractions. For example, to check whether $\frac{3}{4}$ and $\frac{6}{8}$ are equivalent, they may be rewritten as equivalent fractions with the product of 4 and 8 (32) as a common denominator. The cross-product method does not involve a common denominator, but the factors used are identical to those for finding the numerators in the previous method, namely 3×8 and 4×6.

$$\frac{3}{4} \times \frac{8}{8} \qquad \frac{6}{8} \times \frac{4}{4}$$

$$\frac{24}{32} \qquad \frac{24}{32}$$

$$\frac{24}{32} = \frac{24}{32}, \text{ so } \frac{3}{4} = \frac{6}{8}$$

$$\frac{3}{4} \quad \frac{6}{8}$$

$$3 \times 8 \qquad 4 \times 6$$

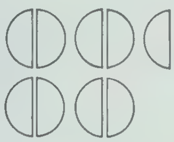
$$24 = 24$$

$$\frac{3}{4} = \frac{6}{8}$$

The operation of division in the partitive sense is used to find the number for part of a set. For instance, $\frac{1}{4}$ of 36 may be found by dividing 36 by 4. In finding $\frac{3}{4}$ of 36, however, two steps are required: the division $36 \div 4 = 9$ followed by the multiplication $3 \times 9 = 27$; thus $\frac{3}{4}$ of 36 is 27.

Changing an improper fraction to a whole number or to a number in mixed form involves the use of measurement

division. For instance, to change $\frac{12}{4}$ to a whole number, since 4 fourths are required for each number, the division $12 \div 4$ implies "How many 4's are in 12?" To change $\frac{9}{2}$ to a mixed number, 2 halves are needed for each whole number, so 9 is divided by 2 and the remainder 1 represents half of another whole.

 $\frac{12}{4} = 3$  $\frac{9}{2} = 4\frac{1}{2}$

In addition and subtraction of fractions with like denominators, the operations are performed using only the numerators. The abstract addition fact $3 + 2 = 5$, or the abstract subtraction fact $3 - 2 = 1$, may be applied in any situation, whether the items are boats, cars, pencils, or fourths, fifths, sixths.

3 fourths + 2 fourths 5 fourths	$\frac{3}{4} + \frac{2}{4} = \frac{5}{4}$	3 fourths − 2 fourths 1 fourth	$\frac{3}{4} - \frac{2}{4} = \frac{1}{4}$

In Unit 15 it is seen that a fraction may be used to represent a ratio. There is one further meaning which can be given to the numerals in a fraction: to show division with two numbers in which the numerator corresponds to the dividend and the denominator to the divisor. By extending the operation into decimal places decimal equivalents may be found for fractions, as shown for $\frac{3}{5}$ and $\frac{3}{4}$. This process may also be used to verify equivalence of fractions. For example, the fractions $\frac{1}{2}$, $\frac{2}{4}$, $\frac{3}{6}$, $\frac{4}{8}$, and $\frac{7}{14}$ are equivalent fractions because the quotient for each is 0.5.

$$\boxed{\tfrac{3}{5}} \quad \begin{array}{r} 0.6 \\ 5\overline{)3.0} \\ 3\,0 \\ \hline 0 \end{array} \quad \frac{3}{5} = 0.6 \qquad\qquad \boxed{\tfrac{3}{4}} \quad \begin{array}{r} 0.75 \\ 4\overline{)3.00} \\ 2\,8 \\ \hline 20 \\ 20 \\ \hline 0 \end{array} \quad \frac{3}{4} = 0.75$$

Teaching Strategies

If the topics in Unit 13 (fractions) and in Unit 15 (ratio) are beyond the scope of the mathematics program for any particular school or group of students, they may be omitted. These topics are included in *Starting Points in Mathematics 6*. It is suggested, however, that the *Problem Solving* lesson on page 286 be considered. It does not involve the use of fractions and it presents an important problem-solving skill.

For the lessons in this unit, it is important that students have opportunities to see illustrations and to handle models in order to bring meaning to the concepts and the operations. For some lessons, the students should use their own materials so that they may have direct experiences of the same kind as those used in presenting the lessons.

Number lines with two or more scales can be used to visually identify equivalent fractions and to compare fractions with unlike denominators. Cardboard strips with equal unit lengths divided into parts may be manipulated for the same purposes (see page T284). Number lines are also effective in dealing with numbers in mixed form and improper fractions. Devices such as these can help the students to acquire better understanding of the concepts involved in fractions.

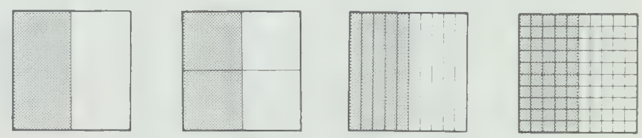
Students may be helped with decimal equivalents for fractions by thinking of money. One quarter (one-fourth of a dollar) is worth 25¢, or \$0.25 ($\frac{1}{4} = 0.25$), and three quarters are worth 75¢, or \$0.75 ($\frac{3}{4} = 0.75$). One half-dollar is worth 50¢, or \$0.50 ($\frac{1}{2} = 0.50$, or 0.5).

Results of the *Checking Up* exercises on page 287 should be examined to discover any needs for reteaching, review, and further practice. The skills involved in Ex. 3-10 are used again in Unit 15 to find equivalent ratios and missing terms in ratios, and any weaknesses in these exercises should receive careful attention at this time.

Models for Halves, Fourths, Tenths, and Hundredths

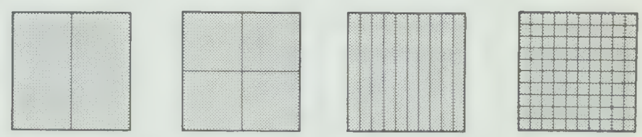
In this unit, equivalent fractions and decimals are introduced. To facilitate this, models for halves, fourths, fifths, and eighths may be prepared by using copies of the decimetre squares on page T393. The squares may be marked and then colored to show, for example, one half, one fourth, two fourths, and so on. Using the decimetre square as the whole enables students to see the same whole divided, for example, into two, four, ten, or one hundred equal parts. This helps in comparing fractions and decimals (A), and in naming equivalent fractions for the number one (B).

A



$\frac{1}{2}$ $\frac{2}{4}$ $\frac{5}{10}$ or 0.5 $\frac{50}{100}$ or 0.50

B



$\frac{2}{2}$ or 1 $\frac{4}{4}$ or 1 $\frac{10}{10}$ or 1 $\frac{100}{100}$ or 1

Materials

objects such as red markers and blue markers for showing sets,
sheets of paper and a blue crayon for showing parts of a whole
cardboard strips prepared for pages T284 and T285
models for $\frac{3}{8}$, $\frac{5}{8}$, $\frac{6}{8}$, $\frac{7}{8}$, $\frac{2}{5}$, $\frac{4}{5}$, and $\frac{1}{2}$
models for wholes, halves, thirds, fourths, fifths, and eighths
a map of Canada for each student
counters, small boxes
models for $\frac{1}{2}$, $\frac{2}{5}$, and $\frac{3}{4}$; models for 0.5, 0.4, and 0.75
real coins or play money (optional)

Vocabulary

fraction	is not equal to ($\neq$)
numerator	like denominators
denominator	unlike denominators
names of fractions, half to tenth, twelfth	improper fraction
prime number	proper fraction
prime factor	mixed form
equivalent fractions	biological
common factors	family tree
lowest terms	ancestors
missing term	generation
cross products	geography

Pages 258-259

LESSON OUTCOME

Write numerals for fractions less than one for part of a whole and part of a set, denominators from 2 to 10 and also 12; draw diagrams to illustrate fractions

Materials

objects such as red markers and blue markers for showing sets, sheets of paper and a blue crayon for showing parts of a whole

Vocabulary

fraction, numerator, denominator, names of fractions less than one with denominators from 2 to 10 and also 12, prime number, prime factor

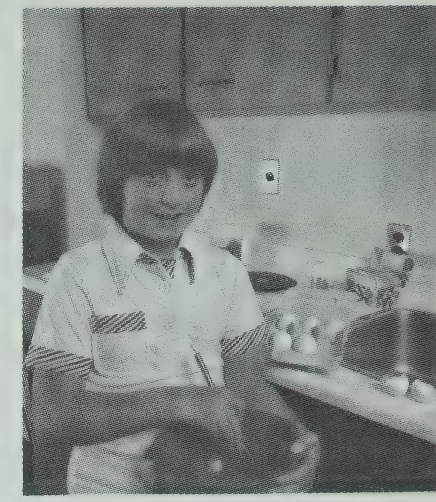
13 FRACTIONS

Writing Fractions

There are 5 eggs left in the carton.
There were 6 eggs in it when it was full.

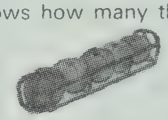
The fraction shows what part of the carton still has eggs. $\frac{5}{6}$ five-sixths

In a fraction, the number above the bar is the **numerator**. The number below the bar is the **denominator**.



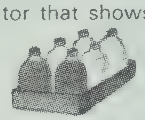
Working Together

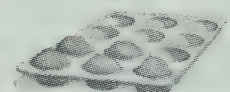
Complete each sentence by using a denominator that shows how many there are in all.



1. $\frac{3}{\blacksquare}$ 5 of the tomatoes are ripe.
2. $\frac{1}{\blacksquare}$ 4 of the pennies shows "heads".

Complete each sentence by using a numerator that shows how many are empty.





3. 4 $\frac{\blacksquare}{6}$ of the bottles are empty.
4. 7 $\frac{\blacksquare}{12}$ of the cups are empty.

Complete each sentence with a fraction.

5. 2 $\frac{\blacksquare}{\blacksquare}$ 3 of the pancakes have butter.
6. 3 $\frac{\blacksquare}{\blacksquare}$ 10 of the apples are green.

258

LESSON ACTIVITY

Before Using the Pages

- Present several examples to enable the students to recall previous work with fractions for part of a whole and part of a set. For example, fold a sheet of paper to illustrate eight equal parts and color three of the parts blue. Ask if the parts are equal, how many parts there are in all, and how many parts are blue. Elicit the fraction "three-eighths" to describe the part of the whole that is blue, and write the numeral $\frac{3}{8}$ on the board. Similarly, display a set of five red markers and seven blue markers. Ask how many markers there are in all and how many of these are red. Elicit the fraction "five-twelfths" to describe the part of the set that is red, and write the numeral $\frac{5}{12}$ on the board. For other examples, have students help to show parts of wholes and parts of sets and write the numerals on the board.

  Ask what name describes such numbers as $\frac{3}{8}$ and $\frac{5}{12}$.

Using the Pages

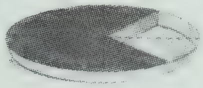
- Have the students note the title at the top of page 258. Draw attention to the photograph and ask how fractions may be involved in the situation suggested. Relate the carton of eggs in the photograph to the fraction $\frac{5}{6}$ in the example. Introduce the terms *numerator* and *denominator* and have students identify these for the fraction $\frac{5}{6}$ and for fractions on the board from the activity in *Before Using the Pages*.

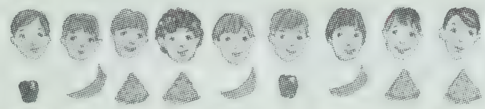
Working Together: Ex. 1 and 2 emphasize that the denominator names the number of equal parts of a whole or the number of objects in a set. For each of these, question the students about the number shown for the numerator and have them explain the number required for the denominator. In a similar manner, discuss Ex. 3 and 4 which emphasize that the numerator names the number of parts or objects that are "special". Ask students to explain their answers for Ex. 5 and 6. As the students write a fraction, ensure that they use a horizontal bar for separating the numerator and denominator.

Exercises

Complete each sentence with a fraction.

1. 
   $\frac{■}{6}$ of the eggs are broken.

2. 
   $\frac{2}{8}$ of the cake has been taken.

3. 

$\frac{2}{9}$ of the boys have apples. $\frac{3}{9}$ of the boys have bananas. $\frac{4}{9}$ of the boys have melon.

Write the fraction

4. with denominator 7 and numerator 5. $\frac{5}{7}$
5. with numerator 4 and denominator 10. $\frac{4}{10}$

Write a sentence with a fraction for these.

6. 5 goats are in a pen.
   2 of them have horns. $\frac{2}{5}$ of the goats have horns.
7. 12 kittens are playing with string.
   5 of them get tangled. $\frac{5}{12}$ of the kittens get tangled.

Do these on a large sheet of paper.

	If	Draw shapes to show
8.	◺ is $\frac{1}{4}$	$\frac{3}{4}$
9.	⏢ is $\frac{1}{3}$	$\frac{2}{3}$
10.	▱ is $\frac{1}{2}$	1

	If	Draw shapes to show
11.	△ is $\frac{1}{3}$	1
12.	□ is $\frac{1}{4}$	1
*13.	⬡ is $\frac{2}{6}$	$\frac{5}{6}$

Answers will vary. Possible shapes are shown on page T369

6 and 10 are factors of 60 because

$60 = 6 \times 10$.

4 and 15 are also factors of 60 because

$60 = 4 \times 15$.

1. Name 2 other factors of 60. The other factors are 1, 2, 3, 5, 12, 20, 30, 60
2. Name 5 factors of 210. Answers will vary. The factors are 1, 2, 3, 5, 6, 7, 10, 14, 15, 21, 30, 35, 42, 70, 105, 210

A whole number greater than 1 that has itself and 1 as its only factors is a **prime number**.

$23 = 1 \times 23$.

This is the only way to show 23 as a product of whole numbers. 23 is a prime number.

Since $24 = 3 \times 8$, 24 is not a prime number.

3. List the prime numbers from 2 to 100. 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97

A factor that is a prime number is a **prime factor**.

List the prime factors

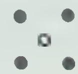
4. of 12. 2, 3
5. of 1386. 2, 3, 7, 11

try this

259

RELATED ACTIVITIES

- For several exercises on page 259, ask students to identify the numerator and the denominator of the fraction named, and to tell what each represents. Display the following example to help students recall the terms and the meaning of each.



4 of the 5 dots are black.

$\frac{4}{5}$ ← numerator
← denominator

- Many of the exercises on pages 258 and 259 may be adapted for naming other fractions. For example, for Ex. 6 on page 258, have students complete the sentence "$\frac{■}{■}$ of the apples are red", and for Ex. 3 on page 259, have them complete the sentence "$\frac{■}{■}$ of the boys do not have bananas".
- You may wish to have students complete the procedure described on page T 379 for finding prime numbers, known as the sieve of Eratosthenes.
- When students play games that involve game boards with numbered squares and/or dice, suggest that the following rules apply.
  1. If the number obtained by tossing the dice is a prime number, the player takes another turn.
  2. If the number in the square on which a player's marker lands is a prime number, the player moves her/his marker back two squares.

Exercises: For Ex. 8-13, you may wish to have the students trace copies of the corresponding shapes from pages T 382-T 385. The shape for Ex. 8 may be obtained by cutting a square along a diagonal; for Ex. 9, a hexagon may be cut in half. Although the students are asked to draw shapes for these, you may wish to have them complete one or more by pasting copies of the appropriate shape on a large sheet of paper. Answers to these exercises can suggest the concept of tiling congruent shapes which is presented in Unit 14. Discuss different solutions for a particular exercise. Note that Ex. 10-12 suggest that $\frac{2}{2}$, $\frac{3}{3}$, $\frac{4}{4}$, and so on, are other names for 1 (one whole). Ex. 13 is starred because the solution requires the use of half the diagram provided.

Try This: These exercises suggest an informal approach to the concepts of prime numbers and prime factors. A prime number has only two factors and these are unequal. Thus, such numbers as 1, 60, and 210 are not prime numbers, and such numbers as 2, 17, and 31 are prime numbers. Have students compare their answers for Ex. 3. For Ex. 4 and 5, ensure that the students do not repeat factors. The prime factors of 12 are 2 and 3, but 12 expressed as a product of prime factors is $2 \times 2 \times 3$.

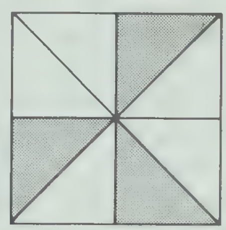
Assessment

Complete each sentence with a fraction.

1.

O		
X	X	X
O	O	X

$\frac{4}{9}$ of the squares show X's.

2. 

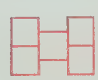
$\frac{3}{8}$ of the picture is gray.

Write the fraction

3. with denominator 6 and numerator 5. $\frac{5}{6}$

If □ is $\frac{1}{5}$,

4. draw a shape to show $\frac{5}{5}$.



Shapes will vary.

LESSON OUTCOME

Write two or more equivalent fractions for diagrams showing part of a whole or part of a set

Materials

a sheet of paper and a crayon for each student, cardboard strips as described in *Before Using the Pages*

Vocabulary

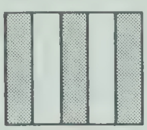
equivalent fractions, common factors

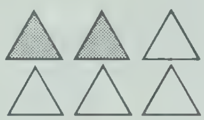
Prerequisite Skills

Write numerals for fractions less than one

Checking Prerequisite Skills

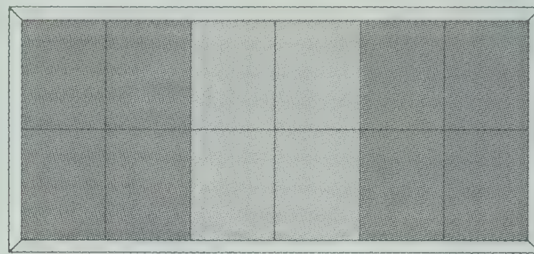
Write the fraction to show how much is shaded.

1. 
$\frac{3}{5}$

2. 
$\frac{2}{6}$

Equivalent Fractions

Ginny covered the bulletin board with construction paper.



She used blue and green to make three parts all the same size.

She used 12 sheets of construction paper.

$\frac{1}{3}$ is blue. $\frac{2}{3}$ is green.
$\frac{4}{12}$ is blue. $\frac{8}{12}$ is green.

$\boxed{\frac{1}{3} = \frac{4}{12}}$

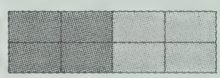
$\frac{1}{3}$ and $\frac{4}{12}$ are **equivalent fractions.**

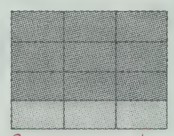
$\boxed{\frac{2}{3} = \frac{8}{12}}$

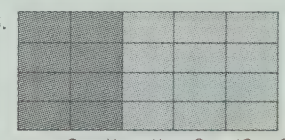
$\frac{2}{3}$ and $\frac{8}{12}$ are also equivalent fractions.

Working Together

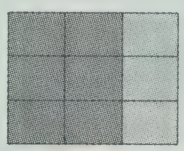
Give the numerators that complete each sentence.

1. 
$\frac{4}{8} = \frac{1}{2}$

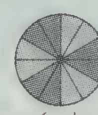
2. 
$\frac{9}{12} = \frac{3}{4}$ $\frac{3}{12} = \frac{1}{4}$

3. 
$\frac{8}{20} = \frac{4}{10}$ $\frac{4}{10} = \frac{2}{5}$ $\frac{12}{20} = \frac{3}{5}$

Give two equivalent fractions for each picture.

4. 
$\frac{6}{9}, \frac{2}{3}; \frac{3}{9}, \frac{1}{3}$

5. 
$\frac{4}{16}, \frac{1}{4}; \frac{12}{16}, \frac{3}{4}$

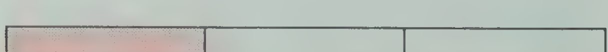
6. 
$\frac{6}{12}, \frac{1}{2}$

260

LESSON ACTIVITY

Before Using the Pages

- Have each student fold a sheet of paper in half, unfold it, and color one half. Have the students fold the same sheet into fourths and note that two fourths are colored. Have them fold the same sheet into eighths and note that four eighths are colored. You may wish to continue the procedure to sixteenths. Summarize that the fractions $\frac{1}{2}$, $\frac{2}{4}$, $\frac{4}{8}$, $(\frac{8}{16})$ represent the same part (one half) of the whole sheet. Develop that 5 of the parts would be colored if the sheet were folded into 10 equal parts.
- In advance of the lesson, prepare several cardboard strips of the same size. Mark one strip into thirds and color one third. Mark and color other strips to show $\frac{2}{6}$, $\frac{3}{9}$, and $\frac{4}{12}$. Display the strips and develop that each fraction names equal parts of the strips.

$\frac{1}{3}$ 

$\frac{2}{6}$, $\frac{3}{9}$, $\frac{4}{12}$ 

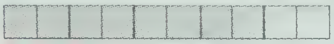
You may also discuss that equal parts of the strips are not colored, and develop that $\frac{2}{3}$, $\frac{4}{6}$, $\frac{6}{9}$, and $\frac{8}{12}$ name the same amount. Tell the students that fractions that represent the same amount have a special name. Ask them to read the title of the lesson on page 260.

Using the Pages

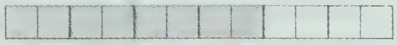
- Ask the students to note that the bulletin board shows three equal sections, two of which are green. Ask them to note, also, that twelve sheets of construction paper were used, eight of which are green. Thus, the same whole suggests both *thirds* and *twelfths*. Have students read the statements below the illustration. Point out that 4 of the 12 sheets are used to cover $\frac{1}{3}$ of the bulletin board ($\frac{1}{3} = \frac{4}{12}$). To cover $\frac{2}{3}$ of the bulletin board, 8 of the 12 sheets are used ($\frac{2}{3} = \frac{8}{12}$).

Exercises

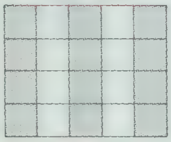
Write two equivalent fractions for each picture. Answers will vary

1. 

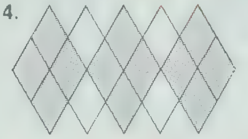
$\frac{8}{10}, \frac{4}{5}; \frac{2}{10}, \frac{1}{5}$

2. 

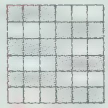
$\frac{8}{12}, \frac{4}{6}; \frac{4}{6}, \frac{2}{3}; \frac{2}{6}, \frac{1}{3}; \frac{4}{12}, \frac{2}{6}$

3. 

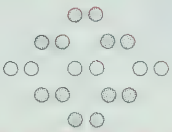
$\frac{12}{20}, \frac{3}{5}; \frac{8}{20}, \frac{2}{5}$

4. 

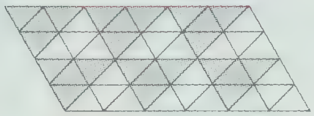
$\frac{12}{16}, \frac{6}{8}; \frac{6}{8}, \frac{3}{4}; \frac{4}{16}, \frac{2}{8}; \frac{2}{8}, \frac{1}{4}$

5. 

$\frac{18}{36}, \frac{6}{12}$

6. 

$\frac{12}{18}, \frac{6}{9}; \frac{6}{9}, \frac{2}{3}; \frac{6}{18}, \frac{3}{9}; \frac{3}{9}, \frac{1}{3}$

7. 

$\frac{24}{48}, \frac{6}{12}$

*8. 

$\frac{8}{12}, \frac{2}{3}; \frac{4}{12}, \frac{1}{3}; \frac{9}{12}, \frac{3}{4}; \frac{3}{12}, \frac{1}{4}$

Here are four ways to show 30 as a product.

1 × 30 = 30   2 × 15 = 30   3 × 10 = 30   5 × 6 = 30

Here are four ways to show 24 as a product.

1 × 24 = 24   2 × 12 = 24   3 × 8 = 24   4 × 6 = 24

The factors of 30 are 1, 2, 3, 5, 6, 10, 15, 30

The factors of 24 are 1, 2, 3, 4, 6, 8, 12, 24

The **common factors** of 30 and 24 are 1, 2, 3, 6

List all the factors of the numbers in each pair.
Then draw rings around their common factors.

1. 12 and 18   2. 40 and 100   3. 84 and 315   4. 52 and 78

try this

1. 1, 2, 3, 4, 6, 12
   1, 2, 3, 6, 9, 18
2. 1, 2, 4, 5, 8, 10, 20, 40
   1, 2, 4, 5, 10, 20, 25, 50, 100
3. 1, 2, 3, 4, 6, 7, 12, 14, 21, 28, 42, 84
   1, 3, 5, 7, 9, 15, 21, 35, 45, 63, 105, 315
4. 1, 2, 4, 13, 26, 52
   1, 2, 3, 6, 13, 26, 39, 78

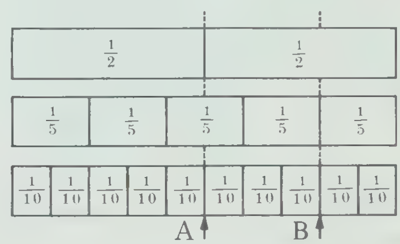
261

RELATED ACTIVITIES

• Have students write several pairs of equivalent fractions by interpreting the picture for Ex. 5 on page 260 in different ways. Some examples are given below. The picture for Ex. 8 on page 261 may also be interpreted in a similar manner.

4 of the 16 shapes are squares.	$\frac{4}{16} = \frac{1}{4}$
4 of the 12 triangles are green.	$\frac{4}{12} = \frac{1}{3}$
8 of the 16 shapes are blue triangles.	$\frac{8}{16} = \frac{1}{2}$
4 of the 12 blue shapes are squares.	$\frac{4}{12} = \frac{1}{3}$
12 of the 16 shapes are triangles.	$\frac{12}{16} = \frac{3}{4}$
8 of the 12 triangles are blue.	$\frac{8}{12} = \frac{2}{3}$

• Fraction strips similar to those described in *Before Using the Pages* are useful for finding equivalent fractions. Nine strips of equal length are marked to show halves, thirds, fourths, fifths, sixths, eighths, ninths, tenths, and twelfths. By placing strips together, students can find equivalent fractions for $\frac{1}{2}, \frac{1}{3}, \frac{2}{3}, \frac{1}{4}, \frac{3}{4}, \frac{1}{5}, \frac{2}{5}, \frac{3}{5}$, and so on.



A $\frac{1}{2} = \frac{5}{10}$ B $\frac{4}{5} = \frac{8}{10}$

Working Together: A diagram that suggests equivalent fractions is provided for each exercise. Note the use of color in the sentences for Ex. 1 and 2 to help students determine the missing numbers. You may wish to have them name equivalent fractions only for the green regions for Ex. 3-6. Then, the exercises may be repeated in terms of the blue regions. Thus, for Ex. 3, the green region generates the following pairs of equivalent fractions: $\frac{8}{20} = \frac{4}{10}$; $\frac{4}{10} = \frac{2}{5}$; $\frac{8}{20} = \frac{2}{5}$. Emphasize that the diagram must be viewed as a whole divided into 20 equal parts, then as a whole divided into 10 equal parts, and finally as a whole divided into 5 equal parts. Then ask, for example, "How many twentieths are green?" Ex. 5 may be interpreted in different ways (see *Related Activities*).

Exercises: Because only two equivalent fractions are to be named for each diagram, it is important to have students explain their answers in terms of the diagrams.

Try This: The concept of common factors is inherent in the use of division to find equivalent fractions (see page 262).

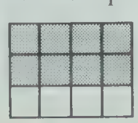
After the students have read the example, discuss the factors of 30. Ask, for example, which is the greatest factor (30) and which is the least factor (1). Have students explain why 7, 8, and 9, for example, are not factors of 30. Emphasize that all the factors of a number must be written before common factors are determined.

It is helpful to write the row of factors of a number to fill both ends of the row simultaneously. For example, for 40 (Ex. 2), write the least and the greatest factors first (1 and 40), the second least and the second greatest factors next, (2 and 20), and so on, to obtain 1, 2, 4, 5, 8, 10, 20, 40. It can be seen that no factors can be written after the pair 5 and 8 because the numbers between 5 and 8 (6 and 7) are not factors of 40.

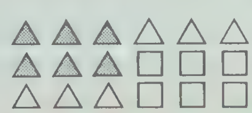
Assessment

Answers will vary.

Write two equivalent fractions for each picture.

1. 

$\frac{8}{12}, \frac{2}{3}; \frac{4}{12}, \frac{1}{3}$

2. 

$\frac{6}{18}, \frac{2}{6}; \frac{12}{18}, \frac{4}{6}; \frac{6}{12}, \frac{1}{2}$

LESSON OUTCOME

Use multiplication or division to find equivalent fractions

Materials

cardboard strips prepared for pages T 284 and T 285

Vocabulary

lowest terms

Prerequisite Skills

Divide by a one-digit number, remainder zero

Checking Prerequisite Skills

Divide.

1. $2\overline{)30}$ 15 **2.** $7\overline{)84}$ 12

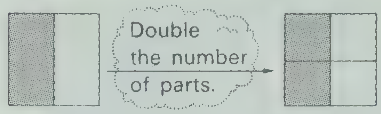
3. $3\overline{)63}$ 21 **4.** $5\overline{)70}$ 14

RELATED ACTIVITIES

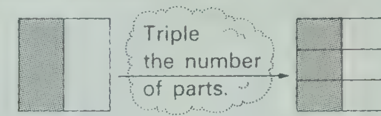
- Give each student a copy of page T 391 on which to complete a multiplication table for the numbers 1 to 9 as factors. Demonstrate how the table can be used to help name equivalent fractions. For example, the row for 2 and the row for 5 can reveal fractions that are equivalent to $\frac{2}{5}$ ($\frac{4}{10}$, $\frac{6}{15}$, $\frac{8}{20}$, . . .). You may wish to have students cut along the rows of the table to enable them to position appropriate strips together.

Finding Equivalent Fractions

Multiplication can be used to find equivalent fractions.

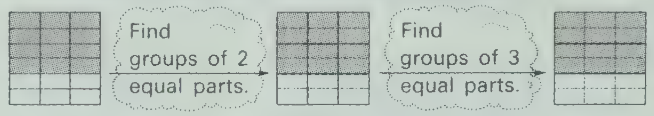


$$\frac{1}{2} \xrightarrow[2 \times 2]{1 \times 2} \frac{2}{4}$$



$$\frac{1}{2} \xrightarrow[2 \times 3]{1 \times 3} \frac{3}{6}$$

Division can be used to find equivalent fractions.



$$\frac{12}{18} \xrightarrow[18 \div 2]{12 \div 2} \frac{6}{9} \xrightarrow[9 \div 3]{6 \div 3} \frac{2}{3}$$

Exercises

Multiply or divide to find equivalent fractions.

1. (1 × 2) (1 × 3) (1 × 4) . . .

$\frac{1}{2}$	$\frac{2}{4}$	$\frac{3}{6}$	$\frac{4}{8}$	$\frac{5}{10}$

(2 × 2) (2 × 3) (2 × 4) . . .

2. (12 ÷ 2) (6 ÷ 3)

$\frac{12}{30}$	$\frac{6}{15}$	$\frac{2}{5}$

(30 ÷ 2) (15 ÷ 3)

3. (3 × 2) (3 × 3) . . .

$\frac{3}{8}$	$\frac{6}{16}$	$\frac{9}{24}$	$\frac{12}{32}$	$\frac{15}{40}$

(8 × 2) (8 × 3) . . .

4. (63 ÷ 3) (? ÷ 7)

$\frac{63}{84}$	$\frac{21}{28}$	$\frac{3}{4}$

(84 ÷ 3) (? ÷ 7)

5. (5 × 2) . . .

$\frac{5}{6}$	$\frac{10}{12}$	$\frac{15}{18}$	$\frac{20}{24}$	$\frac{25}{30}$

(6 × 2) . . .

6. (40 ÷ 2)

$\frac{40}{70}$	$\frac{20}{35}$	$\frac{4}{7}$

(70 ÷ 2)

When there is no whole number that divides both numerator and denominator, the fraction is in **lowest terms.**

Do you know a way to find the lowest terms in just one step?

262

LESSON ACTIVITY

Before Using the Page

- Use the cardboard strips prepared for the preceding lesson to review that $\frac{1}{3}$, $\frac{2}{6}$, $\frac{3}{9}$, and $\frac{4}{12}$ are equivalent fractions. Write the sentence $\frac{1}{3} = \frac{2}{6}$ on the board and lead the students to observe that multiplying each term of $\frac{1}{3}$ by 2 gives $\frac{2}{6}$. Discuss the sentences $\frac{1}{3} = \frac{3}{9}$ and $\frac{1}{3} = \frac{4}{12}$ in a similar way. Ask what fraction is obtained by multiplying each term of $\frac{1}{2}$ by 5, and draw diagrams to illustrate that $\frac{1}{2}$ and $\frac{5}{10}$ are equivalent fractions. Ask whether $\frac{8}{16}$ is equivalent to $\frac{1}{2}$ and have students give reasons for their answers.

Using the Page

- The examples demonstrate that multiplying (dividing) each term of a fraction by the same number gives an equivalent fraction. Diagrams are shown to clarify the procedure. Emphasize that the use of multiplication or division is an efficient method for finding equivalent fractions; however, the number by which the numerator is multiplied (divided) is the same number by which the denominator is multiplied (divided). In the example using division, establish that both the numerator and the denominator of $\frac{12}{18}$ are divided by 2 to get $\frac{6}{9}$, and that in the second step both terms are divided by 3 to get $\frac{2}{3}$.

Exercises: Equivalent fractions for Ex. 1, 3, and 5 are obtained by multiplying each term of the original fraction by 2, 3, 4, and then 5, in sequence. For Ex. 2, 4, and 6, the final fraction obtained is in lowest terms. When the students have completed the three exercises, discuss the meaning of *lowest terms*. Then, discuss how to obtain the lowest terms in one step for each exercise.

Assessment

Multiply or divide to find equivalent fractions.

1. (3 × 2)

$\frac{3}{4}$	$\frac{6}{8}$	$\frac{9}{12}$	$\frac{12}{16}$	$\frac{15}{20}$

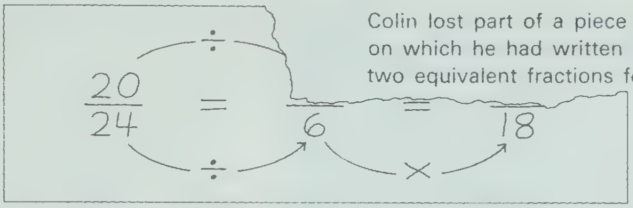
(4 × 2)

2. (48 ÷ 3)

$\frac{48}{72}$	$\frac{16}{24}$	$\frac{12}{18}$

(72 ÷ 3)

Finding the Missing Term



To find the first missing term,

24 6

think ⋯ 24 ÷ 4.

Then, divide 20 by 4.

$\frac{20}{24} = \frac{5}{6}$ (20 ÷ 4, 24 ÷ 4)

To find the second missing term,

6 18

think ⋯ 6 × 3.

Then, multiply 5 by 3.

$\frac{5}{6} = \frac{15}{18}$ (5 × 3, 6 × 3)

Working Together

What factor is used to get the equivalent fraction?

1. $\frac{1}{3} = \frac{5}{15}$ 1 × ■ 5, 3 × ■ 5
2. $\frac{12}{15} = \frac{4}{5}$ 12 ÷ ■ 3, 15 ÷ ■ 3

Find the missing term.

3. $\frac{3}{4} = \frac{■}{16}$ 12
4. $\frac{2}{7} = \frac{6}{■}$ 21
5. $\frac{6}{12} = \frac{■}{2}$ 1
6. $\frac{15}{25} = \frac{3}{■}$ 5

Exercises

Find the missing term.

1. $\frac{2}{3} = \frac{■}{12}$ 8
2. $\frac{1}{4} = \frac{9}{■}$ 36
3. $\frac{7}{42} = \frac{■}{6}$ 1
4. $\frac{21}{36} = \frac{7}{■}$ 12
5. $\frac{5}{8} = \frac{■}{16}$ 10

Complete the equivalent fractions in each chart.

6.

$\frac{■}{5}$ 2	$\frac{16}{■}$ 40	$\frac{4}{10}$	$\frac{■}{30}$ 12	$\frac{8}{■}$ 20

7.

$\frac{■}{10}$ 5	$\frac{9}{■}$ 18	$\frac{■}{2}$ 1	$\frac{8}{16}$	$\frac{12}{■}$ 24

8.

$\frac{■}{12}$ 9	$\frac{24}{32}$	$\frac{18}{■}$ 24	$\frac{■}{48}$ 36	$\frac{3}{■}$ 4

Page 263

LESSON OUTCOME

Find the missing term in two equivalent fractions

Vocabulary
missing term

RELATED ACTIVITIES

- Have students work in pairs for the activities suggested below. They may reverse their roles after several examples.
  1. One student writes a fraction. The other student writes a fraction equivalent to it. The first student identifies the divisor or multiplier applied to each term.
  2. One student writes a fraction. The other student writes the numerator or the denominator for an equivalent fraction. The first student determines the missing term.
- To assist students in finding missing terms, a review of products and factors is appropriate at this time. Have students write problems similar to the following for other students to solve.
  1. The product is 16. One factor is 2. What is the other factor?
  2. When 12 is divided by a number, the quotient is 4. What is the divisor?

LESSON ACTIVITY

Before Using the Page

- Write $\frac{3}{7}$ on the board. Have a student demonstrate the use of multiplication to find an equivalent fraction. For example, multiplying the numerator and denominator by 2 gives $\frac{3}{7} = \frac{6}{14}$. Follow a similar procedure for $\frac{12}{20}$ to review the use of division to find an equivalent fraction, for example, $\frac{12}{20} = \frac{6}{10}$. Have students explain the procedure for each example.
- Write $\frac{3}{5} = \frac{■}{■}$ on the board. Ask the students to think of different fractions that are equivalent to $\frac{3}{5}$. Ask one student to write the numerator of her/his fraction on the board in the appropriate place, for example, $\frac{3}{5} = \frac{18}{■}$. Ask the other students to determine the denominator. Discuss the procedures they used to do this.

Using the Page

- Have students read and help to explain the worked example. Discuss that the numerator and denominator are known as the two terms of a fraction, hence, the expression *missing term* in an example such as $\frac{20}{24} = \frac{■}{6}$.

Working Together: Ex. 1 and 2 emphasize that corresponding terms of two equivalent fractions are related by the same number for multiplication or division. This fact is applied in Ex. 3-6. You may wish to have students draw arrows to indicate the procedure as in Ex. 1 and 2.

Exercises: For Ex. 6 and 7, note that it is not possible to complete the charts from left to right. For example, for Ex. 6, the known fraction, $\frac{4}{10}$, is the third fraction in the chart. In Ex. 8, lead the students to realize that it would be best to complete the last fraction, $\frac{3}{■}$, first because it will be in lowest terms and all the other fractions may be found by using multiplication.

Assessment

Find the missing term.

1. $\frac{3}{7} = \frac{■}{28}$ 12
2. $\frac{7}{10} = \frac{42}{■}$ 60
3. $\frac{18}{36} = \frac{■}{4}$ 2
4. $\frac{12}{30} = \frac{2}{■}$ 5

LESSON OUTCOME

Use cross products to determine whether two fractions are equivalent; solve related word problems

Vocabulary

cross products, is not equal to ($\neq$)

Prerequisite Skills

Multiply whole numbers

Checking Prerequisite Skills

Multiply.

1. 15 × 7 = 105
2. 35 × 8 = 280
3. 14 × 12 = 168
4. 65 × 13 = 845

RELATED ACTIVITIES

- Prepare two sets of cards showing fractions such as $\frac{1}{2}$, $\frac{1}{3}$, $\frac{2}{3}$, $\frac{1}{4}$, $\frac{3}{4}$, $\frac{1}{5}$, and several equivalent fractions ($\frac{2}{4}$, $\frac{3}{6}$, $\frac{4}{8}$, $\frac{5}{10}$ for $\frac{1}{2}$, for example). Have a student shuffle the cards and share them equally with another student to play ''Fraction Snap''. Players turn over their top cards simultaneously. When equivalent fractions are shown, the first player to say ''Snap'' claims the two cards. The player with more cards at the end of a specified time is the winner.

Checking for Equivalent Fractions

Cross products can be used to check whether two fractions are equivalent.

For the fractions $\frac{5}{8}$ and $\frac{15}{24}$, both cross products are 120.

120 120
5 × 24 8 × 15
5 15
8 24

When the cross products for two fractions are equal, the fractions are equivalent.

$\frac{5}{8} = \frac{15}{24}$

For the fractions $\frac{2}{7}$ and $\frac{4}{9}$, the cross products are 18 and 28.

18 28
2 × 9 7 × 4
2 4
7 9

When the cross products for two fractions are not equal, the fractions are not equivalent.

$\frac{2}{7} \neq \frac{4}{9}$

The symbol "$\neq$" means "is not equal to".

Working Together

Find the cross products. Are the fractions equivalent?

1. 14 14 $\frac{1}{2}$, $\frac{7}{14}$ yes
2. 32 35 $\frac{4}{5}$, $\frac{7}{8}$ no
3. 72 72 $\frac{6}{9}$, $\frac{8}{12}$ yes

Exercises

Find the cross products. Are the fractions equivalent?

1. 84 84 $\frac{21}{28}$, $\frac{3}{4}$ yes
2. 18 21 $\frac{2}{7}$, $\frac{3}{9}$ no
3. 72 72 $\frac{12}{36}$, $\frac{2}{6}$ yes
4. 210 210 $\frac{14}{35}$, $\frac{6}{15}$ yes
5. 63 56 $\frac{7}{8}$, $\frac{7}{9}$ no

Use cross products to help you answer the questions.

6. When Shirley was given 32 of the 48 beads, she was told that she was getting $\frac{2}{3}$ of the beads. Was she? yes
7. After cultivating 72 of the 95 rows, the farmer said that $\frac{3}{4}$ of the field was cultivated. Was it? no

LESSON ACTIVITY

Before Using the Page

- Write pairs of equivalent fractions on the board and one or two pairs of non-equivalent fractions.

  $\frac{2}{5}$ and $\frac{8}{20}$ $\frac{2}{3}$ and $\frac{10}{15}$ $\frac{2}{3}$ and $\frac{8}{10}$ $\frac{3}{4}$ and $\frac{18}{24}$

  Ask the students to determine whether the fractions are equivalent and to explain the method used. For example, for $\frac{2}{5}$ and $\frac{8}{20}$, they will likely suggest $2 \times 4 = 8$ and $5 \times 4 = 20$. For $\frac{2}{3}$ and $\frac{8}{10}$, they will likely suggest $2 \times 4 = 8$, and then realize that 3×4 is not equal to 10.

  For the first example, point to the 2 and to the 20 and ask for the product. Then point to the 5 and to the 8 and ask for the product. Have the students find similar products for the remaining pairs of fractions. Lead them to discover that equal products are obtained for equivalent fractions and unequal products are obtained for non-equivalent fractions.

Using the Page

- Introduce the term *cross products*. Have students read the examples and explain in their own words how cross products may be used to test for equivalent fractions. Draw attention to the symbol $\neq$.

Working Together: Ask students to give reasons for their answers.

Exercises: Ask the students to show the cross products to justify their answers.

Assessment

Find the cross products. Are the fractions equivalent?

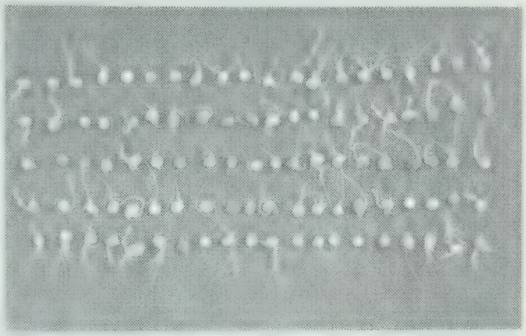
1. 140 140 $\frac{14}{20}$, $\frac{7}{10}$ yes
2. 30 40 $\frac{2}{5}$, $\frac{8}{15}$ no
3. 300 300 $\frac{10}{12}$, $\frac{25}{30}$ yes

Use cross products to help you answer the question.

4. Mary completed 50 of the 75 exercises on the page. She said that she had finished $\frac{2}{3}$ of the page. Was she right? yes

Finding the Missing Term Using Cross Products

Louise placed 100 beans on a damp paper towel. 3 d later, $\frac{3}{4}$ of them had sprouted. What fraction, with 100 as denominator, is equivalent to $\frac{3}{4}$?



To find the value of $\blacksquare$ in $\frac{3}{4} = \frac{\blacksquare}{100}$, use cross products.

$3 \times 100 \qquad 4 \times \blacksquare$

$\frac{3}{4} = \frac{\blacksquare}{100}$ $\qquad 4 \times \blacksquare = 3 \times 100 \qquad 4 \times \blacksquare = 300 \qquad 4 \times 75 = 300$

$300 \div 4 = 75$

In 3 d, 75 beans had sprouted. $\frac{3}{4} = \frac{75}{100}$

Working Together

Write a sentence showing equal cross products.

Example: For $\frac{2}{3} = \frac{10}{\blacksquare}$, write $2 \times \blacksquare = 3 \times 10$.

1. $\frac{4}{5} = \frac{\blacksquare}{10}$ $5 \times \blacksquare = 4 \times 10$
2. $\frac{3}{7} = \frac{12}{\blacksquare}$ $3 \times \blacksquare = 7 \times 12$
3. $\frac{2}{8} = \frac{\blacksquare}{100}$ $8 \times \blacksquare = 2 \times 100$

Find the missing term.

Example: For $5 \times \blacksquare = 8 \times 10$, or 80, divide 80 by 5 to find a value for $\blacksquare$.

4. $2 \times \blacksquare = 3 \times 10$ 15
5. $5 \times \blacksquare = 4 \times 10$ 8
6. $\frac{3}{7} = \frac{12}{\blacksquare}$ 28
7. $\frac{6}{8} = \frac{\blacksquare}{100}$ 75

Exercises

Use cross products to find the missing term.

1. $\frac{1}{3} = \frac{6}{\blacksquare}$ 18
2. $\frac{1}{7} = \frac{\blacksquare}{49}$ 7
3. $\frac{2}{5} = \frac{6}{\blacksquare}$ 15
4. $\frac{2}{3} = \frac{\blacksquare}{9}$ 6
5. $\frac{2}{9} = \frac{16}{\blacksquare}$ 72
6. $\frac{8}{32} = \frac{\blacksquare}{4}$ 1
7. $\frac{25}{30} = \frac{5}{\blacksquare}$ 6
8. $\frac{30}{48} = \frac{\blacksquare}{8}$ 5
9. $\frac{12}{24} = \frac{1}{\blacksquare}$ 2
10. $\frac{14}{18} = \frac{\blacksquare}{9}$ 7
11. $\frac{6}{8} = \frac{9}{\blacksquare}$ 12
12. $\frac{10}{14} = \frac{\blacksquare}{21}$ 15
13. $\frac{12}{15} = \frac{20}{\blacksquare}$ 25
14. $\frac{2}{12} = \frac{\blacksquare}{30}$ 5
15. $\frac{12}{32} = \frac{15}{\blacksquare}$ 40

265

LESSON OUTCOME

Use cross products to find the missing term in two equivalent fractions

Prerequisite Skills

Multiply and divide whole numbers

Checking Prerequisite Skills

Multiply.

1. 8×30 240 **2.** 9×14 126 **3.** 12×25 300

Divide.

4. $4\overline{)300}$ 75 **5.** $14\overline{)210}$ 15 **6.** $18\overline{)90}$ 5

RELATED ACTIVITIES

- Have students complete chains of equivalent fractions similar to the following. Tell the students that they may find the missing numbers in any order for a chain.

1. $\frac{2}{3} = \frac{\blacksquare}{18} = \frac{6}{\blacksquare} = \frac{\blacksquare}{12} = \frac{4}{\blacksquare}$
2. $\frac{3}{\blacksquare} = \frac{\blacksquare}{20} = \frac{12}{\blacksquare} = \frac{9}{12} = \frac{\blacksquare}{28}$

- Have students find the missing factors for exercises similar to the following.

1. $3 \times \square = 4 \times 6$
2. $\square \times 8 = 4 \times 4$
3. $9 \times 4 = \square \times 6$

- Provide exercises similar to the following. Have students write = or ≠ to form true statements.

1. $\frac{2}{3}$ ◯ $\frac{3}{5}$ 2. $\frac{9}{12}$ ◯ $\frac{3}{4}$
3. $\frac{2}{9}$ ◯ $\frac{1}{4}$ 4. $\frac{9}{15}$ ◯ $\frac{3}{5}$

LESSON ACTIVITY

Before Using the Page

- Have students suggest several examples of two equivalent fractions and write them on the board. Review that equivalent fractions have equal cross products.

  Ask the students to find the missing term for $\frac{6}{9} = \frac{\blacksquare}{15}$. Because there is no whole number to relate the denominators 9 and 15 by multiplication or division, develop that the missing term may be found by thinking of cross products.

$$\frac{6}{9} = \frac{\blacksquare}{15}$$
$$9 \times \blacksquare = 6 \times 15$$
$$9 \times \blacksquare = 90$$

  Direct students to consider how the missing factor $\blacksquare$ may be found in the multiplication sentence. Record the division sentence $90 \div 9 = \blacksquare$. Have a student explain the procedure on the board.

Using the Page

- Have a student read the word problem. Emphasize that although the numerator for $\frac{\blacksquare}{100}$ is not yet known, it is known that $\frac{3}{4}$ and $\frac{\blacksquare}{100}$ are equivalent fractions, and thus their cross products must be equal. Have students help to complete the division $4\overline{)300}$ on the board.

Working Together: Ex. 1-3 deal with determining the cross products. Ex. 4 and 5 concentrate on finding the missing term by using division. These skills are applied in completing Ex. 6 and 7. Have the students write solutions using the steps indicated in the worked example.

Exercises: For Ex. 1-4, the necessary calculations can be done mentally by many students. It is important for them to explain the procedure when they have completed the exercises.

Assessment

Use cross products to find the missing term.

1. $\frac{3}{5} = \frac{12}{\blacksquare}$ 20 **2.** $\frac{8}{32} = \frac{\blacksquare}{4}$ 1 **3.** $\frac{10}{12} = \frac{25}{\blacksquare}$ 30

LESSON OUTCOME

Compare fractions with like denominators; compare fractions with unlike denominators by writing equivalent fractions with like denominators

Materials

models for $\frac{3}{8}$, $\frac{5}{8}$, $\frac{6}{8}$, $\frac{7}{8}$, $\frac{2}{5}$, $\frac{4}{5}$, and $\frac{1}{2}$, prepared from copies of the squares on page T 393

Vocabulary

like denominators, unlike denominators, common denominators

Prerequisite Skills

Find the missing term in two equivalent fractions

Checking Prerequisite Skills

Find the missing term.

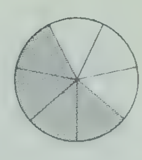
1. $\frac{2}{3} = \frac{\blacksquare}{9}$ 6
2. $\frac{3}{4} = \frac{\blacksquare}{20}$ 15
3. $\frac{7}{25} = \frac{21}{\blacksquare}$ 75
4. $\frac{2}{7} = \frac{10}{\blacksquare}$ 35

Background

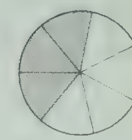
The concept of *least common denominator* is introduced at a later level. At this time, a common denominator for two fractions with unlike denominators is found by multiplying their denominators. The result may be the least common denominator, but not necessarily. For example, to compare $\frac{2}{3}$ and $\frac{3}{5}$, their equivalent fractions $\frac{10}{15}$ and $\frac{9}{15}$ are compared; for $\frac{3}{8}$ and $\frac{4}{10}$, $\frac{30}{80}$ and $\frac{32}{80}$ are compared.

Comparing Fractions

To compare fractions with like denominators, compare the numerators.



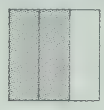
$4 > 3$, so $\frac{4}{7} > \frac{3}{7}$.



Like denominators are often called **common denominators**.

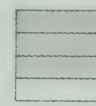
4 is greater than 3, so $\frac{4}{7}$ is greater than $\frac{3}{7}$.

To compare fractions with unlike denominators, first write equivalent fractions with like denominators.



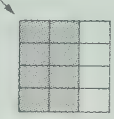
$\frac{2}{3}$ becomes $\frac{8}{12}$

$\frac{3}{4}$ becomes $\frac{9}{12}$

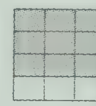


Then, compare the numerators.

$\frac{8}{12}$



$\frac{8}{12} < \frac{9}{12}$, so $\frac{2}{3} < \frac{3}{4}$.



$\frac{9}{12}$

$\frac{8}{12}$ is less than $\frac{9}{12}$, so $\frac{2}{3}$ is less than $\frac{3}{4}$.

Here is a way to find like denominators for two fractions.

For $\frac{3}{5}$ and $\frac{4}{7}$, the unlike denominators are 5 and 7.

$\frac{3}{5} \xrightarrow{5 \times 7} \frac{\quad}{5 \times 7}$ $\qquad$ $\frac{4}{7} \xrightarrow{7 \times 5} \frac{\quad}{7 \times 5}$

Since $5 \times 7 = 7 \times 5$, the two new denominators are alike.

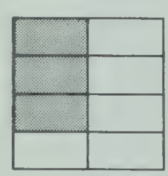
$\frac{3}{5} \underset{5 \times 7}{\overset{3 \times 7}{=}} \frac{21}{35}$ $\qquad$ $\frac{4}{7} \underset{7 \times 5}{\overset{4 \times 5}{=}} \frac{20}{35}$ $\qquad$ $\frac{21}{35} > \frac{20}{35}$, so $\frac{3}{5} > \frac{4}{7}$.

266

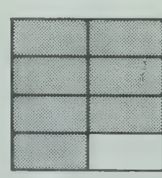
LESSON ACTIVITY

Before Using the Pages

- Display a model of $\frac{3}{8}$ and a model of $\frac{7}{8}$ and have students identify the numbers represented. Ask which number is greater and have students explain their answers.



$\frac{3}{8}$



$\frac{7}{8}$

Repeat the procedure using models of $\frac{5}{8}$ and $\frac{6}{8}$. Ask the students how they can tell which of two fractions is greater without referring to the models. They will likely suggest that they can compare the numerators. Have them compare $\frac{2}{5}$ and $\frac{4}{5}$ and use the models to check their answer. Repeat the procedure for $\frac{1}{2}$ and $\frac{3}{8}$. In this case, although 3 is greater than 1, $\frac{3}{8}$ is less than $\frac{1}{2}$. Ask students to suggest why the procedure of checking the numerators cannot apply in this last example. Lead them to realize that the numerators reveal which fraction is greater only if the denominators are identical. Have students express $\frac{1}{2}$ as $\frac{4}{8}$ and then compare $\frac{4}{8}$ and $\frac{3}{8}$.

Using the Pages

- Lead the students through the three examples on page 266. For $\frac{4}{7}$ and $\frac{3}{7}$, emphasize that the denominators are identical. For $\frac{2}{3}$ and $\frac{3}{4}$, emphasize that the denominators are different and that the fractions in each pair must first be expressed as equivalent fractions so that like denominators are obtained. Draw attention to the diagrams that show $\frac{2}{3}$ as $\frac{8}{12}$, and $\frac{3}{4}$ as $\frac{9}{12}$. Emphasize that $\frac{8}{12}$ represents the same part of the whole as $\frac{2}{3}$; thus, they are different names for the same number. Similarly, $\frac{3}{4}$ and $\frac{9}{12}$ are different names for the same number.

  The third example demonstrates a procedure for obtaining like denominators for two fractions without using

Working Together

For the fractions in each pair, give equivalent fractions with like denominators.
Answers may vary

1. $\frac{1}{2}, \frac{4}{9}$ $\frac{9}{18}, \frac{8}{18}$ 2. $\frac{2}{3}, \frac{5}{7}$ $\frac{14}{21}, \frac{15}{21}$ 3. $\frac{3}{4}, \frac{4}{6}$ $\frac{9}{12}, \frac{8}{12}$

Use >, <, or = to make true statements.

4. $\frac{1}{2}$ ◯ $\frac{4}{9}$ > 5. $\frac{2}{3}$ ◯ $\frac{5}{7}$ <

Exercises

For each of these, show equivalent fractions with like denominators. Then, use >, <, or = to make a true statement.

Example: $\frac{4}{9} = \frac{28}{63}$

$\frac{3}{7} = \frac{27}{63}$

$\frac{28}{63} > \frac{27}{63}$ so $\frac{4}{9} > \frac{3}{7}$

Equivalent fractions may vary

1. $\frac{3}{4}$ ◯ $\frac{4}{7}$ > $\frac{21}{28}, \frac{16}{28}$ 2. $\frac{6}{16}$ ◯ $\frac{3}{8}$ = $\frac{6}{16}, \frac{6}{16}$
3. $\frac{1}{3}$ ◯ $\frac{2}{5}$ < $\frac{5}{15}, \frac{6}{15}$ 4. $\frac{3}{7}$ ◯ $\frac{4}{10}$ > $\frac{30}{70}, \frac{28}{70}$
5. $\frac{10}{25}$ ◯ $\frac{6}{15}$ = $\frac{2}{5}, \frac{2}{5}$ 6. $\frac{2}{9}$ ◯ $\frac{1}{5}$ > $\frac{10}{45}, \frac{9}{45}$
7. $\frac{7}{12}$ ◯ $\frac{2}{3}$ < $\frac{7}{12}, \frac{8}{12}$ 8. $\frac{7}{9}$ ◯ $\frac{5}{6}$ < $\frac{14}{18}, \frac{15}{18}$

List from least to greatest.

*9. $\frac{2}{7}, \frac{3}{7}, \frac{3}{8}$ $\frac{2}{7}, \frac{3}{8}, \frac{3}{7}$ *10. $\frac{4}{12}, \frac{6}{15}, \frac{6}{14}$ $\frac{4}{12}, \frac{6}{15}, \frac{6}{14}$

List from greatest to least.

*11. $\frac{3}{4}, \frac{3}{5}, \frac{4}{5}$ $\frac{4}{5}, \frac{3}{4}, \frac{3}{5}$ *12. $\frac{3}{7}, \frac{4}{8}, \frac{5}{9}$ $\frac{5}{9}, \frac{4}{8}, \frac{3}{7}$

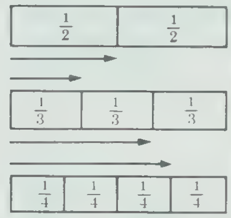
Pies are often cut into sixths or eighths.

13. Which is more, $\frac{1}{6}$ of a pie or $\frac{1}{8}$ of a pie? $\frac{1}{6}$
14. Which is more, $\frac{4}{6}$ of a pie or $\frac{5}{8}$ of a pie? $\frac{4}{6}$
15. Which is more, 2 pieces of a pie that is cut into sixths or 3 pieces of a pie that is cut into eighths? the 3 pieces
*16. $\frac{5}{6}$ of the blueberry pie has been eaten. $\frac{7}{8}$ of the apple pie has been eaten. Of which pie is there more left? the blueberry pie

267

RELATED ACTIVITIES

• Have students use the fraction strips described in *Related Activities* on page T 285, for a visual comparison of two given fractions. For each comparison, have them write one or two sentences as shown below.



$\frac{1}{2} > \frac{1}{3}$, $\frac{1}{3} < \frac{1}{2}$

$\frac{2}{3} < \frac{3}{4}$, $\frac{3}{4} > \frac{2}{3}$

• For the given set of fractions, have students compare each fraction with $\frac{1}{2}$ and write each in the appropriate column of a chart similar to the following.

$\frac{1}{3}, \frac{3}{5}, \frac{2}{4}, \frac{5}{9}, \frac{2}{3}$

$\frac{6}{12}, \frac{5}{8}, \frac{3}{7}, \frac{5}{6}, \frac{4}{8}$

$\frac{2}{9}, \frac{7}{10}, \frac{8}{15}, \frac{5}{12}, \frac{4}{7}$

Less than $\frac{1}{2}$	Equal to $\frac{1}{2}$	Greater than $\frac{1}{2}$
$\frac{1}{3}$	$\frac{2}{4}$	$\frac{3}{5}$

diagrams. The new denominator, 35, is the product of the unlike denominators 5 and 7. After discussing the steps shown, develop similar steps on the board for $\frac{2}{3}$ and $\frac{3}{4}$ of the previous example.

$$\frac{2}{3} = \frac{2 \times 4}{3 \times 4} = \frac{8}{12} \qquad \frac{3}{4} = \frac{3 \times 3}{4 \times 3} = \frac{9}{12}$$

Working Together: Have students follow the preceding format to obtain equivalent fractions. However, keep in mind that answers may vary because the concept of least common denominator is not dealt with at this time. For example, for Ex. 3, some students may express each fraction with a denominator of 24, others may use 12. Have each method shown on the board.

Exercises: Draw attention to the fact that each of Ex. 9-12 involves three fractions which will have to be expressed with like denominators. You may need to work one of these exercises on the board with the students to provide them with an example.

Ex. 16 involves the comparison of two fractions. Note that it is not necessary to use subtraction to answer the question "Of which pie is there more left?" By determining that $\frac{5}{6}$ is less than $\frac{7}{8}$, students can reason that less blueberry pie has been eaten and thus more of it is left. However, some students may intuitively think of comparing $\frac{1}{6}$ and $\frac{1}{8}$, rather than $\frac{5}{6}$ and $\frac{7}{8}$.

Assessment

Show equivalent fractions with like denominators. Then use >, <, or = to make a true statement.

1. $\frac{2}{3}$ ◯ $\frac{5}{8}$ > 2. $\frac{9}{15}$ ◯ $\frac{6}{10}$ = 3. $\frac{3}{4}$ ◯ $\frac{4}{5}$ <

Solve. $\frac{16}{24}$ $\frac{15}{24}$ $\frac{54}{90}$ $\frac{54}{90}$ $\frac{15}{20}$ $\frac{16}{20}$

4. Which is more, $\frac{5}{6}$ of a pie or $\frac{7}{8}$ of a pie? $\frac{7}{8}$

LESSON OUTCOME

Express a whole number as an improper fraction; express a number in mixed form as an improper fraction

Materials

models for wholes, halves, thirds, fourths, fifths, and eighths prepared from copies of the squares on page T 393

Vocabulary

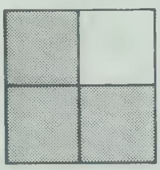
improper fraction, proper fraction, mixed form

Prerequisite Skills

Write fractions for numbers less than one; simplify an expression involving multiplication and addition

Checking Prerequisite Skills

Write the fraction.

1.  ▪ is shaded. $\frac{3}{4}$

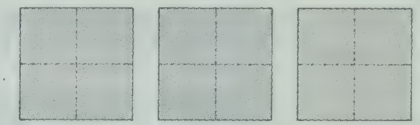
2. The numerator is 5 and the denominator is 8. $\frac{5}{8}$

Complete.

3. $(4 \times 3) + 2$ 14
4. $(5 \times 6) + 1$ 31
5. $(3 \times 9) + 4$ 31
6. $(2 \times 12) + 5$ 29

Changing to Improper Fractions

A whole number can be written as an **improper fraction**.



3 tiles or $\frac{12}{4}$ tiles

In an improper fraction, the numerator is greater than or equal to the denominator.

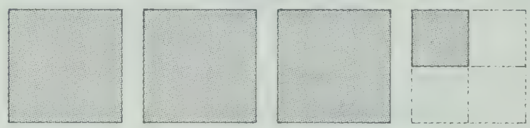
In a **proper fraction**, the denominator is greater than the numerator.

To find how many fourths there are in the whole number 3, multiply

3×4 fourths = 12 fourths or $\frac{12}{4}$.

3 groups of 4 fourths

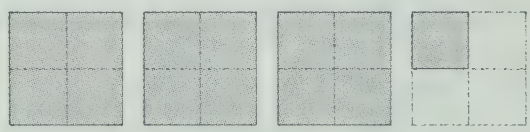
A number in **mixed form** can represent a whole amount together with some part of a whole.



$3\frac{1}{4}$ tiles

3 and $\frac{1}{4}$

It can also be written as an improper fraction.



$\frac{13}{4}$ tiles

For a number in mixed form, change the whole number first.

For $3\frac{1}{4}$, 3 and $\frac{1}{4} = \frac{12}{4}$ and $\frac{1}{4}$, or $\frac{13}{4}$.

Change the whole number first.

$3\frac{1}{4}$

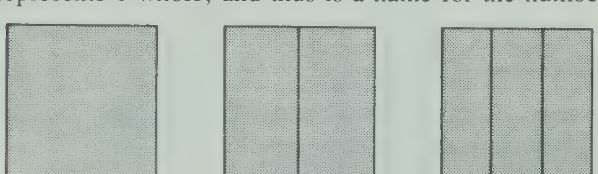
Multiply, then add.

268

LESSON ACTIVITY

Before Using the Pages

- Develop that fractions may be used to name whole numbers. For example, use models to show that each of $\frac{2}{2}$, $\frac{3}{3}$, and $\frac{4}{4}$ represents 1 whole, and thus is a name for the number 1.



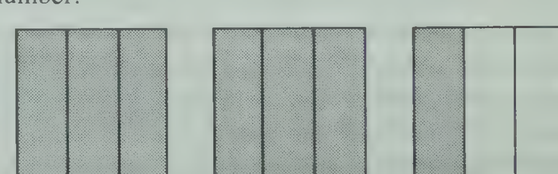
1 $\qquad \frac{2}{2} \qquad \frac{3}{3}$

Similarly, develop that 2 wholes may be thought of as 4 halves, 6 thirds, 8 fourths, and so on, and 5 wholes may be thought of as 10 halves, 15 thirds, and 20 fourths. Write numerals on the board to show different names for the same number.

$1 = \frac{2}{2} = \frac{3}{3} = \frac{4}{4} \qquad 5 = \frac{10}{2} = \frac{15}{3} = \frac{20}{4}$

You may wish to discuss that such numerals as 5 and $\frac{5}{1}$ (5 wholes) represent the same number.

- Use models to lead students to recall such numbers as $2\frac{1}{3}$, $1\frac{2}{5}$, and $4\frac{3}{8}$. For example, display two models of wholes and a model of one-third and ask how many wholes there are and what part of a whole is shown. Write the numeral $2\frac{1}{3}$ on the board and review that it is read "two and one-third". Mark the wholes into thirds and have a student count the thirds. Emphasize that $2\frac{1}{3}$ and $\frac{7}{3}$ are different names for the same number.



$2\frac{1}{3}$ or $\frac{7}{3}$

Using the Pages

- The worked examples introduce the terms *improper fraction*, *proper fraction*, and *mixed form*. Point out that the

Working Together

Multiply to write improper fractions for the whole number.

3×2 3×3 3×4 ...

1.	3	$\frac{\blacksquare}{2}$ 6	$\frac{\blacksquare}{3}$ 9	$\frac{\blacksquare}{4}$ 12	$\frac{\blacksquare}{5}$ 15
2.	7	$\frac{\blacksquare}{2}$ 14	$\frac{\blacksquare}{3}$ 21	$\frac{\blacksquare}{5}$ 35	$\frac{\blacksquare}{8}$ 56

Change the whole number first. Then write each of these as an improper fraction.

Example: $2\frac{7}{8}$ is $\frac{16}{8}$ and $\frac{7}{8}$, or $\frac{23}{8}$.

$\frac{3}{3}$ and $\frac{2}{3}$, or $\frac{5}{3}$ $\frac{10}{2}$ and $\frac{1}{2}$, or $\frac{11}{2}$ $\frac{18}{6}$ and $\frac{5}{6}$, or $\frac{23}{6}$

3. $1\frac{2}{3}$ 4. $5\frac{1}{2}$ 5. $3\frac{5}{6}$

Multiply, then add to change each of these to an improper fraction.

Example: For $6\frac{4}{7}$, write $\frac{6 \times 7 + 4}{7}$, or $\frac{46}{7}$

6. $4\frac{1}{8}$ $6\ \frac{4 \times 8 + 1}{8}$, or $\frac{33}{8}$

7. $1\frac{3}{10}$ $\frac{1 \times 10 + 3}{10}$, or $\frac{13}{10}$

8. $7\frac{3}{5}$ $\frac{7 \times 5 + 3}{5}$, or $\frac{38}{5}$

Exercises

Complete.

1.	2	$\frac{\blacksquare}{2}$ 4	$\frac{\blacksquare}{4}$ 8	$\frac{\blacksquare}{7}$ 14	$\frac{\blacksquare}{12}$ 24
2.	5	$\frac{\blacksquare}{2}$ 10	$\frac{\blacksquare}{5}$ 25	$\frac{\blacksquare}{6}$ 30	$\frac{\blacksquare}{10}$ 50

Change to improper fractions.

3. $1\frac{1}{2}$ $\frac{3}{2}$ 4. $4\frac{1}{5}$ $\frac{21}{5}$ 5. $3\frac{3}{4}$ $\frac{15}{4}$ 6. $7\frac{2}{3}$ $\frac{23}{3}$

7. $1\frac{3}{7}$ $\frac{10}{7}$ 8. $9\frac{3}{8}$ $\frac{75}{8}$ 9. $2\frac{1}{2}$ $\frac{5}{2}$ 10. $6\frac{5}{8}$ $\frac{53}{8}$

11. $2\frac{5}{9}$ $\frac{23}{9}$ 12. $8\frac{1}{6}$ $\frac{49}{6}$ 13. $1\frac{4}{5}$ $\frac{9}{5}$ 14. $3\frac{7}{12}$ $\frac{43}{12}$

15. $5\frac{6}{7}$ $\frac{41}{7}$ 16. $12\frac{1}{4}$ $\frac{49}{4}$ 17. $4\frac{2}{3}$ $\frac{14}{3}$ 18. $2\frac{1}{3}$ $\frac{7}{3}$

Complete.

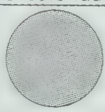
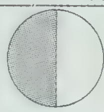
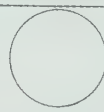
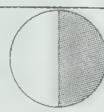
19. 

$3\frac{3}{8}$ oranges = $\frac{\blacksquare}{8}$ oranges   27

20. 

$2\frac{3}{4}$ waffles = $\frac{\blacksquare}{4}$ waffles   11

On some calendar pages you will see this information.

New moon  on the 2nd	First quarter  on the 9th	Full moon  on the 16th	Third quarter  on the 24th

1. Explain why the terms "first quarter" and "third quarter" are used as shown.

How many of these terms can you explain? Answers will vary

2. quarter note   3. halfback   4. quarterfinal
5. half brother   6. half-mast   7. quarter horse
8. half nelson   9. halfhearted   10. quarterly

PROBLEM SOLVING

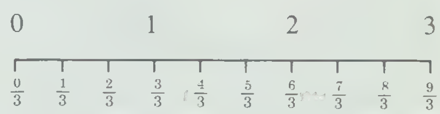
269

RELATED ACTIVITIES

- Some students may be able to suggest and explain other terms similar to those in Ex. 2-10 of the *Problem Solving* feature.
- Ask students to write the first ten terms for number patterns similar to the following, to emphasize that whole numbers may be named by using fractions.

$$1, \frac{2}{2}, \frac{3}{3}, \frac{4}{4}, \ldots$$
$$4, \frac{8}{2}, \frac{12}{3}, \frac{16}{4}, \ldots$$

- Provide students with copies of unmarked number lines from page T 389. Ask them to mark the lines to show whole numbers above the lines and fractions below, as shown for thirds. By referring to the number line, students can rename improper fractions as numbers in mixed form and vice versa.



$\frac{4}{3} = 1\frac{1}{3}$ $2\frac{2}{3} = \frac{8}{3}$

fractions involved in the preceding lessons were proper fractions. For some of the numerals on the board from the activities for *Before Using the Pages*, have students classify each as a proper fraction, an improper fraction, or a number in mixed form.

Ask students to explain how the diagrams for $3\frac{1}{4}$ and $\frac{13}{4}$ help to express $3\frac{1}{4}$ as the improper fraction $\frac{13}{4}$. Then discuss the procedure at the bottom of page 268, and note that the use of multiplication and addition is an efficient method because it does not rely on a diagram.

Working Together: Ex. 1 and 2 deal with expressing a whole number as an improper fraction. Ex. 3-5 apply that skill in expressing a number in mixed form as an improper fraction. Discuss the example provided. Ex. 6-8 emphasize the use of multiplication and addition to obtain an improper fraction. Color is shown in the example to help identify the steps. Use other similar examples as required.

Exercises: Ensure that the students take care in writing numerals for fractions. The numerator and denominator are separated by a horizontal bar and, in mixed form, the whole number is written so that it is separate from and larger than the numerals for the numerator and denominator.

Problem Solving: Terms such as *half* and *quarter* are encountered in situations outside the mathematics classroom. Some of these are presented here. Provide opportunities for the students to consult dictionaries and discuss the terms over a period of several days.

Assessment

Complete.

1.	3	$\frac{\blacksquare}{2}$ 6	$\frac{\blacksquare}{5}$ 15	$\frac{\blacksquare}{8}$ 24	$\frac{\blacksquare}{10}$ 30

Change to improper fractions.

2. $3\frac{4}{7}$ $\frac{25}{7}$ 3. $1\frac{2}{3}$ $\frac{5}{3}$ 4. $10\frac{3}{5}$ $\frac{53}{5}$

LESSON OUTCOME

Express an improper fraction as a whole number or as a number in mixed form

Materials

models for fourths and other fractions prepared from copies of the squares on page T 393

Vocabulary

biological, family tree, ancestors, generation

Prerequisite Skills

Divide whole numbers

Checking Prerequisite Skills

Divide.

1. $3\overline{)47}$ 15 R2 **2.** $7\overline{)180}$ 25 R5

3. $12\overline{)205}$ 17 R1 **4.** $10\overline{)169}$ 16 R9

Changing Improper Fractions

Eddie estimates that he was able to watch 21 quarters of football during the season. How many whole games is 21 quarters equivalent to?

Each football game has 4 quarters.

Eddie has watched about $\frac{21}{4}$ games.

To change the improper fraction $\frac{21}{4}$ to a number in mixed form,

think of $\frac{21}{4}$ as $\frac{20}{4}$ and $\frac{1}{4}$, or 5 and $\frac{1}{4}$. $5\frac{1}{4}$

OR

divide to find how many groups of 4 fourths there are in 21.

$$\begin{array}{r} 5 \\ 4\overline{)21} \\ \underline{20} \\ 1 \end{array}$$

5 whole games

1 quarter of another game

$5\frac{1}{4}$

There is as much playing time in $5\frac{1}{4}$ football games as there is in 21 quarters.

Working Together

Show the quotient and remainder as a number in mixed form.

Example: For $\begin{array}{r} 4 \\ 7\overline{)31} \\ \underline{28} \\ 3 \end{array}$, write $4\frac{3}{7}$.

1. $\begin{array}{r} 3 \\ 5\overline{)16} \\ \underline{15} \\ 1 \end{array}$ $3\frac{1}{5}$ **2.** $\begin{array}{r} 5 \\ 3\overline{)17} \\ \underline{15} \\ 2 \end{array}$ $5\frac{2}{3}$ **3.** $\begin{array}{r} 6 \\ 8\overline{)53} \\ \underline{48} \\ 5 \end{array}$ $6\frac{5}{8}$

Divide the numerator by the denominator. Show the quotient and remainder as a number in mixed form.

Example: For $\frac{23}{6}$,

divide $\begin{array}{r} 3 \\ 6\overline{)23} \\ \underline{18} \\ 5 \end{array}$ and write $3\frac{5}{6}$.

4. $\frac{24}{5}$ $4\frac{4}{5}$ **5.** $\frac{20}{3}$ $6\frac{2}{3}$

6. $\frac{64}{8}$ 8 **7.** $\frac{53}{12}$ $4\frac{5}{12}$

Sometimes you get a whole number.

270

LESSON ACTIVITY

Before Using the Pages

- Display models of two wholes and three-fourths and ask what number is shown. Have a student write the numeral $2\frac{3}{4}$ on the board. Ask why it is described as a number in mixed form. Establish that it names a number of wholes and part of another whole. Review the use of multiplication and addition to express $2\frac{3}{4}$ as $\frac{11}{4}$.
- Display 13 fourths cut from models of fourths. Have students arrange the fourths to show the number of wholes that can be formed and the number of fourths for part of a whole. Emphasize that $\frac{13}{4}$ and $3\frac{1}{4}$ are names for the same number. Use other examples as required. Then ask how the answers can be obtained without the use of models or diagrams. Have students write examples on the board to demonstrate their ideas.

Using the Pages

- Have a student read the word problem to introduce the situation. Discuss that $\frac{20}{4}$ (rather than $\frac{16}{4}$, $\frac{12}{4}$, $\frac{8}{4}$, and $\frac{4}{4}$) is the best improper fraction to be expressed as a whole number for $\frac{21}{4}$. Since $\frac{20}{4}$ is another name for 5, $\frac{21}{4}$ is equal to $5\frac{1}{4}$. Discuss the use of division as an efficient method for determining a number in mixed form. Emphasize that the remainder indicates the number of parts of a whole left over and the dividend indicates the number of equal parts in all.

Working Together: Ex. 1-3 deal with interpreting the divisor, quotient, and remainder in a division as a number in mixed form. This skill is applied in completing Ex. 4-7. Note that the remainder for Ex. 6 is zero. Discuss that there are no parts of a whole left over when the remainder is zero: the result is a whole number.

Exercises: Remind the students to take care in writing a horizontal bar for each fraction.

Exercises

Write each improper fraction as a number in mixed form or as a whole number.

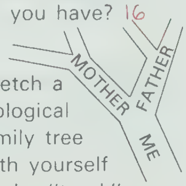
1. $\frac{11}{2}$ $5\frac{1}{2}$ 2. $\frac{4}{3}$ $1\frac{1}{3}$ 3. $\frac{81}{9}$ 9 4. $\frac{13}{4}$ $3\frac{1}{4}$ 5. $\frac{41}{10}$ $4\frac{1}{10}$
6. $\frac{41}{12}$ $3\frac{5}{12}$ 7. $\frac{20}{7}$ $2\frac{6}{7}$ 8. $\frac{27}{8}$ $3\frac{3}{8}$ 9. $\frac{36}{6}$ 6 10. $\frac{12}{5}$ $2\frac{2}{5}$
11. $\frac{33}{4}$ $8\frac{1}{4}$ 12. $\frac{17}{10}$ $1\frac{7}{10}$ 13. $\frac{32}{3}$ $10\frac{2}{3}$ 14. $\frac{23}{12}$ $1\frac{11}{12}$ 15. $\frac{64}{7}$ $9\frac{1}{7}$
16. $\frac{59}{5}$ $11\frac{4}{5}$ 17. $\frac{144}{12}$ 12 18. $\frac{130}{7}$ $18\frac{4}{7}$ 19. $\frac{112}{10}$ $11\frac{2}{10}$ 20. $\frac{223}{8}$ $27\frac{7}{8}$

Write a number in mixed form to complete each sentence.

21. There are 30 d in June. There are $\frac{30}{7}$ weeks or ▒ weeks in June. $4\frac{2}{7}$
22. 12 eggs fill a carton. 91 eggs will fill $\frac{91}{12}$ cartons or ▒ cartons. $7\frac{7}{12}$
23. 4 wheels make up a set. 23 wheels will make up ▒ sets of wheels. $5\frac{3}{4}$
24. 5 riders make up a carload. 33 riders will make up ▒ carloads. $6\frac{3}{5}$
25. 3 lemons are needed for each recipe. With 8 lemons you could make ▒ recipes. $2\frac{2}{3}$
26. Each guest was served an eighth of a pie. There were 21 guests, so ▒ pies were used. $2\frac{5}{8}$
27. The restaurant sold 151 "quarter chickens". This would be ▒ "whole chickens". $37\frac{3}{4}$
28. 101 quarter-dollars are worth ▒ whole dollars. $25\frac{1}{4}$

Each person has one pair of *biological* parents.

1. How many biological grandparents do you have? 4
2. How many biological great-grandparents do you have? 8
3. How many biological great-great-grandparents do you have? 16
4. Sketch a biological family tree with yourself as the "trunk".



Answers will vary.

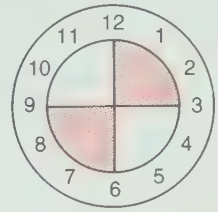
5. If you continue the number pattern that shows how many biological *ancestors* you had in each *generation*, how many would there be 20 generations ago? 1 048 576
6. If each pair of grandparents had 3 children and each of these children had 3 children, how many cousins would you have? 12

PROBLEM SOLVING

271

RELATED ACTIVITIES

- Select exercises from page 85 and have students show the quotient and remainder as a number in mixed form.
- Have students label a copy of the large circle on page T383 as a dial clock face. Have them mark the circle to show fourths and refer to the diagram to answer questions similar to the following.



Through how many quarter-hours does the minute hand move

1. in $1\frac{1}{4}$ h?
2. in $2\frac{2}{4}$ h?
3. from 03:45 to 08:00?

The following times are measured in quarter-hours. Write each as a number in mixed form.

4. $\frac{7}{4}$ h 5. $\frac{14}{4}$ h 6. $\frac{93}{4}$ h

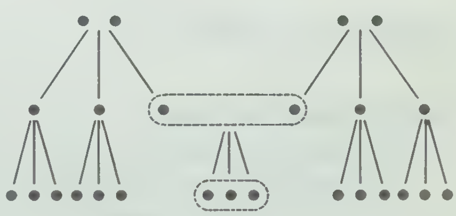
Problem Solving: For Ex. 1-4, the students are directed toward a strategy of finding a pattern with the help of a diagram. Once the pattern is discovered, it is useful in solving Ex. 5. For example, the student represents the present generation. One generation ago, there would be 2 ancestors in that generation. Two generations ago, there would be 4, or 2×2, ancestors in that generation. Three generations ago, there would be 8, or $2 \times 2 \times 2$, ancestors, and four generations ago there would be 16, or $2 \times 2 \times 2 \times 2$, ancestors. (The generation indicates the number of times 2 is used as a factor for the number of ancestors.) The answer for Ex. 5, then, would be found by using 2 as a factor twenty times.

Some students may suggest that the answer to Ex. 6 is 18. If so, remind them that they are included as one of the children, and each of their parents is a child of one pair of grandparents. Thus, the following family tree suggests that they would have 12 cousins.

(two pairs of grandparents)

(parents)

(children)



Assessment

Write each improper fraction as a number in mixed form or as a whole number.

1. $\frac{37}{6}$ $6\frac{1}{6}$ 2. $\frac{121}{11}$ 11 3. $\frac{43}{5}$ $8\frac{3}{5}$ 4. $\frac{88}{9}$ $9\frac{7}{9}$

Write a number in mixed form to complete the sentence.

5. 2 eggs are needed for each recipe. With 9 eggs you could make ▒ recipes. $4\frac{1}{2}$

Pages 272-273

LESSON OUTCOME

Add two proper fractions or two numbers in mixed form with like denominators, regrouping

Materials

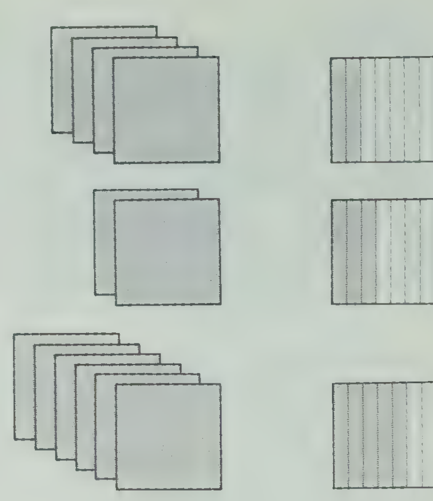
models for wholes and fourths prepared from copies of the squares on page T 393

Adding Fractions

Add $4\frac{2}{7}$ and $2\frac{3}{7}$.

$$\begin{array}{r} 4\frac{2}{7} \\ 2\frac{3}{7} \\ \hline 6\frac{5}{7} \end{array}$$

Add sevenths.
Then add ones.



The sum of $4\frac{2}{7}$ and $2\frac{3}{7}$ is $6\frac{5}{7}$.

Add $2\frac{4}{5}$ and $1\frac{3}{5}$.

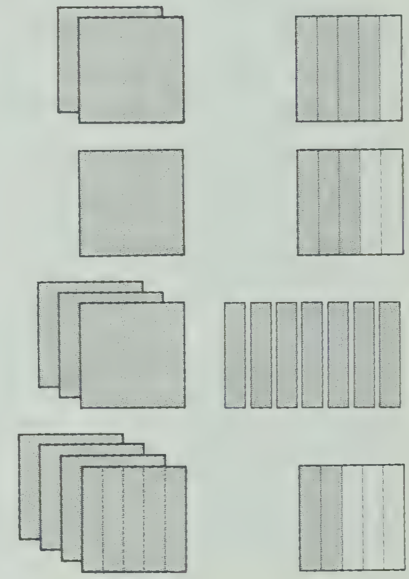
$$\begin{array}{r} 2\frac{4}{5} \\ 1\frac{3}{5} \\ \hline 3\frac{7}{5} \end{array}$$

Add fifths.
Then add ones.

or

$4\frac{2}{5}$

Remember that $\frac{7}{5} = 1\frac{2}{5}$, so $3\frac{7}{5} = 4\frac{2}{5}$.



The sum of $2\frac{4}{5}$ and $1\frac{3}{5}$ is $4\frac{2}{5}$.

272

LESSON ACTIVITY

Before Using the Pages

- Write several exercises on the board to review the procedure of expressing an improper fraction as a number in mixed form or as a whole number.

  $\frac{7}{3}$ $\quad$ $\frac{13}{9}$ $\quad$ $\frac{24}{3}$ $\quad$ $\frac{15}{7}$ $\quad$ $\frac{32}{5}$

- Use whole models and models cut into four equal parts. Have one student represent $1\frac{1}{4}$ with models. Have another student represent $2\frac{2}{4}$. Join the two groups of models. Ask how many fourths and how many wholes there are and then ask what number is represented. Repeat the procedure for $1\frac{1}{4}$ and $2\frac{3}{4}$, leading the students to suggest regrouping the 4 fourths as 1 more whole. Then have students demonstrate the sum of $1\frac{3}{4}$ and $1\frac{2}{4}$ and explain the necessary regrouping.

Using the Pages

- Page 272 presents two examples of addition of fractions. The first, $4\frac{2}{7} + 2\frac{3}{7}$, involves no regrouping. Relate the fractions to the diagrams provided and note the order of carrying out the addition — add the sevenths, then add the ones.

  The second example involves regrouping, but a similar order is used to add the numbers — add the fifths, then add the ones. Draw attention to the explanation in the "thought cloud" at the bottom of page 272 and relate the explanation to the corresponding diagrams showing the regrouping.

Working Together: Note that at this time it is not implied that fractions in sums are to be shown in lowest terms. Thus, the sum $\frac{4}{6}$ is appropriate for Ex. 1. Ex. 1 and 2 concern sums less than one. Ex. 3 and 4 emphasize the order of adding numbers in mixed form. Ex. 5-7 provide the sums and students are to regroup to obtain either a proper fraction or a

Working Together

Add the fractions.

1. $\frac{1}{6} + \frac{3}{6} = \frac{4}{6}$

2. $\frac{5}{8} + \frac{2}{8}$ $\frac{7}{8}$

Add the fractions. Then add the whole numbers.

3. $3\frac{2}{9} + 2\frac{4}{9} = 5\frac{6}{9}$

4. $3\frac{1}{4} + 7\frac{1}{4}$ $10\frac{2}{4}$

Regroup the sum so that it has no improper fraction.

5. $5\frac{2}{3} + 3\frac{2}{3} = 8\frac{4}{3}$ $9\frac{1}{3}$

6. $1\frac{4}{6} + 8\frac{3}{6} = 9\frac{7}{6}$ $10\frac{1}{6}$

7. $4\frac{1}{4} + 2\frac{3}{4} = 6\frac{4}{4}$ 7

Add. Regroup the sum when it shows an improper fraction.

8. $1\frac{1}{5} + 3\frac{4}{5}$ 5

9. $5\frac{5}{12} + 9\frac{2}{12}$ $14\frac{7}{12}$

10. $4\frac{5}{9} + 4\frac{7}{9}$ $9\frac{3}{9}$

Exercises

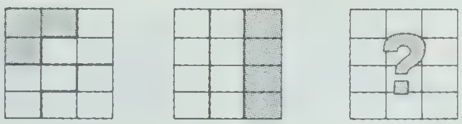
Add. Regroup the sum when it shows an improper fraction.

1. $\frac{7}{9} + \frac{6}{9}$ $1\frac{4}{9}$
2. $4\frac{1}{3} + 5\frac{2}{3}$ 10
3. $1\frac{6}{8} + 2\frac{7}{8}$ $4\frac{5}{8}$
4. $2\frac{1}{5} + \frac{2}{5}$ $2\frac{3}{5}$
5. $7\frac{7}{12} + 5\frac{5}{12}$ 13
6. $4\frac{5}{6} + 3$ $7\frac{5}{6}$
7. $\frac{11}{12} + \frac{6}{12}$ $1\frac{5}{12}$
8. $3\frac{1}{2} + 7\frac{1}{2}$ 11
9. $2\frac{3}{8} + 5\frac{6}{8}$ $8\frac{1}{8}$
10. $1\frac{3}{9} + 5\frac{4}{9}$ $6\frac{7}{9}$
11. $1\frac{2}{7} + 10\frac{5}{7}$ 12
12. $11\frac{7}{10} + 8\frac{6}{10}$ $20\frac{3}{10}$

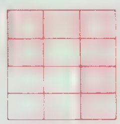
Solve.

13. Clark had $1\frac{1}{2}$ bags of groceries. Molly had $2\frac{1}{2}$ bags of groceries. How many full bags of groceries could they have in all? 4

14. The carpenter estimated that one job would take $1\frac{3}{4}$ workdays and the other job would take $2\frac{3}{4}$ workdays. How many workdays would both jobs take? $4\frac{2}{4}$



$\frac{1}{4}$ + $\frac{1}{3}$ =

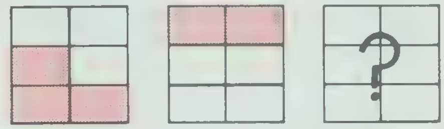


try this

1. How would you complete the picture?
2. How would you complete the addition sentence? $\frac{7}{12}$

RELATED ACTIVITIES

- You may wish to present other diagrams similar to those in the *Try This* feature.



$\frac{1}{2}$ + $\frac{1}{3}$ =

- Some students may need further practice in regrouping fractions as follows.

$1\frac{7}{4} = 2\frac{■}{4}$ $\quad 2\frac{8}{5} = ■\frac{3}{5}$

whole number as in Ex. 7. It is important to discuss the given steps as well as those that the students complete. For Ex. 8-10, ask which sums required regrouping and have students explain why the regrouping was necessary.

Exercises: Remind the students to take care in writing numerals for fractions so that their meaning is clear, for example, careless writing of $2\frac{3}{4}$ might look like $\frac{23}{4}$.

Try This: These exercises enable students to consider the sum of two fractions having unlike denominators. The diagrams can help them to discover that the sum of fourths and thirds is expressed as twelfths.

Assessment

Add. Regroup the sum when it shows an improper fraction.

1. $\frac{6}{10} + \frac{5}{10}$ $1\frac{1}{10}$
2. $4\frac{3}{8} + 2\frac{2}{8}$ $6\frac{5}{8}$
3. $2\frac{4}{5} + 3\frac{3}{5}$ $6\frac{2}{5}$
4. $1\frac{1}{3} + 3\frac{2}{3}$ 5
5. $4\frac{4}{7} + 3$ $7\frac{4}{7}$

Pages 274-275

LESSON OUTCOME

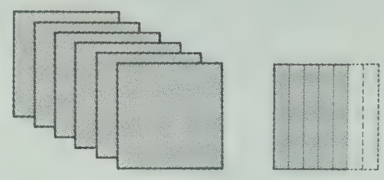
Subtract proper fractions or numbers in mixed form with like denominators, regrouping

Materials
models for wholes, halves, thirds, and other fractions prepared from copies of the squares on page T 393

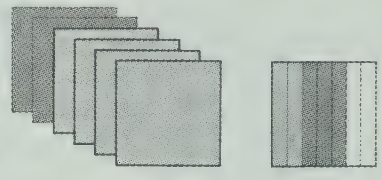
Subtracting Fractions

Subtract $2\frac{3}{7}$ from $6\frac{5}{7}$.

Subtract sevenths.
Then subtract ones.



$$\begin{array}{r} 6\frac{5}{7} \\ 2\frac{3}{7} \\ \hline 4\frac{2}{7} \end{array}$$

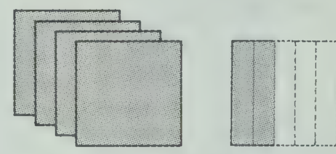


$6\frac{5}{7} - 2\frac{3}{7} = 4\frac{2}{7}$

Subtract $1\frac{3}{5}$ from $4\frac{2}{5}$.

First, look at the fractions in the mixed numbers.

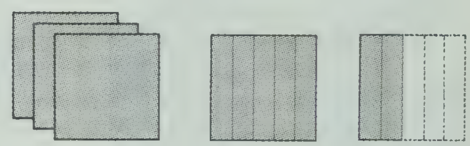
$\frac{2}{5} - \frac{3}{5}$?



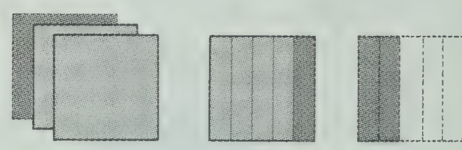
Then, regroup $4\frac{2}{5}$ as $3\frac{7}{5}$.

$4\frac{2}{5}$ becomes $3\frac{7}{5}$

$$\begin{array}{r} 1\frac{3}{5} \\ \hline \end{array} \qquad \begin{array}{r} 1\frac{3}{5} \\ \hline 2\frac{4}{5} \end{array}$$



Next, subtract fifths. Then subtract ones.



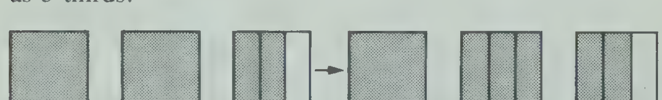
$4\frac{2}{5} - 1\frac{3}{5} = 2\frac{4}{5}$

274

LESSON ACTIVITY

Before Using the Pages

- Review that 1 whole can be thought of as $\frac{2}{2}$, $\frac{3}{3}$, $\frac{4}{4}$, and so on. Then use models to rename a number such as $2\frac{2}{3}$ by regrouping 1 whole as 3 thirds. For this example, a model for 1 whole may be marked on the opposite side to show thirds. The model may then be reversed to regroup 1 whole as 3 thirds.



$2\frac{2}{3}$ $\qquad$ $1\frac{5}{3}$

Use several examples to establish the procedure, emphasizing that each shows different names for the same number. Then encourage the students to complete exercises, similar to the following, without the use of models.

$4\frac{3}{5} = 3\frac{■}{5}$ $\qquad$ $5 = 4\frac{■}{2}$ $\qquad$ $1\frac{2}{7} = \frac{■}{7}$ $\qquad$ $3\frac{1}{4} = ■\,\frac{5}{4}$

- Cut a few models of thirds so that the strips may be manipulated to demonstrate subtraction. Write the exercise $3\frac{2}{3} - 1\frac{1}{3}$ in vertical form on the board. Ask the students what they think the result will be and have them use models to check their answer. Repeat the procedure for $3\frac{1}{3} - 1\frac{2}{3}$, leading the students to suggest regrouping $3\frac{1}{3}$ as $2\frac{4}{3}$ to facilitate the subtraction.

Using the Pages

- The first of two examples on page 274 presents subtraction with no regrouping. Note the order of performing the subtraction — subtract the sevenths, then subtract the ones. Ask why regrouping is not required in this example. For the second example, it is necessary to regroup in order to subtract. Have students explain how they can tell that regrouping is required. Emphasize that $4\frac{2}{5}$ and $3\frac{7}{5}$ are different names for the same number and that $3\frac{7}{5}$ is appropriate for this situation. Relate the steps to the accompanying diagrams.

Working Together

Subtract the fractions. Then subtract the whole numbers.

1. $\begin{array}{r} 3\frac{4}{5} \\ -1\frac{1}{5} \\ \hline 2\frac{3}{5} \end{array}$

2. $1\frac{7}{9} - \frac{2}{9}$ $1\frac{5}{9}$

Regroup $4\frac{1}{8}$ as $3\frac{9}{8}$. Then subtract.

3. $\begin{array}{r} 4\frac{1}{8} \\ -1\frac{7}{8} \\ \hline 2\frac{2}{8} \end{array}$

4. $4\frac{1}{8} - 3\frac{5}{8}$ $\frac{4}{8}$

Regroup 3 as 2 and a fraction. Then subtract.

5. $\begin{array}{r} 3 \\ -1\frac{1}{4} \\ \hline 1\frac{3}{4} \end{array}$

6. $3 - \frac{5}{6}$ $2\frac{1}{6}$

Subtract. Add to check.

7. $\begin{array}{r} 6\frac{1}{4} \\ -3\frac{3}{4} \\ \hline 2\frac{2}{4} \end{array}$

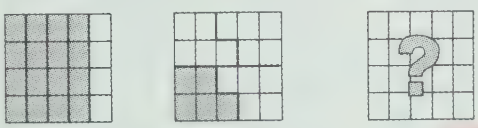
8. $4\frac{2}{3} - 1\frac{1}{3}$ $3\frac{1}{3}$

9. $7\frac{3}{10} - 4\frac{5}{10}$ $2\frac{8}{10}$

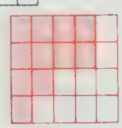
Exercises

Subtract. Add to check.

1. $\begin{array}{r} 5\frac{1}{3} \\ -2\frac{2}{3} \\ \hline 2\frac{2}{3} \end{array}$
2. $\begin{array}{r} 2\frac{1}{4} \\ -\frac{3}{4} \\ \hline 1\frac{2}{4} \end{array}$
3. $\begin{array}{r} 8\frac{2}{7} \\ -3\frac{5}{7} \\ \hline 4\frac{4}{7} \end{array}$
4. $\begin{array}{r} 4\frac{3}{6} \\ -4\frac{1}{6} \\ \hline \frac{2}{6} \end{array}$
5. $\begin{array}{r} 6\frac{2}{9} \\ -5\frac{4}{9} \\ \hline \frac{7}{9} \end{array}$
6. $\begin{array}{r} 2\frac{5}{12} \\ -\frac{11}{12} \\ \hline 1\frac{6}{12} \end{array}$
7. $6\frac{3}{5} - 3\frac{4}{5}$ $2\frac{4}{5}$
8. $5\frac{11}{12} - 1\frac{5}{12}$ $4\frac{6}{12}$
9. $10\frac{5}{9} - 4\frac{8}{9}$ $5\frac{6}{9}$
10. $6 - 5\frac{1}{2}$ $\frac{1}{2}$
11. $1\frac{1}{7} - \frac{6}{7}$ $\frac{2}{7}$
12. $12\frac{3}{8} - 9\frac{7}{8}$ $2\frac{4}{8}$
13. $3\frac{4}{7} - 2$ $1\frac{4}{7}$
14. $7\frac{1}{10} - 5\frac{7}{10}$ $1\frac{4}{10}$
15. $10 - 7\frac{3}{12}$ $2\frac{9}{12}$
16. $7 - 4\frac{1}{5}$ $2\frac{4}{5}$
17. $3\frac{7}{10} - 2\frac{9}{10}$ $\frac{8}{10}$
18. $9\frac{1}{4} - 7\frac{2}{4}$ $1\frac{3}{4}$
19. $5\frac{1}{8}$ of 7 pies were eaten. How many pies are left? $1\frac{7}{8}$
20. $1\frac{2}{3}$ of 3 bags of nuts were eaten. How many bags are left? $1\frac{1}{3}$



$\frac{4}{5} \quad - \quad \frac{1}{4} \quad =$



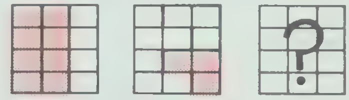
1. How would you complete the picture?
2. How would you complete the subtraction sentence? $\frac{11}{20}$

try this

275

RELATED ACTIVITIES

- For enrichment, you may wish to have some students express fractions in lowest terms for the answers of appropriate exercises on page 275.
- Provide other diagrams similar to those in the *Try This* feature.



$\frac{2}{3} \quad - \quad \frac{1}{4} \quad =$

- Provide practice in renaming numbers as follows.

$3 = 2\frac{\blacksquare}{3}$ $\qquad 5 = \blacksquare\frac{4}{4}$

$3\frac{1}{8} = \frac{\blacksquare}{8}$ $\qquad 7\frac{3}{5} = \blacksquare\frac{8}{5}$

Working Together: Ex. 1-6 lead students gradually through the steps in subtracting fractions. Ask how to determine whether regrouping is required in Ex. 1 and 2. Ask a student to explain the regrouping of $4\frac{1}{8}$ as $3\frac{9}{8}$ for Ex. 3 and 4. Discuss the different ways of regrouping the same minuend, 3, for Ex. 5 and 6. For Ex. 7-9, review how to determine whether regrouping will be required. Note that the students are asked to use addition to check their subtractions for Ex. 7-9.

Exercises: At this time it is not implied that a fraction obtained in a difference is to be expressed in lowest terms. For example, the difference $4\frac{6}{12}$ is appropriate for Ex. 8.

Try This: These exercises enable students to explore subtraction of fractions having unlike denominators. The diagrams help to show that $\frac{4}{5}$ and $\frac{1}{4}$ must be expressed as equivalent fractions with like denominators.

Assessment

Subtract. Add to check.

1. $\begin{array}{r} 6\frac{4}{7} \\ -2\frac{1}{7} \\ \hline 4\frac{3}{7} \end{array}$
2. $\begin{array}{r} 3\frac{1}{5} \\ -\frac{3}{5} \\ \hline 2\frac{3}{5} \end{array}$
3. $\begin{array}{r} 4\frac{3}{8} \\ -1\frac{7}{8} \\ \hline 2\frac{4}{8} \end{array}$
4. $10\frac{4}{9} - 3\frac{7}{9}$ $6\frac{6}{9}$
5. $5\frac{1}{4} - 2$ $3\frac{1}{4}$

OBJECTIVE

Demonstrate competence in writing equivalent fractions and in adding and subtracting fractions; solve related word problems

Materials

a map of Canada for each student

Vocabulary

geography

British Columbia $\frac{6}{15}$

Alberta $\frac{3}{7}$

Saskatchewan $\frac{4}{12}$

Manitoba $\frac{4}{8}$

Ontario $\frac{4}{7}$

Quebec $\frac{3}{6}$

New Brunswick $\frac{3}{12}$

Nova Scotia $\frac{5}{10}$

Prince Edward Island $\frac{6}{18}$

Newfoundland $\frac{4}{12}$

Practice

How well do you know your *geography*? Answer each question. Make a guess if you are not sure. Then use a map to help you check your work.

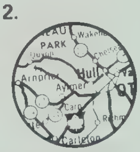
1. What fraction shows how many provinces have more than one word in their name? $\frac{4}{10}$
2. Answer is given at the left. For the name of each province, what fraction shows how many letters are vowels?
3. What fraction shows how many provinces share a land border with the United States? $\frac{7}{10}$
4. What fraction shows how many provinces border on any of the five Great Lakes? $\frac{1}{10}$
5. What fraction shows how many Great Lakes border on Canada? $\frac{4}{5}$
6. What fraction shows how many provinces border on the Pacific Ocean? $\frac{1}{10}$
7. What fraction shows how many provinces are west of Ontario? $\frac{4}{10}$
8. What fraction shows how many provinces are islands? $\frac{2}{10}$
9. There are three Maritime Provinces. What fraction shows how many of these are islands? $\frac{1}{3}$
10. There are four Atlantic Provinces. What fraction shows how many of these are islands? $\frac{2}{4}$

Can you find these on a map? Look for clues.

1.

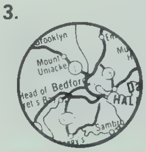
British Columbia

2.



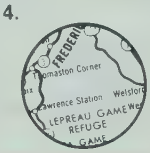
Ontario and Quebec

3.



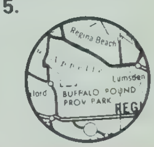
Nova Scotia

4.



New Brunswick

5.



Saskatchewan

276

LESSON ACTIVITY

Before Using the Pages

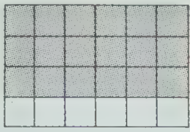
- Briefly review some of the concepts presented in the previous lessons of this unit. Have students write examples of equivalent fractions, proper fractions, improper fractions, and numbers in mixed form on the board. Review that two equivalent fractions have equal cross products. Ask students to identify the numerator and the denominator for several fractions. Review the use of multiplication and addition to express a number in mixed form as an improper fraction, and the use of division to carry out the reverse process.

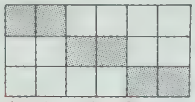
Using the Pages

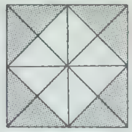
- Ask a student to read the sentences at the top of page 276. Have several maps of Canada available for those students who will require them. The maps may also be required for reference in the *Problem Solving* feature.

Problem Solving: The students are to identify the province(s) for the area indicated in each exercise. Some may be able to do so from only the information shown; others may need to refer to a map. A discussion of the answers will enable students to describe the methods they used to identify each area.

Write two equivalent fractions for each picture.

11.  $\frac{18}{24}, \frac{3}{4}$; $\frac{6}{24}, \frac{1}{4}$

12.  $\frac{6}{18}, \frac{1}{3}$; $\frac{12}{18}, \frac{2}{3}$

13.  $\frac{8}{16}, \frac{1}{2}$; $\frac{2}{4}, \frac{1}{2}$

Complete the equivalent fractions in each chart.

14.

$\frac{1}{4}$	$\frac{2}{■}$ 8	$\frac{■}{12}$ 3	$\frac{4}{■}$ 16	$\frac{■}{20}$ 5

15.

$\frac{■}{3}$ 2	$\frac{■}{6}$ 4	$\frac{6}{9}$	$\frac{8}{■}$ 12	$\frac{■}{15}$ 10

Find the missing term in each pair of equivalent fractions.

16. $\frac{3}{5} = \frac{■}{10}$ 6 **17.** $\frac{2}{4} = \frac{6}{■}$ 12 **18.** $\frac{7}{8} = \frac{■}{32}$ 28 **19.** $\frac{15}{36} = \frac{5}{■}$ 12 **20.** $\frac{2}{6} = \frac{■}{9}$ 3

Use >, <, or = to make true statements.

21. $\frac{5}{6} ● \frac{2}{3}$ > **22.** $\frac{7}{10} ● \frac{3}{4}$ < **23.** $\frac{6}{12} ● \frac{1}{2}$ = **24.** $\frac{3}{7} ● \frac{4}{9}$ < **25.** $\frac{5}{12} ● \frac{3}{8}$ >

Write each of these as an improper fraction.

26. $1\frac{3}{4}$ $\frac{7}{4}$ **27.** $7\frac{5}{8}$ $\frac{61}{8}$ **28.** $10\frac{4}{5}$ $\frac{54}{5}$

Write each of these as a number in mixed form.

29. $\frac{43}{5}$ $8\frac{3}{5}$ **30.** $\frac{35}{12}$ $2\frac{11}{12}$ **31.** $\frac{110}{9}$ $12\frac{2}{9}$

Add.

32. $\frac{3}{8} + \frac{2}{8}$ = $\frac{5}{8}$

33. $5\frac{1}{7} + \frac{3}{7}$ = $5\frac{4}{7}$

34. $3\frac{2}{3} + 2$ = $5\frac{2}{3}$

35. $1\frac{5}{6} + 1\frac{5}{6}$ = $3\frac{4}{6}$

36. $2\frac{2}{3} + 1\frac{1}{3}$ 4

37. $5\frac{2}{4} + 3\frac{3}{4}$ $9\frac{1}{4}$

Subtract.

38. $\frac{4}{5} - \frac{2}{5}$ = $\frac{2}{5}$

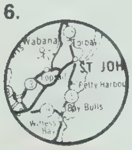
39. $7\frac{1}{2} - 2\frac{1}{2}$ = 5

40. $3 - 1\frac{7}{10}$ = $1\frac{3}{10}$

41. $6\frac{2}{9} - 1\frac{8}{9}$ = $4\frac{3}{9}$

42. $4\frac{1}{3} - 3\frac{2}{3}$ $\frac{2}{3}$

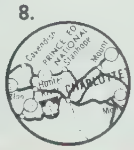
43. $10\frac{3}{8} - 6\frac{5}{8}$ $3\frac{6}{8}$

6. 

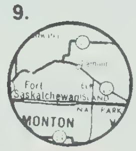
Newfoundland

7. 

Manitoba

8. 

Prince Edward Island

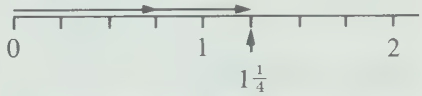
9. 

Alberta

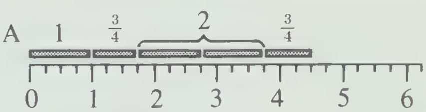
PROBLEM SOLVING

RELATED ACTIVITIES

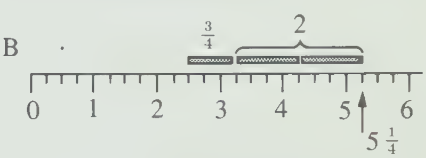
- Have students adapt Ex. 2 on page 276 for their names and for the name of their school, street, or city.
- Addition of fractions with like denominators may be illustrated on a number line. For example, for $\frac{3}{4} + \frac{2}{4}$, a number line is marked in fourths. (Use copies of page T 389.)



Some students may benefit from using fraction strips (and whole number strips, where appropriate) to demonstrate addition and subtraction on a classroom number line. The example for number line A illustrates $1\frac{3}{4} + 2\frac{3}{4} = 4\frac{2}{4}$ as $1 + \frac{3}{4} + 2 + \frac{3}{4}$. Note that for addition, the strips can be placed along the number line in any order and the sum is always the same. For example, the sum can be found by adding the whole numbers first $(1 + 2 + \frac{3}{4} + \frac{3}{4})$.

A 

For showing subtraction on the number line, number strips are required only for the subtrahend as shown on number line B.

B 

$5\frac{1}{4} - 2\frac{3}{4} = 2\frac{2}{4}$

Pages 278-279

LESSON OUTCOME

Find the number for part of a set when the number of the set is a multiple of the denominator of the fraction; solve related word problems

Materials

counters, small boxes

Prerequisite Skills

Write a fraction to represent part of a set; find the missing term in two equivalent fractions; divide whole numbers

Checking Prerequisite Skills

Write a fraction to show how much is shaded.

1.

$\frac{3}{5}$

2.

$\frac{4}{8}$

Find the missing term.

3. $\frac{2}{5} = \frac{■}{15}$ 6 **4.** $\frac{8}{12} = \frac{■}{15}$ 10

Divide.

5. $8\overline{)96}$ 12 **6.** $12\overline{)180}$ 15

Another Look at Finding the Missing Term

There are 24 students in Bill's class.
He estimated that $\frac{3}{4}$ of them have brown eyes.
How many are there in $\frac{3}{4}$ of 24?

To find the missing term in $\frac{3}{4} = \frac{■}{24}$,

divide 24 by 4

$\frac{3}{4} \xrightarrow{4 \times ■} \frac{■}{24}$ (6)

to find this value.

Then multiply 6 by 3 to find this value.

$\frac{3}{4} \overset{3 \times 6}{\underset{4 \times 6}{=}} \frac{18}{24}$

OR

divide 24 by 4

$4\overline{)24}$ 6

to find $\frac{1}{4}$ of 24.

Then multiply 6 by 3

$3 \times 6 = 18$

to find $\frac{3}{4}$ of 24.

There are 18 in $\frac{3}{4}$ of 24.

278

LESSON ACTIVITY

Before Using the Pages

- Have the students use counters to form a set and then find the number for a part of the set. For example, have a student form a set of 6 counters and have another student place one half of the set in a box. Repeat the procedure for one third of the set and then for two thirds of the set. For each example, have students state the number of counters placed in the box and write statements on the board.

  $\frac{1}{2}$ of 6 is 3. $\frac{1}{3}$ of 6 is 2. $\frac{2}{3}$ of 6 is 4.

  Use a similar procedure to show, for example, $\frac{3}{4}$ of 12, $\frac{2}{4}$ of 12, and $\frac{1}{4}$ of 12. Ask how the answers can be determined without using counters or drawing a diagram. For example, ask how to find $\frac{1}{5}$ of 20, and then ask how that answer can be used to find $\frac{2}{5}$ of 20, $\frac{3}{5}$ of 20, and so on.

Using the Pages

- Have a student read the word problem to introduce the situation. The worked example illustrates two procedures for solving the problem. Discuss that $\frac{3}{4}$ of 24 suggests equivalent fractions for which one term is missing, $\frac{3}{4} = \frac{■}{24}$. Have students help to explain the procedure for finding the missing term. Emphasize that division is used first to find the appropriate factor (6) which is then multiplied by the numerator (3). For the second approach, point out that dividing 24 by 4 gives $\frac{1}{4}$ of 24, or 6, and that value is multiplied by 3 to find $\frac{3}{4}$ of 24.

Working Together: Ex. 1 and 2 establish the procedure for finding the number for part of a set when the fraction has a numerator of 1. The procedure is applied in Ex. 3-6. Ex. 7 and 8 establish the procedure for a numerator greater than 1, and this is applied in Ex. 9-12.

Working Together

Complete.

1. To find $\frac{1}{4}$ of a number, divide that number by 4

2. To find $\frac{1}{7}$ of a number, divide that number by 7

Find each of these.

3. $\frac{1}{3}$ of 18 6 4. $\frac{1}{8}$ of 56 7 5. $\frac{1}{6}$ of 36 6 6. $\frac{1}{2}$ of 32 16

Complete.

7. To find $\frac{3}{4}$ of a number, divide that number by 4 and then multiply by 3

8. To find $\frac{7}{12}$ of a number, divide that number by 12 and then multiply by 7

Find each of these.

9. $\frac{2}{3}$ of 15 10 10. $\frac{4}{7}$ of 42 24 11. $\frac{2}{5}$ of 65 26 12. $\frac{7}{10}$ of 80 56

Exercises

Find each of these.

1. $\frac{1}{5}$ of 40 8 2. $\frac{1}{9}$ of 36 4 3. $\frac{1}{10}$ of 100 10 4. $\frac{1}{7}$ of 84 12 5. $\frac{1}{12}$ of 72 6

6. $\frac{3}{4}$ of 16 12 7. $\frac{2}{7}$ of 28 8 8. $\frac{7}{8}$ of 32 28 9. $\frac{4}{5}$ of 30 24 10. $\frac{2}{9}$ of 81 18

11. $\frac{5}{7}$ of 49 35 12. $\frac{3}{10}$ of 230 69 13. $\frac{7}{9}$ of 135 105 14. $\frac{3}{8}$ of 96 36 15. $\frac{7}{12}$ of 312 182

There are 24 students in the class.

16. Gail estimated that $\frac{1}{2}$ of them have brown eyes. How many are there in $\frac{1}{2}$ of 24? 12

17. Mel estimated that $\frac{2}{3}$ of them have brown eyes. How many are there in $\frac{2}{3}$ of 24? 16

18. Ned estimated that $\frac{1}{6}$ of them have blue eyes. How many are there in $\frac{1}{6}$ of 24? 4

19. Connie estimated that $\frac{5}{8}$ of them have eye color that is not blue. How many are there in $\frac{5}{8}$ of 24? 15

Use division for each of these. Write the quotient and remainder as a number in mixed form.

Example: For $\frac{1}{5}$ of 17, use $5\overline{)17}$ 3 R2

Write "$\frac{1}{5}$ of 17 is $3\frac{2}{5}$".

20. Find $\frac{1}{3}$ of 14. 21. Find $\frac{1}{12}$ of 113.

$\frac{1}{3}$ of 14 is $4\frac{2}{3}$ $\frac{1}{12}$ of 113 is $9\frac{5}{12}$

279

Exercises: Note that Ex. 20 and 21 involve remainders in the division process. Draw attention to the example shown to the right of Ex. 20 and 21.

Assessment

Find each of these.

1. $\frac{1}{5}$ of 30 6 2. $\frac{3}{10}$ of 120 36

3. $\frac{5}{8}$ of 88 55 4. $\frac{4}{9}$ of 108 48

Solve.

5. There are 28 students in the class. Val estimated that $\frac{3}{4}$ of them take a bus to school. How many are there in $\frac{3}{4}$ of 28? 21

RELATED ACTIVITIES

- Students having difficulty may need more experience using counters to find part of a set as described in *Before Using the Pages*.
- For enrichment, provide exercises similar to the following.

Which is greater,

1. $\frac{2}{3}$ of 15 or $\frac{3}{4}$ of 12?
2. $\frac{2}{5}$ of 20 or $\frac{3}{7}$ of 21?
3. $\frac{3}{4}$ of 24 or $\frac{2}{5}$ of 45?

- Provide information similar to the following about the students in your class and have them find the number of students for the fraction named.

There are 32 students in our class.

$\frac{3}{4}$ of them have brown hair.

$\frac{1}{8}$ of them eat lunch at school.

$\frac{1}{2}$ of them have older brothers or sisters.

$\frac{3}{16}$ of them were at a different school last year.

Pages 280-281

LESSON OUTCOME

Express a fraction as a decimal by first writing an equivalent fraction with a denominator of 10 or 100, for halves, fourths, and fifths

Materials

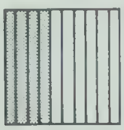
models for $\frac{1}{2}$, $\frac{2}{5}$, and $\frac{3}{4}$ prepared from copies of the squares on page T 393; models for 0.5, 0.4, and 0.75 prepared from copies of pages T 394 and T 395

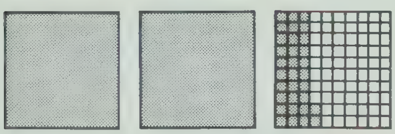
Prerequisite Skills

Write decimals; find the missing term in two equivalent fractions

Checking Prerequisite Skills

Write the decimal.

1.  0.4

2.  2.32

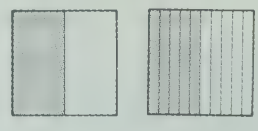
3. seven-tenths 0.7
4. four and three-hundredths 4.03

Find the missing term.

5. $\frac{3}{4} = \frac{■}{100}$ 75   6. $\frac{2}{5} = \frac{■}{10}$ 4

Equivalent Fractions and Decimals

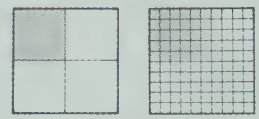
The fraction $\frac{1}{2}$ is equivalent to the decimal 0.5.



$\frac{1}{2} = \frac{5}{10}$ (1 × 5, 2 × 5) or 0.5

A factor of 5 will change halves to tenths.

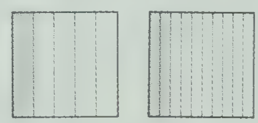
The fraction $\frac{1}{4}$ is equivalent to the decimal 0.25.



$\frac{1}{4} = \frac{25}{100}$ (1 × 25, 4 × 25) or 0.25

A factor of 25 will change fourths to hundredths.

The fraction $\frac{1}{5}$ is equivalent to the decimal 0.2.



$\frac{1}{5} = \frac{2}{10}$ (1 × 2, 5 × 2) or 0.2

A factor of 2 will change fifths to tenths.

280

LESSON ACTIVITY

Before Using the Pages

- Review that the numerator of a fraction names the number of equal parts of a whole that are special and the denominator names the number of equal parts in all. For example, $\frac{2}{5}$ represents 2 of 5 equal parts and $\frac{3}{10}$ represents 3 of 10 equal parts. Then have students recall that a decimal can be written to represent the number when the number of equal parts, represented by the denominator, is 10, 100, or 1000. Have the students write the following as decimals, explaining why a one-place decimal or a two-place decimal is used.

$$\frac{4}{10},\ \frac{25}{100},\ 3\frac{2}{10},\ 5\frac{2}{100},\ 1\frac{5}{10}$$

- Display a model for $\frac{1}{2}$ and ask what fraction is represented. Ask if there is a decimal that represents the same part of the whole as $\frac{1}{2}$. To help the students decide, display a model for 0.5 beside the model for $\frac{1}{2}$. Use a similar procedure for $\frac{3}{4}$ and 0.75 and for $\frac{2}{5}$ and 0.4. Emphasize that such numerals as $\frac{3}{4}$ and 0.75 are different names for the same number.

Using the Pages

- Have a student read the title at the top of page 280. Point out that the term *equivalent* is now used to describe a fraction and a decimal that represent the same amount. Discuss the three examples, noting that diagrams are helpful in each case but they are not necessary. Have students help to explain the procedure of obtaining an equivalent fraction for each example. Pay particular attention to the choice of 10 or 100 as the denominator of the equivalent fraction, and to the information in the "thought clouds".

Working Together: After the students complete Ex. 1, present the example $\frac{3}{4} = \frac{■}{10}$ and discuss why it is not appropriate for obtaining the decimal equivalent to $\frac{3}{4}$. For Ex. 2, some students may suggest $\frac{2}{5} = \frac{■}{100}$ as an alternative approach. Discuss that the simplest decimal numeral is 0.4 as opposed to 0.40 or 0.400. Ex. 3 introduces a number greater than 1. For Ex. 4-7, the information shown in the "thought

Working Together

Complete.

1. $\frac{3}{4} = \frac{\blacksquare}{100}$ 75 or the decimal 0.75.
2. $\frac{2}{5} = \frac{\blacksquare}{10}$ 4 or the decimal .04
3. $1\frac{1}{2} = 1\frac{\blacksquare}{10}$ 5 or the decimal .15

Give a decimal that is equivalent to each of these.

4. $\frac{4}{5}$ 0.8 5. $\frac{2}{4}$ 0.5
6. $4\frac{1}{4}$ 4.25 7. $3\frac{3}{5}$ 3.6

Exercises

Write a decimal that is equivalent to each of these.

1. $\frac{3}{5}$ 0.6 2. $3\frac{1}{10}$ 3.1 3. $\frac{3}{4}$ 0.75
4. $2\frac{1}{4}$ 2.25 5. $5\frac{1}{2}$ 5.5 6. $\frac{3}{10}$ 0.3
7. $7\frac{3}{4}$ 7.75 8. $1\frac{4}{5}$ 1.8 9. $6\frac{5}{10}$ 6.5
10. $2\frac{1}{5}$ 2.2 11. $\frac{1}{2}$ 0.5 12. $3\frac{2}{5}$ 3.4
13. $1\frac{2}{4}$ 1.5 14. $\frac{1}{5}$ 0.2 15. $\frac{7}{10}$ 0.7
*16. $\frac{1}{8}$ 0.125 *17. $\frac{3}{8}$ 0.375 *18. $1\frac{5}{8}$ 1.625

For each situation, write a sentence that uses a decimal.

19. Mandy lives 1 km from the store. She walked a fourth of the way home and rode the rest of the way with a neighbor.
20. Mandy bought a carton holding 1 L of milk. She drank half of the milk from the carton.
*21. Mandy bought 1 kg of ground beef. She used about $\frac{1}{8}$ of it for a hamburger.

Divide. Use as many zeros as you need in the dividend.

1. $2\overline{)23.1}$ 11.55 2. $8\overline{)16.12}$ 2.015 3. $4\overline{)17.2}$ 4.3
4. $5\overline{)38.1}$ 7.62 5. $6\overline{)1.5}$ 0.25 6. $10\overline{)12.3}$ 1.23
7. $8\overline{)60.8}$ 7.6 8. $2\overline{)8.11}$ 4.055 9. $5\overline{)2}$ 0.4
10. $4\overline{)3}$ 0.75 11. $12\overline{)9}$ 0.75 12. $10\overline{)7}$ 0.7
13. $2\overline{)19}$ 9.5 14. $8\overline{)13}$ 1.625 15. $10\overline{)37}$ 3.7

Example:

$$\begin{array}{r} 5.75 \\ 4\overline{)23.00} \\ \underline{20} \\ 3\,0 \\ \underline{2\,8} \\ 20 \\ \underline{20} \\ 0 \end{array}$$

KEEPING SHARP

19. Mandy walked 0.25 km and rode 0.75 km with a neighbor
20. Mandy drank 0.5 L of the milk from the carton
21. Mandy used 0.125 kg of ground beef for a hamburger

281

RELATED ACTIVITIES

• Have the students list fractions and their decimal equivalents to enable them to note patterns.

A chart showing these equivalents should be displayed for several days to enable students to refer to it, and thus encourage incidental learning.

$\frac{1}{2} = 0.5$	$\frac{1}{4} = 0.25$	$\frac{1}{5} = 0.2$
	$\frac{2}{4} = 0.50$	$\frac{2}{5} = 0.4$
	$\frac{3}{4} = 0.75$	$\frac{3}{5} = 0.6$
		$\frac{4}{5} = 0.8$

clouds'' on page 280 is particularly helpful. Ask questions such as ''What is the denominator of the fraction in Ex. 4?'', ''What denominator would you suggest for the equivalent fraction?'', and ''What factor will change fifths to tenths?'' Note that for a number in mixed form (Ex. 3, 6, and 7) the whole number remains the same and is shown to the left of the decimal point.

Exercises: Ex. 16-18, and 21 are starred because these extend the concept of the lesson to thousandths in expressing eighths as equivalent decimals.

Keeping Sharp: These exercises help to maintain skills in dividing, particularly when more zeros are needed in the dividend to terminate the quotient. This skill will be applied in the lesson on pages 282 and 283, which introduces the procedure of dividing the numerator of a fraction by the denominator to find an equivalent decimal.

Assessment

Write a decimal that is equivalent to each of these.

1. $\frac{4}{5}$ 0.8 **2.** $2\frac{1}{2}$ 2.5 **3.** $5\frac{3}{4}$ 5.75 **4.** $\frac{9}{10}$ 0.9

Pages 282-283

LESSON OUTCOME

Divide the numerator of a fraction by the denominator to express a fraction as a decimal

Materials

real coins or play money (optional)

Prerequisite Skills

Divide using zeros to show more decimal places in the dividend, quotient terminating by the third decimal place

Checking Prerequisite Skills

Divide. Use as many zeros as you need in the dividend.

1. $2\overline{)9}$ 4.5
2. $4\overline{)2}$ 0.5
3. $10\overline{)36}$ 3.6
4. $12\overline{)147}$ 12.25

Changing Fractions to Decimals by Dividing

Each quarter is $\frac{1}{4}$ of a dollar.

27 quarters are $\frac{27}{4}$ dollars.

To change $\frac{27}{4}$ to show whole dollars

divide and show the number in mixed form,

```
    6
4)27        6 whole dollars
  24                            6 3/4 dollars
   3        3 quarters of
            another dollar
```

OR

divide and show a decimal.

```
   6.75
4)27.00
  24
   3 0
   2 8
     20
     20
      0
```

$\frac{27}{4}$ dollars is the same as $6\frac{3}{4}$ dollars.

$\frac{27}{4}$ dollars is the same as 6.75 dollars or $6.75.

Here are other divisions that show how to change a fraction to a decimal.

Always divide the numerator by the denominator.

For $\frac{1}{2}$,

```
   0.5
2)1.0
  1 0
    0
```

For $\frac{1}{8}$,

```
   0.125
8)1.000
  8
   20
   16
    40
    40
     0
```

For $\frac{81}{12}$,

```
     6.75
12)81.00
   72
    9 0
    8 4
      60
      60
       0
```

$\frac{1}{2} = 0.5$ $\frac{1}{8} = 0.125$ $\frac{81}{12} = 6.75$

282

LESSON ACTIVITY

Before Using the Pages

- Review that a fraction such as $\frac{3}{5}$ may be expressed as an equivalent decimal by following the procedure presented in the previous lesson.

$$\frac{3}{5} \underset{5 \times 2}{\overset{3 \times 2}{=}} \frac{\blacksquare}{10} \qquad \frac{3}{5} = \frac{6}{10} \text{ or } 0.6$$

Have the students use a similar procedure for other fractions such as $\frac{1}{2}$, $\frac{1}{4}$, $\frac{3}{4}$, and $\frac{4}{5}$. Summarize the results in a chart (columns A and B). When this has been done, extend the chart (column C) and have the students complete the indicated divisions, using zeros to show more places in the dividend to terminate the quotient. Compare the results obtained in column C with the numbers shown in columns A and B. This activity will enable the students to discover that a fraction can be expressed as an equivalent decimal by dividing the numerator of the fraction by the denominator.

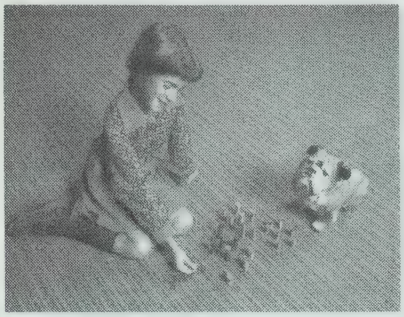
A	B	C
$\frac{1}{2}$	0.5	$2\overline{)1.0}$ (0.5)
$\frac{1}{4}$	0.25	$4\overline{)1}$
$\frac{3}{4}$	0.75	$4\overline{)3}$
$\frac{3}{5}$	0.6	$5\overline{)3}$
$\frac{4}{5}$	0.8	$5\overline{)4}$

Using the Pages

- The worked example recalls the use of division to express an improper fraction, $\frac{27}{4}$, as a number in mixed form, $6\frac{3}{4}$. Then the division process is extended so that a terminating decimal quotient is obtained. You may wish to use real coins or play money to demonstrate that 27 quarters may be regrouped as 6 dollars and 3 quarters, which is the same as $6\frac{3}{4}$ dollars ($6.75). The procedure of the previous lesson can be used to express $6\frac{3}{4}$ as 6.75. However, if the division process is extended to the right of the decimal point, the equivalent decimal can be obtained.

Exercises

Karen opened her bank. She counted 234 pennies, 163 nickels, 177 dimes, 53 quarters, and 19 fifty-cent coins.



Use division to find how much Karen's coins are worth.

1. Each penny is $\frac{1}{100}$ of a dollar. 234 pennies are $\frac{234}{100}$ dollars. How much are 234 pennies worth? $2.34

2. Each nickel is $\frac{1}{20}$ of a dollar. 163 nickels are $\frac{163}{20}$ dollars. How much are 163 nickels worth? $8.15

3. Each dime is $\frac{1}{10}$ of a dollar. 177 dimes are $\frac{177}{10}$ dollars. How much are 177 dimes worth? $17.70

4. Each quarter is $\frac{1}{4}$ of a dollar. 53 quarters are $\frac{53}{4}$ dollars. How much are 53 quarters worth? $13.25

5. Each fifty-cent coin is $\frac{1}{2}$ of a dollar. 19 fifty-cent coins are $\frac{19}{2}$ dollars. How much are 19 fifty-cent coins worth? $9.50

6. Karen's goal is $100 for a new bicycle. Is she halfway to her goal yet? yes

Divide the numerator by the denominator to change each fraction to a decimal. Use as many zeros in the dividend as you need.

7\. $\frac{7}{2}$ 3.5 8. $\frac{3}{4}$ 0.75 9. $\frac{23}{10}$ 2.3 10. $\frac{3}{5}$ 0.6 11. $\frac{17}{4}$ 4.25 12. $\frac{1}{8}$ 0.125

13\. $\frac{58}{10}$ 5.8 14. $\frac{4}{5}$ 0.8 15. $\frac{1}{2}$ 0.5 16. $\frac{227}{4}$ 56.75 17. $\frac{1}{5}$ 0.2 18. $\frac{7}{8}$ 0.875

19\. $\frac{1}{4}$ 0.25 20. $\frac{38}{5}$ 7.6 21. $\frac{5}{8}$ 0.625 22. $\frac{2}{5}$ 0.4 23. $\frac{60}{8}$ 7.5 24. $\frac{3}{8}$ 0.375

Three more examples are shown at the bottom of page 282. You may wish to have the students try one or more of these independently and then check their work with the examples on the page. Emphasize that such numbers as $\frac{81}{12}$ and 6.75 are different names for the same number.

Exercises: Before the students begin, have one student read the statements above and below the photograph. Point out that Ex. 1-6 refer to the coins from Karen's bank.

Assessment

Divide the numerator by the denominator to change each fraction to a decimal.

1. $\frac{3}{5}$ 0.6 **2.** $\frac{7}{4}$ 1.75 **3.** $\frac{67}{10}$ 6.7 **4.** $\frac{29}{8}$ 3.625

RELATED ACTIVITIES

• Students may help to prepare a set of domino cards for use in reinforcing the concept of equivalent fractions and decimals. For assistance and for checking accuracy, the students may refer to the chart of equivalent fractions and decimals suggested in *Related Activities* on page T 305.

3.6	$\frac{3}{4}$

0.75	$\frac{1}{2}$

0.5	$\frac{5}{8}$

0.625

• Provide students with the following chart.

$\frac{1}{2} = 0.5$

$\frac{1}{4} = 0.25$

$\frac{1}{5} = 0.2$

$\frac{1}{8} = 0.125$

Have them refer to the chart to find the decimal equivalents for fractions as shown.

Example

1\. $\frac{3}{4} = 0.25 + 0.25 + 0.25$
$= 0.75$

or

$\frac{3}{4} = 3 \times 0.25$
$= 0.75$

1\. $\frac{3}{4}$ 2. $\frac{5}{8}$ 3. $\frac{3}{5}$

4\. $\frac{7}{8}$ 5. $\frac{6}{5}$ 6. $\frac{9}{4}$

Pages 284-285

LESSON OUTCOME

Compare fractions using their decimal equivalents; add and subtract fractions and their decimal equivalents, and then compare the results

Prerequisite Skills

Add and subtract decimals

Checking Prerequisite Skills

Add.

1. 1.8	**2.** 8.75	**3.** 3.125
2.4	1.50	1.375
4.2	10.25	4.500

Subtract.

4. 4.4	**5.** 6.00	**6.** 4.125
1.6	3.25	1.875
2.8	2.75	2.250

Using Decimals to Work with Fractions

Each carton of milk can hold 1 L of milk. One carton is $\frac{2}{5}$ full. The other is $\frac{3}{8}$ full. Which carton has more milk? How much milk is there in the two cartons?

$\frac{2}{5} = 0.4$

$\frac{3}{8} = 0.375$

$0.4 > 0.375$, so

$\frac{2}{5} > \frac{3}{8}$

The carton that is $\frac{2}{5}$ full has more milk.

Add.
$$\begin{array}{r} 0.400 \\ \underline{0.375} \\ 0.775 \end{array}$$

There are 0.775 L of milk in the two cartons.

0.775 is a little greater than 0.750.

$0.75 = \frac{3}{4}$

If all the milk could be poured into one carton, that carton would be a little more than $\frac{3}{4}$ full.

284

LESSON ACTIVITY

Before Using the Pages

- Write a few exercises on the board to review the use of division to change a fraction to a decimal.

  $\frac{5}{8}$ $\quad$ $\frac{3}{5}$ $\quad$ $\frac{50}{8}$ $\quad$ $\frac{31}{5}$

  Have students show and explain their work on the board.

- Have the students recall that two fractions with unlike denominators may be compared by changing the fractions to equivalent fractions having like denominators. Use this procedure to compare $\frac{5}{8}$ and $\frac{3}{5}$. Then ask how the results of the first activity would enable them to compare $\frac{5}{8}$ and $\frac{3}{5}$ without finding equivalent fractions. Lead the students to suggest that their decimal equivalents may be compared. Have the students compare $\frac{50}{8}$ and $\frac{31}{5}$ from the first activity by comparing the equivalent decimals. For example,

  $\frac{50}{8} = 6.25$ and $\frac{31}{5} = 6.2$.
  Since $6.25 > 6.20$, then $\frac{50}{8} > \frac{31}{5}$.

  Review that such numerals as 6.2 and 6.20 represent the same number.

Using the Pages

- The worked example demonstrates that two fractions may be compared by comparing their equivalent decimals. Establish that it is necessary to compare $\frac{2}{5}$ and $\frac{3}{8}$ to answer the question "Which carton has more milk?" Have students show the divisions on the board to obtain the decimals 0.4 and 0.375. To help students in comparing 0.4 and 0.375, review that 0.4 and 0.400 represent the same number.

  Addition is required to answer the question "How much milk is there in the two cartons?" Addition of the fractions $\frac{2}{5}$ and $\frac{3}{8}$ presents the difficulty of unlike denominators. Point out that it is convenient to find the sum of the equivalent decimals. The example concludes by comparing the amount of milk in all to a carton that is a little more than $\frac{3}{4}$ full. If necessary, show the division $4\overline{)3.00}$ with quotient 0.75 on the board.

Exercises

Will >, <, or = make the statement true?
Use decimals to help you decide.

1. $\frac{3}{5} \bigcirc \frac{5}{8}$ <
2. $\frac{9}{10} \bigcirc \frac{7}{8}$ >
3. $\frac{11}{4} \bigcirc \frac{14}{5}$ <
4. $\frac{18}{5} \bigcirc \frac{36}{10}$ =
5. $\frac{36}{5} \bigcirc \frac{29}{4}$ <
6. $\frac{12}{8} \bigcirc \frac{15}{10}$ =
7. $\frac{54}{5} \bigcirc \frac{86}{8}$ >
8. $\frac{30}{4} \bigcirc \frac{61}{8}$ <

Add the fractions. Then add the equivalent decimals. Compare the sums.

9. $1\frac{3}{4}$ 1.75 | $+2\frac{2}{4}$ 2.50 | $4\frac{1}{4}$ 4.25
10. $4\frac{7}{8}$ 4.875 | $+7\frac{5}{8}$ 7.625 | $12\frac{4}{8}$ 12.500
11. $8\frac{3}{5}$ 8.6 | $+1\frac{4}{5}$ 1.8 | $10\frac{2}{5}$ 10.4
12. $3\frac{7}{10}$ 3.7 | $+5\frac{5}{10}$ 5.5 | $9\frac{2}{10}$ 9.2
13. $7\frac{1}{2}$ 7.5 | $+2\frac{1}{2}$ 2.5 | 10 10.0
14. $9\frac{3}{8}$ 9.375 | $+6\frac{6}{8}$ 6.750 | $16\frac{1}{8}$ 16.125
15. $3\frac{2}{5}$ 3.4 | $+1\frac{3}{5}$ 1.6 | 5 5.0
16. $10\frac{3}{4}$ 10.75 | $+9\frac{3}{4}$ 9.75 | $20\frac{1}{2}$ 20.50

Subtract the fractions. Then subtract the equivalent decimals. Compare the differences.

17. $4\frac{1}{10}$ 4.1 | $-2\frac{7}{10}$ 2.7 | $1\frac{4}{10}$ 1.4
18. 2 2.0 | $-1\frac{1}{2}$ 1.5 | $\frac{1}{2}$ 0.5
19. $8\frac{3}{8}$ 8.375 | $-3\frac{7}{8}$ 3.875 | $4\frac{4}{8}$ 4.500
20. $10\frac{1}{4}$ 10.25 | $-6\frac{2}{4}$ 6.50 | $3\frac{3}{4}$ 3.75
21. $1\frac{2}{5}$ 1.4 | $-\frac{4}{5}$ 0.8 | $\frac{3}{5}$ 0.6
22. $3\frac{5}{8}$ 3.625 | $-3\frac{1}{8}$ 3.125 | $\frac{4}{8}$ 0.500
23. $6\frac{1}{4}$ 6.25 | $-1\frac{3}{4}$ 1.75 | $4\frac{2}{4}$ 4.50
24. $12\frac{1}{5}$ 12.2 | $-7\frac{2}{5}$ 7.4 | $4\frac{4}{5}$ 4.8

Use the equivalent decimals to find each sum or difference. Give a fraction that is equivalent, or almost equivalent, to your decimal result.

1.50 + 1.75 = 3.25 2.25 − 1.50 = 0.75

*25. $1\frac{1}{2} + 1\frac{3}{4}$ $3\frac{1}{4}$
*26. $2\frac{1}{4} - 1\frac{1}{2}$ $\frac{3}{4}$
*27. $3\frac{1}{4} + \frac{5}{8}$ $3\frac{7}{8}$
*28. $1\frac{1}{8} - \frac{3}{4}$ $\frac{3}{8}$
*29. $\frac{4}{5} + \frac{3}{4}$ $1\frac{11}{20}$
*30. $1\frac{7}{8} - \frac{2}{5}$ $1\frac{19}{40}$

0.80 + 0.75 = 1.55 1.875 − 0.400 = 1.475

*27 3.250 + 0.625 = 3.875 *28 1.125 − 0.750 = 0.375

How many times will the minute hand of a clock be directly over the hour hand in 24 h? 22

PROBLEM SOLVING

285

RELATED ACTIVITIES

- Have students order the following fractions from least to greatest by using their decimal equivalents.

$\frac{3}{8}, \frac{2}{5}, \frac{1}{4}, \frac{3}{5}, \frac{1}{2}, \frac{5}{8}$

- Have students use decimal equivalents to find the missing fraction in each sentence. The first exercise is completed as an example.

1. $\frac{1}{4} + \frac{1}{2} + \frac{1}{8} + \blacksquare = 1$

$$\begin{array}{r} 0.250 \\ 0.500 \\ +\ 0.125 \\ \hline 0.875 \end{array} \qquad \begin{array}{r} 1.000 \\ -\ 0.875 \\ \hline 0.125 \end{array} \qquad 0.125 = \frac{1}{8}$$

2. $\frac{1}{4} + \frac{2}{5} + \frac{1}{4} + \blacksquare = 1$
3. $\frac{3}{4} + \frac{3}{4} + \frac{1}{2} + \blacksquare = 2\frac{1}{4}$
4. $\frac{4}{5} + \frac{1}{8} + \blacksquare + \frac{1}{2} + \frac{1}{5} = 2\frac{1}{2}$

Exercises: Before the students begin, review the instructions for each group of exercises to ensure that they understand what is required. You may wish to complete on the board one exercise from each group with the students. Ex. 25-30 involve unlike denominators. By working with equivalent decimals, students may discover an approach for adding fractions with unlike denominators. A reference list of fractions and equivalent decimals would be helpful. (See *Related Activities* on page T 305.)

Problem Solving: After the students have had an opportunity to consider the problem, have them share their ideas with the rest of the class. Some students may need to refer to a demonstration dial clock to understand that in a 24-hour period, for example, from noon one day to noon the next day, the minute hand is directly over the hour hand only 22 times. The diagram serves as a reminder that the hands do not coincide at the five-minute marks such as 5:25 or 6:30. They will coincide at noon, at midnight, and at times between 1:05 and 1:10, 2:10 and 2:15, 3:15 and 3:20, 4:20 and 4:25, 5:25 and 5:30, 6:30 and 6:35, 7:35 and 7:40, 8:40 and 8:45, 9:45 and 9:50, and 10:50 and 10:55. Note that the second time the hands coincide at noon is counted in the next 24-hour period.

Assessment

Use >, <, or = to make a true statement. Use decimals to help you decide.

1. $\frac{3}{4} \bigcirc \frac{7}{8}$ <
2. $\frac{6}{4} \bigcirc \frac{9}{6}$ =

Add the fractions. Then add the equivalent decimals. Compare the sums.

3. $2\frac{3}{4}$ 2.75 | $+1\frac{2}{4}$ 1.50 | $4\frac{1}{4}$ 4.25
4. $3\frac{4}{5}$ 3.8 | $+4\frac{3}{5}$ 4.6 | $8\frac{2}{5}$ 8.4

Subtract the fractions. Then subtract the equivalent decimals. Compare the differences.

5. $3\frac{1}{4}$ 3.25 | $-1\frac{3}{4}$ 1.75 | $(1\frac{1}{2})$ $1\frac{2}{4}$ 1.50
6. 4 4.000 | $-1\frac{1}{8}$ 1.125 | $2\frac{7}{8}$ 2.875

OBJECTIVE

Select the necessary information for solving a problem, and then solve the problem

RELATED ACTIVITIES

- Have students rewrite some of the word problems in this unit to show too much information. Display together different versions of a problem for the students to compare.

Choosing the Information Needed

Sometimes a problem gives more information than is needed.

Byron is 148.4 cm tall. Last year he was 140.5 cm tall. Last year Sandra was 147.7 cm tall. Now she is 153.2 cm tall. Who is taller now? How much taller?

Here is the information needed.
Byron is 148.4 cm tall.
Sandra is 153.2 cm tall.
Here is the solution.
153.2 - 148.4 = 4.8
Sandra is 4.8 cm taller.

For each problem, copy only the information you need. Then solve the problem. Answers are shown below.

1. Rosalie was 145.6 cm tall last year. Now she is 3.7 cm taller. Blake is 6.2 cm taller than he was last year when he was 143.8 cm tall. Who was taller last year? How much taller?
2. Last year Eva was 146.2 cm tall. Since then she has grown 5.9 cm. Irv has grown 8.1 cm since last year. Now he is 151.2 cm tall. Who has grown more since last year? How much more?
3. Julius has grown 6.8 cm since last year when he was 144.5 cm tall. Nora has grown 5.3 cm since last year and now is 147.6 cm tall. Who is taller now? How much taller?
4. Last year Ben was 140.3 cm tall and Phyllis was 141.5 cm tall. This year Phyllis is 145.3 cm tall and Ben is 148.0 cm tall. Since last year who has grown more? How much more?
5. Last year Gil was 149.7 cm tall and Chip was 147.1 cm tall. Since then Chip has grown 2.0 cm more than Gil and now is just 0.6 cm shorter than Gil. Gil is 155.5 cm tall now. How much has Gil grown since last year?
6. Last year when she was 148.7 cm tall, Jo was 4.8 cm taller than Sue. Jo has grown 4.6 cm and Sue has grown 7.8 cm since then. Now who is taller? How much taller?
7. Lyn is 153.4 cm tall, which is 4.3 cm taller than Joy. Last year Lyn was 146.5 cm tall. Since then she has grown 1.7 cm more than Joy. Joy was 2.6 cm shorter than Lyn last year but has grown 5.2 cm since then. Lyn has grown 6.9 cm this past year. Now Joy is 149.1 cm tall. How tall was Joy last year?

PROBLEM SOLVING

286

1. Rosalie was 145.6 cm tall last year.
   Blake was 143.8 cm tall last year.
   145.6 - 143.8 = 1.8
   Rosalie was 1.8 cm taller last year.
2. Eva has grown 5.9 cm.
   Irv has grown 8.1 cm.
   8.1 - 5.9 = 2.2
   Irv has grown 2.2 cm more than Eva.
3. Julius is now (6.8 + 144.5) cm, or 151.3 cm, tall.
   Nora is now 147.6 cm tall.
   151.3 - 147.6 = 3.7
   Julius is 3.7 cm taller now than Nora.

LESSON ACTIVITY

Using the Page

- Have students read the title of the lesson and the introductory statement at the top of the page. Then have the students read the word problem silently. Ask what word in the question "Who is taller now?" indicates what information is needed, and thus what information is not needed to solve the problem. Draw attention to the solution shown, noting that the necessary facts have been named first.
- For the exercises, emphasize that it is important to read each problem carefully, more than once, to be certain about the question and the facts needed to answer it. After they have completed the problems, have the students explain how they decided which facts were needed for solving each problem.

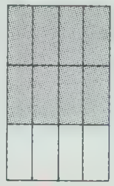
4. Ben has grown (148.0 - 140.3) cm, or 7.7 cm, since last year.
   Phyllis has grown (145.3 - 141.5) cm, or 3.8 cm, since last year.
   7.7 - 3.8 = 3.9
   Since last year, Ben has grown 3.9 cm more than Phyllis.
5. Last year Gil was 149.7 cm tall.
   This year Gil is 155.5 cm tall.
   155.5 - 149.7 = 5.8
   Gil has grown 5.8 cm since last year.
6. Last year Sue was (148.7 - 4.8) cm, or 143.9 cm, tall.
   Sue is now (143.9 + 7.8) cm, or 151.7 cm, tall.
   Jo is now (148.7 + 4.6) cm, or 153.3 cm, tall.
   153.3 - 151.7 = 1.6
   Jo is 1.6 cm taller than Sue now.
7. Joy is now 149.1 cm tall.
   Joy grew 5.2 cm this year.
   149.1 - 5.2 = 143.9
   Last year Joy was 143.9 cm tall.

Checking Up

Write two equivalent fractions for each picture.

1. 

$\frac{2}{4}, \frac{1}{2}$

2. 

$\frac{8}{12}, \frac{2}{3}, \frac{4}{12}, \frac{1}{3}$

Answers will vary

Write two fractions that are equivalent to each of these.

3. $\frac{3}{5}$ $\frac{6}{10}, \frac{12}{20}$
4. $\frac{7}{8}$ $\frac{14}{16}, \frac{28}{32}$
5. $\frac{6}{18}$ $\frac{1}{3}, \frac{12}{36}$
6. $\frac{9}{12}$ $\frac{3}{4}, \frac{18}{24}$

Find the missing term in each pair of equivalent fractions.

7. $\frac{1}{5} = \frac{■}{35}$ 7
8. $\frac{5}{6} = \frac{15}{■}$ 18
9. $\frac{2}{9} = \frac{■}{36}$ 8
10. $\frac{7}{12} = \frac{35}{■}$ 60

Use >, <, or = to make true statements.

11. $\frac{4}{5}$ ● $\frac{2}{5}$ >
12. $\frac{5}{8}$ ● $\frac{3}{5}$ >
13. $\frac{4}{6}$ ● $\frac{6}{9}$ =
14. $\frac{3}{7}$ ● $\frac{5}{12}$ >

Write each of these as an improper fraction.

15. $5\frac{1}{2}$ $\frac{11}{2}$
16. $2\frac{2}{3}$ $\frac{8}{3}$
17. $4\frac{7}{8}$ $\frac{39}{8}$
18. $1\frac{5}{12}$ $\frac{17}{12}$

Write each of these as a number in mixed form.

19. $\frac{11}{4}$ $2\frac{3}{4}$
20. $\frac{31}{5}$ $6\frac{1}{5}$
21. $\frac{13}{9}$ $1\frac{4}{9}$
22. $\frac{55}{10}$ $5\frac{5}{10}$

Write each of these as a decimal.

23. $\frac{3}{4}$ 0.75
24. $\frac{2}{5}$ 0.4
25. $\frac{5}{8}$ 0.625
26. $\frac{25}{4}$ 6.25

Add.

27. $2\frac{1}{5}$ + $4\frac{3}{5}$ = $6\frac{4}{5}$
28. $1\frac{3}{4}$ + $2\frac{3}{4}$ = $4\frac{2}{4}$
29. $5\frac{5}{8}$ + $3\frac{7}{8}$ = $9\frac{4}{8}$
30. $6\frac{4}{10} + 5\frac{6}{10}$ 12
31. $3\frac{7}{12} + 6\frac{9}{12}$ $10\frac{4}{12}$

Subtract.

32. $3\frac{5}{7}$ − $1\frac{2}{7}$ = $2\frac{3}{7}$
33. 5 − $2\frac{5}{6}$ = $2\frac{1}{6}$
34. $8\frac{1}{3}$ − $4\frac{2}{3}$ = $3\frac{2}{3}$
35. $6\frac{3}{8} - 1\frac{7}{8}$ $4\frac{4}{8}$
36. $10\frac{7}{9} - 4\frac{8}{9}$ $5\frac{8}{9}$

Find each of these.

37. $\frac{1}{6}$ of 42 7
38. $\frac{2}{3}$ of 15 10
39. $\frac{3}{7}$ of 56 24
40. $\frac{7}{12}$ of 60 35

287

Page 287

OBJECTIVE

Demonstrate an understanding of the concepts and skills presented in this unit

RELATED ACTIVITIES

- Have students write fractions for sets of objects arranged in the classroom. Consider these examples: a set of pencils, some of which have erasers; a stack of books, some of which are mathematics books; a group of students, some of whom are girls.
- Have the students determine the number of minutes in certain parts of an hour, for example, $\frac{3}{4}$ of an hour. Ask what part of an hour is represented by 10 min, 20 min, 40 min, and so on.

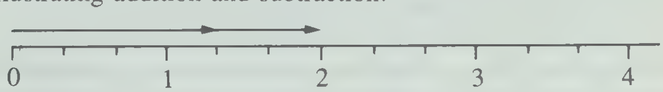
Skills	Exercises	Related Pages
Write equivalent fractions for diagrams	1, 2	T284-T285
Use multiplication or division to find equivalent fractions	3-6	T286
Find the missing term in two equivalent fractions	7-10	T287, T289
Compare fractions	11-14	T290-T291 T308-T309
Express a number in mixed form as an improper fraction	15-18	T292-T293
Express an improper fraction as a number in mixed form	19-22	T294-T295
Express a fraction as a decimal	23-26	T304-T307
Add fractions	27-31	T296-T297
Subtract fractions	32-36	T298-T299
Find the number for part of a set	37-40	T302-T303

Comments

If students are having difficulty with concepts in this unit, have them work with models or draw pictures to represent fractions. Working with number patterns can help them to develop an understanding of equivalent fractions.

$$\frac{2}{5}, \frac{4}{10}, \frac{6}{15}, \frac{8}{20}, \frac{10}{25}, \frac{12}{30}$$

Number lines marked to show fractions can be useful in illustrating addition and subtraction.



$$1\tfrac{1}{3} + \tfrac{2}{3} = 1\tfrac{3}{3}$$
$$= 2$$

When the students have completed their work, you may wish to discuss different answers for Ex. 3-6.

Unit 14 Overview

Motion Geometry

Congruency of shapes is studied in this unit by means of motions. In Unit 9, congruent angles and congruent figures are identified by tracings. In this unit, tracings are used to identify three kinds of motions: the slide, the flip, and the turn. Students use these three motions to construct congruent figures and also to derive other polygons by repeating given shapes. This repetition of one or more shapes leads to the concept of tiling which is found in many real-life applications. The students also learn how to enlarge pictures by using grids with larger squares, to reduce them by using grids with smaller squares, and to distort them with grids composed of rectangles or parallelograms. Skills in operations with whole numbers and decimals are maintained in one *Keeping Sharp* feature and may be evaluated in the four sets of exercises on pages 314 and 315.

Unit Outcomes

- identify one figure as the slide image of another figure for a given slide arrow
- draw the slide image of a given figure for a given slide arrow, with a grid and without a grid
- identify one figure as the flip image of another figure for a given flip line
- draw the flip image of a given figure for a given flip line, with a grid and without a grid
- identify one figure as the turn image of another figure for a given turn angle
- draw the turn image of a given figure for a given turn angle, with a grid and without a grid
- turn or flip triangles to build regular polygons
- fit congruent shapes together so that they share congruent sides
- make a tiling pattern using one shape
- copy a picture from a square grid onto a grid with squares of a different size and onto grids with rectangles or parallelograms
- use two or more shapes to create tiling patterns

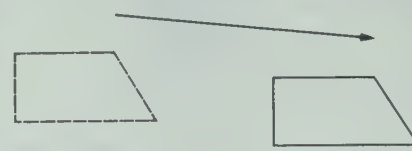
Background

The term *congruent* is used to describe polygons that have the same size and the same shape. Their sides match point for point and their corresponding angles formed by pairs of adjacent sides are also the same size. In contrast, polygons which have corresponding angles of the same size, but sides which are not the same length, are described as being *similar* in shape. This unit includes one lesson showing how to use square grids to copy figures that are congruent and similar. The major emphasis, however, is given to a study of congruency which is determined by noting whether a tracing of a shape can be moved to match another shape. This is known as *transformation geometry*, or informally, as *motion geometry*. This approach is particularly effective with young children since it uses visual and manipulative techniques, rather than abstract reasoning, to verify the congruency of shapes.

Three basic transformations are studied: the *translation* (slide), the *reflection* (flip), and the *rotation* (turn). The less formal terms in parentheses are used in *Starting Points in Mathematics* since they are more meaningful to students at this level.

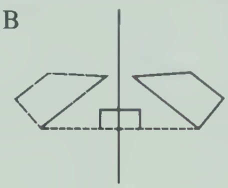
All the motions in this unit occur in one plane, in other words, on a flat surface. A slide can occur along a straight line and the orientation of the shape is not changed. Both the direction and the length of a slide may be indicated by a slide arrow (A). Each point of a shape moves the same distance and in the same direction for a slide. Therefore, measuring the distance between corresponding vertices may be used to check the congruency of the shapes.

A

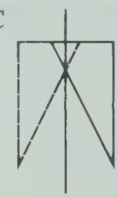


A shape can be flipped about a line, and the image is the reverse of the original shape. The term *mirror image* is sometimes used to describe it. A flip changes the orientation of a shape by reversing it. The flip line may be located outside the shape (B), inside the shape (C), or on the shape — either along one side (D), or at a vertex (E).

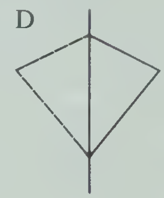
B



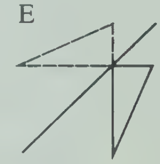
C



D



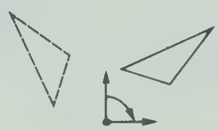
E



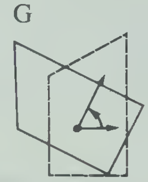
For a flip, it should be pointed out that corresponding points, particularly vertices, are equal distances from the flip line, and the line joining them is perpendicular to the flip line, that is, it intersects the flip line at right angles (B).

A shape can be turned using any point as a turn center. Again, the turn center may be outside the shape (F), inside the shape (G), or on the shape (H and I).

F

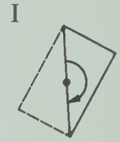


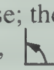
G



H

I



The direction and amount of rotation is indicated by a turn arrow which shows both the turn center and the turn angle. The orientation of the shape is altered according to the amount of rotation. It should be pointed out that for two different turns with the same turn center a shape may have two identical turn images. One turn is counterclockwise and the other is clockwise; the sum of the turn angles is 360° (a complete rotation). Thus,  and  give the same result.

The introductory lessons involve flip lines and turn centers that are outside the shapes. Later, in building polygons from triangles, flip lines and turn centers on shapes are encountered.

The tiling of surfaces with a variety of congruent shapes dates from very early times. Some of the most artistic arrangements were created by the Moors who occupied large areas of Spain from the ninth to the fifteenth centuries. Intricate tiling patterns remain to this day in courtyards and patios and in mosaics on the walls of palaces and cathedrals. Tiling is achieved by using squares and rectangles because their shapes generally leave the fewest uncovered spaces. Parquet floors, found in many buildings today, usually have patterns using squares and rectangles, such as those shown in the illustrations.

Other polygons, particularly regular hexagons and isosceles right-angled triangles, are used frequently, although spaces sometimes remain along the edges of rectangular areas. Honey bees use a form of tiling with hexagons when they build a honeycomb inside a rectangular form. The bees simply fill in the spaces along the edges with more wax.

In this unit a method is presented for copying pictures by means of grids. All students can benefit from experiences using this useful technique. The most common grid is composed of squares, but rectangles and parallelograms may also be used. Congruent copies can be made if the units in the two grids are the same size and shape. Enlargements or reductions are possible if the grid units are similar in shape but different in size. For example, copying from a grid of small squares onto a grid of larger squares will produce an enlargement. Distortions can also be made in pictures if the shape of the grid on the paper for copying is different from the original grid. For instance, copying from a square grid onto a rectangular grid will distort a picture, and the elongation will be either vertical or horizontal according to the orientation of the rectangular grid.

Teaching Strategies

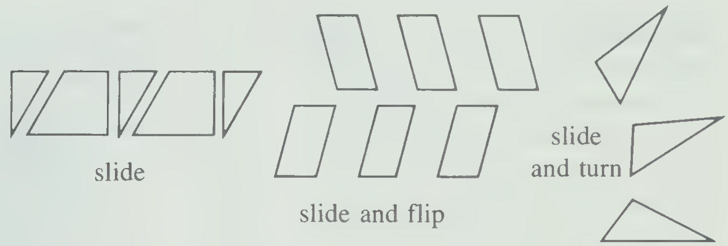
This unit on motion geometry is not dependent upon a background of skills and concepts associated with numbers and operations with them. For this reason, it may be used at almost any time in the year's program to provide a change for the students. The practice exercises on pages 301, 314, and 315 do involve number operations, however, and their use is dependent upon the levels attained by the students.

It is suggested that the students be organized into groups of ten or fewer students so that the teacher may observe and work closely with them as they carry out the transformations indicated in the lessons. While one group of students is engaged in these activities, others may be working in other areas of the curriculum or may be completing some of the exercises on pages 301, 314, and 315. Other sets of practice exercises are provided on pages 332–337.

Tracings are used to test all the transformations in this unit, but alternative methods may also be used in some cases. In connection with slides on grids, the slide arrows may be described in terms of how many units to the right or to the left and how many units up or down the tracing is to be moved. Since all points of a shape slide the same distance and in the same direction, congruency of the image and the shape may be checked by counting units in two directions for each vertex. For slides on plain paper, a ruler, or a strip of paper marked to show the length of the slide arrow, may be used to check the distances between corresponding vertices of the shape and its image. The orientation of these measuring tools must remain the same as that of the slide arrow.

Counting and measuring may also be used to check flip images of shapes. With this type of transformation, the distance from any vertex to the flip line is equal to the perpendicular distance from the flip line to the corresponding vertex on the other side. A semitransparent mirror may also be used to check flip images, as described on pages 182 and 183.

The study of motion geometry can be correlated with work in other subjects of the curriculum. In physical education a student can demonstrate slides and turns, and two students working together can assume "mirror" postures to represent flip images. Transformation of shapes can be used to create interesting artistic designs. Students can design gift-wrapping paper, wallpaper, and fabric patterns and reproduce them in colors, using several kinds of media. Simple transformations may involve only one kind of motion, such as a slide; more complex patterns may be based on two or more different motions, such as a slide and a turn, or a slide and a flip. Students may find it interesting to examine commercially prepared designs on paper and fabric and to describe how the designs are created in terms of motion geometry.



A table or a mathematics center in one area of the classroom is recommended to keep a supply of materials for the students to use. There should be an ample supply of graph paper and tracing paper, models of polygons for the lesson on pages 304 and 305, and parquetry blocks for making colorful designs such as the one on page 311. Parquetry blocks are usually found in kindergartens, but they are useful in connection with the work in this unit, especially tiling. Other materials in the center could be samples of wallpaper and fabric, pieces of ceramic tile, and plexiglass mirrors.

Materials

copies of page T397, tracing paper, plain paper, a straight edge, and a sharp pencil for each student
overhead projector and transparent acetate marked with a square grid, large sheets of graph paper, a sheet of clear acetate
pins (optional)
copies of Ex. 5 – 8 on page 291 (optional)
a sheet of plain paper and carbon paper for each student
copies of Ex. 5 – 8 on page 295 (optional)
large isosceles right-angled triangle cut from Bristol board and labeled ACB as shown on page 302
two congruent scalene triangles cut from Bristol board and crayons; tracing paper or construction paper, scissors, crayons, and a straight edge for each student
copies of pages T399 and T400 for each student (optional)
parquetry blocks similar to those shown on page 311 (optional)
copies of the polygons on pages T382 – T385 (optional)

Vocabulary

slide	turn angle
slide arrow	turn image
slide image	decagon
congruent shapes	tiling
flip	grid
flip line	distorted
flip image	midpoint
turn	clockwise (optional)
turn center	counterclockwise (optional)

Pages 288-289

LESSON OUTCOME

Identify one figure as the slide image of another figure for a given slide arrow

Materials

copies of page T 397, tracing paper, a straight edge, and a sharp pencil for each student; overhead projector and transparent acetate marked with a square grid or a large sheet of graph paper; a sheet of clear acetate

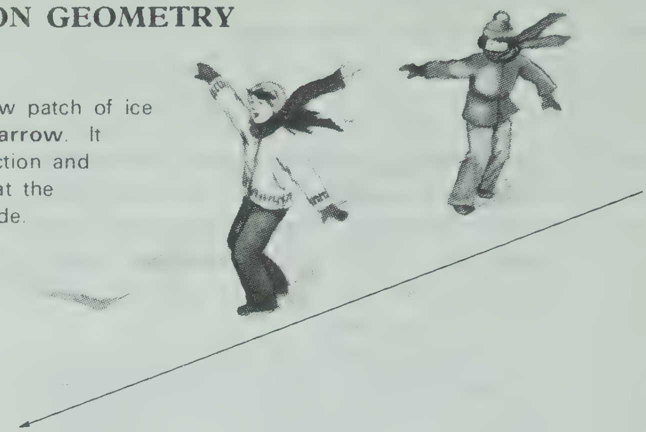
Vocabulary

slide, slide arrow, slide image, congruent shapes

14 MOTION GEOMETRY

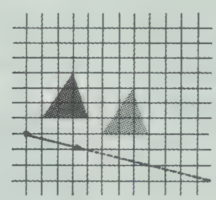
Slides

The long, narrow patch of ice is like a **slide arrow**. It shows the direction and the distance that the children can slide.

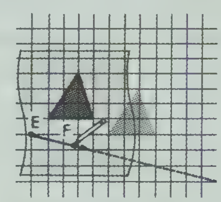


Working Together

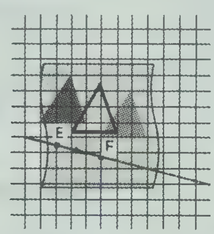
Use graph paper. Copy the two shapes and the slide arrow. Then extend the slide arrow.



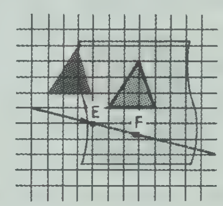
Place tracing paper on your drawing. Trace the red shape and mark points E and F as shown.



Slide the tracing so that points E and F move along the path of the slide arrow until . . .



. . . point E is on the tip of the arrow. The tracing of the red shape matches the blue shape. The blue shape is the **slide image** of the red shape.



288

LESSON ACTIVITY

Before Using the Pages

- Introduce the concept of a slide by having the students move shapes on a flat surface according to your instructions. For example, ask them to place their mathematics books on their desks. Direct them to slide the books to the right, to the left, forward, backward, toward one corner of the desk, to one edge of the desk, and so on. Point out that they must not turn the book. Develop that a slide involves direction and distance. Ask what would have to be specified for all the students to slide the books to the same position on their desks (the starting point, the direction of the slide, and the length of the slide).

Using the Pages

- The example at the top of page 288 introduces the *slide arrow* to indicate both the direction and the length of a slide. For the situation illustrated, emphasize that the children do not turn. Ask students to name other examples of slides; for instance, you slide a drawer open or shut, and you may slide a window to open or shut it.

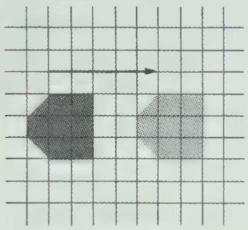
Working Together: The procedure of sliding a tracing according to a given slide arrow is best demonstrated on an overhead projector. In advance of the lesson, copy the red shape, the blue shape, and the slide arrow on a transparency marked with a grid. An alternative method is to copy the diagram on a large sheet of graph paper and tape the paper to the chalkboard. Have the students compare the copy with the illustration on page 288 to ensure that the copy is correct. Emphasize that the line containing the slide arrow is to be extended far enough so that at least one more identical slide arrow can be shown. Marking points E and F and extending the slide arrow ensures that students do not turn the tracing, and that they slide the tracing for the given direction and distance. Tape the transparency in place on the overhead projector. Place a sheet of clear acetate over the grid. Demonstrate the procedure of tracing the red shape and sliding the tracing to match the blue shape. Repeat the slide motion two or three times to familiarize the

Exercises

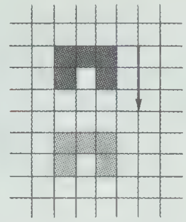
Copy the two shapes and the slide arrow on graph paper. Then use tracing paper to test whether the blue shape is the slide image of the red shape for the slide arrow shown.

The slide arrow shows the direction and distance.

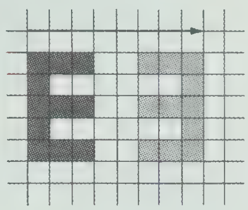
1. yes



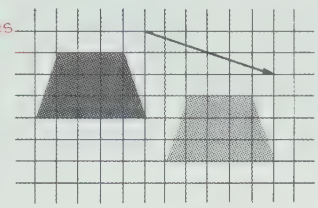
2. no



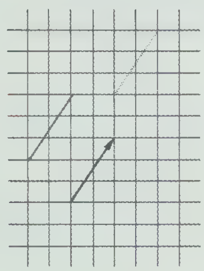
3. no



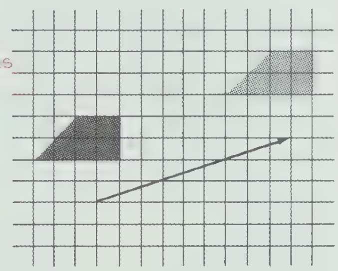
4. yes



5. no



6. yes



Use graph paper.

*7. Draw two congruent shapes and a slide arrow. Have a friend use tracing paper to tell whether one is the slide image of the other.
Answers will vary

Congruent means same size and shape.

289

RELATED ACTIVITIES

• For Ex. 2, 3, and 5 on page 289, ask students to explain why the blue shape is not the slide image of the red shape; for example, the slide arrow may be the wrong length. Some students may intuitively recognize that the two shapes in Ex. 3 cannot be matched by a slide because they have different orientations.

• Have students describe the slide arrows on page 289 using the format "right (left), up (down)". For example, the slide arrow for Ex. 4 can be described as "right 6, down 2", and for Ex. 2, as "right 0, down 3". Note that this order corresponds to that used in naming points on a grid with ordered pairs of numbers.

• For Ex. 1-6 on page 289, students may experience difficulty if they focus on the shape rather than on the slide arrow when sliding each tracing. For example, the shapes in Ex. 2 are congruent, but they do not match for the given slide arrow. Encourage the students to pay particular attention to the length of the slide arrow for each exercise.

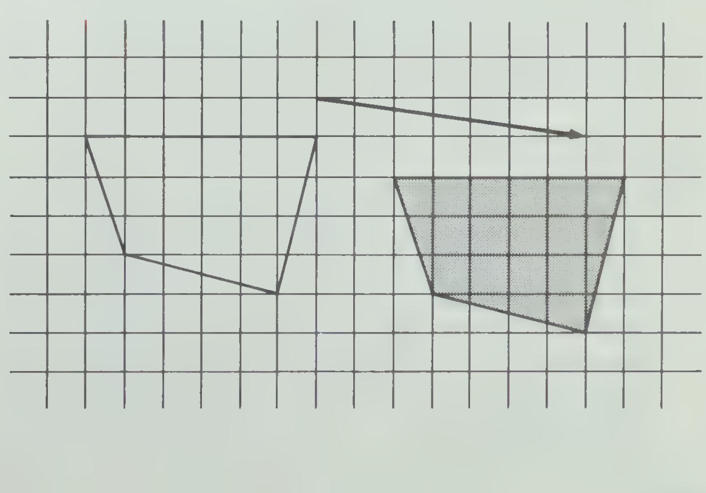
students with the procedure. Then provide graph paper (copies of page T 397) and tracing paper for them to try the procedure. Point out that the blue shape is the *slide image* of the red shape because the tracing of the red shape matches the blue shape. Discuss that the slide image would be in a different position if the slide arrow were of a different length and/or in a different direction.

Exercises: It is essential that the students work on a flat surface and that they extend the slide arrows as shown in *Working Together* on page 288. For these reasons, they are directed to copy the diagrams onto graph paper. Since this will require extra care, allow them ample time so that they will not be rushed. You may prefer to prepare copies and duplicate them for the students. For Ex. 7, point out what is meant by *congruent shapes*. (Congruent angles were encountered on page 174.)

Assessment

Copy the two shapes and the slide arrow on graph paper. Then use tracing paper to test whether the gray shape is the slide image of the white shape for the slide arrow shown.

1. no



Pages 290-291

LESSON OUTCOME

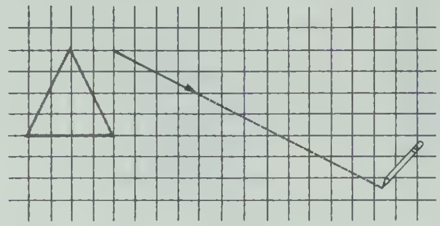
Draw the slide image of a given figure for a given slide arrow, with a grid and without a grid

Materials

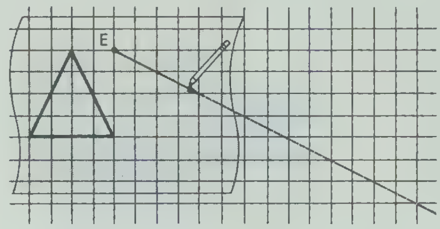
copies of page T 397, tracing paper, plain paper, a straight edge, and a sharp pencil for each student; pins (optional); overhead projector and transparent acetate marked with a square grid (optional); large sheets of graph paper; copies of Ex. 5-8 on page 291 (optional)

Drawing Slide Images

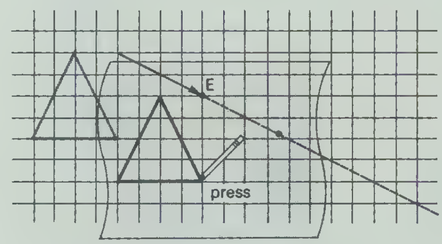
Draw the slide image of the triangle for the slide arrow shown.



Extend the slide arrow.

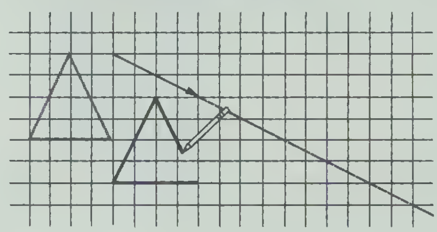


Use tracing paper. Copy the triangle. Mark the end point (E) and one other point of the slide arrow.



Slide the tracing so the two points move along the path of the slide arrow. Stop when the end point has moved to the tip of the arrow.

Mark the vertices with a pin or a sharp pencil.



Remove the tracing paper and draw the slide image.

290

LESSON ACTIVITY

Before Using the Pages

- Use the overhead projector as described in the previous lesson, or use a large sheet of graph paper. Review the procedure of tracing a shape and sliding the tracing according to a given slide arrow. Use an example for which a slide image is obtained, that is, the tracing of the first shape matches the second shape for the given slide arrow.

Using the Pages

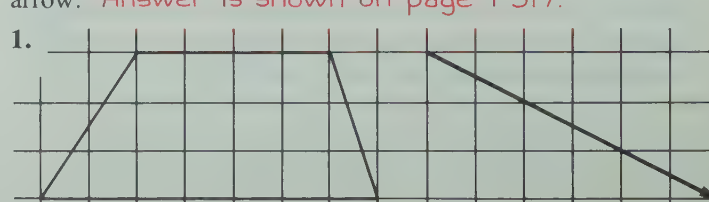
- In advance of the lesson, copy the red triangle and the slide arrow from the example on page 290 onto a large sheet of graph paper. Present the problem of drawing the slide image and ask how this can be done. The students will likely suggest the use of tracing paper. Have them demonstrate their suggestions on the graph paper. Then draw their attention to the steps outlined on page 290. Provide graph paper (use copies of page T 397) and tracing paper for them to try the procedure. If they are not using pins to mark the vertices of the slide image, ensure that they use sharp pencils and press firmly so that the indentations show on the graph paper under the tracing paper.

Exercises: Provide the students with copies of page T 397 on which to copy the shapes and slide arrows for Ex. 1-4. Because the shapes for Ex. 5-8 are not on a grid, you may wish to prepare copies of the shapes for the students. Note that for Ex. 8 the slide image will overlap the original shape.

Assessment

Copy the shape and the slide arrow on graph paper. Use tracing paper to help you draw the slide image of the shape for the slide arrow. Answer is shown on page T 317.

1.

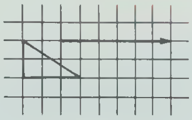


Exercises

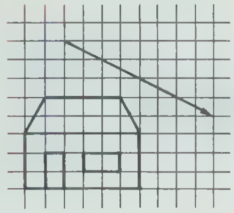
Copy each shape and the slide arrow on graph paper. Use tracing paper to help you draw the slide image of the shape for the slide arrow shown.

Answers are shown on page T369

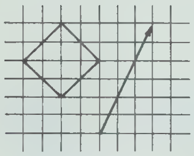
1.



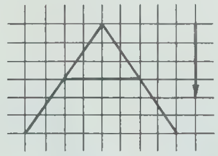
2.



3.

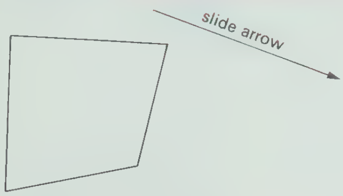


4.

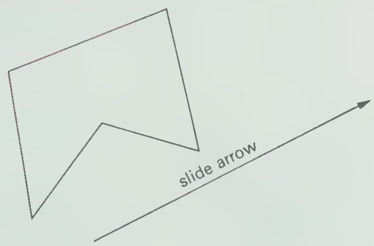


Copy each shape and the slide arrow on plain paper. Use tracing paper to help you draw the slide image of the shape. Answers are shown on page T370

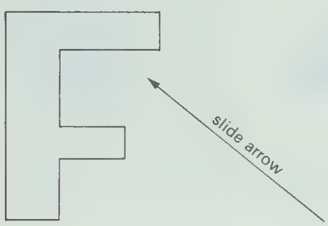
5.



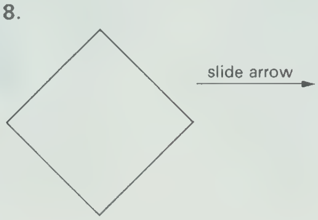
6.



7.



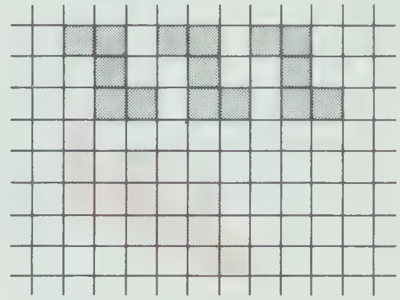
8.



291

RELATED ACTIVITIES

- Have the students create slide patterns on strips of graph paper. They may wish to color the patterns.



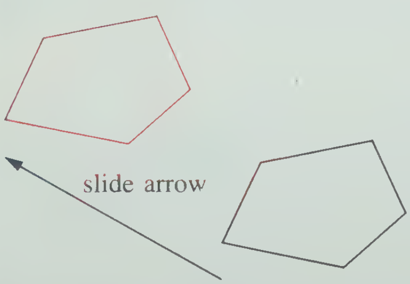
- Have the students examine samples of wallpaper and fabric for patterns and designs based on slides.
- Demonstrate slide patterns by typing the same letter or numeral several times in sequence on a typewriter. Note that the key on the typewriter does not slide to create the pattern; the typewriter slides the paper a fixed distance to the left each time the key is pressed. The distance (slide arrow) may be changed through the use of the space bar on the typewriter.

Y Y Y Y Y Y

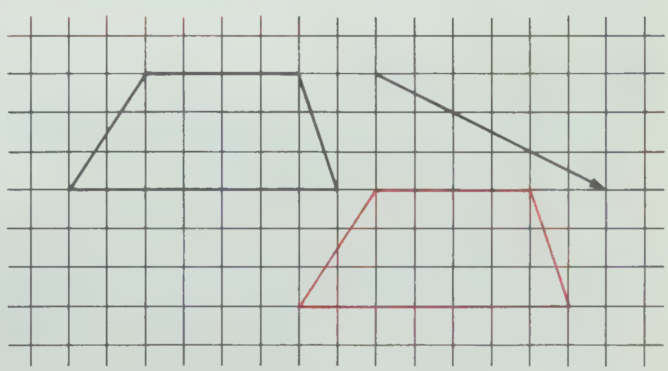
3 3 3 3 3 3

Copy the shape and the slide arrow on plain paper. Use tracing paper to help you draw the slide image of the shape for the slide arrow.

2.



Ex. 1 of Assessment



Pages 292-293

LESSON OUTCOME

Identify one figure as the flip image of another figure for a given flip line

Materials

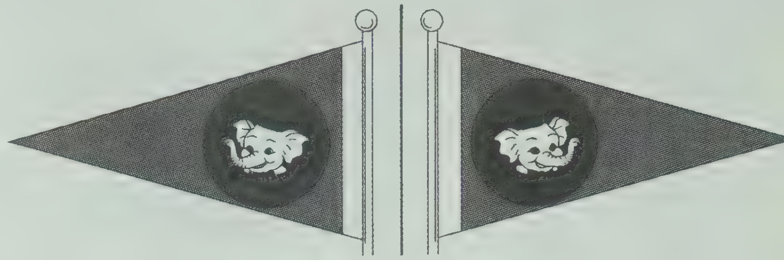
copies of page T397, tracing paper, a straight edge, and a sharp pencil for each student; a sheet of plain paper and carbon paper for each student; overhead projector or a large sheet of graph paper

Vocabulary

flip, flip line, flip image

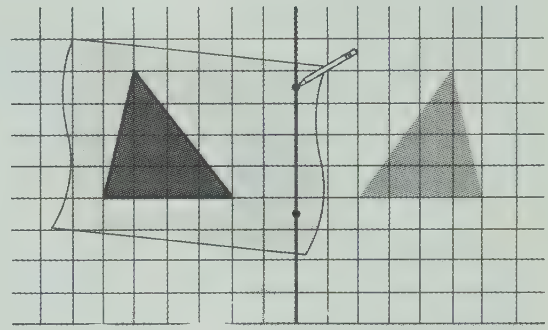
Flips

The flag in a west wind is the **flip image** of itself in an east wind.



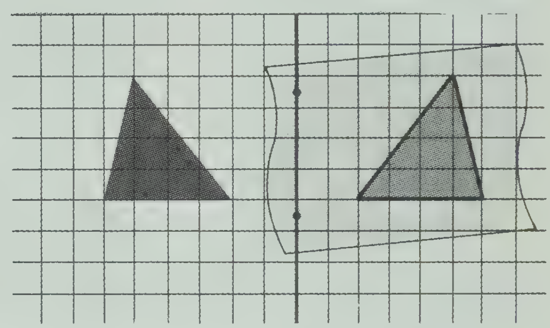
Working Together

Use tracing paper. Trace the red shape and the two points shown on the **flip line**.



Flip the tracing over. Match the two points on the tracing with those on the flip line.

The tracing of the red shape matches the blue shape. The blue shape is the flip image of the red shape.

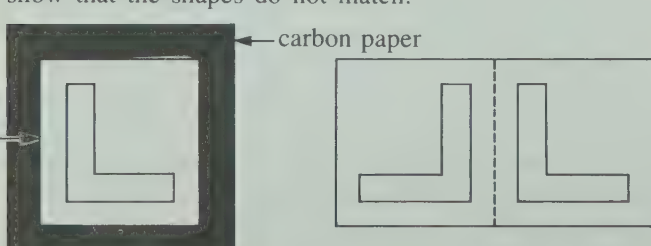


292

LESSON ACTIVITY

Before Using the Pages

- Have each student complete the following activity or demonstrate the activity for the students. Fold a piece of paper in half and place it on top of a sheet of carbon paper with the carbon side up. Draw a letter such as L, J, or F on the paper. Then unfold the paper to show the drawing and the carbon copy. Ask whether a tracing of the letter L can slide to match the carbon copy. Trace the letter on tracing paper and ask a student to demonstrate the procedure to show that the shapes do not match.



Ask how the tracing can be moved so that the shapes will match. Lead the students to suggest flipping the tracing paper. They can also demonstrate that the shapes will match by flipping half the page over the fold to match the other half.

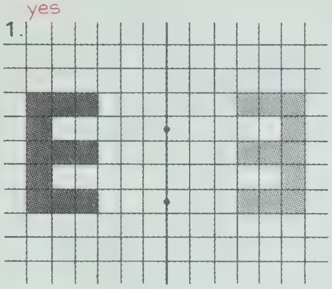
Using the Pages

- The example at the top of page 292 introduces the term *flip image*. The illustration shows that the direction of a flip image is reversed in relation to the original shape. For example, if a flag points to the left, the flip image points to the right. You may wish to have the students trace one of the flags and flip the tracing to match the other flag. This can motivate a discussion of the need to mark two reference points on the flip line. These reference points ensure that the image will be the same distance from the flip line as the original shape. They also ensure that the image is directly opposite the original shape, with no sliding upward or downward during the flip motion. Have students suggest other examples of flips, such as flipping the pages of a book and flipping a pancake on a griddle.

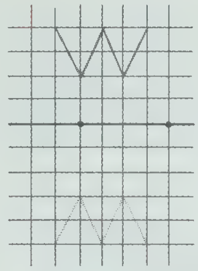
Exercises

Copy each picture on graph paper. Use tracing paper to test whether the blue shape is the flip image of the red shape for the flip line shown.

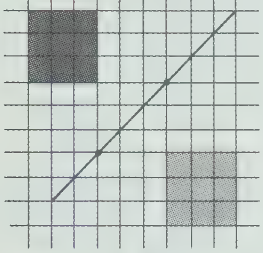
1. yes



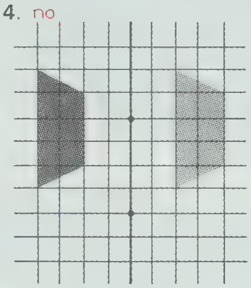
2. no



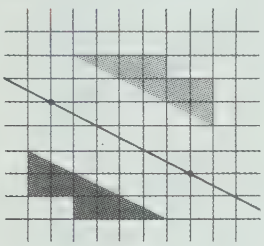
3. yes



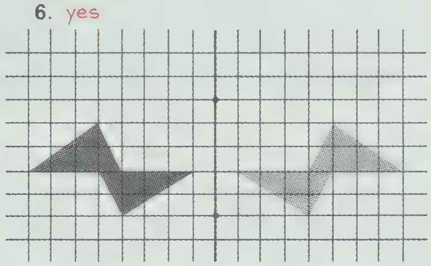
4. no



5. no



6. yes



Use graph paper.

*7. Draw two congruent shapes and a flip line. Have a friend use tracing paper to tell whether one is the flip image of the other. Answers will vary.

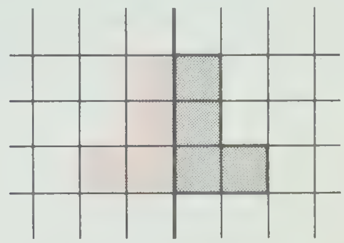
293

RELATED ACTIVITIES

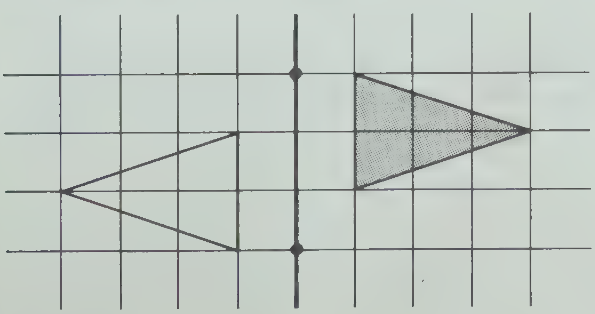
• The *Try This* feature on pages 182 and 183 demonstrates the use of a semitransparent plexiglass mirror in studying line symmetry. Such a mirror is useful at this time to determine whether one shape is the flip image of another shape for a given flip line. Have the students test for flip images by placing the mirror along the flip line for each of Ex. 1-6 on page 293 and for the example at the top of page 292.

• Have students work in pairs facing one another, so that one student represents an object and the other student represents the flip image. Have them place a metre stick on the floor to represent the flip line. For each position taken by the "object", the "image" assumes the appropriate position. For example, if the "object" raises the left arm, the "image" raises the right arm.

• Provide exercises similar to those on page 293 such that the flip line is on the shape. An example is shown below. To prepare students for this concept, have them repeat the activity in *Before Using the Pages* so that one segment of the letter L is the folded edge of the paper.



Working Together: Adapt the method described in *Working Together* on pages T314 and T315 to demonstrate flip images on the overhead projector or on a large sheet of graph paper. Then provide tracing paper for the students to try the procedure. Draw attention to the *flip line* and discuss the importance of marking two reference points on that line. This helps to position the tracing correctly. You may wish to use a diagram similar to the following to emphasize the necessity of marking two points on the flip line. The gray shape is not the flip image of the white shape.



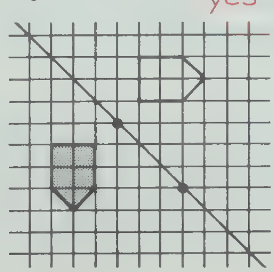
When the students have finished, ask them to use their tracings to demonstrate that the blue shape on page 292 is not a slide image of the red shape.

Exercises: Because it is necessary to work on a flat surface to obtain accurate results, the students are directed to copy the diagrams onto graph paper (copies of page T397). For Ex. 7, review the meaning of *congruent shapes*.

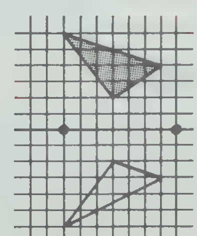
Assessment

Copy the picture on graph paper. Use tracing paper to test whether the gray shape is the flip image of the white shape for the flip line shown.

1. yes



2. no



LESSON OUTCOME

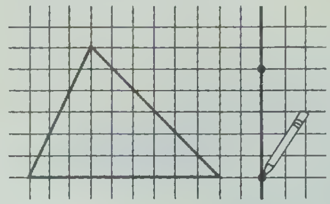
Draw the flip image of a given figure for a given flip line, with a grid and without a grid

Materials

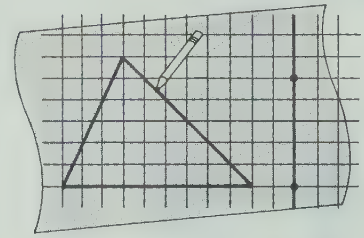
copies of page T397, tracing paper, plain paper, a straight edge, and a sharp pencil for each student; pins (optional); overhead projector (optional) or large sheets of graph paper; copies of Ex. 5-8 on page 295 (optional)

Drawing Flip Images

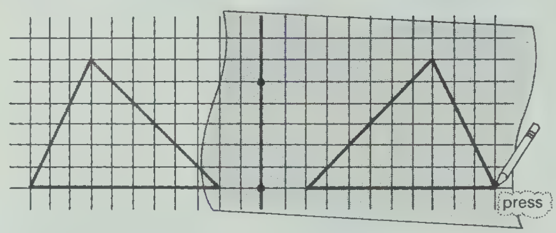
Draw the flip image of the triangle for the flip line shown.



Mark two points on the flip line.

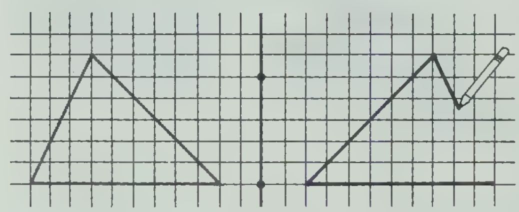


Use tracing paper.
Copy the two points on the flip line.
Copy the triangle.



Flip the tracing over. Match the two points on the tracing with those on the flip line.

Mark the vertices with a pin or a sharp pencil.



Remove the tracing paper and draw the flip image.

294

LESSON ACTIVITY

Before Using the Pages

- Use the overhead projector or a large sheet of graph paper. Review the procedure for tracing a shape and flipping the tracing for a given flip line to match the shape with its flip image. Emphasize that the tracing paper is flipped; it must not slide or turn.
- In advance of the lesson, copy the red triangle and the flip line from the example on page 294 onto a large sheet of graph paper. Ask how to find the flip image of the triangle. Have several students demonstrate their suggestions. Then draw the students' attention to the steps outlined on page 294.

Using the Pages

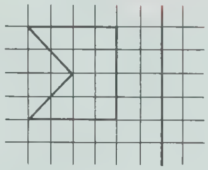
- Provide the students with tracing paper and graph paper (copies of page T 397). Have them copy the red triangle and the flip line for the first diagram on page 294 and follow the procedure for finding the flip image. Remind them to mark the two reference points on the flip line. If they are not using pins to mark the vertices of the flip image, ensure that they use sharp pencils and press firmly so that the indentations show on the graph paper under the tracing paper.

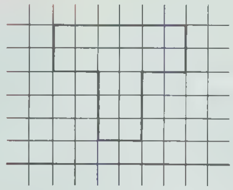
Exercises: Provide the students with tracing paper and graph paper on which to copy and complete Ex. 1-4. Since Ex. 5-8 are to be shown on plain paper, you may wish to prepare copies of the shapes in advance of the lesson. Some students may find it helpful to place the tracing paper so that one edge of the paper matches the flip line. Two reference points can be marked along the edge of the tracing paper. Then, the tracing paper is flipped about that edge and the two reference points are matched.

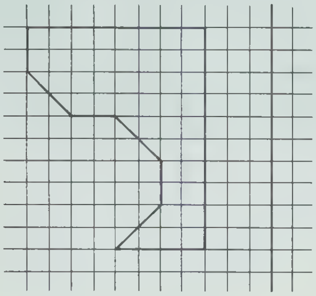
Exercises

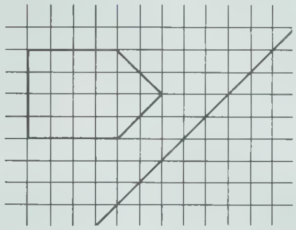
Copy each shape and the flip line on graph paper. Use tracing paper to help you draw the flip image of the shape for the flip line shown.

Answers are shown on page T370.

1. 

2. 

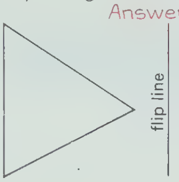
3. 

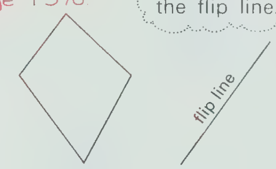
4. 

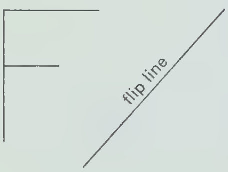
Copy each shape and the flip line on plain paper. Use tracing paper to help you draw the flip image of the shape.

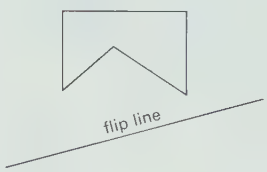
Remember to mark two points on the flip line.

Answers are shown on page T370.

5. 

6. 

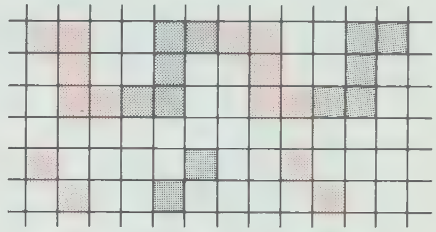
7. 

8. 

295

RELATED ACTIVITIES

- Have the students use a semitransparent plexiglass mirror along the flip lines to check their work for Ex. 1-8 on page 295.
- Ask the students to create flip patterns on strips of graph paper. Color can highlight shapes that are related by a slide motion.

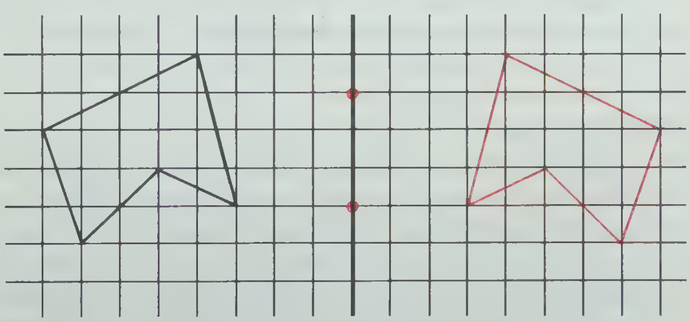


- Have the students examine samples of wallpaper and fabric for patterns and designs based on flips.
- If a flip line follows a line of the grid as in Ex. 1-3 on page 295, for example, some students may be able to draw the flip image of a shape without using tracing paper. The matching vertices for a shape and its image are on opposite sides of the flip line but are the same distance from the flip line. Students may draw the flip image without using tracing paper and then use tracing paper to check their work. A greater challenge is presented when flip lines are diagonal on the grid as in Ex. 4.

Assessment

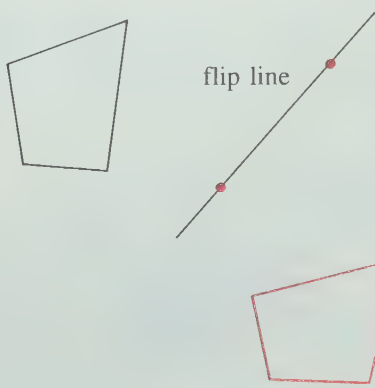
Copy the shape and the flip line on graph paper. Use tracing paper to help you draw the flip image of the shape for the flip line shown.

1.



Copy the shape and the flip line on plain paper. Use tracing paper to help you draw the flip image of the shape.

2.



Pages 296 - 297

LESSON OUTCOME

Identify one figure as the turn image of another figure for a given turn angle

Materials

copies of page T 397, tracing paper, a straight edge, and a sharp pencil for each student; overhead projector and a transparency marked with a grid (optional) or large sheets of graph paper; pins (optional)

Vocabulary

turn, turn centre, turn angle, turn image, clockwise and counter-clockwise (optional)

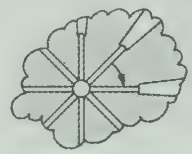
Turns

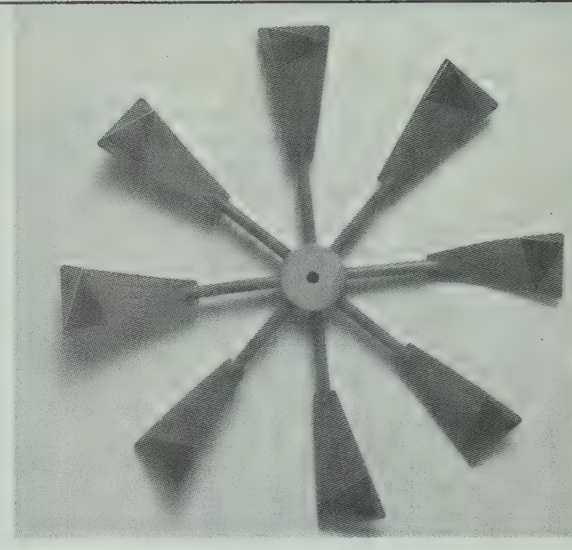
Each sail of the toy windmill is a **turn image** of the other sails.

The hole *through*

is the **turn center**.

Two arms of the windmill suggest a **turn angle**. The turn center is its vertex.





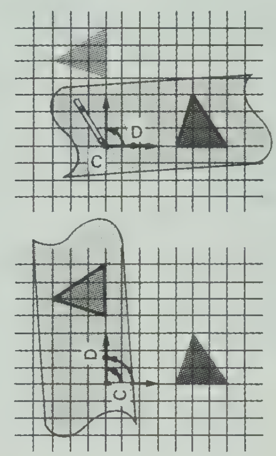
A turn angle shows how much a shape turns about the turn center to go from one position to the next.

Working Together

Use tracing paper. Trace the red shape and mark points C (turn center) and D as shown.

Turn the tracing about point C until point D is on the other ray of the turn angle.

The tracing of the red shape matches the blue shape. The blue shape is the turn image of the red shape.

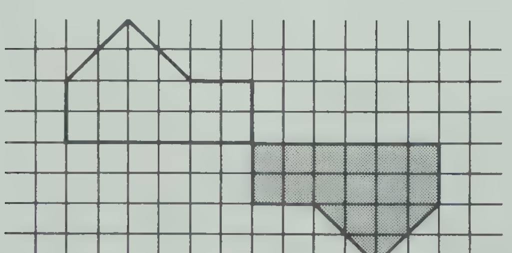


296

LESSON ACTIVITY

Before Using the Pages

- Use a large sheet of graph paper or use the overhead projector and a transparency marked with a grid. Display a diagram similar to the following.



Have students demonstrate that the grey shape is neither the slide image nor the flip image of the white shape. Suggest that there is a way for a tracing of the white shape to match the grey shape. Ask a few students to illustrate a procedure they think would work. Elicit the word *turn* to describe the motion. Repeat the turn motion, using a pin or a sharp pencil on the tracing paper at the turn centre to illustrate the procedure. Note that a tracing of the white shape can be flipped twice to match the grey shape. However, the use of a turn involves only one motion.

- Ask students to name examples of turn motions, such as moving the hands of a clock, turning a key or a knob to wind a clock, and pushing a revolving door.

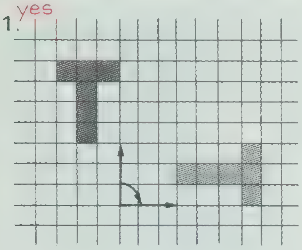
Using the Pages

- The example at the top of page 296 introduces the terms *turn image, turn centre,* and *turn angle*. Most students are familiar with the turning motion of the sails of a windmill. To familiarize the students with the new vocabulary and the concept of a turn, provide them with tracing paper. Have them trace one sail and the centre spool of the toy windmill in the photograph, place the point of a pencil on the turn centre, and turn the tracing to match several of the other sails. You may wish to ask if the turn angle shown in the illustration is acute, right, or obtuse.

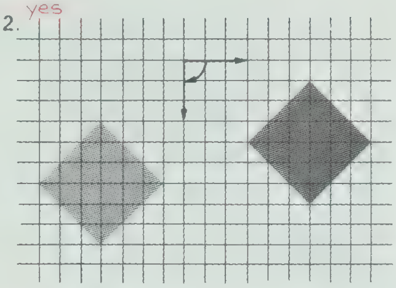
Exercises

Copy each picture on graph paper. Use tracing paper to test whether the blue shape is the turn image of the red shape for the turn angle shown

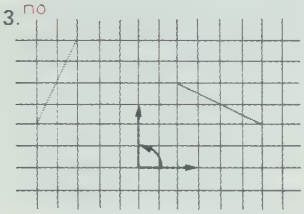
1. yes



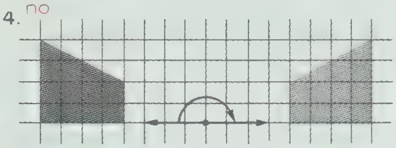
2. yes



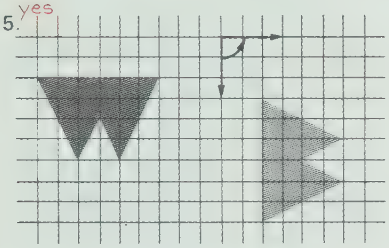
3. no



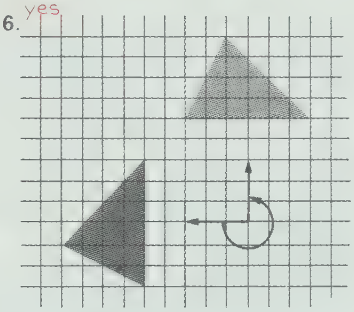
4. no



5. yes



6. yes



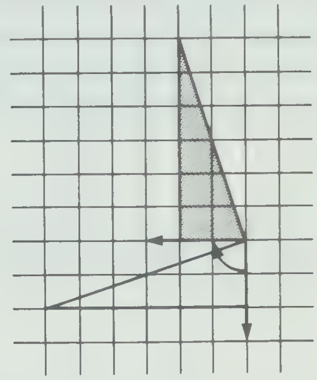
Use graph paper.

*7. Draw two congruent shapes and a turn angle. Have a friend use tracing paper to tell whether one is the turn image of the other. Answers will vary

297

RELATED ACTIVITIES

- Have the students examine samples of fabric and wallpaper for patterns and designs based on turns.
- The exercises on page 297 involve turn centers that are not on the shapes. Provide exercises for which the turn center is a vertex of the shape.



- Some of the exercises on page 297 and on previous pages can be described in terms of a motion other than the one suggested for that lesson. For example, Ex. 3 on page 289 can be described in terms of a flip, Ex. 5 on page 293 in terms of a turn, and Ex. 2 on page 297 in terms of a slide. Tracing paper can help students to determine these alternatives for their copies on graph paper.

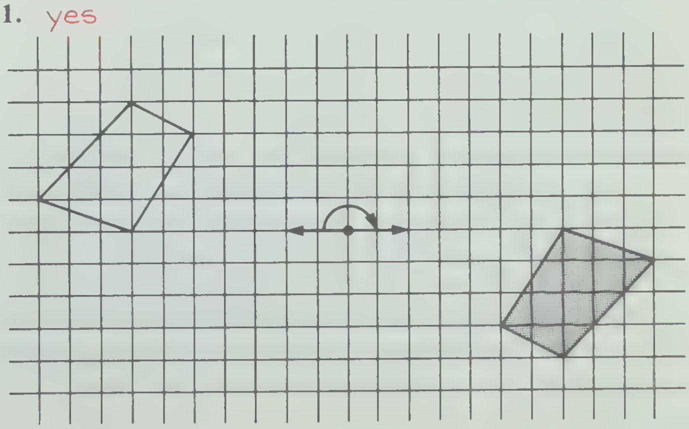
Working Together: The procedure of turning a tracing according to a given turn angle can be demonstrated on an overhead projector, but a large sheet of graph paper may also be used with tracing paper. In advance of the lesson, copy the red shape, the blue shape, and the appropriate ray of the turn angle on a transparency marked with a grid. Demonstrate the procedure shown, using a pencil to hold the turn center in place. Pay particular attention to the direction of the turn. (You may wish to introduce the terms *clockwise* and *counterclockwise*.) Have the students recall that it was necessary to mark two reference points for a slide arrow and for a flip line. For a turn, the reference points are the turn center and another point on the appropriate ray. Repeat the turn motion several times to familiarize the students with the procedure. Then provide tracing paper for them to try the procedure.

Exercises: Allow the students sufficient time to copy the pictures on graph paper and to exercise care in tracing the shapes. A sharp pencil is preferable for holding the turn center in place as the paper is turned. Ex. 7 can challenge students in their attempt to show congruent shapes that are turn images for their chosen turn angles.

Assessment

Copy the picture on graph paper. Use tracing paper to test whether the gray shape is the turn image of the white shape for the turn angle shown.

1. yes



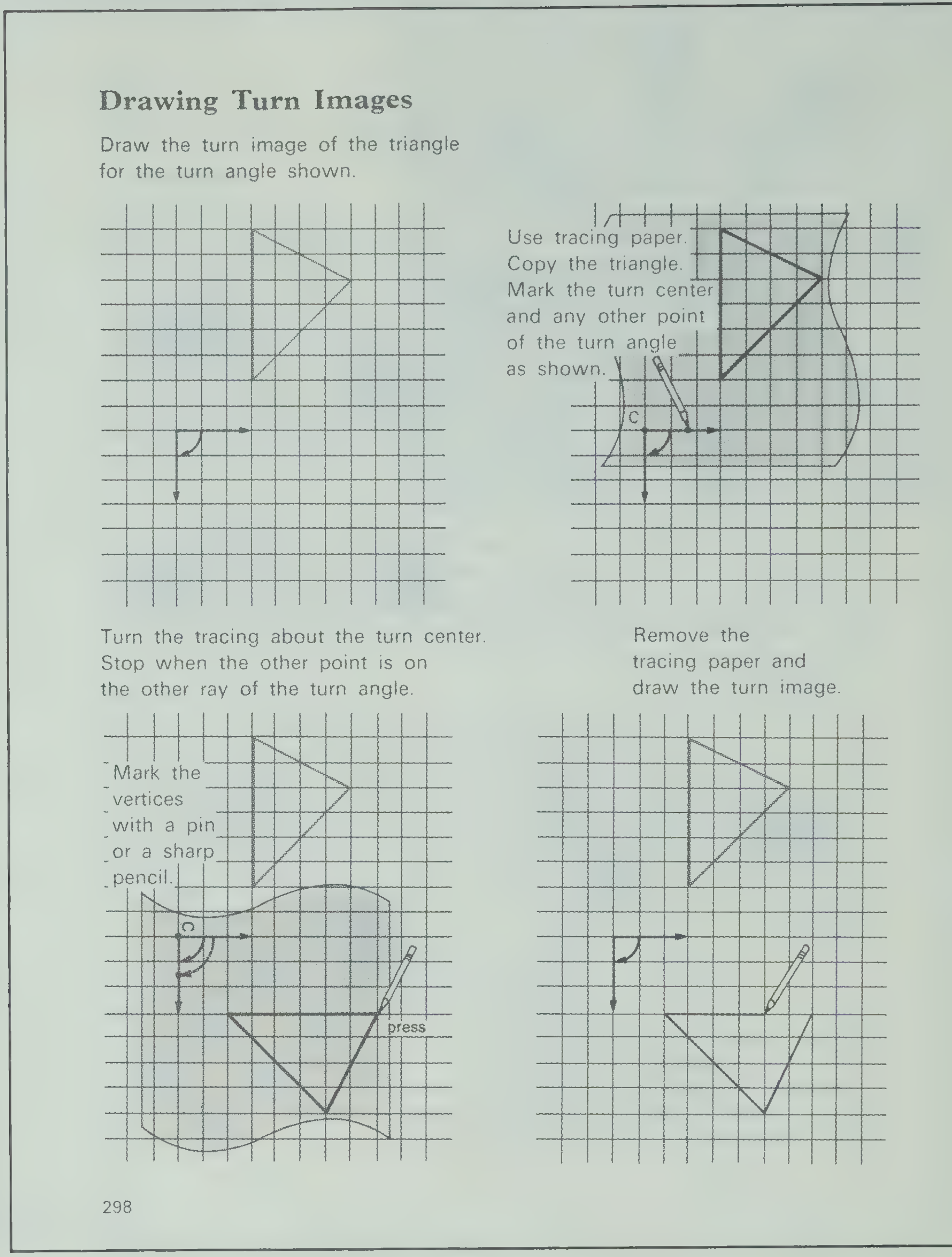
Pages 298-299

LESSON OUTCOME

Draw the turn image of a given figure for a given turn angle, with a grid and without a grid

Materials

copies of page T397, tracing paper, plain paper, a straight edge, and a sharp pencil for each student; overhead projector or a large sheet of graph paper



Drawing Turn Images

Draw the turn image of the triangle for the turn angle shown.

Use tracing paper. Copy the triangle. Mark the turn center and any other point of the turn angle as shown.

Turn the tracing about the turn center. Stop when the other point is on the other ray of the turn angle.

Mark the vertices with a pin or a sharp pencil.

Remove the tracing paper and draw the turn image.

298

LESSON ACTIVITY

Before Using the Pages

- Use the overhead projector or a large sheet of graph paper. Review the procedure of tracing a shape and turning the tracing for a given turn angle to match the shape with its turn image. Review that the turn angle indicates how far to turn the tracing and the turn arrow indicates the direction of the turn.

Using the Pages

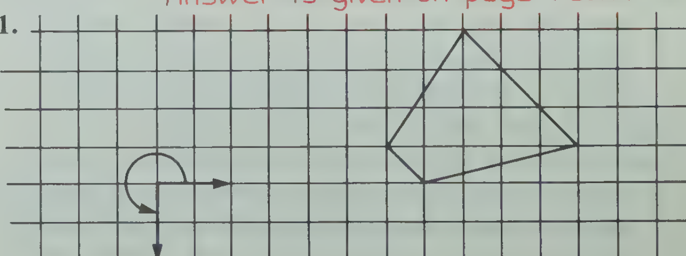
- Because the students have performed similar activities for slides and flips, they will likely be able to suggest the method for drawing a turn image. Discuss the steps outlined on page 298. Provide them with graph paper on which to copy the red triangle, the turn angle, and the turn centre. Have them use tracing paper to find the position of the turn image, marking the vertices using a pin or a sharp pencil.

Exercises: Remind the students that the turn centre is the vertex of the turn angle. Provide them with tracing paper, copies of page T397, and plain paper. Caution them to copy each shape carefully and to allow ample space for drawing the turn image. You may wish to have them copy each shape in the centre of a separate sheet of paper. You may wish to demonstrate one of Ex. 5-8 on a large sheet of plain paper.

Assessment

Copy the shape and the turn angle on graph paper. Use tracing paper to help you draw the turn image of the shape for the turn angle shown. Answer is given on page T325.

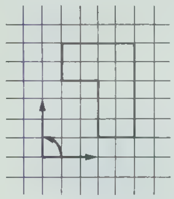
1.

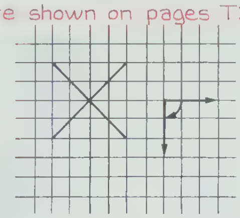


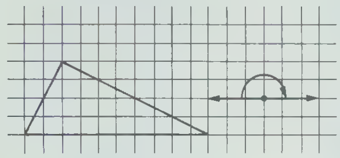
Exercises

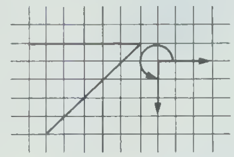
Copy each shape and the turn angle on graph paper. Use tracing paper to help you draw the turn image of the shape for the turn angle shown.

Answers are shown on pages T370 and T371

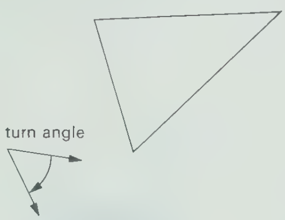
1. 

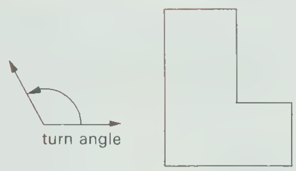
2. 

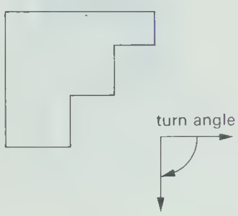
3. 

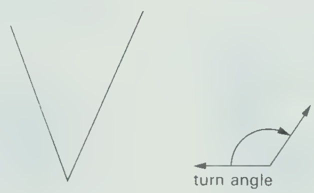
4. 

Copy each shape and the turn angle on plain paper. Use tracing paper to help you draw the turn image of the shape. Answers are shown on page T371

5. 

6. 

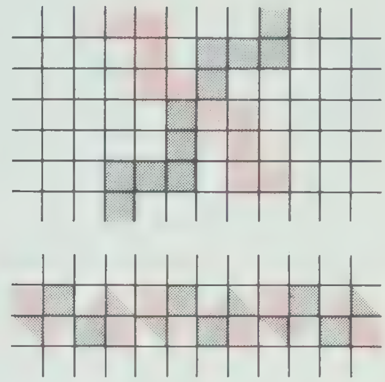
7. 

8. 

299

RELATED ACTIVITIES

• Ask the students to create turn patterns on strips of graph paper. Color can be used to enhance the patterns.

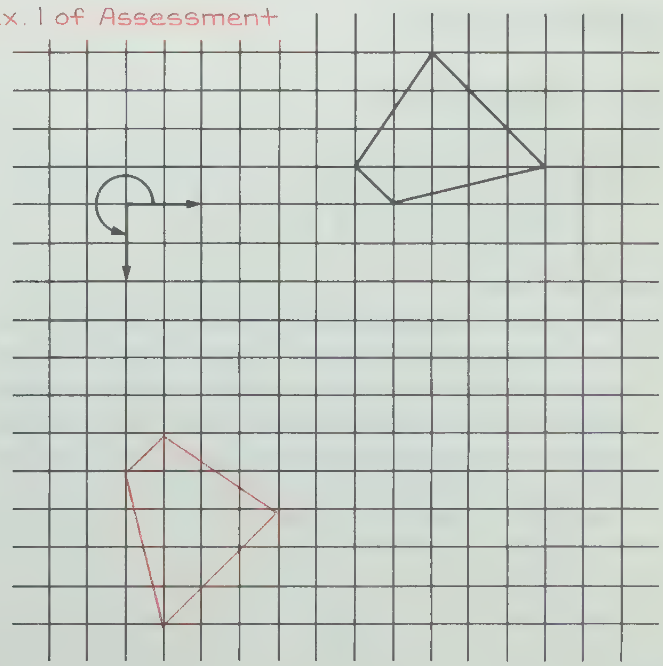


• For enrichment, you may wish to introduce the terms *quarter turn* for a turn angle of 90°, *half turn* for a turn angle of 180°, *three-quarter turn* for a turn angle of 270°, and *complete turn* for a turn angle of 360°. These can be used with the words *clockwise* and *counterclockwise* to describe turn motions that match a shape and its image. For example, for Ex. 6 on page 297, the blue shape is the turn image of the red shape for a three-quarter turn counterclockwise and for a quarter turn clockwise.

Copy the shape and the turn angle on plain paper. Use tracing paper to help you draw the turn image of the shape.

2.

Ex. 1 of Assessment



OBJECTIVE

Demonstrate competence in identifying and drawing images for slides, flips, and turns

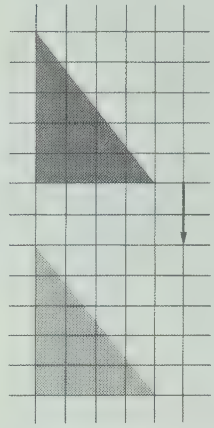
Materials

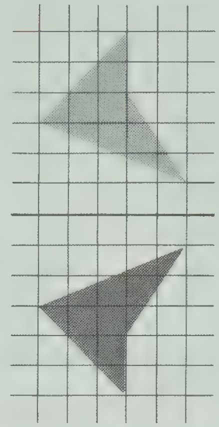
copies of page T 397, tracing paper, plain paper, a sharp pencil, and a straight edge for each student

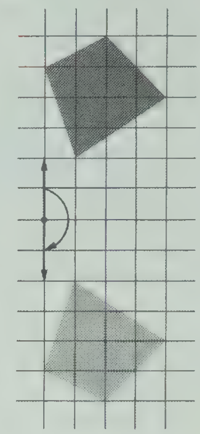
Practice

Copy each picture on graph paper.
Use tracing paper to test whether

1. the blue shape is the slide image of the red shape for the slide arrow shown. no
2. the blue shape is the flip image of the red shape for the flip line shown. yes
3. the blue shape is the turn image of the red shape for the turn angle shown. no



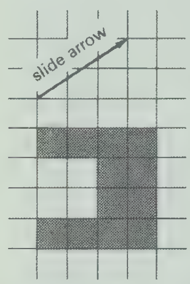


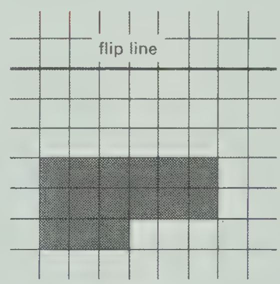


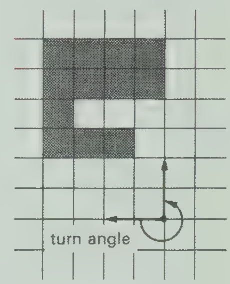
Copy each picture on graph paper.
Then use tracing paper to help you draw

Answers are shown on page T371.

4. the slide image.
5. the flip image.
6. the turn image.







300

LESSON ACTIVITY

Before Using the Pages

- Briefly review the concepts of slide, flip, and turn. Emphasize that for each motion, the shape and its image are congruent. Discuss that flipping a shape for a given flip line is similar to reflecting the shape in a mirror.

Using the Pages

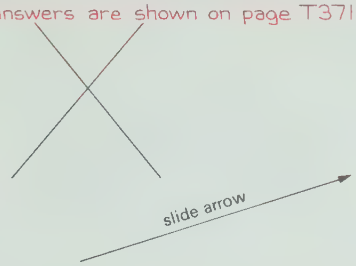
- Provide the students with tracing paper, plain paper, and copies of page T 397. You may prefer to prepare copies of the exercises for the students, leaving sufficient space for them to draw the images for Ex. 4-9. Observe the students as they work, helping those who encounter difficulties. Note that the slide image in Ex. 4 will overlap the original shape for the given slide arrow.

Keeping Sharp: These exercises help to maintain skills in addition, subtraction, and multiplication with whole numbers and with decimals. Before the students begin, you may wish to ask questions such as "How many decimal places will there be in the sum for Ex. 4? in the difference for Ex. 12? in the product for Ex. 22?"

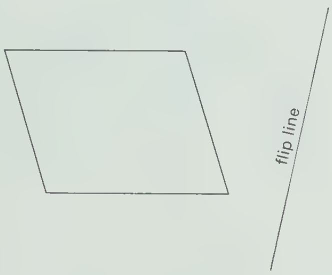
Copy each picture on plain paper. Then use tracing paper to help you draw the image of each shape for the slide arrow, flip line, or turn angle shown.

Answers are shown on page T371

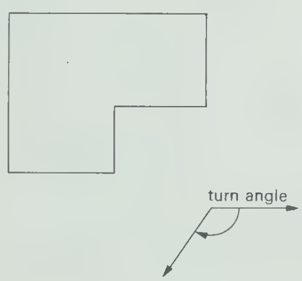
7.



8.



9.



KEEPING SHARP

Add.

1. 36 078 + 46 953 = 83 031	2. 27 845 + 63 795 = 91 640
3. 62.14 + 33.87 = 96.01	4. 36.89 + 17.83 = 54.72
5. 5.095 + 2.298 = 7.393	6. 4.849 + 4.967 = 9.816

Subtract.

7. 82 354 − 17 416 = 64 938	8. 76 001 − 38 523 = 37 478
9. 90.46 − 34.27 = 56.19	10. 82.07 − 25.04 = 57.03
11. 7.005 − 6.168 = 0.837	12. 3.697 − 0.658 = 3.039

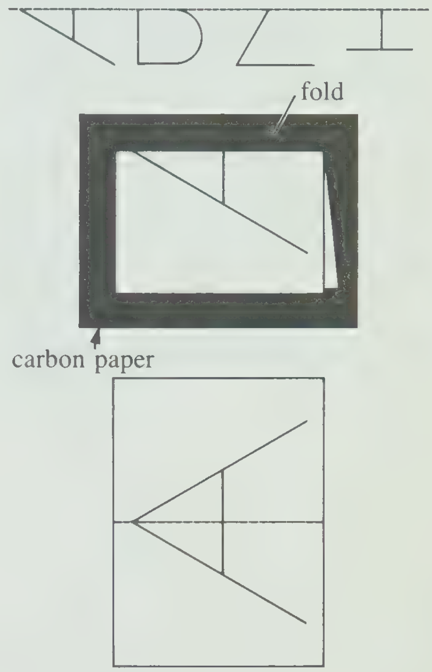
Multiply.

13. 6749 × 23 = 155 227	14. 8015 × 49 = 392 735
15. 387 × 528 = 204 336	16. 406 × 796 = 323 176
17. 197 × 5.9 = 1162.3	18. 2.06 × 81 = 166.86
19. 7.8 × 4.7 = 36.66	20. 3.5 × 2.6 = 9.10
21. 68.5 × 5.9 = 404.15	22. 97.1 × 8.7 = 844.77
23. 49.5 × 20.4 = 1009.80	

301

RELATED ACTIVITIES

- Provide the students with copies of page T396 or geoboards and rubber bands. Have them work in pairs so that one student shows a shape and a flip line, and the other shows the flip image. This activity may be adapted for slides and turns.
- Adapt the activity described in *Before Using the Pages* on page T318 as follows. Have students suggest which letters of the alphabet will be obtained for diagrams, similar to the following, drawn on the folded paper. Have them check their answers by using carbon paper to show the complete letters. The first diagram is completed below as an example.



Pages 302-303

LESSON OUTCOME

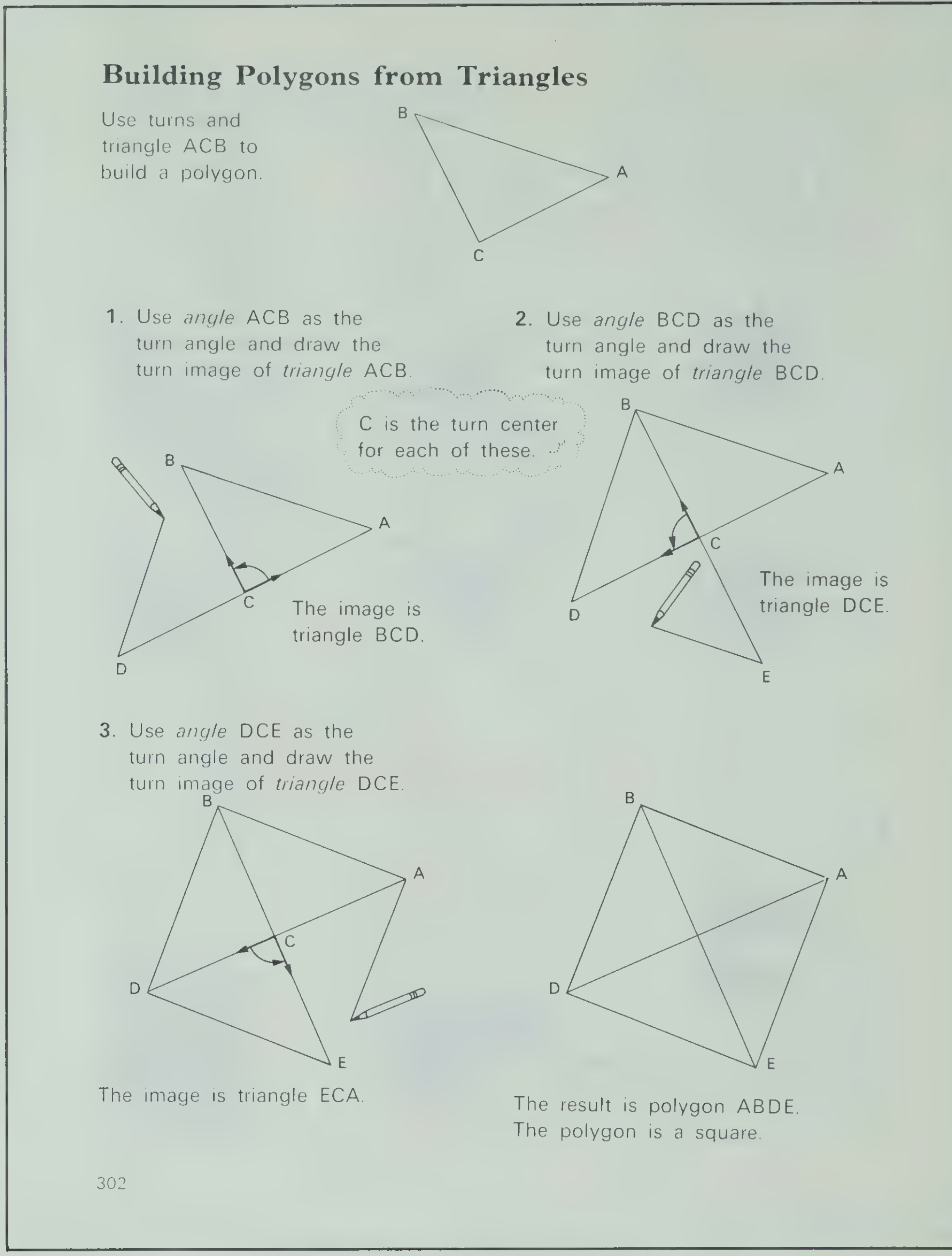
Turn or flip triangles to build regular polygons

Materials

large isosceles right-angled triangle cut from Bristol board and labeled ACB as shown on page 302; large sheet of plain paper; plain paper, tracing paper, a straight edge, and a sharp pencil for each student; pins (optional)

Vocabulary

decagon



Building Polygons from Triangles

Use turns and triangle ACB to build a polygon.

1. Use *angle* ACB as the turn angle and draw the turn image of *triangle* ACB.

C is the turn center for each of these.

The image is triangle BCD.

2. Use *angle* BCD as the turn angle and draw the turn image of *triangle* BCD.

The image is triangle DCE.

3. Use *angle* DCE as the turn angle and draw the turn image of *triangle* DCE.

The image is triangle ECA.

The result is polygon ABDE. The polygon is a square.

302

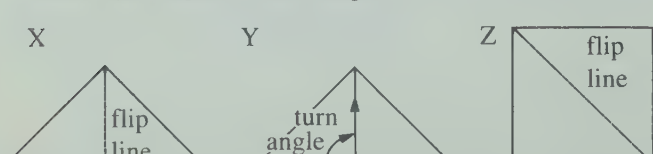
LESSON ACTIVITY

Before Using the Pages

- Display the triangle cut from Bristol board as described in *Materials*. Have students name the shape. Point out that two of the sides are the same length and that the angle formed by these two sides is a right angle.

  Place the shape on a large sheet of plain paper and trace around it. Ask a student to flip the shape, using one of the equal sides as the flip line. Trace around the shape in this position. Have the students note that the result of tracing, flipping, and tracing again is a larger triangular shape (X). Demonstrate that the same result is achieved by tracing, turning, and tracing again (Y). Point out that the turn angle is the right angle of the triangle.

X Y Z



Ask students to trace, flip, and trace the triangle so that the resulting polygon is a square (Z). They can discover that the side opposite the right angle is the flip line. Then ask whether a square can be obtained by using only a turn motion. After they have had an opportunity to consider the problem, ask them to turn to page 302.

Using the Pages

- The example on page 302 demonstrates that turning a triangle as shown can result in a square. Discuss the steps, pointing out that the turn centre and the turn angle are parts of triangle ACB. Then provide plain paper and tracing paper for the students to try the procedure. Caution them to copy triangle ACB with care so that accurate results are obtained.

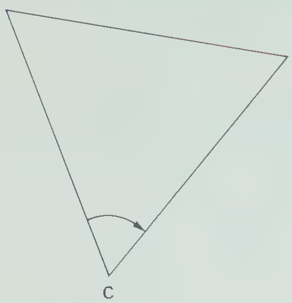
Exercises: You may prefer to provide the students with copies of the diagrams on page 303. If so, make certain that there is sufficient space for them to draw the images. Before the students begin, review the name of a polygon having four sides (quadrilateral), five sides (pentagon), six sides

Exercises

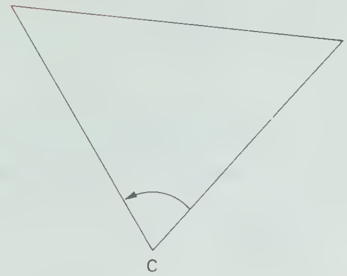
Carefully copy each triangle and use as many turns as needed to build a polygon. Use C as the turn center for the turn angle.

Answers are shown on page T371.

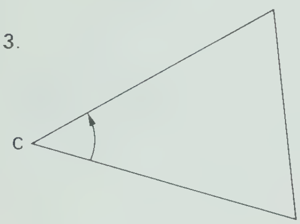
1.



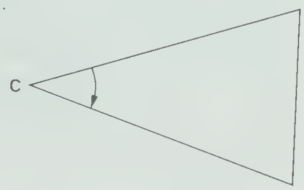
2.



3.



4.

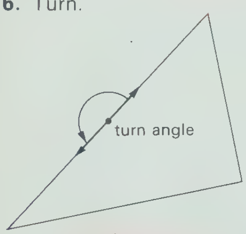


5. Name the polygons you drew for Exercises 1 to 4.
hexagon, pentagon, octagon, decagon

Copy each triangle. Draw the suggested flip or turn image. Name the polygon that results.

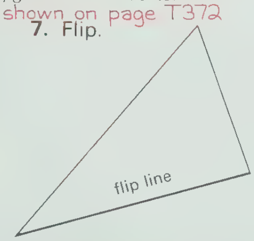
Answers are shown on page T372

6. Turn.



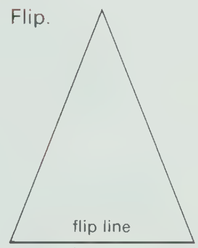
quadrilateral (rectangle)

7. Flip.



quadrilateral (kite)

8. Flip.



quadrilateral (rhombus)

303

RELATED ACTIVITIES

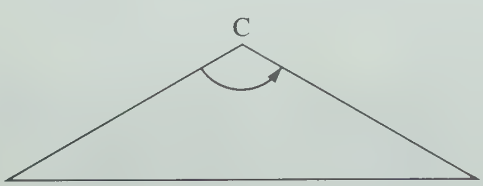
- Students can work Ex. 1-4 and Ex. 6 on page 303 again in the following way. A sheet of tracing paper is placed over Ex. 2, for example. The shape is traced and the tracing is turned for the given turn angle. The original shape is traced again and then the tracing paper is turned through the turn angle again. The motion is repeated as many times as are necessary to build a polygon. A sharp pencil will hold the turn centre in place on the page as the tracing paper is turned.
- Develop that the turn angle for a complete rotation measures 360°. For example, have students use a protractor to measure the turn angle (60°) for the triangle in Ex. 1 on page 303. Use the information to show that a complete turn involves $6 \times 60°$, or 360°, since the polygon built from that triangle has six such angles at the centre. This information can be applied in Ex. 2-4. Students can divide 360 by the number of congruent turn angles at the centre of a polygon to find the measure of the turn angle. They can check the calculation by measuring the turn angle on page 303.

(hexagon), and eight sides (octagon). Some students may be able to suggest the name *decagon* for a ten-sided polygon (Ex. 4), if you remind them that our numeration system is based on tens and hence is described as a decimal system. For more assistance with names of polygons, direct the students to pages 179, 184, and 186. Note that each motion for Ex. 6-8 is performed only once.

Assessment

Copy this triangle.

1.

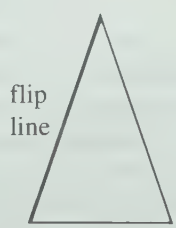


2. With point C as the turn centre for the turn angle, use turns to build a polygon. See diagram on page T372.

3. Name the polygon you drew. equilateral triangle

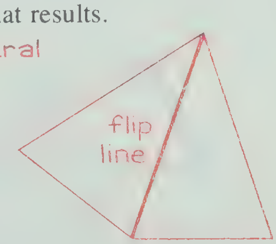
Copy this triangle.

4.



5. Draw the suggested flip image.

6. Name the polygon that results.
quadrilateral



LESSON OUTCOME

Fit congruent shapes together so that they share congruent sides

Materials

two congruent scalene triangles cut from Bristol board and crayons; tracing paper or construction paper, scissors, crayons, and a straight edge for each student

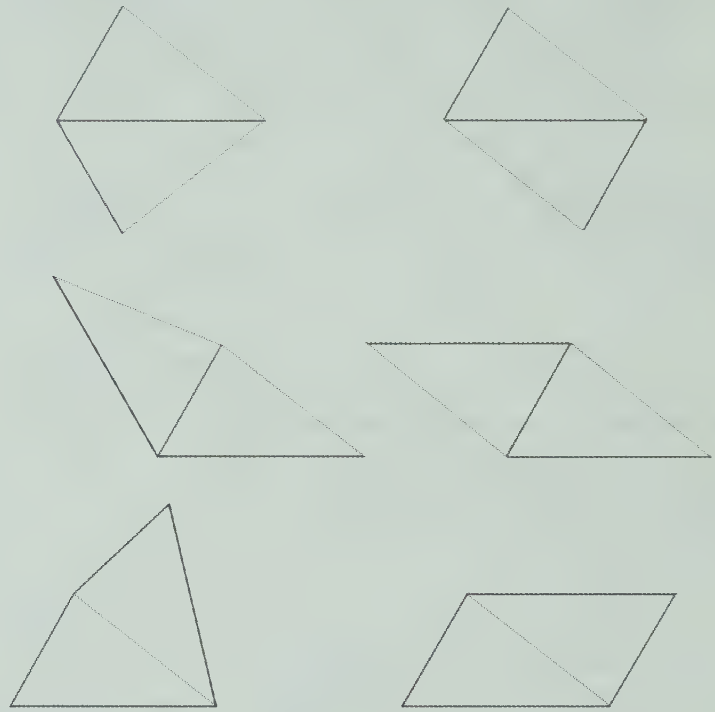
Background

This lesson prepares the students for tiling congruent shapes in the lesson on pages 306 and 307 and provides a visual insight into the problem. It also shows that when congruent shapes are fitted together in different ways, the resulting shapes are not necessarily congruent.

Sharing Congruent Sides

How many different shapes can be made by fitting congruent triangles together along their matching sides?

Color the matching sides in each triangle the same. Then fit the sides with the same color together.



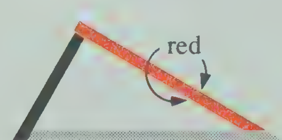
As many as 6 different shapes can be made when congruent triangles are fitted together along matching sides.

304

LESSON ACTIVITY

Before Using the Pages

- Display two congruent scalene triangles cut from Bristol board. Review that congruent figures have the same shape and the same size. Ask whether the two triangles displayed are congruent. Have a student match the two triangles to demonstrate that they are congruent. As the students observe, color each side of one triangle a different color, making certain that the color appears on each side of the cardboard.



Have students help to color corresponding sides of the second triangle in the same way. This will involve a discussion regarding which sides are corresponding, and thus which color should be used.

Ask a student to arrange the two triangular shapes side by side on the chalkboard so that the two red sides touch. Explain that the end points of the sides must match. Trace around the shapes to illustrate the figure that is obtained. Ask for another way to match the two red sides so that a different shape is obtained. (One triangular shape may be flipped.) Trace the outline on the board. Tell the students that today's lesson involves making as many different shapes as possible by matching congruent sides of two congruent shapes.

Using the Pages

- Have the students trace the triangles at the top of page 304 and color the sides as shown. Have them cut out the tracings and use the shapes to copy or to cover the six shapes on the page. After they have completed the work, discuss whether a slide, a flip, or a turn gives the desired result.

Exercises: Review the instructions before the students begin the exercises, to ensure that they understand what is required. It is advisable that different results for each exercise be

Exercises

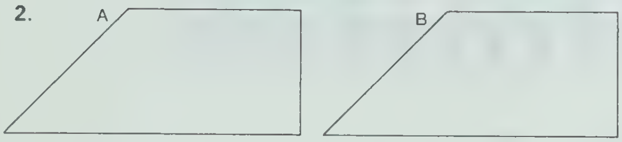
Answers are shown on pages T372

Trace shape A. On another piece of tracing paper trace shape B. Color the matching sides of the congruent shapes the same color. Cut out the shapes and fit them together to make as many different shapes as possible.

1.

The two triangles are congruent.

2.

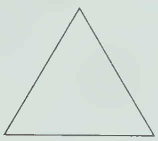


The two trapezoids are congruent.

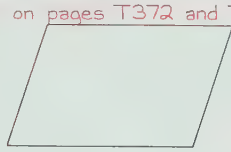
Trace the shape. On another piece of paper, trace it again. The two shapes you traced are congruent. Color their matching sides, cut them out, and fit them together to make as many different shapes as possible.

Answers are shown on pages T372 and T373

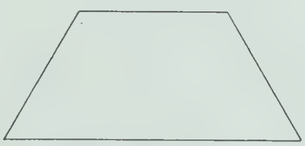
3.



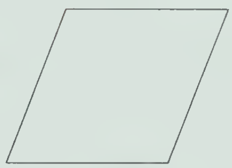
4.



5.



6.

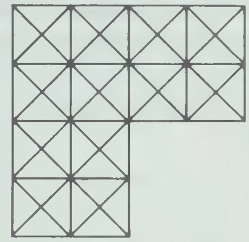


7. When two congruent shapes share a side in Exercises 1 to 6, is one shape the slide, flip, or turn image of the other shape?
A slide is not possible when matching sides of the same color

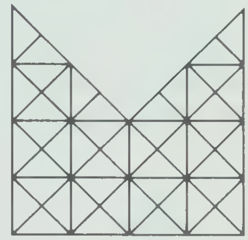
305

RELATED ACTIVITIES

- Prepare the following shapes on copies of page T 397. Have students draw lines to divide each shape into the indicated number of congruent parts. Describe relationships among the parts in terms of slides, flips, and turns.

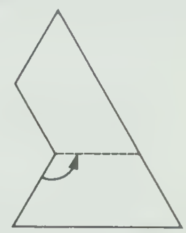


1. 2 parts
2. 3 parts
3. 4 parts
4. 6 parts

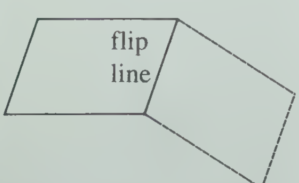


1. 2 parts
2. 3 parts
3. 6 parts
4. 8 parts

- You may wish to have students work exercises on page 305 again so that any pair of congruent sides may be matched, regardless of color. For example, the trapezoid in Ex. 5 has three congruent sides and can generate the following shape by a turn motion.



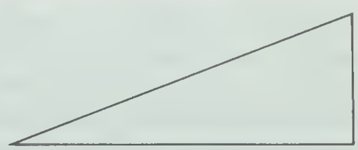
displayed and discussed. For example, for Ex. 3, only one shape is possible; no matter which corresponding sides are arranged to match, the same shape, a rhombus, results, but it is seen in a different position. Some students may realize that this occurs because the original shape is an equilateral triangle. Encourage the students to consider whether each shape for an exercise is a new one or merely a former one oriented differently. Note that although opposite sides of a parallelogram are equal (Ex. 4), the motion of a slide would match congruent sides shown in different colors. For two congruent parallelograms to share matching sides, only a flip or a half turn can be used.



Assessment

Use two copies of this shape. Color the matching sides and fit them together to make as many different shapes as possible.

1.



The different shapes are shown on page T373.

LESSON OUTCOME

Make a tiling pattern using one shape

Materials

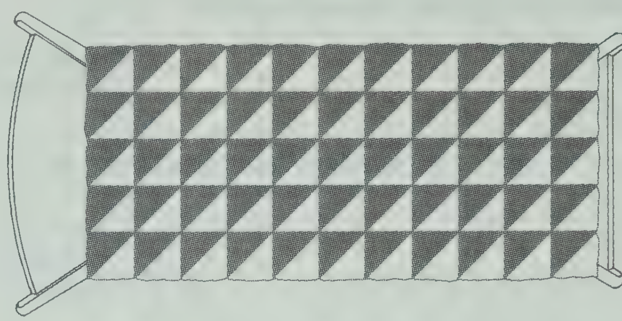
plain paper, tracing paper, a straight edge, and a sharp pencil for each student; copies of the shapes on page T 400 (optional); pins (optional)

Vocabulary

tiling

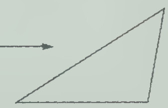
Tiling with Congruent Shapes

The patches in the quilt show triangles.
All the triangles are congruent.

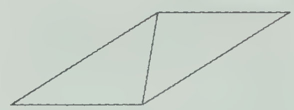


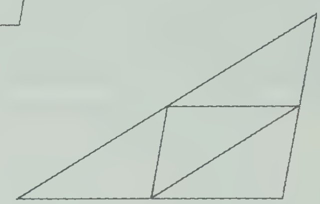
Working Together

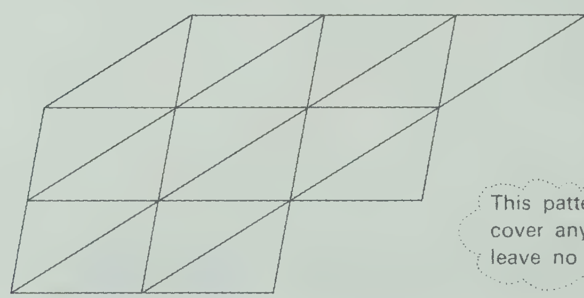
Draw a triangle like this one. →



Then use tracing paper to help you draw each of these.







This pattern could cover any region and leave no spaces.

306

LESSON ACTIVITY

Before Using the Pages

- If possible, refer to examples in the classroom to develop the concept of tiling. For instance, draw attention to the ceiling tiles and/or floor tiles, which are likely square or rectangular. Discuss that identical shapes can be placed together so that they do not overlap, nor do they leave any spaces; this process is called *tiling*. Squares and rectangles are usually used in practical applications of tiling. Some students may be able to suggest another shape that they have seen in a tiling pattern.

Using the Pages

- Discuss the illustration at the top of page 306. Have the students note that the pattern in the quilt shows congruent triangles. Some students may point out examples of slides, flips, and turns in the pattern. Suggest that a similar design can be prepared by using a triangle and tracing it several times so that congruent sides are shared.

Working Together: Contrast the triangle shown with the one in the quilt. The one in the quilt is an isosceles triangle and has a right angle; this one is a scalene triangle and has no right angle. Nevertheless, it can be used to form a pattern that leaves no spaces.

Provide the students with plain paper and tracing paper to try the procedure. Ensure that they use a sharp pencil or a pin to mark vertices through the tracing paper. Some students will probably produce patterns with spaces as a result of using flip images exclusively. Emphasize that the shapes must not overlap and that there should be no spaces. Suggest the turn motion to those who require assistance.

Exercises: Provide the students with sufficient plain paper and tracing paper to complete these exercises. The shapes in Ex. 1 - 6 also appear on page T 400. You may prefer to give the students copies of these for making their patterns. Allow time for them to attempt patterns that leave no spaces. Ask students to identify the given shapes in these exercises. When Ex. 4 is completed, the pattern may be related to the honeycomb pattern.

Exercises

Possible patterns are shown on pages T373 and T374.

Trace the shape. Use it to make a pattern by placing congruent sides together. Show the shape at least 9 times in the pattern.

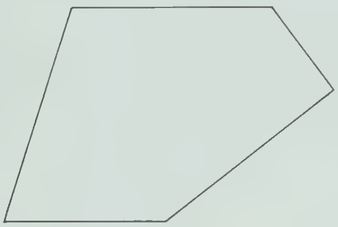
The shapes should touch but not overlap.

1.

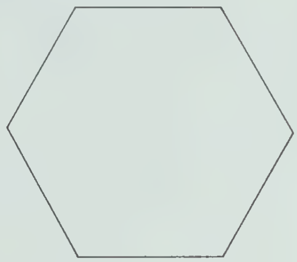
2.

Do your patterns leave any spaces?

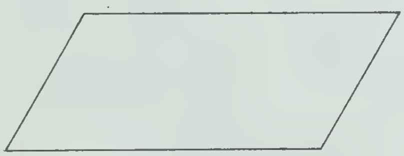
3.



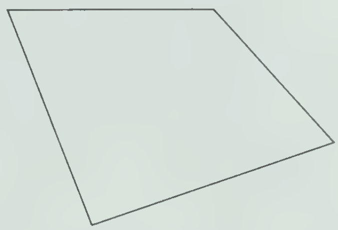
4.



5.



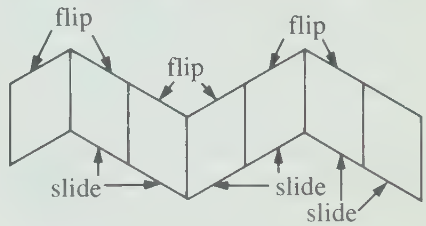
6.



RELATED ACTIVITIES

• For some students, it may be preferable to extend the work of this lesson over two or three days to enable them to work with copies of the shapes rather than with tracing paper. If a shape is traced several times and the tracings are cut out, they can be arranged to form a pattern with no spaces. Then the shapes can be pasted on construction paper to display the pattern.

• Have the students use parquetry blocks similar to those shown on page 311. Have them trace around one block to create a pattern based on a combination of slides, flips, and turns. As an alternative, have students use the blocks to form patterns for other students to describe.

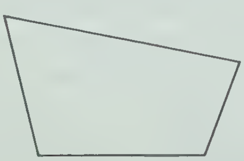


• Congruent felt shapes can be used very effectively on a flannel board to demonstrate tiling patterns because the shapes remain in position once they are arranged.

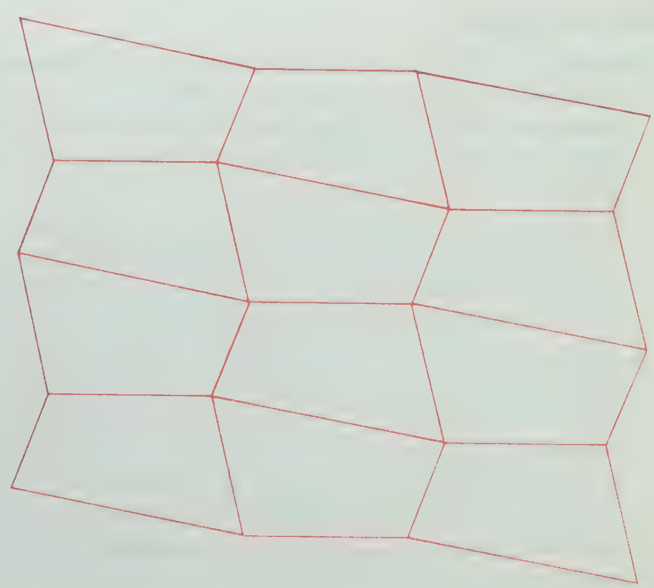
Assessment

Use this shape to make a pattern by placing congruent sides together. Try to show a pattern without spaces. Use the shape at least 9 times in the pattern.

1.



Ex. 1 of Assessment



LESSON OUTCOME

Copy a picture from a square grid onto a grid with squares of a different size and onto grids with rectangles or parallelograms

Materials

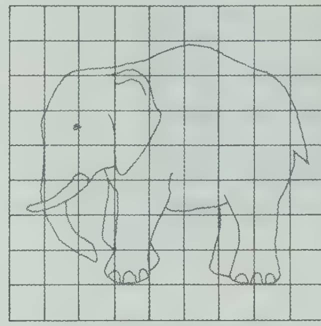
overhead projector and transparent acetate marked with a square grid (optional); one copy of each of the pages T 397, T 399, and T 400 for each student; plain paper and a straight edge for each student

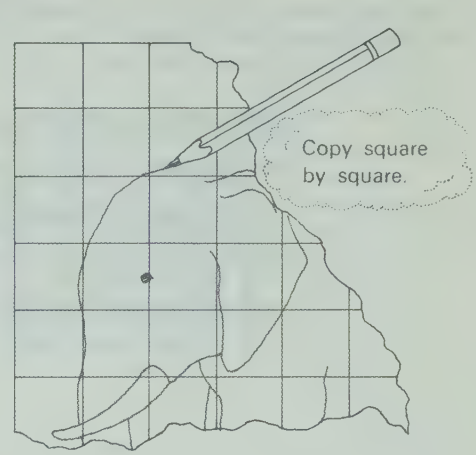
Vocabulary

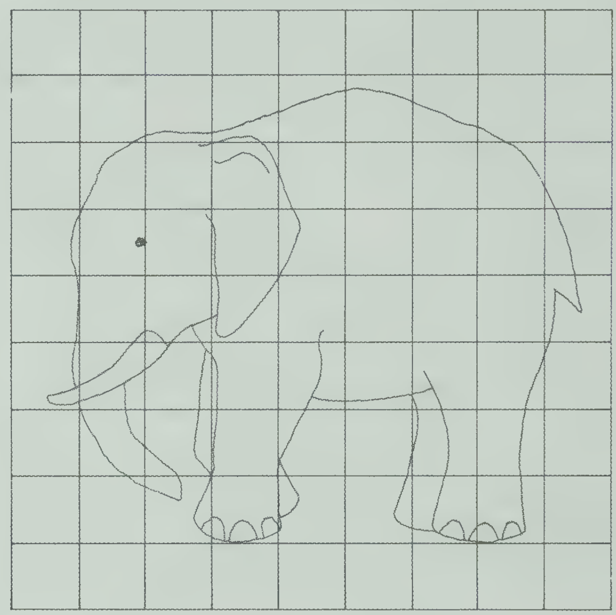
grid, distorted, midpoint

Copying Pictures Using Grids

Copy this picture of the elephant on a grid of a different size.







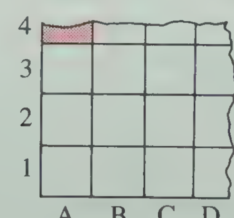
308

LESSON ACTIVITY

Using the Pages

- Approach this lesson as one of exploration and enjoyment. Although the only skill required is the ability to match points and join them in the same way as in the original shape, it provides the basis for drawing scale diagrams on a grid.

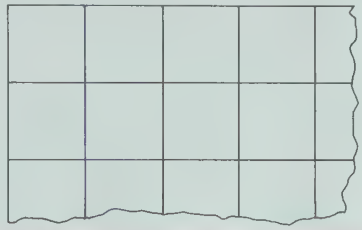
  If you wish, prepare a copy of the picture of the elephant on transparent acetate marked with a square grid. Have the students trace the shape on page 308 at the same time as you trace the shape on the overhead projector. Discuss that a grid with smaller squares produces a smaller image, and that a grid with squares larger than the original grid produces a larger image. This concept can be demonstrated by moving the projector closer to or farther from the screen and adjusting the focus. Point out that all the images have the same shape but are different sizes. Emphasize that each part of the elephant is shown in the corresponding square on each grid. For example, the elephant's tusk ends in the first square of the fourth row. Because the students have studied ordered pairs previously, they may label the rows and columns of squares with letters and numbers, and use ordered pairs to identify regions. For the following way of labeling the grid, the elephant's tusk ends in the region identified by (A,4).



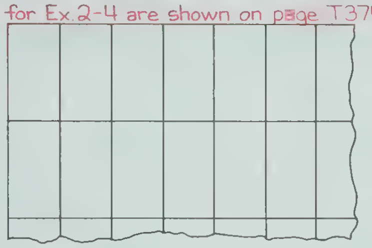
Exercises: Before the students begin, discuss the four grids shown. Ask how they are alike and how they are different. For the grids in Ex. 2-4, ask students to describe the image that will likely be obtained. For example, the image for Ex. 2 will not be the same size as the original shape, and it will be *distorted*; that is, it will appear narrower and taller.

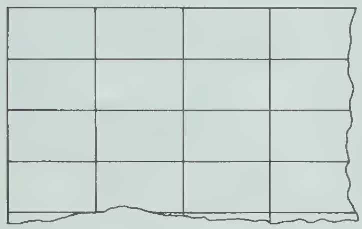
Exercises

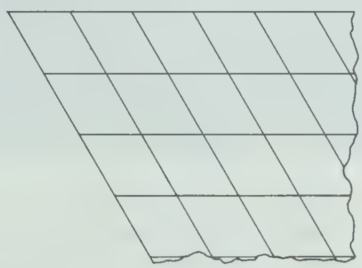
Copy the elephant on grids that look like these.

1. 

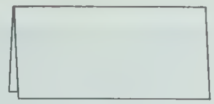
Pictures for Ex. 2-4 are shown on page T374

2. 

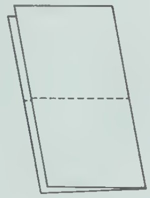
3. 

*4. 

Fold a piece of paper.



Open it. Fold it again so that one half of the first fold line matches the other half.



Open the paper.

1. What kinds of angles are formed by the two fold lines? right angles
2. What word completes the following sentence?
   The fold lines are __?__ to each other. perpendicular

Draw a line segment on a piece of paper.

Fold the paper so that one end point matches the other end point.

Open the paper.

3. What kinds of angles are formed by the fold line and line segment? right angles
4. The fold line and line segment are __?__ to each other. perpendicular
5. Why is **midpoint** a good word to use for the point where the fold line and line segment meet?

try this

The fold line divides the line segment in half.

309

RELATED ACTIVITIES

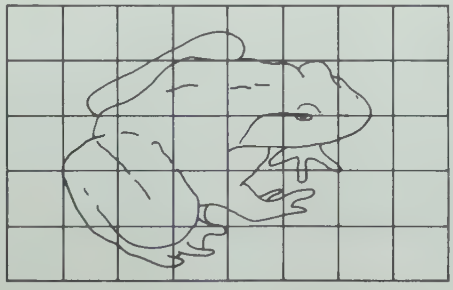
- Ask the students to name examples of distorted images such as those seen in curved mirrors at a fair, a reflection in either side of a spoon or in the surface of a kettle, and words or pictures on inflated balloons.
- Have the students draw distorted images of their favorite cartoon characters using copies of page T 399 or T 400.
- Ask the students to draw an angle of 60° on a piece of paper. Have them fold the paper so that one ray of the angle matches the other ray, and then unfold the paper. Have them measure each angle formed by the fold line and a ray.
- Give each student a copy of the large triangle on page T 383. Have them fold the shape, matching the end points for one side. Have them unfold the shape and then fold it again, matching the end points of another side. Repeat the procedure for the third side. The three fold lines should intersect at one point inside the triangle.

Provide each student with a copy of page T 397 for Ex. 1, a copy of page T 399 for Ex. 2 and 3, and a copy of page T 400 for Ex. 4. Ex. 4 will challenge some of the students because the grid has no vertical lines.

Try This: These exercises provide a review of the concept of perpendicular lines in activities involving paper folding. The procedure is used to locate the *midpoint* of a line segment.

Assessment

Copy this picture on a grid of a different size.

1. 

Pages 310-311

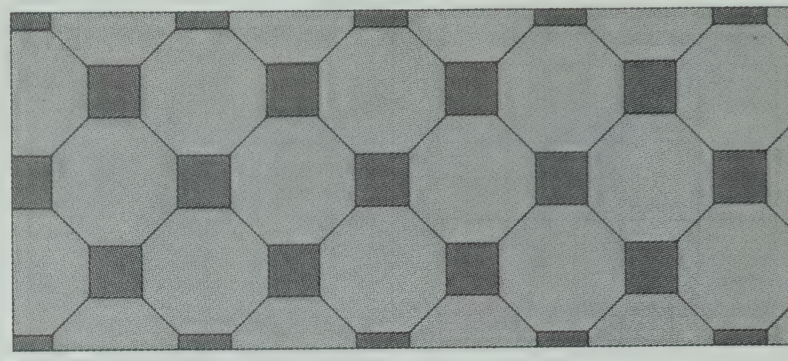
OBJECTIVE

Use two or more shapes to create tiling patterns

Materials

parquetry blocks similar to those shown on page 311 (optional); copies of the polygons on pages T 382-T 385 (optional), tracing paper, scissors, a straight edge, and crayons for each student

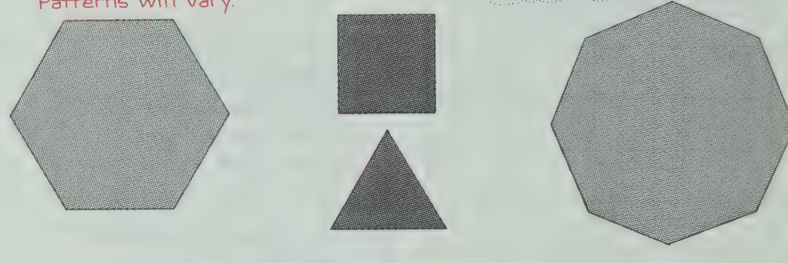
Working with Models



Sometimes two or more different shapes are used for tiling a floor.

1. Try to make a tiling pattern that uses two or more of these shapes.
   Patterns will vary.

Remember, a tiling pattern should leave no spaces.



2. Try to make a tiling pattern that uses two or more of these shapes. Patterns will vary.

   triangle (any kind) quadrilateral (any kind)

   pentagon hexagon octagon

PROBLEM SOLVING

3. Color the tiling patterns that you have made.

310

LESSON ACTIVITY

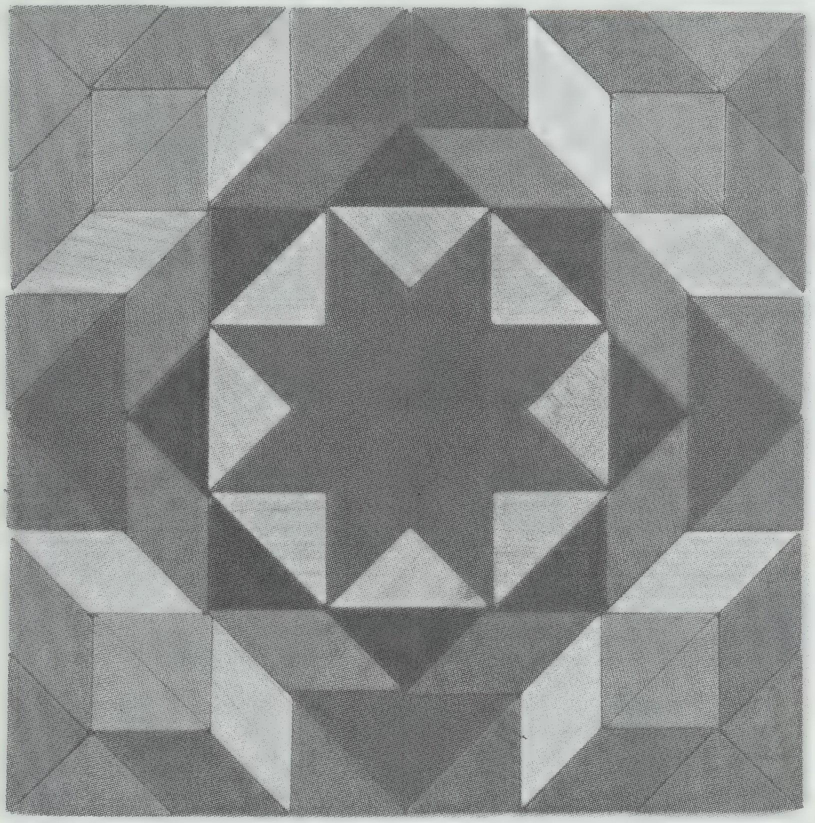
Using the Pages

- Begin with a discussion of the pattern illustrated at the top of page 310. Ask how many different shapes are used to form the pattern and have students identify the shapes (square, octagon). Point out that the pattern leaves no spaces and the shapes do not overlap; thus, the pattern is a tiling pattern. Some students may have seen a similar pattern on a floor.

Exercises: Discuss the design on page 311. Have students name the shapes of the parquetry blocks and determine the number of different shapes used in the design. (There are only two different shapes: an isosceles right triangle and a rhombus.) Discuss the use of color to highlight other shapes and symmetry in the design. Students may be able to describe parts of the design in terms of slides, flips, and turns.

- Provide the students with tracing paper and, if necessary, review the procedure for tracing shapes to create a tiling pattern. If parquetry blocks are available, have students use them to form their patterns. As an alternative, have them use copies of polygons on pages T 382 to T 385. For Ex. 2, they may choose the shapes to draw and copy.

4. What patterns could you make? Answers will vary



RELATED ACTIVITIES

- Have students find examples of tiling patterns and other geometric patterns for a classroom collection and display. The examples may be photographs of floor tiles or wall tiles, or samples of fabric and wallpaper.
- Display patterns created by the students for Ex. 1-3 on page 310. The patterns can motivate a discussion of shapes and motions as well as symmetry.
- Have students use a semitransparent plexiglass mirror on the design on page 311 to find lines of symmetry. Have them trace one shape in the design on tracing paper, and then turn the tracing paper while holding the center of the design in place with a sharp pencil. This will allow them to observe turn patterns. Tracing paper can also be used to find examples of slides and flips in the design.
- Students may find it interesting to note the variety of patterns created by a simple kaleidoscope. They can demonstrate the principle of such an instrument by using two rectangular mirrors and one or more parquetry blocks. The mirrors are placed on their edges to form an angle and the blocks are placed within the angle formed. By varying the size of the angle and/or the arrangement of blocks, the pattern is changed. Two mirrors may be placed in a similar fashion on the photograph on page 311, so that the vertex of the angle is at the center of the design.

OBJECTIVE

Demonstrate an understanding of the concepts and skills presented in this unit

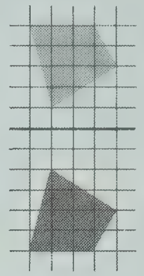
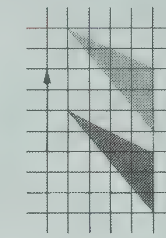
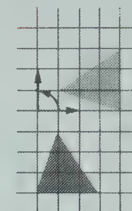
Materials

copies of page T 397, tracing paper, plain paper, a straight edge, and a sharp pencil for each student; a copy of page T 399 or T 400 for each student (optional)

Checking Up

Use tracing paper to test whether

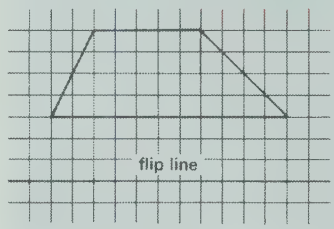
1. the blue shape is the flip image of the red shape for the flip line shown. no
2. the blue shape is the slide image of the red shape for the slide arrow shown. yes
3. the blue shape is the turn image of the red shape for the turn angle shown. yes

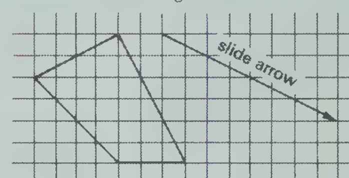
Copy each picture on graph paper.
Then use tracing paper to help you draw

Answers are shown on page T375.

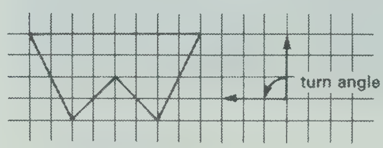
4. the flip image.



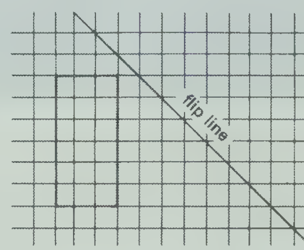
5. the slide image.



6. the turn image.



7. the flip image.



312

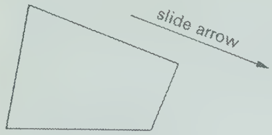
Skills	Exercises	Related Pages
Identify one figure as the flip image of another figure for a given flip line	1	T 318-T 319
Identify one figure as the slide image of another figure for a given slide arrow	2	T 314-T 315
Identify one figure as the turn image of another figure for a given turn angle	3	T 322-T 323
Draw the flip image of a given figure for a given flip line, with a grid and without a grid	4, 7, 10	T 320-T 321
Draw the slide image of a given figure for a given slide arrow, with a grid and without a grid	5, 8	T 316-T 317
Draw the turn image of a given figure for a given turn angle, with a grid and without a grid	6, 9, 11	T 324-T 325
Turn a triangle to build a regular polygon and identify the polygon	12, 13	T 328-T 329
Make a tiling pattern using one shape	14	T 332-T 333
Copy a picture from a square grid onto a grid with squares of a different size and onto grids with rectangles or parallelograms	15	T 334-T 335

Comments

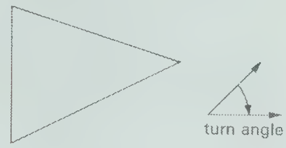
Provide the students with sufficient tracing paper, graph paper, and plain paper to complete these exercises. For Ex. 15, you may wish to give them a choice of using a copy of page T 397, T 399, or T 400. The students should not be rushed in these exercises. Allow them ample time to work carefully at tracing shapes and manipulating the tracings.

Copy each picture on plain paper.
Then use tracing paper to help you draw
Answers are shown on page T375.

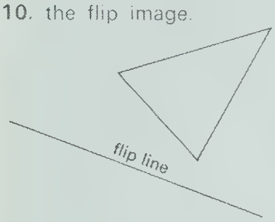
8. the slide image.



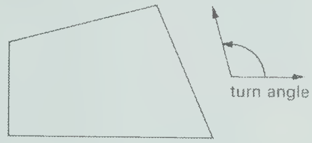
9. the turn image.



10. the flip image.



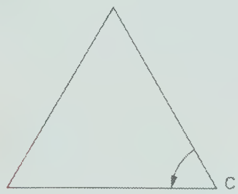
11. the turn image.



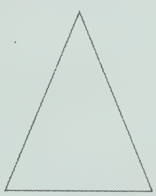
Copy this triangle.
With C as the turn center
for the turn angles,
Answer is shown on page T375

12. use turns to build a polygon.

13. Name the polygon you drew. hexagon



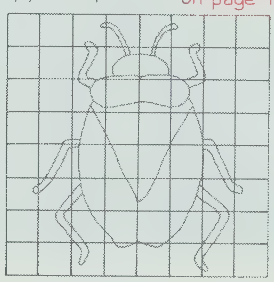
Use this triangle
and tracing paper.



14. Draw a pattern that leaves no spaces. Use the triangle at least 9 times in your pattern.
One pattern is shown on page T375.

Using a grid of a different size,

15. copy this picture. One picture is shown on page T375.

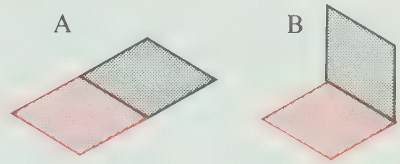


313

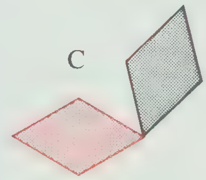
RELATED ACTIVITIES

- Have students use parquetry blocks similar to those in the photograph on page 311 to show slides, flips, and turns for one shape at a time. For example, for a rhombus, use a red block and a blue block to show a slide (A), a flip (B), and a turn (C).

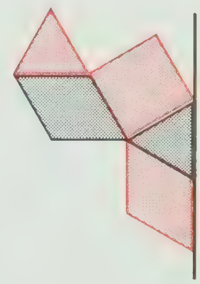
A B



C



Then have students work in pairs so that one student makes a pattern on one side of a line on a piece of paper. The other student shows the flip image of the pattern, using the given line as a flip line.



Pages 314-315

OBJECTIVE

Demonstrate competence in addition, subtraction, multiplication, and division; solve related word problems

Checking Skills

Add.

1. 3754 + 3032 = 6786
2. $5612 + 846 = $6458
3. $198.67 + 373.15 = $571.82
4. 98 548 + 9 268 = 107 816
5. 17.54 + 83.02 + 96.58 = 197.14
6. 4 376 + 21 396 + 5 312 = 31 084
7. 5 804 + 812 + 10 272 + 5 624 = 22 512
8. 46.118 + 79.408 + 18.698 + 30.648 = 174.872

9. $70.97 + $60.87 $131.84
10. 1858.9 + 2455.9 4314.8
11. 91 072 + 83 462 + 42 196 216 730
12. 769 + 36 465 37 234
13. 1.051 + 4.351 + 2.751 + 3.658 11.811
14. 70.44 + 33.38 + 89.62 193.44
15. $9508 + $6667 $16 175
16. 0.931 + 9.623 + 0.914 11.468
17. 6705 + 221 + 9831 + 53 16 810

Solve.

18. Brian bought 1.56 kg of cheddar cheese and 1.73 kg of soft cheese. How much cheese did he buy? 3.29 kg
19. Libby bought items at the grocery store that cost $1.56, $2.79, $0.88, and $3.15. How much did she pay? $8.38

Subtract.

1. 6539 − 4231 = 2308
2. $27.80 − 7.32 = $20.48
3. 902.57 − 658.44 = 244.13
4. 7.846 − 0.943 = 6.903
5. $58 542 − 4 863 = $53 679
6. 91 103 − 79 487 = 11 616
7. 8.12 − 6.14 = 1.98
8. 564.8 − 177.5 = 387.3
9. 41 037 − 5 708 = 35 329
10. 70 035 − 1 069 = 68 966

11. 81.4 − 27.9 53.5
12. 67 036 − 52 496 14 540
13. $8030 − $689 $7341
14. 7.7685 − 6.9287 0.8398
15. 60 000 − 375 59 625
16. 75.246 − 36.789 38.457
17. 10 090 − 736 9354
18. 4.612 − 1.889 2.723
19. $945.21 − $435.66 $509.55
20. 99 243 − 9 555 89 688

Solve.

21. Sharon went to the store with $9.87. She spent $7.36. How much money did she have left? $2.51
22. There are 245 g of cereal in one box and 350 g of cereal in a larger box. How much more cereal is in the larger box? 105 kg

314

LESSON ACTIVITY

Using the Pages

- The exercises are presented in four sections, each section dealing with one operation. Also, each section involves operating with whole numbers and with decimals, including amounts of money. Addition and subtraction exercises shown in the horizontal form require students to align the numerals correctly according to place value.
- It would be preferable to have the students complete a few exercises each day for several days. The exercises may be assigned in different ways, according to the needs of the students. Some suggestions are given below.
  1. Assign only the addition exercises to review the skills with that operation. Then continue in a similar manner with the other operations.
  2. Assign a few exercises from one section along with a few from the section for the inverse operation, for example, exercises for addition and subtraction. Skill with one operation can reinforce work with the inverse operation.
  3. Assign exercises involving whole numbers first so that difficulties with the algorithms may be resolved before decimals are encountered.
  4. Assign a few exercises from each section at a time so that skills in all operations are practiced together.

Review concepts that students find difficult by adapting the lessons and activities on the appropriate pages.

Multiply.

1. 75
8
600

2. $24
36
$864

3. 217
49
10 633

4. 94.8
8
758.4

5. 6.071
6
36.426

6. $3.90
472
$1840.80

7. 104
846
87 984

8. 6.9
8.9
61.41

9. 3.05
5
15.25

10. 745
9.75
7263.75

11. 9 × $1974 $17 766
12. 8.6 × 5.6 48.16
13. 478 × 237 113 286
14. 79 × 4369 345 151
15. 3 × 3.189 9.567
16. 6 × 9.8 58.8
17. 315 × 612 192 780
18. 8.9 × 2.8 24.92
19. 63 × $43.75 $2756.25
20. 5 × 79 124 395 620
21. 7.8 × 3.6 28.08
22. 1.7 × 7.5 12.75

Solve.

23. Each kilogram of apples costs $0.89. How much will 15 kg of apples cost? $13.35

24. Al bought 7 boxes of crackers. There were 375 g of crackers in each box. How heavy were the crackers he bought? 2625 kg

Divide.

1. 6)9.6 1.6
2. 5)$480 $ 96
3. 40)12 360 309
4. 21)16.380 0.780
5. 88)82.72 0.94
6. 45)27 855 619
7. 56)$53.76 $ 0.96
8. 82)53 136 648
9. 34)127 874 3761
10. 93)803.52 8.64

11. 506 160 ÷ 60 8436
12. 9.217 ÷ 13 0.709
13. $181.48 ÷ 52 $3.49
14. 43 316 ÷ 68 637
15. $69 450 ÷ 75 $926
16. 920.4 ÷ 39 23.6

Divide. Use more zeros when needed.

17. 5)2 0.4
18. 16)37.6 2.35
19. 28)$21 $ 0.75
20. 8)6 0.75
21. 64)28.8 0.45
22. 35)6.3 0.18

23. 152.1 ÷ 52 2.925
24. 99 ÷ 72 1.375
25. 34.5 ÷ 92 0.375
26. $147 ÷ 42 $3.50

Solve.

27. Sylvia bought 6 kg of hamburger for $16.38. How much did each kilogram of hamburger cost? $2.73

28. Oscar bought 12 oranges for $4.68. Each orange cost the same amount. How much did each orange cost? $0.39

315

RELATED ACTIVITIES

- Students may be asked to complete one or more of the following for extra practice and review.

1. Use addition to check the answers for several subtraction exercises.
2. Use multiplication (and addition where appropriate) to check the answers for selected division exercises.
3. Estimate the products for a selection of multiplication exercises.
4. Write the word name for each addend in a selection of addition exercises.
5. Tell the place value of each digit in one or more numerals of an exercise.
6. Write the three or four addends of an addition exercise in order from least to greatest.
7. Write the fractions equivalent to the decimals in a selection of exercises.

Unit 15 Overview

Ratio

The first lesson of the unit introduces the comparison of two numbers by a ratio and students are shown how to represent a ratio by using a colon between the numerals and by writing them in the form of a fraction. Equivalent ratios are developed in the same manner as for equivalent fractions, namely, by patterns, by multiplication and division of the terms, and by cross products. The concepts of a ratio are then applied to situations where constant rates apply. Toward the end of the unit, ratios are presented with 100 as the second term, along with the word *percent* and the symbol for it. The *Problem Solving* lesson presents situations in which numbers are approximated and then compared by means of ratios using simpler terms. Another *Problem Solving* feature extends the concepts and skills of chess beyond those in the basic lesson on ratios.

Prerequisite Skills

- write numerals for fractions
- multiply to find equivalent fractions
- use multiplication or division to find the missing term in two equivalent fractions
- write the cross products for two equivalent fractions
- use cross products to find the missing term in two equivalent fractions
- write decimal hundredths

Unit Outcomes

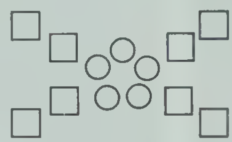
- write a ratio using words, using the symbol :, and using fraction notation
- use patterns or multiplication to find equivalent ratios
- use multiplication or division to find the missing term in two equivalent ratios
- use cross products to find the missing term in two equivalent ratios
- use patterns or multiplication to find equivalent rates
- find the missing term in two equivalent rates
- use cross products to find the missing term in two equivalent rates
- associate a percent with a ratio that compares an amount to 100 ; express the relationship ''n to 100'' using words, using the symbol :, as a fraction, as a decimal, and as a percent (halves, fourths, fifths, tenths)
- solve word problems involving ratios and rates
- use ratios to estimate lengths

Background

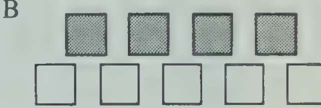
There are a number of ways in which numbers may be compared. The difference of two numbers can be found by subtracting one from the other and determining how much greater (less) one is compared with the other. For example, the statement $8 - 5 = 3$ establishes the difference of 8 and 5 as 3 and leads to such statements as $8 > 5$ and $5 < 8$.

Certain numbers may be compared in terms of multiplication, if one is a multiple of the other. For instance, 40 and 8 may be compared by stating that 40 is 8 times 5 ($8 \times 5 = 40$). An inverse form of comparison of these two numbers involves a fraction, such as 5 is one-eighth of 40, or $\frac{1}{8}$ of 40 is 5. Two numbers may also be compared in a *ratio* which uses them in one expression. There are two accepted symbolic forms for writing a ratio: one separates the numerals by a colon, as in 5:8; and the other uses the fractional form $\frac{5}{8}$. Both of these are interpreted as comparing the number 5 with the number 8 and they are read ''five to eight''. The numerator of $\frac{5}{8}$ and the first named number of 5:8 indicate that the 5 is being compared to the 8, not vice versa. For instance, in diagram A there are 5 circles and 8 squares, so the ratio of the number of circles to the number of squares is 5 to 8 (5:8, $\frac{5}{8}$). The ratio of the number of squares to the number of circles is 8 to 5 (8:5, $\frac{8}{5}$).

A



B



In previous work with fractions to denote part of a whole or part of a set, the numerator and the denominator represented parts of the same unit or set. For the fraction $\frac{4}{9}$ in reference to diagram B, the 4 refers to the number of gray squares and the 9 to the number of squares in the whole set. In a ratio expressed in fractional form, the two numerals (numerator and denominator) usually refer to different sets. For B, two sets may be considered — a set of 4 gray squares and a set of 5 white squares. The ratio of the number of gray squares to the number of white squares is 4 to 5 (4:5, $\frac{4}{5}$), and the ratio of the number of white squares to the number of gray squares is 5 to 4 (5:4, $\frac{5}{4}$). However, the ratio 4 to 9 (4:9, $\frac{4}{9}$) compares part of the set to the whole set, namely, the number of gray squares to the number of squares in all.

Ratios may be expressed in equivalent forms using the same techniques as with fractions. Refer to the Overview for Unit 13, paying particular attention to the parts dealing with equivalent fractions. Number sequences, multiplication, and division may be used to obtain equivalent ratios. In the ratio table shown, the numbers for adults may be extended by increasing each number in the sequence by 2, and the numbers of students may be extended by increasing each number by 25. The missing terms in the ratio table may also be found by using multiplication and division. For instance, 8 (adults) is 4 times the unit of 2 (adults), so the number of students supervised by 8 adults is 4×25, or 100. To find the fifth ratio in the table, the two numbers in the ratio, 2 and 25, may be multiplied by 5 to obtain the numbers 10 and 125. The ratios 2:25, 4:50, 6:75, and so on, all represent the same ratio and the symbol = may be used between them in pairs to show that they are equivalent.

2 adults to supervise 25 students

2	4	6	8	10
25	50	75	100	125

2:25 = 4:50 2:25 = 6:75 4:50 = 10:125

$$\frac{2}{25} = \frac{4}{50} \qquad \frac{2}{25} = \frac{6}{75} \qquad \frac{4}{50} = \frac{10}{125}$$

Equivalent ratios may be checked and missing terms in equivalent ratios may be found in the same manner as with fractions. In the example shown, the cross products (300) are the same. The missing term in the equivalent ratios 2:25 and 8:■ can be found by using cross products and the operations of multiplication and division.

$$\frac{4}{50} = \frac{6}{75}$$

$$\frac{2}{25} = \frac{8}{\blacksquare}$$

$$2 \times \blacksquare = 25 \times 8$$
$$2 \times \blacksquare = 200$$
$$\blacksquare = 200 \div 2$$
$$\blacksquare = 100$$

There is little difference between the concepts of ratio and rate. The term *ratio* may be applied to comparison of abstract numbers, whereas the term *rate* is almost always used in applications in real life. A ratio might relate to a single instance and it might not be logical to extend it to equivalent forms, but a rate may be extended. For example, if 225 cars and 75 trucks proceed through an intersection over a certain period of time, the ratio of the number of cars to the number of trucks is 225 to 75 or, in its simplest equivalent form, 3 to 1. But it would not be logical to conclude that over a longer or a shorter period of time the same ratio would prevail. In contrast, a machine that fills 2 jars every 11 seconds operates at a constant rate, and the rate may be extended in a table, as shown. It is logical to extend rate tables to any desired number of multiples.

number of jars	2	4	6	...	16	18	20
number of seconds	11	22	33	...	88	99	110

Probably the most frequently encountered ratio in real life is the one we call *percent*. The term *percent* means "out of 100", or "compared to 100", from the Latin words *per* meaning through and *centum* meaning hundred. Ratios which have 2, 4, 5, 10, 20, 25, and 50 as their second terms may be expressed as percents by finding equivalent forms for them with 100 as the second term. As with the other forms for showing a ratio, there are two terms in showing a percent, although the numeral 100 appears as the symbol %. Thus, $\frac{65}{100}$ is equivalent to 65%, in which 65 represents the first term and %, the second term.

Teaching Strategies

Since the first lesson in the unit involves the numbers of pieces in the game of chess, it would be advisable to have a checkerboard and checkers and a chess set in the classroom before Unit 15 is begun. Students who know the games may be encouraged to play them in their spare time or as extracurricular activities. It may even be possible to organize a small tournament in these games. Familiarity with the names of the various chess pieces beforehand would help the students to concentrate on the mathematical aspects of the lesson.

Equivalent ratios may be found by completing number sequences which list multiples of the terms. For example, equivalent ratios for $\frac{2}{3}$ have multiples of 2 for their first terms and corresponding multiples of 3 for their second terms. Therefore, in preparation for the work on equivalent ratios, students may be asked to write sequences showing multiples of numbers. A complete table of these would, of course, appear very much like the products in a multiplication table. From rows of these multiples, a ratio table of equivalent ratios can be constructed very quickly. For example, the ratio $\frac{3}{5}$ uses corresponding number pairs from the sequences for 3 and 5. It is obvious that if corresponding multiples of two numbers are used, the equivalent forms of the ratios are actually obtained by either multiplying or dividing the terms by the same number.

2 4 6 . . . 18

$\boxed{3\ \ 6\ \ 9\ \ldots\ 27}$

4 8 12 . . . 36

$\boxed{5\ \ 10\ \ 15\ \ldots\ 45}$

6 12 18 . . . 54

3	6	9	12	...	27
5	10	15	20	...	45

The ratio $\frac{12}{20}$ is actually the fourth multiple of $\frac{3}{5}$; that is, each term of $\frac{3}{5}$ is multiplied by 4 to obtain $\frac{12}{20}$. Similarly, the ratio $\frac{24}{40}$ may be expressed in an equivalent form by dividing each term by a common factor of 24 and 40. Thus, $\frac{24}{40} = \frac{12}{20}$ (division by 2), $\frac{24}{40} = \frac{6}{10}$ (division by 4), and $\frac{24}{40} = \frac{3}{5}$ (division by 8). Any of these approaches will strengthen students' understanding of ratios and will also reinforce their awareness of the relationships and structure of our numeration system.

For the lesson on pages 320 and 321, it is suggested that slides of different areas of the classroom be projected onto a screen so that ratios may be established between sizes on the screen and actual sizes of the objects pictured. If such slides are not available, some of the students may have recent slides of themselves from which their real and projected heights may be compared and expressed as ratios.

Finding missing terms in equivalent ratios by means of cross products involves the two operations of multiplication and division. For example, to find the missing term in $\frac{9}{6} = \frac{6}{\blacksquare}$, the multiplication step is $6 \times 6 = 36$, and then the division step is $36 \div 9 = 4$. Speed and accuracy in completing equivalent ratios are dependent on mastery of basic facts and skills in these operations. Many of the exercises on pages 323 and 327 can be completed at a glance by merely using basic facts. Quick reviews and drills of such facts are recommended.

Some of the more capable students may learn the percents that are equivalent to simple fractions, such as $\frac{1}{2} = 50\%$, $\frac{1}{4} = 25\%$, $\frac{1}{5} = 20\%$, and multiples of these. The work in Unit 13 with fractions and decimal equivalents may be reviewed to develop this link between fractions and percents.

Students might be directed to look for and list places and situations where percents are used, such as banks and sales of merchandise. They might also be directed to watch for instances where sales personnel use rate tables or charts of costs to determine the cost of more than one item. For instance, a service station operator might consult a chart, similar to the one shown, in writing out a statement after changing the oil in a car.

L	Cost
1	\$1.09
2	2.18
3	3.27
4	4.36
5	5.45

If any weaknesses become apparent in the *Checking Up* exercises on page 331, they should be identified carefully since many of the skills are the same as those used in working with fractions. It may be advisable to reteach and review related topics from Unit 13 and Unit 15 at the same time.

Materials

a chess set, a game of checkers; an eight-by-eight section of squared paper for each student (optional)
4 red checkers, 8 black checkers, a mass of 1 hg, a mass of 10 dag
slide projector and slides (optional)
centimetre ruler for each student
several cubes showing 3 yellow faces, 2 red faces, and 1 blue face
a Canadian flag, a dollar bill

Vocabulary

ratio	percent, %
equivalent ratios	hectogram, hg
ratio table	decagram, dag
scale	kilometres per hour, km/h
rate	names of pieces used in chess
equivalent rates	
rate table	rate of exchange

LESSON OUTCOME

Write a ratio using words, using the symbol :, and using fraction notation

Materials

a chess set; a game of checkers; an eight-by-eight section of squared paper for each student (optional)

Vocabulary

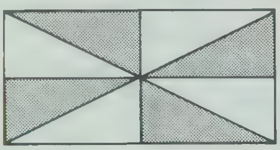
ratio, names of pieces used in chess

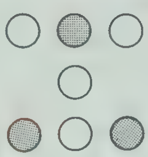
Prerequisite Skills

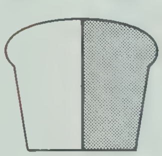
Write numerals for fractions

Checking Prerequisite Skills

Write a fraction to show how much is shaded.

1.  $\frac{4}{8}$

2.  $\frac{3}{7}$

3.  $\frac{1}{2}$

15 RATIO

Writing Ratios

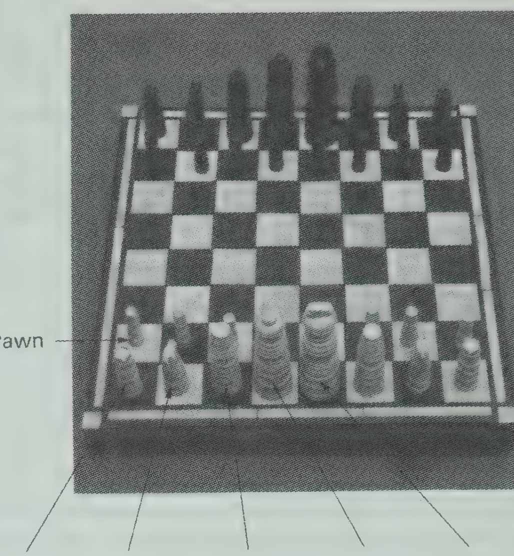
A chess game has 4 knights and 16 pawns.

The **ratio** of knights to pawns can be written in different ways.

4 to 16
knights pawns

4 : 16
knights pawns

$\frac{4}{16}$ knights / pawns



Working Together

Give the first number for each ratio.

1. 7 to 9 7 **2.** 10 to 5 10

3. the number of rooks for a player to the number of rooks in a game 2

Give the second number for each ratio.

4. 1 out of 4 4 **5.** 6 out of 6 6

6. the number of bishops for a player to the number of bishops for another player 2

Complete.

7.	5 to 6	5 : 6	$\frac{5}{6}$
8.	4 to 12	? 4:12	? $\frac{4}{12}$
9.	? 11 to 1	11:1	? $\frac{11}{1}$
10.	? 10 to 10	? 10:10	$\frac{10}{10}$

316

LESSON ACTIVITY

Before Using the Pages

- Display a set of checkers and have students arrange the checkers on the checkerboard for a game. Establish that there are 12 red checkers and 12 black checkers at the beginning of the game. Have two students play the game. As checkers are captured, have students describe the number of red checkers to the number of black checkers that are on the checkerboard. You may wish to show the ratios in a list on the chalkboard as the game progresses, without referring to the term *ratio*.

red checkers	to	black checkers
12	to	12
11	to	12
11	to	9
.		
.		
.		
0	to	4

Using the Pages

- Ask what game is shown in the photograph. For students who are not familiar with the game, it would be advisable to display a chess set and to establish that there are 32 pieces. Have them refer to the photograph to help in identifying the different pieces by name. Ask questions such as ''How many knights are there?'' and ''How many rooks are there?'' Draw attention to the headings at the top of page 316 and tell the students that the numbers of objects in two sets can be compared by a *ratio*. Point out that the ratio ''4 to 16'' compares the number of *knights* to the number of *pawns*. Emphasize the importance of associating the first number in the ratio with the first set named and the second number with the second set named. For example, the ratio ''4 to 16'' is not the same as the ratio ''16 to 4''; the latter compares the number of pawns to the number of knights. Note the different ways of writing the ratio: 4 to 16, 4:16, and $\frac{4}{16}$. Each is read ''four to sixteen''.

Working Together: Ex. 1-3 deal with identifying the first number in a ratio. Ex. 4-6 deal with identifying the second

Exercises

Complete.

1.	2 to 3	2:3	$\frac{2}{3}$
2.	3 to 4	3:?4	? $\frac{3}{4}$
3.	4 ?to 6	4:6	? $\frac{4}{6}$
4.	8 ?to 1	8:1?	$\frac{8}{1}$
5.	5 ?to 5	5:5	? $\frac{5}{5}$

Write each as a ratio in two other ways.

6. 2 out of 7 2:7, $\frac{2}{7}$

7. 3:5 3 to 5, $\frac{3}{5}$

8. 8 to 12 8:12, $\frac{8}{12}$

9. $\frac{9}{9}$ 9:9, 9 to 9

10. 1 out of 3 1:3, $\frac{1}{3}$

11. 7:10 7 to 10, $\frac{7}{10}$

12. 15 to 100 15:100, $\frac{15}{100}$

13. $\frac{0}{14}$ 0:14, 0 to 14

14. 4 out of 4 4:4, $\frac{4}{4}$

15. 15:8 15 to 8, $\frac{15}{8}$

For each of these, write a ratio in three ways.

Examples: 4 moves out of 9 moves
4 out of 9, 4:9, $\frac{4}{9}$

2 players for 1 game
2 for 1, 2:1, $\frac{2}{1}$

16. a score of 4 games to 3 games 4 to 3, 4:3, $\frac{4}{3}$

17. a score of 0 games to 6 games 0 to 6, 0:6, $\frac{0}{6}$

18. winning 8 games out of 8 games 8 out of 8; 8:8; $\frac{8}{8}$

19. 2 players out of 26 players in the final competition 2 out of 26; 2:26; $\frac{2}{26}$

20. the number of rooks in a chess game to the number of players in the game 4 to 2, 4:2, $\frac{4}{2}$

21. the number of pawns for a player to the number of knights for a player 8 to 2, 8:2, $\frac{8}{2}$

22. the number of kings in a chess game to the number of pawns in the game 2 to 16, 2:16, $\frac{2}{16}$

23. the number of queens in a chess game to the number of players in the game 2 to 2, 2:2, $\frac{2}{2}$

In a chess game, a knight's move is 2 squares along a row or column and then 1 square to the left or right. A knight can move over other chess pieces, but cannot land on a square already occupied except to capture an opponent's piece.

1. How many first moves are possible for a knight when the board is set up for a game as shown in the photograph? 2

2. For each of the possible first moves for a knight, how many moves are possible for the player's second move with the same knight? 3 for one, 5 for the other

3. How many possible positions are there for a knight after 2 moves? 6

PROBLEM SOLVING

RELATED ACTIVITIES

- Have students work with balance scales and such objects as pieces of new chalk, erasers, keys, washers, nails, scissors, corks, pennies, and so on. Have them find, for example, the number of nails that will balance one pair of scissors and write the corresponding ratio.
- Have students write ratios to compare numbers for examples similar to the following.

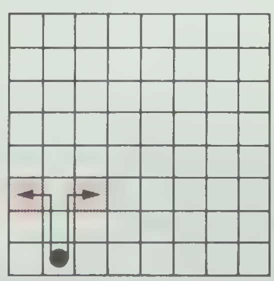
1. the number of girls in the class to the number of boys
2. the number of teachers to the number of students in the class or in the school
3. the number of doors to the number of windows in the classroom
4. the number of Grade 4 teachers to the number of Grade 5 teachers
5. the number of students in the class who walk to school to the number of students who ride
6. the number of vowels to the number of consonants in a student's name

number in a ratio. Ex. 7-10 involve the three forms for writing a ratio. Point out that for a ratio written as a fraction, the first number is the numerator and the second number is the denominator.

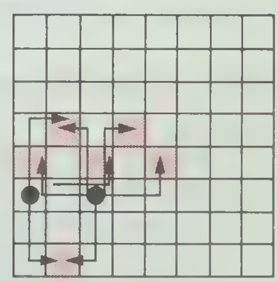
Exercises: Students may refer to the photograph on page 316 or to a chess set for assistance with Ex. 20-23.

Problem Solving: Students can be grouped for these exercises so that in each group there is at least one student familiar with the game of chess. As an alternative, discuss the introductory statements with the students before they begin the exercises. Demonstrate the L-shaped move for a knight on a chess board. You may wish to give each student an eight-by-eight section of squared paper cut from copies of page T 397, to help in determining the answers. Diagram A shows the two possible first moves for one knight. Diagram B shows the possible second moves from those first positions. Note that one of the possible moves for a knight is a move back to the original position.

A



B



Assessment

Write each as a ratio in two other ways.

1. 4 to 7 4:7 $\frac{4}{7}$ **2.** 2:15 2 to 15 $\frac{2}{15}$ **3.** $\frac{17}{100}$ 17 to 100 17:100

Write a ratio in three ways.

4. a score of 5 games to 4 games 5 to 4 5:4 $\frac{5}{4}$

LESSON OUTCOME

Use patterns or multiplication to find equivalent ratios; solve related word problems

Materials

4 red checkers, 8 black checkers, a mass of 1 hg, a mass of 10 dag

Vocabulary

equivalent ratios, ratio table, hectogram, hg, decagram, dag

Prerequisite Skills

Multiply to find equivalent fractions

Checking Prerequisite Skills

Find equivalent fractions.

3 × 2   3 × 3

1.	$\frac{3}{5}$	$\frac{■}{■}$ 6/10	$\frac{■}{■}$ 9/15	$\frac{■}{■}$ 12/20	$\frac{■}{■}$ 15/25

5 × 2   5 × 3

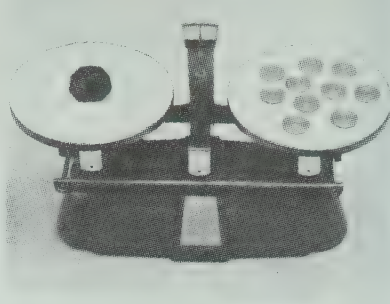
6 × 2   6 × 3

2.	$\frac{6}{7}$	$\frac{■}{■}$ 12/14	$\frac{■}{■}$ 18/21	$\frac{■}{■}$ 24/28	$\frac{■}{■}$ 30/35

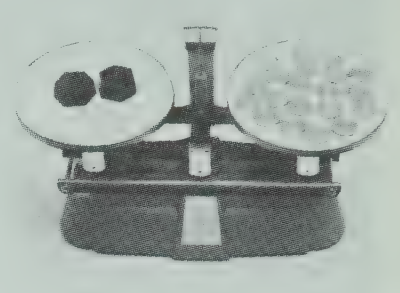
7 × 2   7 × 3

Writing Equivalent Ratios

1 hg (hectogram) is the same mass as 10 dag (decagrams).



1 : 10



2 : 20

The ratios 1:10 and 2:20 are **equivalent ratios**.

This **ratio table** shows equivalent ratios.

If both numbers in a ratio are multiplied by the same number to give another ratio, the ratios are equivalent.

		1 × ■2	1 × ■3	1 × ■4	1 × ■5
hectograms	1	2	3	4	5
decagrams	10	20	30	40	50
		10 × ■2	10 × ■3	10 × ■4	10 × ■5

1 hg: 10 dag   2 hg: 20 dag   3 hg: 30 dag   4 hg: 40 dag   5 hg: 50 dag

Each of these ratios shows the ratio of the mass in hectograms to an equal mass in decagrams.

318

LESSON ACTIVITY

Before Using the Pages

- Display a group of 4 red checkers and a group of 8 black checkers. Ask for the ratio of the number of red checkers to the number of black checkers. Have three students show on the board different ways of writing the ratio.

  4 to 8   4:8 $\frac{4}{8}$

  Rearrange the checkers to show 4 groups with 1 red checker and 2 black checkers in each group. Develop that this arrangement shows there is 1 red checker for every 2 black checkers. Establish that the ratio 4:8 is the same as the ratio 1:2. Lead the students to recognize that the ratios 4:8 and 1:2 correspond to the equivalent fractions $\frac{4}{8}$ and $\frac{1}{2}$. Ask what name can describe such ratios as 4:8, 3:6, 2:4, and 1:2, and have students explain their answers.

Using the Pages

- Draw attention to the title of the lesson on page 318 and have a student read the statement below it. Note the symbols hg for *hectogram* and dag for *decagram*. Display an example of each mass. Ask students to identify the hectogram masses and the decagram masses in the photographs. Have them relate each photograph to the corresponding ratio. Ask how many decagrams would balance 3 hectograms. Discuss the *ratio table* and have students explain how to obtain other ratios equivalent to 1:10. Emphasize that multiplying each number in a ratio by the same factor gives an equivalent ratio.

Working Together: These exercises emphasize that equivalent ratios are found by using the same procedure as for equivalent fractions: when one term of a ratio is multiplied by a number, the other term of the ratio is multiplied by that same number. Students may complete patterns to find equivalent ratios for Ex. 4 and 5.

Exercises: Ex. 7 and 8 are starred because the simplest form of the ratio is not provided. Students can think of number patterns or multiplication facts to help complete the exercises. Note that ratio tables are suggested for Ex. 13 and 14 in the same way as they are shown for Ex. 9-12.

Working Together

Complete.

1. $\frac{1}{4} \xrightarrow{1 \times 2} \frac{■}{■}$, 4×2 — 2, 8

2. $\frac{4}{7} \xrightarrow{4 \times 3} \frac{■}{■}$, $7 \times ■$ — 3 — 12, 21

3. $\frac{9}{2} \xrightarrow{■ \times 4} \frac{■}{■}$, $■ \times ■$ — 9; 2, 4 — 36, 8

4. $\frac{2}{5} = \frac{4}{10} = \frac{6}{15} = \frac{■}{■} = \frac{■}{■} = \frac{■}{■} = \frac{14}{35}$ — $\frac{8}{20}$ $\frac{10}{25}$ $\frac{12}{30}$

5. $\frac{7}{10} = \frac{14}{20} = \frac{21}{30} = \frac{■}{■} = \frac{■}{■} = \frac{■}{■} = \frac{■}{■}$ — $\frac{28}{40}$ $\frac{35}{50}$ $\frac{42}{60}$ $\frac{49}{70}$

Exercises

Complete. Use patterns or multiplication.

1. $\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{■}{■} = \frac{■}{■} = \frac{■}{■} = \frac{7}{14}$ — $\frac{4}{8}$ $\frac{5}{10}$ $\frac{6}{12}$

2. $\frac{5}{4} = \frac{10}{8} = \frac{15}{12} = \frac{■}{■} = \frac{■}{■} = \frac{■}{■} = \frac{■}{■}$ — $\frac{20}{16}$ $\frac{25}{20}$ $\frac{30}{24}$ $\frac{35}{28}$

3. $\frac{3}{3} = \frac{6}{6} = \frac{9}{9} = \frac{■}{■} = \frac{■}{■} = \frac{■}{■} = \frac{■}{■}$ — $\frac{12}{12}$ $\frac{15}{15}$ $\frac{18}{18}$ $\frac{21}{21}$

4. $\frac{7}{6} = \frac{14}{12} = \frac{21}{18} = \frac{■}{■} = \frac{■}{■} = \frac{■}{■} = \frac{■}{■}$ — $\frac{28}{24}$ $\frac{35}{30}$ $\frac{42}{36}$ $\frac{49}{42}$

5. $\frac{4}{1} = \frac{8}{2} = \frac{12}{3} = \frac{■}{■} = \frac{■}{■} = \frac{■}{■} = \frac{■}{■}$ — $\frac{16}{4}$ $\frac{20}{5}$ $\frac{24}{6}$ $\frac{28}{7}$

6. $\frac{3}{8} = \frac{6}{16} = \frac{9}{24} = \frac{■}{■} = \frac{■}{■} = \frac{■}{■} = \frac{■}{■}$ — $\frac{12}{32}$ $\frac{15}{40}$ $\frac{18}{48}$ $\frac{21}{56}$

*7. $\frac{■}{■} = \frac{■}{■} = \frac{■}{■} = \frac{8}{28} = \frac{10}{35} = \frac{12}{42} = \frac{■}{■}$ — $\frac{2}{7}$ $\frac{4}{14}$ $\frac{6}{21}$; $\frac{14}{49}$

*8. $\frac{■}{■} = \frac{■}{■} = \frac{27}{15} = \frac{36}{20} = \frac{45}{25} = \frac{■}{■} = \frac{■}{■}$ — $\frac{9}{5}$ $\frac{18}{10}$; $\frac{54}{30}$ $\frac{63}{35}$

Complete these ratio tables.

9.

millilitres	1000	2000	? 3000	? 4000	? 5000
litres	1	2	? 3	? 4	? 5

10.

decimetres	1	? 2	? 3	4	? 5
centimetres	10	? 20	? 30	40	? 50

11.

years	100	? 200	? 300	? 400	? 500
centuries	1	? 2	? 3	? 4	? 5

12.

minutes	1	? 2	? 3	? 4	? 5
seconds	60	? 120	? 180	? 240	? 300

Use a ratio table to solve each of these.

13. 1 can of frozen juice and 3 cans of water are needed to make lemonade. How many cans of water are needed for 7 cans of frozen juice? 21

14. 500 mL of flour are needed to make 1 cake. How many millilitres of flour are needed to make 4 cakes? 2000

319

RELATED ACTIVITIES

- Ask students to read some of the completed exercises aloud to emphasize that the fraction $\frac{4}{1}$, for instance, is read "four to one" when it represents a ratio.
- Have students draw pictures to match a list of equivalent ratios. For example, suggest that 2 eggs are needed to make 1 cake.

Eggs → $\frac{2}{1} = \frac{4}{2} = \frac{6}{3} = \frac{8}{4} = \frac{10}{5}$
Cakes →

- Have the students measure the length of their paces to the nearest centimetre. Then have them write a ratio table.

paces	1				
centimetres	44				

- If students completed the first activity in *Related Activities* on page T 345, they may use their results to complete ratio tables similar to the following.

pairs of scissors	1	2	3
nails	5	10	15

However, some students may be able to answer the questions without the use of tables. Keep in mind that there are two ways to show a ratio table, as indicated below for Ex. 13.

cans of frozen juice	1
cans of water	3

cans of water	3
cans of frozen juice	1

Assessment

Complete. Use patterns or multiplication.

1. $\frac{5}{8} = \frac{10}{■} = \frac{■}{24} = \frac{■}{■} = \frac{■}{■} = \frac{■}{■} = \frac{■}{■}$ — 16; 15; $\frac{20}{32}$ $\frac{25}{40}$ $\frac{30}{48}$ $\frac{35}{56}$

Complete the ratio table.

2.

days	7	14	21	28	35
weeks	1	2	3	4	5

Solve.

3. There are 6 pencils in one pack. How many pencils are there in 4 packs? 24

Pages 320-321

LESSON OUTCOME

Use multiplication or division to find the missing term in two equivalent ratios; solve related word problems

Materials

slide projector and slides (optional)

Prerequisite Skills

Use multiplication or division to find the missing term in two equivalent fractions

Checking Prerequisite Skills

Find the missing term.

1. $\frac{2}{3} = \frac{\blacksquare}{9}$ 6 **2.** $\frac{1}{5} = \frac{8}{\blacksquare}$ 40

3. $\frac{6}{42} = \frac{\blacksquare}{7}$ 1 **4.** $\frac{8}{18} = \frac{4}{\blacksquare}$ 9

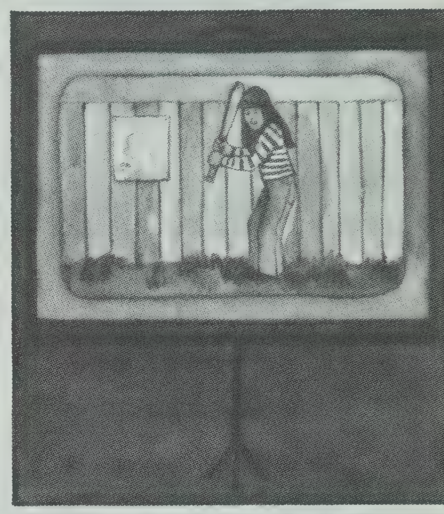
Finding the Missing Term in Equivalent Ratios

An object that is 3 cm tall on the screen has a real height of 8 cm.

The ratio of height on the screen to real height is $\frac{3}{8}$.

The height of the fence on the screen is 48 cm. What is its real height?

The ratio of the height of the fence on the screen to its real height is $\frac{48}{\blacksquare}$.



To find the real height of the fence, write $\frac{3}{8} = \frac{48}{\blacksquare}$.

To find the missing term, think

$$3 \xrightarrow{3 \times 16} 48$$

Then multiply 8 and 16.

$$\frac{3}{8} = \frac{48}{128} \qquad 3 \times 16,\ 8 \times 16$$

The real height of the fence is 128 cm.

Here is an example of how the missing term can be found by division.

$$\frac{12}{18} = \frac{\blacksquare}{3}$$

To find the missing term, think

$$18 \xrightarrow{18 \div 6} 3$$

Then, divide 12 by 6.

$$\frac{12}{18} = \frac{2}{3} \qquad 12 \div 6,\ 18 \div 6$$

320

LESSON ACTIVITY

Before Using the Pages

- In advance of this lesson, you may wish to take a few slides of familiar objects or of different areas in the classroom. One or more slides can be projected on a screen at a predetermined distance from the projector. Students can then measure the heights of objects on the screen and their real heights in the classroom, noting the equivalent ratios.

Using the Pages

- Students used multiplication and division on page 263 to find the missing term in two equivalent fractions. In this lesson, the same procedure is used to find the missing term in two equivalent ratios.

  Ask a student to read the introductory statements that introduce the ratio 3 to 8 and to relate the statements to the illustration. You may wish to develop a ratio table on the board and have students explain the ratios.

height on the screen in centimetres	3	6	9
real height in centimetres	8	16	24

Direct the students' attention to the illustration once again and have a student read the statements above it. The real height of the fence can be found by extending the ratio table; but if the students suggest this, ask for a way of finding the real height without having to determine any of the other equivalent ratios. Establish that $\frac{3}{8}$ and $\frac{48}{\blacksquare}$ are equivalent ratios and that the missing term can be determined by thinking of multiplication. Discuss the example at the bottom of page 320 to demonstrate that the missing term can be found by division.

Working Together: These exercises emphasize that each term of a ratio must be multiplied (divided) by the same number, in finding the missing term for two equivalent ratios. Ask students to explain their work and to read the equivalent ratios aloud.

Working Together

Complete each of these.

1. 2 × 4; $\frac{2}{3} = \frac{8}{■}$ 12; 3 × 4
2. 1 × ■ 3; $\frac{1}{6} = \frac{■}{18}$ 3; 6 × 3
3. 1 × ■ 2; $\frac{1}{2} = \frac{2}{■}$ 4; 2 ■ × ■ 2
4. $\frac{3}{5} = \frac{■}{10}$ 6
5. 10 ÷ 5; $\frac{10}{35} = \frac{2}{■}$ 7; 35 ÷ 5
6. 3 ÷ 3; $\frac{3}{12} = \frac{1}{■}$ 4; 12 ÷ ■ 3
7. 72 ■ ÷ ■ 8; $\frac{72}{40} = \frac{■}{5}$ 9; 40 ÷ ■ 8
8. $\frac{27}{21} = \frac{■}{7}$ 9

Exercises

Complete each of these.

1. $\frac{1}{2} = \frac{■}{12}$ 6
2. $\frac{5}{6} = \frac{35}{■}$ 42
3. $\frac{35}{40} = \frac{■}{8}$ 7
4. $\frac{8}{10} = \frac{4}{■}$ 5
5. $\frac{7}{10} = \frac{70}{■}$ 100
6. $\frac{2}{9} = \frac{■}{27}$ 6
7. $\frac{6}{8} = \frac{■}{4}$ 3
8. $\frac{4}{12} = \frac{■}{3}$ 1
9. $\frac{18}{10} = \frac{■}{5}$ 9
10. $\frac{9}{9} = \frac{3}{■}$ 3
11. $\frac{5}{4} = \frac{■}{28}$ 35
12. $\frac{2}{3} = \frac{10}{■}$ 15
13. $\frac{100}{100} = \frac{■}{5}$ 5
14. $\frac{20}{12} = \frac{5}{■}$ 3
15. $\frac{1}{2} = \frac{■}{16}$ 8
16. $\frac{10}{1} = \frac{30}{■}$ 3
17. $\frac{45}{72} = \frac{■}{8}$ 5
18. $\frac{3}{4} = \frac{■}{100}$ 75
19. $\frac{6}{42} = \frac{■}{7}$ 1
20. $\frac{3}{2} = \frac{24}{■}$ 16

Use = or ≠ to make true statements.

21. $\frac{3}{8} \bigcirc \frac{6}{16}$ =
22. $\frac{1}{16} \bigcirc \frac{1}{4}$ ≠
23. $\frac{40}{7} \bigcirc \frac{10}{3}$ ≠
24. $\frac{6}{45} \bigcirc \frac{2}{15}$ =
25. $\frac{24}{8} \bigcirc \frac{6}{2}$ =

For the slide shown on the screen, write a pair of equivalent ratios. Then solve the problems.

26. The girl is 51 cm tall on the screen. What is her real height? 136 cm
27. The baseball bat is 33 cm long on the screen. What is its real length? 88 cm
28. The poster is 15 cm wide on the screen. What is its real width? 40 cm
*29. The poster is 24 cm long on the screen. What is its real perimeter? 256 cm

321

Exercises: Before the students begin the exercises, discuss the procedure that would be used to answer Ex. 21-25. Also, point out that Ex. 26-29 refer to the information in the worked example on page 320. Ex. 29 is starred because information must be obtained from Ex. 28 and the solution requires more than one step. Some students may find the perimeter for the poster on the screen first (78 cm) and then work with the equivalent ratios $\frac{3}{8}$ and $\frac{78}{■}$. The solution may also be found by finding the missing term for $\frac{3}{8} = \frac{24}{■}$ first and then finding the perimeter of the real poster.

Assessment

Complete.

1. $\frac{5}{9} = \frac{30}{■}$ 54
2. $\frac{42}{21} = \frac{■}{1}$ 2
3. $\frac{2}{7} = \frac{■}{63}$ 18

Write a pair of equivalent ratios and solve the problem.

4. The ratio of height on the screen to real height is $\frac{1}{3}$. The height of the fence on the screen is 45 cm. What is the real height of the fence? $\frac{1}{3} = \frac{45}{■}$ 135 cm

RELATED ACTIVITIES

- Have students write equivalent ratios to solve problems similar to the following.
  1. There are 12 months in one year. How many years are in 168 months?
  2. There are 7 days in one week. How many days are in 19 weeks?
- For enrichment, problems similar to the following may be assigned.
  1. A row of 10 pennies is 19 cm long. How many pennies are there in a row 133 cm long? How long is the row of pennies having a value of $2.00?
  2. A stack of 7 pennies has a height of 1 cm. What is the height of a stack of pennies having a value of $1.05? What would be the value of a stack of pennies 1 m in height?
  3. Two dimes placed side by side measure 36 mm. How many dimes are there in a row 90 mm long? in a row 18 cm long? How long is a row of 12 dimes? How long is a row of dimes having a value of $5.00?

LESSON OUTCOME

Use cross products to find the missing term in two equivalent ratios; solve related word problems

Materials

centimetre ruler for each student

Vocabulary

scale

Finding the Missing Term Using Cross Products

The scale on the map shows that 1 cm on the map represents 32 km. On the map, the distance between Louisbourg and Sable Island is 6 cm. What is the distance between these places in kilometres?

The ratio of the map distance in centimetres to the real distance in kilometres is $\frac{1}{32}$.

The ratio of the distance between Louisbourg and Sable Island to the real distance is

$$\frac{6}{■}.$$

To find the real distance, write $\frac{1}{32} = \frac{6}{■}$ and use cross products.

$$1 \times ■ \qquad 32 \times 6$$

$$\frac{1}{32} = \frac{6}{■}$$

$$1 \times ■ = 32 \times 6$$

$$1 \times ■ = 192$$

$$1 \times \boxed{192} = 192$$

The distance between Louisbourg and Sable Island is 192 km.

Here is another example of finding the missing term.

$$\frac{6}{8} = \frac{■}{12}$$

Use cross products.

$$6 \times 12 \qquad 8 \times ■$$

$$\frac{6}{8} = \frac{■}{12}$$

$$6 \times 12 = 8 \times ■$$

$$72 = 8 \times ■$$

$$72 \div 8 = \boxed{9}$$

$$72 = 8 \times \boxed{9}$$

$$\frac{6}{8} = \frac{9}{12}$$

Working Together

Write the cross products.

1. $\frac{3}{5} = \frac{9}{15}$ 3 × 15 5 × 9
2. $\frac{12}{16} = \frac{18}{24}$ 12 × 24 16 × 18

Use cross products to complete these.

3. $\frac{28}{32} = \frac{■}{40}$ 35
4. $\frac{16}{12} = \frac{8}{■}$ 6

322

LESSON ACTIVITY

Before Using the Pages

- Ask students to suggest pairs of equivalent fractions, for example, $\frac{4}{6}$ and $\frac{10}{15}$, and pairs of fractions that are not equivalent, for example, $\frac{5}{8}$ and $\frac{4}{7}$. Write the fractions on the board and have students demonstrate that cross products are equal for equivalent fractions and not equal for other fractions. Have them use cross products to find the missing term in two equivalent fractions.

$$\frac{15}{25} = \frac{■}{15}$$

$$\frac{6}{14} = \frac{12}{■}$$

- Review that a ratio can be written using fraction notation. For example, the ratio of the number of girls to the number of boys in the class might be shown as $\frac{14}{18}$. Ask the students whether cross products can be used to find the missing term in two equivalent ratios, for example, $\frac{14}{18} = \frac{■}{9}$.

Using the Pages

- Begin with a discussion of the map on page 323. Have students identify the provinces shown and read the names of several places on the map. Draw attention to the *scale* and have students explain its significance and how it is used to find real distances. Point out that the scale ''1 cm to 32 km'' can be expressed as the ratio $\frac{1}{32}$. Ask questions such as ''What distance is represented by a length of two centimetres?'' and ''How many centimetres represent a distance of ninety-six kilometres?''

  Have a student read the word problem at the top of page 322. You may wish to have the students use their centimetre rulers to check that the distance between Louisbourg and Sable Island is 6 cm on the map. Discuss with the students each step of the worked example to find the real distance between these two places. Emphasize that equivalent ratios have equal cross products. A second example is provided to demonstrate the use of cross products. Note that the missing term for $\frac{1}{32} = \frac{6}{■}$ in the first example can be found in one step by using only

Exercises

Complete each of these.

1. $\frac{2}{4} = \frac{\blacksquare}{8}$ 4
2. $\frac{16}{12} = \frac{4}{\blacksquare}$ 3
3. $\frac{7}{21} = \frac{\blacksquare}{6}$ 2
4. $\frac{9}{6} = \frac{6}{\blacksquare}$ 4
5. $\frac{4}{10} = \frac{\blacksquare}{15}$ 6
6. $\frac{70}{63} = \frac{\blacksquare}{45}$ 50
7. $\frac{9}{9} = \frac{7}{\blacksquare}$ 7
8. $\frac{40}{48} = \frac{5}{\blacksquare}$ 6
9. $\frac{4}{12} = \frac{5}{\blacksquare}$ 15
10. $\frac{70}{60} = \frac{\blacksquare}{12}$ 14
11. $\frac{6}{14} = \frac{9}{\blacksquare}$ 21
12. $\frac{32}{40} = \frac{\blacksquare}{25}$ 20
13. $\frac{24}{21} = \frac{\blacksquare}{7}$ 8
14. $\frac{75}{100} = \frac{\blacksquare}{4}$ 3
15. $\frac{3}{3} = \frac{2}{\blacksquare}$ 2
16. $\frac{44}{40} = \frac{33}{\blacksquare}$ 30
17. $\frac{10}{5} = \frac{\blacksquare}{9}$ 18
18. $\frac{6}{12} = \frac{5}{\blacksquare}$ 10
19. $\frac{25}{40} = \frac{\blacksquare}{24}$ 15
20. $\frac{42}{24} = \frac{21}{\blacksquare}$ 12

Use = or ≠ to make true statements.

21. $\frac{4}{16} \bigcirc \frac{2}{8}$ =
22. $\frac{4}{9} \bigcirc \frac{8}{12}$ ≠
23. $\frac{3}{6} \bigcirc \frac{4}{10}$ ≠
24. $\frac{12}{6} \bigcirc \frac{6}{3}$ =
25. $\frac{8}{8} \bigcirc \frac{6}{6}$ =

For each of these, use the map to find the real distance. Answers are given to the nearest kilometre.

26. West Point to Yarmouth 448 km
27. Halifax to Saint John 189 km
28. Dartmouth to Sydney 288 km
29. Lunenburg to Sable Island 314 km
30. Moncton to Saint John 125 km
31. Charlottetown to Pictou 64 km

323

RELATED ACTIVITIES

- Provide the students with maps of your community or province. Have them find actual distances between various points on the maps by using the scales.
- For enrichment, give each student a copy of the large rectangle on page T 382. Have the students measure the length and the width of the rectangle and find the perimeter and the area. Have them fold and cut the rectangle into four identical rectangles. For one of the rectangles, have them measure the length and the width and find the perimeter and the area. The results may be shown in a chart. They may use their results to write ratios comparing lengths, widths, perimeters, and areas of the two rectangles. Then they may use cross products to determine whether any two of the ratios are equivalent.

Shape	Length (cm)	Width (cm)	Perimeter (cm)	Area (cm^2)
Large rectangle	16			
Small rectangle	8			

multiplication. However, the second example requires two steps.

$$\frac{1}{32} = \frac{6}{\blacksquare} \qquad 1 \times 6 \qquad 32 \times 6$$

$$\frac{6}{8} = \frac{\blacksquare}{12}$$
$$6 \times 12 = 8 \times \blacksquare$$
$$72 \div 8 = 9$$

Working Together: Ex. 1 and 2 deal with writing cross products. This skill is applied for finding the missing term of the equivalent ratios in Ex. 3 and 4.

Exercises: Draw attention to the instructions for Ex. 21-25. Ask a student to explain a procedure for determining the correct symbol. For Ex. 26-31, point out that it will be necessary to use a ruler to measure the number of centimetres between specified points. Note that the real distances are not the driving or the sailing distances.

Assessment

Complete.

1. $\frac{6}{20} = \frac{3}{\blacksquare}$ 10
2. $\frac{18}{10} = \frac{\blacksquare}{15}$ 27

Use = or ≠ to make true statements.

3. $\frac{12}{14} \bigcirc \frac{6}{20}$ ≠
4. $\frac{4}{9} \bigcirc \frac{8}{18}$ =

Find the real distance.

5. The scale for a map is 1 cm to 45 km. On the map, the distance between Toronto and London is 4 cm. 180 km

Pages 324-325

LESSON OUTCOME

Use patterns or multiplication to find equivalent rates; solve related word problems

Vocabulary

rate, equivalent rates, rate table, kilometres per hour, km/h

Prerequisite Skills

Write ratios using fraction notation; use patterns or multiplication to write equivalent ratios

Checking Prerequisite Skills

Write the ratio.

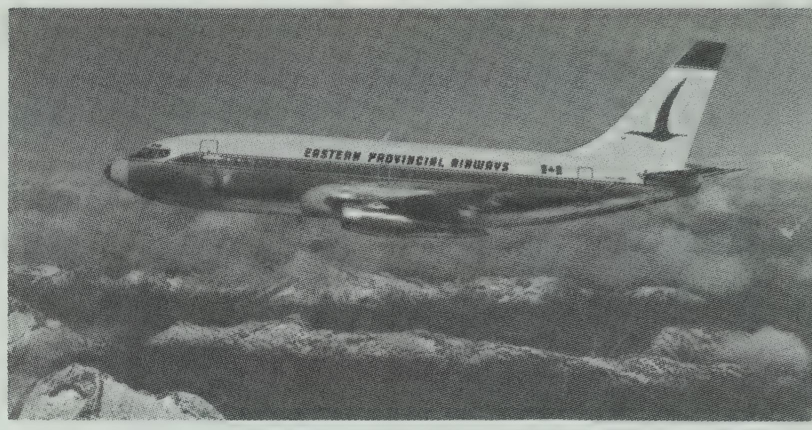
1. a score of 3 games to 8 games $\frac{3}{8}$
2. 4 players for 1 game $\frac{4}{1}$

Complete. Use patterns or multiplication.

3. $\frac{2}{5} = \frac{4}{10} = \frac{6}{15} = \frac{8}{20} = \frac{10}{25} = \frac{12}{30}$
4. $\frac{4}{9} = \frac{8}{18} = \frac{12}{27} = \frac{16}{36} = \frac{20}{45} = \frac{24}{54}$

Writing Equivalent Rates

This plane can travel at a **rate** of 900 km in 1 h which is a speed of 900 km/h (kilometres per hour).



The rates 900:1 and 1800:2 are **equivalent rates**.

This **rate table** shows equivalent rates.

If both numbers in a rate are multiplied by the same number to give another rate, the rates are equivalent.

900 × 2 900 × 3 900 × 4 900 × 5

kilometres	900	1800	2700	3600	4500
hours	1	2	3	4	5

1 × 2 1 × 3 1 × 4 1 × 5

900 km in 1 h — 1800 km in 2 h — 2700 km in 3 h — 3600 km in 4 h — 4500 km in 5 h

This rate shows the speed of the plane in kilometres per hour.

324

LESSON ACTIVITY

Using the Pages

- Begin with a discussion of the photograph on page 324. Some students may recognize that the type of aircraft shown is a 737. Have a student read the statement above the photograph. Draw attention to the symbol km/h for *kilometres per hour*. Ask how far the plane would travel in two hours if the speed is 900 km/h. Have a student read the statement below the photograph and explain the expression *equivalent rates* in her/his own words. Introduce the *rate table*, pointing out that it is similar to a ratio table. Develop that the rate "900 km in 1 h" can be expressed as a ratio in the form of a fraction, $\frac{900}{1}$. This can help students to understand the statements in the "thought clouds" above and below the table.

Exercises: When the students have completed Ex. 1-4, it would be advisable to have them interpret the patterns as rates. For example, for Ex. 1, a student might suggest that a rate of 2 books read in 3 months is equivalent to a rate of 4 books read in 6 months, and so on. Ex. 4 is starred because the simplest form of the rate is not given and must be determined from the pattern. Note that rate tables are to be used to solve Ex. 9-14. In this way, students can use patterns or multiplication to extend and complete the table as far as necessary to solve each problem. For example, for Ex. 9, the table would show from 1 h to 4 h. Remind the students that a speed of 5 km/h is a rate of 5 km in 1 h.

Keeping Sharp: These exercises provide practice in computation without written work. The students are to write only the result of performing the operations from left to right in each exercise.

Exercises

Complete each of these patterns.

1. $\frac{2}{3} = \frac{4}{6} = \frac{6}{9} = \frac{8}{12}, \frac{10}{15}, \frac{12}{18}, \frac{14}{21}$
2. $\frac{1}{5} = \frac{2}{10} = \frac{3}{15} = \frac{4}{20}, \frac{5}{25}, \frac{6}{30}, \frac{7}{35}$
3. $\frac{8}{7} = \frac{16}{14} = \frac{24}{21} = \frac{32}{28}, \frac{40}{35}, \frac{48}{42}, \frac{56}{49}$

*4. $\frac{9}{4} = \frac{18}{8} = \frac{27}{12} = \frac{36}{16} = \frac{45}{20} = \frac{54}{24}, \frac{63}{28}$

5.

metres	7	14	?21	?28	?35
seconds	2	4	?6	?8	?10

6.

litres	15	?30	?45	?60	?75
hours	4	?8	?12	?16	?20

Answers will vary for Ex. 5-8.

7.

years	1	?2	?3	?4	?5
grams	10	?20	?30	?40	?50

8.

minutes	2	?4	?6	?8	?10
steps	15	?30	?45	?60	?75

Use a rate table to solve each of these.

9. Percy walked at a speed of 5 km/h. How far did he walk in 4 h? 20 km
10. Meg ran 16 km at a rate of 2 km in 9 min. How long did she run? 72 min
11. Hazel read 55 pages every 2 h. How long did it take her to read 440 pages? 16 h
12. Records are on sale for \$15 for 4 records. How much do 20 records cost? \$75
13. A small record turns 45 times each minute. How many times does it turn in 4 min? 180
14. A large record turns 33 times each minute. How long does it take for the record to turn 231 times? 7 min

Write only the results.

1. $2 \times 3 \times 5 \div 10 + 12 - 6 + 7$ 16
2. $4 - 1 + 0 + 52 - 50 + 9 - 4$ 10
3. $16 \div 4 \times 6 \div 8 \times 100 + 43 - 3$ 340
4. $54 \div 9 \times 7 \div 6 + 58 + 300 - 100$ 265
5. $39 + 10 + 7000 - 6000 - 5 + 1$ 1045
6. $6 \times 6 \div 9 \times 8 + 8 + 60 - 25$ 75
7. $9642 \times 0 \times 538 + 635 - 5 - 200$ 430
8. $320 + 80 - 300 - 50 + 7 - 0 + 2$ 59
9. $3 \times 200 \div 100 \times 8 \div 6 \times 5 \div 10$ 4
10. $20 \times 9 \div 3 + 8 - 20 + 0 - 8$ 40
11. $7000 - 4000 - 999 - 1 + 200 + 5$ 2205
12. $60 \div 6 \times 2 \div 5 \times 4 - 8 + 9$ 17
13. $1000 \times 6 \div 100 \div 2 \times 30 \times 30 \div 10$ 2700
14. $900 \times 2 \div 3 \times 4 \div 100 \div 8 \div 3$ 1
15. $20 \div 4 \times 50 - 5 + 55 - 275 + 10$ 35

KEEPING SHARP

325

RELATED ACTIVITIES

- Students may find it interesting to measure the rates for various objects such as wind-up toys, electric trains, and battery-operated toy cars. Have them measure the number of centimetres an object travels in 1 min, the number of laps a train completes in 1 min, and so on. Some students may be challenged to express some of the speeds in kilometres per hour. They may also be interested in finding their own rate of walking.
- Have students find the cruising speeds of different types of aircraft and write corresponding rate tables.

Assessment

Complete the patterns.

1. $\frac{7}{4} = \frac{14}{8} = \frac{21}{12} = \frac{\blacksquare}{\blacksquare} = \frac{\blacksquare}{\blacksquare} = \frac{\blacksquare}{\blacksquare} = \frac{\blacksquare}{\blacksquare}$ $\frac{28}{16} = \frac{35}{20} = \frac{42}{24} = \frac{49}{28}$

2.

centimetres	2	4	6	8
minutes	5	10	15	20

Answers may vary.

Use a rate table to solve the problem.

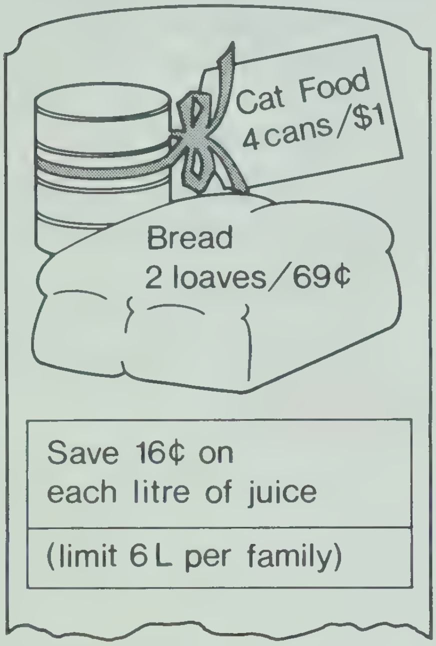
3. Susan counted 8 heartbeats in 5 seconds. How many heartbeats would there be in 30 seconds? 48

LESSON OUTCOME

Find the missing term in two equivalent rates; solve related word problems

RELATED ACTIVITIES

- Paste advertisements of sale items and prices on a large chart. Use the information to generate word problems for students to solve by using equivalent rates.



1. How much for 8 loaves of bread?
2. How much saved on 4 L of juice?

Finding the Missing Term in Equivalent Rates

Kevin rented a bicycle at the rate of $2 for 3 h. How much did he pay for renting a bicycle for 12 h?

The cost of renting a bicycle is at the rate of $2 for 3 h, or $\frac{2}{3}$.

The cost of renting a bicycle for 12 h is $ ■, where

$$\frac{2}{3} = \frac{\blacksquare}{12}.$$

$\frac{2}{3}$ and $\frac{\blacksquare}{12}$ are equivalent rates.

To find the missing term, think

$$3 \xrightarrow{3 \times \blacksquare} 12$$

Then multiply 2 and 4.

$$\frac{2}{3} \overset{2 \times \blacksquare}{\underset{3 \times \blacksquare}{=}} \frac{8}{12}$$

He paid $8 for renting a bicycle for 12 h.

Exercises

Complete each of these.

1. $\frac{3}{4} = \frac{\blacksquare}{12}$ 9
2. $\frac{4}{6} = \frac{2}{\blacksquare}$ 3
3. $\frac{1}{5} = \frac{\blacksquare}{25}$ 5
4. $\frac{5}{6} = \frac{30}{\blacksquare}$ 36
5. $\frac{12}{6} = \frac{2}{\blacksquare}$ 1
6. $\frac{1}{2} = \frac{\blacksquare}{4}$ 2
7. $\frac{8}{8} = \frac{1}{\blacksquare}$ 1
8. $\frac{21}{24} = \frac{7}{\blacksquare}$ 8

Solve.

9. Ned has $6 to rent a bicycle at the rate of $3 for 4 h. For how many hours can he rent the bicycle? 8
10. Julia's family pays $345 each month to rent their house. How much do they pay for 7 months? $2415
11. 5 pens cost $2. How much do 15 pens cost? $6
12. 4 books cost $12. How much do 16 books cost? $48

*13. The cost for each person to visit the museum is 50¢. What is the cost for 5 persons? $2.50

*14. Water flows from a tap at the rate of 25 mL every 3 s. How long would it take to fill a 1 L container? 120 s (2 min)

326

LESSON ACTIVITY

Before Using the Page

- Write a few exercises on the board to review the use of multiplication and division for finding the missing term in two equivalent fractions.

$$\frac{3}{4} \overset{3 \times \boxed{5}}{\underset{4 \times \boxed{5}}{=}} \frac{\blacksquare}{20} \qquad \frac{14}{35} \overset{14 \div \boxed{7}}{\underset{35 \div \boxed{7}}{=}} \frac{2}{\blacksquare}$$

For the completed exercises, have students interpret the equivalent fractions as equivalent rates. For example, a rate of 3 km in 4 min is equivalent to a rate of 15 km in 20 min.

Using the Page

- Have a student read the word problem to introduce the situation. To help students understand why the equation is $\frac{2}{3} = \frac{\blacksquare}{12}$ and not $\frac{2}{3} = \frac{12}{\blacksquare}$, write the following on the board.

$2 for 3 h
$ ■ for 12 h

Work through the example with the students, pointing out how the same procedure has been used with equivalent fractions and equivalent ratios.

Exercises: Note that Ex. 13 and 14 are starred because they require extra steps in changing units of measurement, for example, cents to dollars, before writing the equivalent rates and interpreting the value of the missing term.

Assessment

Complete.

1. $\frac{10}{7} = \frac{60}{\blacksquare}$ 42
2. $\frac{3}{8} = \frac{\blacksquare}{72}$ 27
3. $\frac{18}{36} = \frac{3}{\blacksquare}$ 6

Solve.

4. A bicycle wheel has 32 spokes. How many spokes are on 4 bicycle wheels? 128

Finding the Missing Term Using Cross Products

Esther came from Australia to visit her grandparents. For 10 Australian dollars, she received 13 Canadian dollars. How many Canadian dollars did she receive for 40 Australian dollars?

The rate of exchange for Australian dollars to Canadian dollars was $\frac{10}{13}$.

The rate of exchange for 40 Australian dollars was $\frac{40}{\blacksquare}$.

To find the number of Canadian dollars received, write

$$\frac{10}{13} = \frac{40}{\blacksquare}$$

and use cross products.

$$10 \times \blacksquare = 13 \times 40$$

$$\frac{10}{13} = \frac{40}{\blacksquare}$$

$$10 \times \blacksquare = 13 \times 40$$

$$10 \times \blacksquare = 520$$

$520 \div 10 = 52$

$$10 \times \boxed{52} = 520$$

Esther received 52 Canadian dollars for 40 Australian dollars.

Exercises

Complete each of these.

1. $\frac{4}{8} = \frac{6}{\blacksquare}$ 12
2. $\frac{18}{30} = \frac{\blacksquare}{25}$ 15
3. $\frac{24}{15} = \frac{\blacksquare}{10}$ 16
4. $\frac{6}{18} = \frac{7}{\blacksquare}$ 21
5. $\frac{45}{36} = \frac{\blacksquare}{16}$ 20
6. $\frac{56}{64} = \frac{21}{\blacksquare}$ 24
7. $\frac{9}{12} = \frac{\blacksquare}{28}$ 21
8. $\frac{63}{49} = \frac{54}{\blacksquare}$ 42

Solve.

9. Pierre went to France to visit his aunt. For each dollar, he received 4 francs. How much would he pay for 16 francs? $4
10. 4 baseballs cost $20. How much do 3 baseballs cost? $15
11. 7 pencils cost 84¢. Monica has 72¢. How many pencils can she buy? 6
12. Kay can ride 4 km in 6 min on her bicycle. How long would it take her to ride 8 km? 12 min

*13. The bus travels at a speed of 50 km/h. How long would it take to go 35 km? 0.7 h (42 min)

*14. Tim has learned to type 25 words a minute. How long would it take him to type 40 words? 1.6 min

Page 327

LESSON OUTCOME

Use cross products to find the missing term in two equivalent rates; solve related word problems

Vocabulary

rate of exchange

Prerequisite Skills

Write the cross products for two equivalent fractions; use cross products to find the missing term in two equivalent fractions

Checking Prerequisite Skills

Write the cross products.

1. $\frac{6}{10} = \frac{3}{5}$ 6×5 10×3 **2.** $\frac{9}{21} = \frac{6}{14}$ 9×14 21×6

Use cross products to complete these.

3. $\frac{10}{4} = \frac{\blacksquare}{22}$ 55 **4.** $\frac{12}{8} = \frac{54}{\blacksquare}$ 36

RELATED ACTIVITIES

- Have students work in pairs to find the number of sit-ups, toe touches, or other exercises they can do in 10 s. Then have them use cross products to find the number of exercises they can do in 30 s.
- Have students find the rates of exchange for Canadian dollars to other currencies and use the information to write and solve word problems.

LESSON ACTIVITY

Before Using the Page

- Begin with a brief discussion of traveling from one country to another and having to exchange the currency of one country for the currency of the other. Where possible, have students tell about such experiences.

Using the Page

- Have a student read the word problem at the top of the page. Point out that $\frac{10}{13}$ can be described as the *rate of exchange* for Australian dollars to Canadian dollars. Discuss that $\frac{10}{13}$ and $\frac{40}{\blacksquare}$ are equivalent rates and work through the solution showing the use of cross products. This would be an opportune time to summarize that cross products are equal for equivalent fractions, equivalent ratios, and equivalent rates. The students will recognize this because rates and ratios can be expressed in fraction notation.

Exercises: Ex. 13 and 14 are starred because the missing term in each is a decimal. A solution for Ex. 13 is shown.

$$\frac{50}{1} = \frac{35}{\blacksquare}$$

$$50 \times \blacksquare = 1 \times 35$$
$$50 \times \blacksquare = 35$$
$$50 \times 0.7 = 35$$

$$\begin{array}{r} 0.7 \\ 50\overline{)35.0} \\ 35\ 0 \\ \hline 0 \end{array}$$

It would take 0.7 h, or 42 min, for the bus to go 35 km. Some students may suggest using the following equivalent rates for Ex. 13.

$$\frac{50}{60} = \frac{35}{\blacksquare}$$

Assessment

Complete.

1. $\frac{10}{25} = \frac{8}{\blacksquare}$ 20 **2.** $\frac{12}{20} = \frac{\blacksquare}{10}$ 6 **3.** $\frac{63}{45} = \frac{14}{\blacksquare}$ 10

Solve.

4. In 12 strokes Ann can swim 8 m. How many metres would she swim in 33 strokes? 22

LESSON OUTCOME

Associate a percent with a ratio that compares an amount to 100; express the relationship "n to 100" using words, using the symbol :, as a fraction, as a decimal, and as a percent (halves, fourths, fifths, tenths)

Materials

several cubes showing 3 yellow faces, 2 red faces, and 1 blue face

Vocabulary

percent (%)

Prerequisite Skills

Find the missing term in two equivalent fractions; write decimal hundredths; write ratios using fraction notation

Checking Prerequisite Skills

Complete.

1. $\frac{3}{25} = \frac{12}{\blacksquare}$ 100 **2.** $\frac{3}{4} = \frac{\blacksquare}{100}$ 75

Write the decimal.

3. seven-hundredths 0.07

4. forty-hundredths 0.40

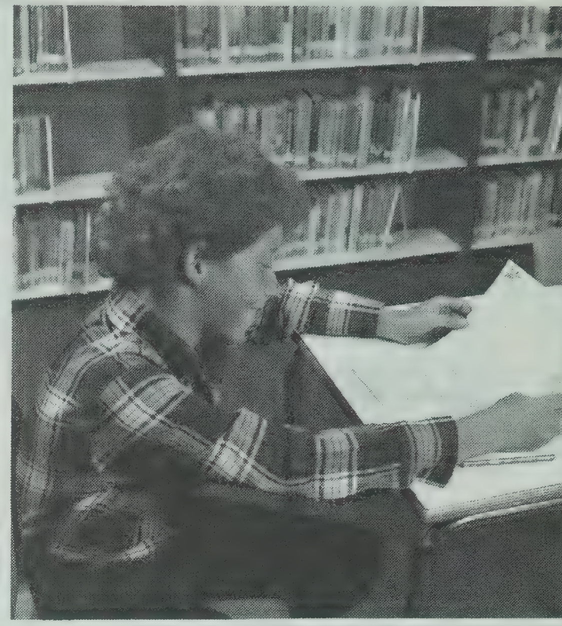
5. sixty-three hundredths 0.63

Show each ratio in three ways.

6.	1 to 5	1:5	$\frac{1}{5}$
7.	3 to 4	3:4	$\frac{3}{4}$
8.	7 to 10	7:10	$\frac{7}{10}$
9.	16 to 100	16:100	$\frac{16}{100}$

Writing Percents

Richard's mark on a mathematics test was 81 out of 100.



Richard's mark can be written as a ratio.

number of marks received

81 : 100

number of possible marks

$$\frac{81}{100}$$

Richard's mark can be written as a **percent**.

81%

A percent shows how many out of 100.

Working Together

Complete.

1. $\frac{65}{100} = \blacksquare$ % 65

2. $\frac{7}{10} = \frac{\blacksquare}{100} = \blacksquare$ % 70 70

3. $\frac{3}{5} = \frac{\blacksquare}{100} = \blacksquare$ % 60 60

4.	7 out of 100	7:100	$\frac{7}{100}$	0.07	7%
5.	43 out of 100	? 43:100	? $\frac{43}{100}$	? 0.43	? 43%
6.	3 out ? of 100	? 3:100	? $\frac{3}{100}$	? 0.03	3%

Write as a percent.

7. 8 out of 10 80%
8. 1 out of 4 25%
9. 2:5 40%
10. $\frac{1}{2}$ 50%
11. 0.75 75%

328

LESSON ACTIVITY

Before Using the Pages

- Write the ratio "73 out of 100" on the board and have a student read it aloud. Tell the students that the ratio represents a mark obtained on a test. Ask what 73 represents and what 100 represents. Then ask for other ways in which the same information can be shown. Elicit the forms 73:100 and $\frac{73}{100}$. Then lead the students to suggest the decimal 0.73. Finally, tell the students that there is another way to express the mark obtained when the second term in a ratio is 100. If no student suggests showing the mark as a percent, have the students turn to page 328.

Using the Pages

- Ask a student to read the statement at the top of page 328. Have the students note the different forms for writing Richard's mark. Introduce the term *percent*, and, if you wish, tell the students that the word originated from the Latin *per centum* meaning "out of a hundred". Thus, 81% means "81 out of 100".

  Draw attention to the symbol % for "percent" and ask in what way it suggests 100. The students will likely suggest that the oblique line and the two small 0's suggest the digits 1, 0, and 0 for 100. Return to the example presented in *Before Using the Pages* and ask a student to write $\frac{73}{100}$ as a percent.

  Ask the students if a mark such as 40 out of 50 can be written as a percent. Because a percent shows how many out of 100, the missing term for $\frac{40}{50} = \frac{\blacksquare}{100}$ will give the number needed to write the percent. If possible, lead the students to suggest this.

Working Together: For Ex. 1, the given fraction has a denominator of 100. Use other similar examples and have students interpret each as a mark, for instance, 65 out of 100. Ex. 2 and 3 give fractions for which the denominators are not 100 and indicate that the missing term for an equivalent fraction must be found first. For Ex. 4-6, emphasize that each exercise shows different ways of

Exercises

Complete this chart.

1.	12 out of 100				
2.	78 out of 100	78:100	$\frac{78}{100}$	0.78	78%
3.	4 out of 100	4:100	$\frac{4}{100}$	0.04	4%
4.	3 out of 100	3:100	$\frac{3}{100}$	0.03	3%
5.	98 out of 100	98:100	$\frac{98}{100}$	0.98	98%
6.	9 out of 100	9:100	$\frac{9}{100}$	0.09	9%

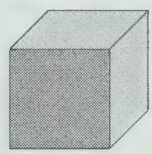
Write each of these as a percent.

7. 15:100 15%
8. 1:100 1%
9. $\frac{6}{100}$ 6%
10. $\frac{24}{100}$ 24%
11. 0.72 72%
12. 0.05 5%
13. 9:10 90%
14. 4:5 80%
15. $\frac{3}{4}$ 75%
16. $\frac{1}{2}$ 50%
17. $\frac{7}{10}$ 70%
18. 0.1 10%

Write a percent for each of these.

19. On a test, Lara's mark was 3 out of 5. 60%
20. 1 of every 2 students tried out for the track team. 50%
21. The sales tax was 7¢ for every 100¢. 7%
22. The skates were on sale with a discount of 20¢ on every 100¢. 20%
23. Paula received 6¢ interest for every dollar she had in the bank. 6%
24. Alfred paid 12¢ interest for every dollar he borrowed from the bank. 12%

Martin tossed a cube with 3 yellow faces, 2 red faces, and 1 blue face 50 times.



The top face was red for 21 tosses.

The ratio for the number of times the top face was red to the number of tosses was 21:50.

$\frac{21}{50} = \frac{42}{100}$

The top face was red for 42% of the tosses.

Toss a cube like Martin's cube 50 times and complete the following chart.

Or use 6 papers (3 yellow, 2 red, 1 blue) in a box.

Answers will vary.

Color	Tally	Number	Ratio	Percent
Yellow	?	?	?	?
Red	?	?	?	?
Blue	?	?	?	?

try this

329

expressing the same information. Although the order followed for completing the chart may vary, students will likely find it easiest to work from left to right. For Ex. 7-11, the procedure indicated in Ex. 2 and 3 can be followed.

Exercises: Pay particular attention to Ex. 23 and 24 because the value of a dollar must be thought of as 100¢.

Try This: Students may work individually or in small groups to complete this investigation. Before they begin, you may wish to have them guess which color for the top face will be obtained most frequently.

Assessment

Write each of these as a percent.

1. 88:100 88% **2.** $\frac{2}{100}$ 2% **3.** $\frac{4}{5}$ 80%

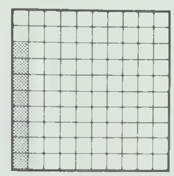
4. 0.63 63% **5.** 0.9 90%

RELATED ACTIVITIES

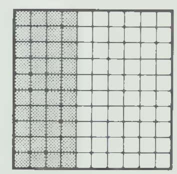
- Students who enjoyed the *Try This* feature may be interested in tossing two regular dice 50 or 100 times and recording the results in a chart. For each toss, the sum of the two numbers is found. The possible sums are 2, 3, 4, ..., 12. The completed chart will indicate which sum occurred most frequently.

Sum	Tally	Number	Ratio	Percent
2				
3				
4				

- To help students having difficulty with percents, use models of hundredths (less than one) and have students write the numbers represented as fractions, as decimals, and as percents. Then repeat the procedure using models of halves, fourths, fifths, and tenths.



$\frac{8}{100}$, 0.08, 8%



$\frac{40}{100}$, 0.40, 40%

Page 330

OBJECTIVE

Use ratios to estimate lengths

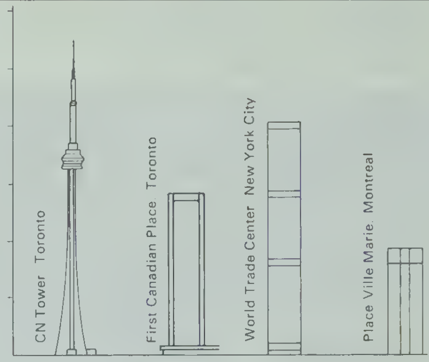
Materials

a Canadian flag, a dollar bill

RELATED ACTIVITIES

- Students can measure the height of their desks and use the result with the ratio in Ex. 6 or 7 to estimate the height of the classroom.
- Discuss the results for Ex. 13. Have students use several different circular objects, large and small, to note that the ratio is the same for each.

Estimating with Ratios



The CN Tower is about twice as tall as the First Canadian Place.

$$\frac{\text{height of CN Tower}}{\text{height of First Canadian Place}} = \frac{2}{1}$$

The First Canadian Place is about half as tall as the CN Tower.

$$\frac{\text{height of First Canadian Place}}{\text{height of CN Tower}} = \frac{1}{2}$$

The CN Tower is 554 m tall.

Using $\frac{2}{1} = \frac{554}{\blacksquare}$ or $\frac{1}{2} = \frac{\blacksquare}{554}$, $2 \times \blacksquare = 554$ $\quad 2 \times 277 = 554$

A good estimate for the height of the First Canadian Place is 277 m.

1. Estimate a ratio that compares the height of the CN Tower to the height of the World Trade Center. $\frac{6}{4}$
2. Estimate a ratio that compares the height of the World Trade Center to the height of the CN Tower. $\frac{4}{6}$
3. Estimate a ratio that compares the heights of the CN Tower and Place Ville Marie. $\frac{6}{2}$
4. Estimate a ratio that compares the heights of the First Canadian Place and the World Trade Center. $\frac{3}{4}$
5. The CN Tower is 554 m tall. Use this fact and your estimated ratios to estimate the heights of the World Trade Center and Place Ville Marie.
   about 369 m   about 185 m

Estimate a ratio that compares

Answers will vary.

6. the height of your classroom to the height of your desk.
7. the height of your desk to the height of your classroom.
8. the width and length of the Canadian flag.
9. the length and width of a one-dollar bill.
10. the length of your hand and the length of your arm.
11. the length of your shoe and the length of your hand.
12. the lengths of your shortest and longest fingers.
13. the circumference of a circle to its diameter.

PROBLEM SOLVING

330

LESSON ACTIVITY

Using the Page

- Have students read the names of the buildings represented in the diagram. Ask which building is the tallest and which building is about half the height of that one. Have students read the introductory statement and the two ratios comparing the heights of the CN Tower and First Canadian Place. Draw attention to the unmarked scale along the vertical edge of the diagram and ask how it is helpful in comparing the heights of these two buildings. Suggest that if the scale is marked 1, 2, 3, 4, 5, 6, the height of the CN Tower to the height of First Canadian Place is about 6 to 3, which is the same as 2 to 1. Point out that if the height of the CN Tower is known to be 554 m, an estimate of the height of First Canadian Place can be obtained by finding the missing term for $\frac{2}{1} = \frac{554}{\blacksquare}$, or $\frac{1}{2} = \frac{\blacksquare}{554}$. Have students help to explain the procedure.

Exercises: For Ex. 1-4, have students use words and numerals to write the ratios as shown in the example. Have them use the unmarked scale of the diagram to help in determining the ratios. These ratios will be required for Ex. 5.

Students may need assistance with Ex. 6-13. Discuss ways of estimating the ratios. For example, lengths of string and a circular object may be used for Ex. 13. A Canadian flag will be required for Ex. 8, and a dollar bill for Ex. 9. Have the students write the words and the numerals to establish the order of the comparison for each ratio.

Checking Up

Ways of expressing the ratios for Ex 1-15 will vary

Write as a ratio in another way.

1. 3 out of 4 3:4
2. 5 to 7 5:7
3. 9 in 6 9:6
4. 9 to 10 9:10
5. 4 in 6 4:6
6. 8 out of 8 8:8
7. The score was 7 to 3. 7:3
8. Greg won 3 games out of 5. 3:5
9. Each student has 2 pencils. 1:2

Write as a rate in two ways.

10. 2 for 1 2:1 , $\frac{2}{1}$
11. 8 in 12 8:12 ; $\frac{8}{12}$
12. 6 for 6 6:6 ; $\frac{6}{6}$
13. The machine makes 7 boxes every 2 min. 7:2 ; $\frac{7}{2}$
14. The boat travels at 15 km/h. 15:1 ; $\frac{15}{1}$
15. 4 books cost \$9. 4:9 ; $\frac{4}{9}$

Complete. Use patterns or multiplication.

16. $\frac{7}{8} = \frac{14}{16} = \frac{21}{24} = \frac{\blacksquare 28}{\blacksquare 32}\ \frac{\blacksquare 35}{\blacksquare 40}\ \frac{\blacksquare 42}{\blacksquare 48}\ \frac{\blacksquare 49}{\blacksquare 56}$
17. $\frac{5}{2} = \frac{10}{4} = \frac{15}{6} = \frac{\blacksquare 20}{\blacksquare 8}\ \frac{\blacksquare 25}{\blacksquare 10}\ \frac{\blacksquare 30}{\blacksquare 12}\ \frac{\blacksquare 35}{\blacksquare 14}$
18. $\frac{2}{5} = \frac{\blacksquare 6}{15}$
19. $\frac{20}{35} = \frac{4}{\blacksquare 7}$
20. $\frac{4}{20} = \frac{\blacksquare 5}{25}$
21. $\frac{9}{12} = \frac{3}{\blacksquare 4}$
22. $\frac{4}{8} = \frac{6}{\blacksquare 12}$

Use cross products to complete each of these.

23. $\frac{6}{2} = \frac{3}{\blacksquare 1}$
24. $\frac{8}{12} = \frac{\blacksquare 12}{18}$
25. $\frac{9}{8} = \frac{\blacksquare 18}{16}$
26. $\frac{14}{16} = \frac{\blacksquare 21}{24}$
27. $\frac{5}{50} = \frac{4}{\blacksquare 40}$

Write a percent for each of these.

28. 34:100 34%
29. $\frac{2}{5}$ 40%
30. 0.1 10%
31. 1:2 50%
32. $\frac{3}{4}$ 75%
33. Max won 1 of the 4 games. 25%
34. Nora swims 3 d out of 5 d. 60%
35. The tax is 5¢ per dollar. 5%

Solve.

36. There are 12 paintbrushes in each box. Norman's class needs 36 paintbrushes. How many boxes of paintbrushes should they buy? 3
37. On a map, 1 cm represents 30 km. The distance between Edmonton and Saskatoon on the map is 16 cm. What is the real distance between these cities? 480 km
38. Elena can skate at a speed of 7 m in 2 s. How far can she skate in 8 s? 28 m
39. 2 posters cost \$5. How much do 10 posters cost? \$25

331

Page 331

OBJECTIVE

Demonstrate an understanding of the concepts and skills presented in this unit

RELATED ACTIVITIES

- Have students search newspapers and magazines for examples of the uses of percents. These may be cut out and displayed for discussion.

Skills	Exercises	Related Pages
Write a ratio	1-9	T 344-T 345
Write a rate	10-15	T 352-T 353
Write equivalent ratios or rates	16, 17	T 346-T 347 T 352-T 353
Find the missing term in two equivalent ratios or rates	18-22	T 348-T 349 T 354
Use cross products to find the missing term in two equivalent ratios or rates	23-27	T 350-T 351 T 355
Write a percent	28-35	T 356-T 357
Solve word problems	36-39	

Comments

Use the results of the students' work to determine which concepts need to be reviewed. Group the students for review according to their needs. Pay particular attention to the meaning of each number in a ratio or a rate. This emphasis will help students in writing equivalent ratios or rates for finding a missing term. Further work in writing ratio tables and rate tables will be helpful.

OBJECTIVE

Demonstrate competence in adding and subtracting whole numbers

Skill Practice–Adding and Subtracting Whole Numbers

Add.

1. 1132 2264 3396	2. 6543 1234 7777	3. 7722 1275 8997	4. 8176 1823 9999	5. 7085 1914 8999
6. 12 345 98 765 111 110	7. 33 445 66 556 100 001	8. 54 637 78 695 133 332	9. 99 887 77 889 177 776	10. 20 089 58 154 78 243
11. 4 086 73 452 662 78 200	12. 3 486 87 773 9 931 101 190	13. 45 023 77 1 900 47 000	14. 64 112 87 877 9 796 161 785	15. 383 45 674 7 098 53 155

16. 7784 + 2203 9987
17. 7181 + 673 7854
18. 9909 + 8991 18 900
19. 45 836 + 1 452 + 673 47 961
20. 2 785 + 12 111 + 10 877 25 773
21. 90 807 + 393 + 4 777 95 977
22. 123 + 45 678 + 9 012 54 813
23. 50 505 + 1 020 + 8 090 59 615
24. 17 181 + 32 819 + 25 000 75 000
25. 4909 + 4099 + 409 9417
26. 753 + 1357 + 7057 9167

Subtract.

27. 8878 4567 4311	28. 9945 7814 2131	29. 6789 5678 1111	30. 4867 1234 3633	31. 9098 7083 2015
32. 14 872 6 864 8 008	33. 35 674 34 684 990	34. 73 792 37 297 36 495	35. 99 091 85 019 14 072	36. 55 855 6 964 48 891

37. 8532 − 4372 4160
38. 9517 − 18 9499
39. 7123 − 1237 5886
40. 5505 − 4606 899
41. 8710 − 811 7899
42. 6056 − 3447 2609
43. 43 000 − 40 015 2985
44. 27 005 − 17 006 9999
45. 58 085 − 996 57 089
46. 70 077 − 6 382 63 695
47. 87 078 − 10 887 76 191
48. 69 163 − 12 088 57 075
49. 70 403 − 60 085 10 318
50. 31 031 − 13 999 17 032
51. 40 004 − 30 555 9449
52. 30 445 − 24 556 5889
53. 70 543 − 67 453 3090
54. 11 025 − 10 976 49
55. 41 078 − 39 988 1090
56. 30 476 − 9 576 20 900
57. 80 781 − 72 272 8509
58. 74 770 − 7 477 67 293
59. 4023 − 224 3799
60. 10 001 − 1 110 8891
61. 49 987 − 1 998 47 989
62. 7887 − 6996 891
63. 34 567 − 5 678 28 889
64. 70 070 − 69 985 85
65. 11 678 − 10 789 889
66. 30 609 − 4 080 26 529
67. 5905 − 4915 990
68. 71 082 − 2 993 68 089
69. 81 726 − 72 837 8889
70. 9119 − 9023 96
71. 10 001 − 2 002 7999
72. 55 667 − 44 558 11 109

332

The skills required for the exercises on this page are indicated in the following chart.

Skills	Exercises
Add two four-digit numbers with no regrouping	1-5
Add two five-digit numbers with regrouping	6-10
Add more than two numbers with regrouping, sums with five or six digits	11-15
Write two or three addends in vertical form and add, with and without regrouping, sums with four or five digits	16-26
Subtract numbers with no regrouping, minuends with four digits	27-31
Subtract numbers with regrouping, minuends with five digits	32-36
Write two numbers in vertical form and subtract with regrouping for zero in one or more places in the minuend, minuends with four or five digits	37-72

Page 333

OBJECTIVE

Demonstrate competence in adding and subtracting decimals

Skill Practice–Adding and Subtracting Decimals

Add.

1. 13.4	2. 46.3	3. 75.9	4. 81.9	5. 35.8
11.7	27.8	10.1	9.7	58.3
25.1	74.1	86.0	91.6	94.1
6. 13.85	7. 49.91	8. 66.36	9. 29.11	10. 90.97
27.15	11.09	43.75	88.55	37.05
41.00	61.00	110.11	117.66	128.02
11. 0.832	12. 1.683	13. 13.982	14. 59.833	15. 22.108
5.796	1.314	1.018	47.733	0.968
8.884	0.772	6.075	8.093	1.732
15.512	3.769	21.075	115.659	24.808

16. 1.4 + 2.7 4.1
17. 9.9 + 1.8 11.7
18. 4.8 + 5.7 + 6.6 17.1
19. 15.7 + 8.4 + 0.4 24.5
20. 11.07 + 13.17 + 9.87 34.11
21. 99.09 + 9.99 + 1.97 111.05
22. 0.86 + 0.07 + 1.31 2.24
23. 17.81 + 8.11 + 0.87 26.79
24. 7.087 + 8.118 + 10.045 25.250
25. 1.991 + 9.225 + 3.796 15.012
26. 99.095 + 17.011 + 8.885 124.991
27. 50.054 + 0.505 + 5.005 55.564

Subtract.

28. 31.35	29. 48.84	30. 17.81	31. 78.78	32. 64.08
2.34	24.48	16.99	29.89	32.09
29.01	24.36	0.82	48.89	31.99
33. 71.735	34. 35.947	35. 4.555	36. 10.047	37. 0.876
7.173	26.158	2.678	1.708	0.088
64.562	9.789	1.877	8.339	0.788
38. 43.785	39. 34.486	40. 1.616	41. 1.188	42. 0.847
5.723	14.975	0.777	0.864	0.708
38.062	19.511	0.839	0.324	0.139
43. 13.133	44. 15.084	45. 3.911	46. 11.055	47. 60.066
7.066	9.175	3.818	2.118	40.404
6.067	5.909	0.093	8.937	19.662

48. 7.4 − 6.8 0.6
49. 8.4 − 7.8 0.6
50. 1.5 − 0.6 0.9
51. 8.59 − 7.67 0.92
52. 0.87 − 0.29 0.58
53. 7.08 − 1.89 5.19
54. 15.832 − 11.837 3.995
55. 25.558 − 20.551 5.007
56. 73.495 − 0.986 72.509
57. 21.998 − 13.009 8.989
58. 80.083 − 0.883 79.200
59. 70.007 − 9.369 60.638

The skills required for the exercises on this page are indicated in the following chart.

Skills	Exercises
Add two decimals showing tenths	1-5
Add two decimals showing hundredths	6-10
Add decimals showing thousandths, three addends	11-15
Write two or three decimals in vertical form and add	16-27
Subtract decimals showing hundredths	28-32
Subtract decimals showing thousandths	33-47
Write two decimals in vertical form and subtract, some minuends with one or more zeros	48-59

OBJECTIVE

Demonstrate competence in multiplying whole numbers

Skill Practice – Multiplying Whole Numbers

Multiply.

1. 35	2. 37	3. 38	4. 47	5. 58	6. 67
3	4	5	6	7	8
105	148	190	282	406	536
7. 77	8. 84	9. 97	10. 43	11. 57	12. 63
9	6	7	4	7	8
693	504	679	172	399	504

13. 2 × 17 34 14. 5 × 72 360 15. 8 × 68 544 16. 7 × 93 651 17. 4 × 87 348

18. 6 × 166 996 19. 3 × 394 1182 20. 6 × 247 1482 21. 4 × 336 1344 22. 7 × 478 3346

23. 5 × 1234 6170 24. 6 × 2468 14 808 25. 8 × 12 837 102 696 26. 3 × 3994 11 982 27. 9 × 87 778 790 002

28. 482	29. 597	30. 1732	31. 48 184	32. 76 018
40	80	90	70	50
19 280	47 760	155 880	3 372 880	3 800 900
33. 189	34. 32	35. 73	36. 784	37. 558
700	4000	8000	600	90
132 300	128 000	584 000	470 400	50 220

38. 49	39. 94	40. 73	41. 57	42. 83	43. 55
49	94	37	75	38	27
2401	8836	2701	4275	3154	1485

44. 38 × 93 3534 45. 72 × 15 1080 46. 89 × 96 8544 47. 37 × 86 3182 48. 19 × 91 1729

49. 123	50. 789	51. 863	52. 1891	53. 7787
32	89	63	89	78
3936	70 221	54 369	168 299	607 386

54. 27 × 1284 34 668 55. 38 × 1574 59 812 56. 67 × 6788 454 796 57. 94 × 9876 928 344

58. 484	59. 7877	60. 6983	61. 5047
484	768	432	347
234 256	6 049 536	3 016 656	1 751 309

62. 556 × 7890	63. 667 × 8762	64. 726 × 9048	65. 778 × 2859
66. 414 × 72	67. 618 × 435	68. 978 × 6000	69. 785 × 42
70. 293 × 577	71. 379 × 2022	72. 817 × 1357	73. 416 × 3003
74. 887 × 778	75. 237 × 6677	76. 348 × 7788	77. 697 × 10 102
78. 357 × 246	79. 468 × 876	80. 917 × 7437	81. 827 × 60 606

334

The skills required for the exercises on this page are indicated in the following chart.

Skills	Exercises
Multiply a two-digit number by a one-digit number	1-17
Multiply a three-digit number by a one-digit number	18-22
Multiply a four-digit or a five-digit number by a one-digit number	23-27
Multiply by a multiple of 10, 100, and 1000, multiplicands with up to five digits	28-37
Multiply a two-digit number by a two-digit number	38-48
Multiply by a two-digit number, multiplicands with three or four digits	49-57
Multiply by a three-digit number, multiplicands with three or four digits	58-65
Multiply by a three-digit number, multiplicands with up to five digits	66-81

62. 4 386 840 63. 5 844 254 64. 6 568 848
65. 2 224 302 66. 29 808 67. 268 830
68. 5 868 000 69. 32 970 70. 169 061
71. 766 338 72. 1 108 669 73. 1 249 248
74. 690 086 75. 1 582 449 76. 2 710 224
77. 7 041 094 78. 87 822 79. 409 968
80. 6 819 729 81. 50 121 162

OBJECTIVE

Demonstrate competence in multiplying decimals

Skill Practice–Multiplying Decimals

Multiply.

1. 3.2 8 25.6	**2.** 4.7 7 32.9	**3.** 3.6 6 21.6	**4.** 9.7 5 48.5	**5.** 6.8 4 27.2	**6.** 5.7 9 51.3
7. 0.81 5 4.05	**8.** 0.93 4 3.72	**9.** 7.07 7 49.49	**10.** 8.24 9 74.16	**11.** 9.08 6 54.48	**12.** 9.68 3 29.04
13. 0.096 2 0.192	**14.** 0.746 7 5.222	**15.** 8.884 8 71.072	**16.** 4.087 6 24.522	**17.** 4.409 5 22.045	**18.** 1.033 4 4.132

19. 9 × 7.709 69.381 **20.** 4 × 9.098 36.392 **21.** 5 × 7.947 39.735 **22.** 8 × 5.206 41.648

23. 8.47 27 228.69	**24.** 9.09 47 427.23	**25.** 3.78 87 328.86	**26.** 6.72 38 255.36	**27.** 5.96 65 387.40	**28.** 6.47 74 478.78
29. 5.59 37 206.83	**30.** 9.86 76 749.36	**31.** 8.76 65 569.40	**32.** 7.65 57 436.05	**33.** 6.54 48 313.92	**34.** 0.93 39 36.27

35. 91 × 4.08 371.28 **36.** 83 × 7.09 588.47 **37.** 77 × 8.83 679.91 **38.** 49 × 1.88 92.12

39. 5.77 × 89 513.53 **40.** 3.07 × 73 224.11 **41.** 4.94 × 44 217.36 **42.** 6.96 × 35 243.60

43. 246 4.563 1122.498	**44.** 748 7.637 5712.476	**45.** 472 9.808 4629.376	**46.** 687 7.068 4855.716

47. 0.03 × 1000 30 **48.** 7.74 × 100 774 **49.** 1000 × 0.06 60 **50.** 10 × 0.9 9

51. 1.007 × 0.1 0.1007 **52.** 0.79 × 1000 790 **53.** 42 × 0.001 0.042 **54.** 771 × 0.0001 0.0771

55. 3.2 0.7 2.24	**56.** 4.7 0.8 3.76	**57.** 3.6 0.5 1.80	**58.** 9.7 0.6 5.82	**59.** 6.8 0.9 6.12	**60.** 5.7 0.4 2.28
61. 34.7 0.7 24.29	**62.** 47.3 0.5 23.65	**63.** 79.9 0.8 63.92	**64.** 12.7 0.6 7.62	**65.** 31.7 0.9 28.53	**66.** 43.8 0.4 17.52
67. 74.8 7.3 546.04	**68.** 73.2 8.1 592.92	**69.** 62.5 5.3 331.25	**70.** 58.7 3.7 217.19	**71.** 29.3 3.7 108.41	**72.** 81.9 2.9 237.51

73. 88.7 × 46.3 4106.81 **74.** 49.1 × 81.8 4016.38 **75.** 52.2 × 64.3 3356.46 **76.** 65.5 × 39.8 2606.90

77. 436.2 × 27.3 11 908.26 **78.** 949.9 × 49.7 47 210.03 **79.** 124.8 × 84.2 10 508.16 **80.** 248.1 × 36.9 9154.89

The skills required for the exercises on this page are indicated in the following chart.

Skills	Exercises
Multiply a one-place decimal by a one-digit whole number	1-6
Multiply a two-place decimal by a one-digit whole number	7-12
Multiply a three-place decimal by a one-digit whole number	13-22
Multiply a two-place decimal and a two-digit whole number	23-42
Multiply a three-place decimal and a three-digit whole number	43-46
Multiply with 0.001, 0.01, 0.1, 10, 100, or 1000 as a factor	47-54
Multiply a one-place decimal by a one-place decimal to 0.9	55-66
Multiply two one-place decimals	67-80

OBJECTIVE

Demonstrate competence in dividing whole numbers

Skill Practice–Dividing Whole Numbers

Divide. Write the quotient and the remainder.

1. 7)37 5 R2	2. 8)96 12	3. 6)59 9 R5	4. 3)89 29 R2	5. 4)68 17	6. 2)19 9 R1
7. 6)44 7 R2	8. 9)39 4 R3	9. 8)72 9	10. 7)50 7 R1	11. 5)50 10	12. 9)98 10 R8

13. 7)546 78	14. 8)552 69	15. 9)235 26 R1	16. 6)448 74 R4
17. 3)320 106 R2	18. 5)244 48 R4	19. 8)990 123 R6	20. 9)450 50
21. 6)9766 1627 R4	22. 8)9106 1138 R2	23. 9)2408 267 R5	24. 5)8508 1701 R3
25. 4)9002 2250 R2	26. 6)7774 1295 R4	27. 7)4930 704 R2	28. 8)7787 973 R3
29. 9)10 486 1 165 R1	30. 7)35 872 5 124 R4	31. 5)77 085 15 417	32. 3)10 101 3 367
33. 4)80 296 20 074	34. 6)13 240 2 206 R4	35. 8)79 305 9 913 R1	36. 9)88 188 9 798 R6
37. 10)580 58	38. 20)260 13	39. 40)8960 224	40. 50)2100 42
41. 90)8820 98	42. 30)31 090 1 036 R10	43. 70)11 060 158	44. 80)33 600 420
45. 33)231 7	46. 19)618 32 R10	47. 62)4590 74 R2	48. 37)1478 39 R35
49. 82)59 040 720	50. 69)28 151 407 R68	51. 51)318 303 6 241 R12	52. 88)363 260 4 127 R84
53. 44)320 7 R12	54. 60)487 8 R7	55. 29)900 31 R1	56. 78)644 8 R20
57. 53)3180 60	58. 32)1795 56 R3	59. 45)3628 80 R28	60. 65)9575 147 R20
61. 77)80 003 1 039	62. 14)44 044 3 146	63. 28)60 004 2 143	64. 83)23 646 284 R74
65. 56)23 530 420 R10	66. 86)71 680 833 R42	67. 72)70 002 972 R18	68. 35)70 105 2 003
69. 47)189 314 4 027 R45	70. 74)603 990 8 162 R2	71. 13)184 048 14 157 R7	72. 91)637 784 7 008 R56
73. 16)549 027 34 314 R3	74. 67)810 012 12 089 R49	75. 94)684 928 7 286 R44	76. 23)456 780 19 860

336

The skills required for the exercises on this page are indicated in the following chart.

Skills	Exercises
Divide a two-digit number by a one-digit number	1-12
Divide a three-digit number by a one-digit number	13-20
Divide a four-digit number by a one-digit number	21-28
Divide a five-digit number by a one-digit number	29-32
Divide a five-digit number by a one-digit number, some quotients with zero in one or more places	33-36
Divide by a multiple of ten from 10 to 90, dividends with up to five digits	37-44
Divide by a two-digit number, dividends with up to six digits, some quotients with zero in one or more places	45-76

OBJECTIVE

Demonstrate competence in dividing a decimal or a whole number by a whole number, decimal quotients

Skill Practice–Dividing Decimals

Divide. Show a decimal quotient for each.

7.6 1. 8)60.8	5.9 2. 9)53.1	4.8 3. 3)14.4	5.8 4. 2)11.6	5.7 5. 5)28.5
6.94 6. 7)48.58	9.14 7. 3)27.42	6.83 8. 6)40.98	7.27 9. 5)36.35	1.92 10. 4)7.68
6.852 11. 2)13.704	3.916 12. 4)15.664	9.237 13. 7)64.659	8.317 14. 6)49.902	5.174 15. 8)41.392
1.63 16. 5)8.15	2.488 17. 9)22.392	3.4 18. 4)13.6	2.66 19. 7)18.62	5.573 20. 6)33.438

0.43 21. 4)1.72	0.912 22. 3)2.736	0.84 23. 8)6.72	0.653 24. 5)3.265	0.9 25. 7)6.3
0.53 26. 7)3.71	0.806 27. 2)1.612	5.94 28. 9)53.46	0.921 29. 5)4.605	0.207 30. 4)0.828
0.48 31. 8)3.84	0.28 32. 3)0.84	0.743 33. 6)4.458	0.456 34. 6)2.736	0.12 35. 9)1.08

4.35 36. 2)8.7	3.225 37. 8)25.8	0.65 38. 4)2.6	4.615 39. 8)36.92	6.12 40. 5)30.6
7.445 41. 8)59.56	0.038 42. 5)0.19	7.775 43. 2)15.55	4.985 44. 6)29.91	0.425 45. 8)3.4
7.825 46. 4)31.3	2.975 47. 8)23.8	0.135 48. 4)0.54	1.504 49. 5)7.52	8.85 50. 8)70.8

2.5 51. 4)10	2.2 52. 5)11	7.125 53. 8)57	4.875 54. 8)39	0.25 55. 4)1	3.5 56. 6)21
1.6 57. 5)8	3.5 58. 2)7	9.25 59. 4)37	6.4 60. 5)32	1.25 61. 8)10	8.5 62. 8)68
5.75 63. 4)23	5.625 64. 8)45	0.75 65. 8)6	4.75 66. 4)19	0.8 67. 5)4	5.875 68. 8)47

3.61 69. 18)64.98	4.9 70. 81)396.9	0.928 71. 37)34.336	2.7 72. 63)170.1
5.37 73. 75)402.75	0.67 74. 58)38.86	6.04 75. 24)144.96	7.96 76. 12)95.52
0.406 77. 42)17.052	1.9 78. 79)150.1	0.867 79. 94)81.498	0.134 80. 86)11.524

5.15 81. 34)175.1	0.648 82. 15)9.72	9.13 83. 40)365.2	6.06 84. 25)151.5	2.675 85. 12)32.1
3.625 86. 56)203	0.2 87. 45)9	7.7 88. 10)77	6.25 89. 92)575	4.8 90. 35)168

337

The skills required for the exercises on this page are indicated in the following chart.

Skills	Exercises
Divide a one-place decimal by a one-digit number, quotients greater than one	1-5
Divide a decimal with up to three decimal places by a one-digit number, dividends with up to five digits, quotients greater than 1	6-20
Divide a decimal with up to three decimal places by a one-digit number, quotients less than 1	21-35
Divide a one-place decimal or a two-place decimal by a one-digit number, using zeros in the dividend, quotients terminating by the third decimal place	36-50
Divide a whole number by a one-digit whole number, using zeros in the dividend, quotients terminating by the third decimal place	51-68
Divide a decimal with up to three decimal places by a two-digit number, quotients terminating by the third decimal place	69-85
Divide a whole number with up to three digits by a two-digit whole number, quotients terminating by the third decimal place	86-90

Answers

Page 27 (Exercises)

1. 6 + 3 = 9
   3 + 6 = 9
   9 − 3 = 6
   9 − 6 = 3

2. 7 − 4 = 3
   7 − 3 = 4
   3 + 4 = 7
   4 + 3 = 7

3. 7 + 7 = 14
   14 − 7 = 7

4. 13 − 8 = 5
   13 − 5 = 8
   5 + 8 = 13
   8 + 5 = 13

5. 1 + 4 = 5
   4 + 1 = 5
   5 − 4 = 1
   5 − 1 = 4

6. 11 − 9 = 2
   11 − 2 = 9
   2 + 9 = 11
   9 + 2 = 11

7. 3 − 0 = 3
   3 − 3 = 0
   3 + 0 = 3
   0 + 3 = 3

8. 6 + 0 = 6
   0 + 6 = 6
   6 − 0 = 6
   6 − 6 = 0

9. 7 + 4 = 11
   4 + 7 = 11
   11 − 4 = 7
   11 − 7 = 4

10. 8 + 4 = 12
    4 + 8 = 12
    12 − 4 = 8
    12 − 8 = 4

11. 14 − 6 = 8
    14 − 8 = 6
    8 + 6 = 14
    6 + 8 = 14

12. 7 + 3 = 10
    3 + 7 = 10
    10 − 3 = 7
    10 − 7 = 3

13. 8 + 8 = 16
    16 − 8 = 8

14. 16 − 7 = 9
    16 − 9 = 7
    9 + 7 = 16
    7 + 9 = 16

15. 8 − 8 = 0
    8 − 0 = 8
    0 + 8 = 8
    8 + 0 = 8

16. 5 + 6 = 11
    6 + 5 = 11
    11 − 6 = 5
    11 − 5 = 6

17. 4 + 9 = 13
    9 + 4 = 13
    13 − 9 = 4
    13 − 4 = 9

18. 14 − 9 = 5
    14 − 5 = 9
    5 + 9 = 14
    9 + 5 = 14

19. 12 − 6 = 6
    6 + 6 = 12

20. 3 + 8 = 11
    8 + 3 = 11
    11 − 8 = 3
    11 − 3 = 8

Page 62 (Problem Solving)

3.

	Agnes	Beth	Charles	Doug	Ellen
Frank	Agnes Frank	Beth Frank	Charles Frank	Doug Frank	Ellen Frank
Gary	Agnes Gary	Beth Gary	Charles Gary	Doug Gary	Ellen Gary
Helen	Agnes Helen	Beth Helen	Charles Helen	Doug Helen	Ellen Helen

4.

	1	2	3	4	5	6
1	1,1	1,2	1,3	1,4	1,5	1,6
2	2,1	2,2	2,3	2,4	2,5	2,6
3	3,1	3,2	3,3	3,4	3,5	3,6
4	4,1	4,2	4,3	4,4	4,5	4,6
5	5,1	5,2	5,3	5,4	5,5	5,6
6	6,1	6,2	6,3	6,4	6,5	6,6

5.

	rabbit	elf	boy	giant
jolly	jolly rabbit	jolly elf	jolly boy	jolly giant
angry	angry rabbit	angry elf	angry boy	angry giant
silly	silly rabbit	silly elf	silly boy	silly giant
friendly	friendly rabbit	friendly elf	friendly boy	friendly giant
foolish	foolish rabbit	foolish elf	foolish boy	foolish giant
bashful	bashful rabbit	bashful elf	bashful boy	bashful giant
gentle	gentle rabbit	gentle elf	gentle boy	gentle giant

6.

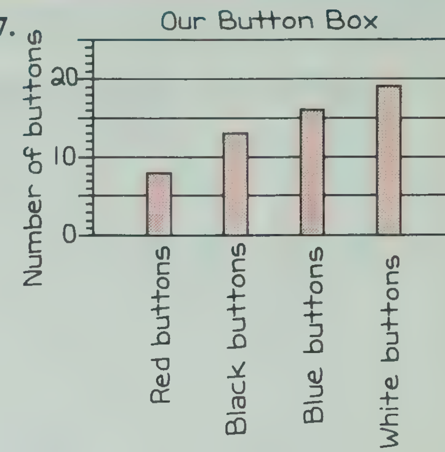
	a	e	i	o	u
a		ae	ai	ao	au
b	ba	be	bi	bo	bu
c	ca	ce	ci	co	cu
d	da	de	di	do	du
e	ea		ei	eo	eu
f	fa	fe	fi	fo	fu
g	ga	ge	gi	go	gu
h	ha	he	hi	ho	hu
i	ia	ie		io	iu
j	ja	je	ji	jo	ju
k	ka	ke	ki	ko	ku
l	la	le	li	lo	lu
m	ma	me	mi	mo	mu
n	na	ne	ni	no	nu
o	oa	oe	oi		ou
p	pa	pe	pi	po	pu
q	qa	qe	qi	qo	qu
r	ra	re	ri	ro	ru
s	sa	se	si	so	su
t	ta	te	ti	to	tu
u	ua	ue	ui	uo	
v	va	ve	vi	vo	vu
w	wa	we	wi	wo	wu
x	xa	xe	xi	xo	xu
y	ya	ye	yi	yo	yu
z	za	ze	zi	zo	zu

Page 68 (Working Together)

6. Things in Our Sewing Cabinet

Pins	/////////
Needles	//
Buttons	⊙⊙⊙⊙⊙⊙⊙⊙
Snaps	⊛⊛⊛⊛⊛⊛
Each picture stands for 10 items.	

7.



Page 69 (Exercises)

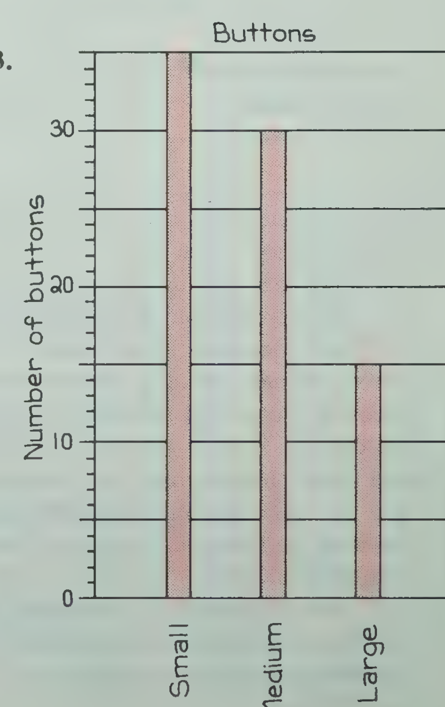
1.

Colors of Buttons	
Yellow	⊙⊙⊙⊙⊙⊙ 6
Red	⊙⊙⊙⊙⊙⊙⊙⊙⊙⊙ 10
Green	⊙⊙⊙⊙⊙⊙⊙⊙⊙⊙⊙⊙⊙⊙⊙⊙ 16
Blue	⊙⊙⊙⊙⊙⊙⊙⊙ 8
Each picture stands for 2 buttons.	

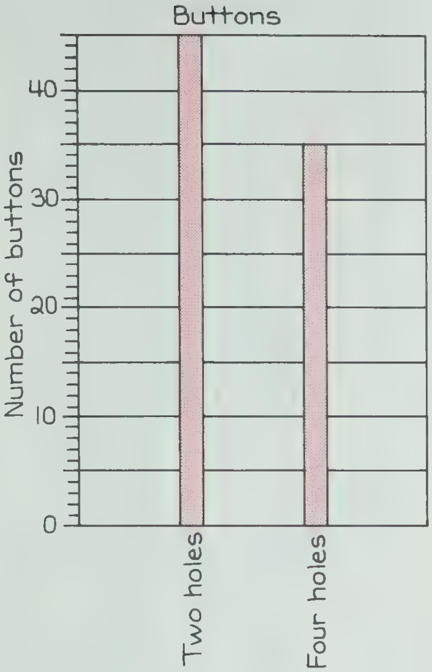
2.

Sizes of Buttons	
Large	⊙⊙⊙ 3
Medium	⊙⊙⊙⊙⊙⊙ 6
Small	⊙⊙⊙⊙⊙⊙⊙ 7
Each picture stands for 5 buttons.	

3.

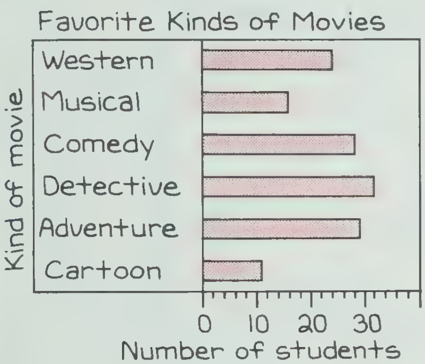


4.

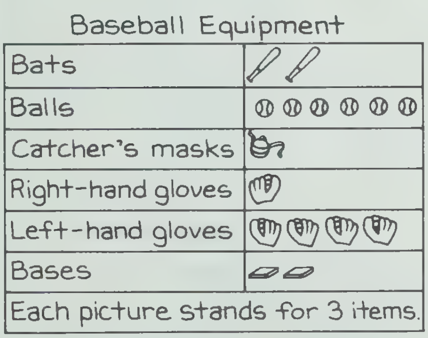


Page T 75 (Assessment)

1.

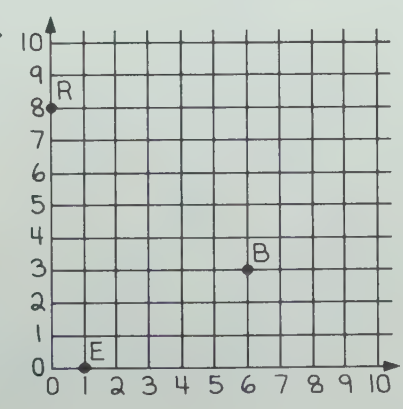


2.



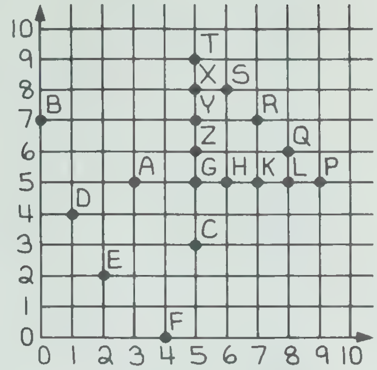
Page 73 (Working Together)

6.-8.



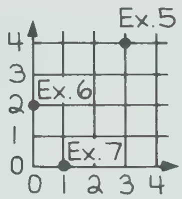
Page 73 (Exercises)

9.-26.



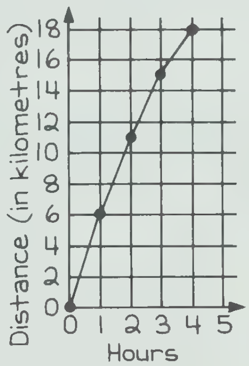
Page T 79 (Assessment)

5.-7.



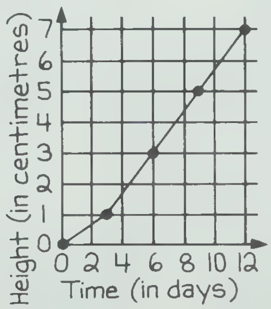
Page 75 (Working Together)

1.-4.

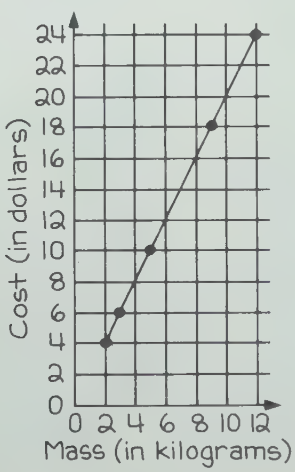


Page 75 (Exercises)

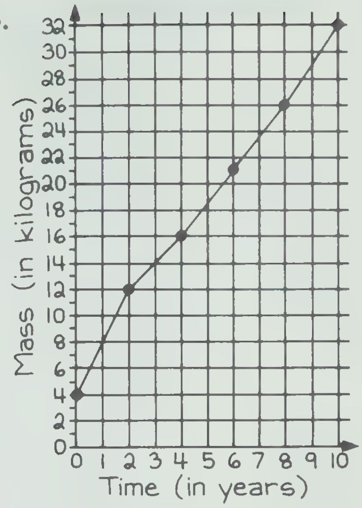
1.



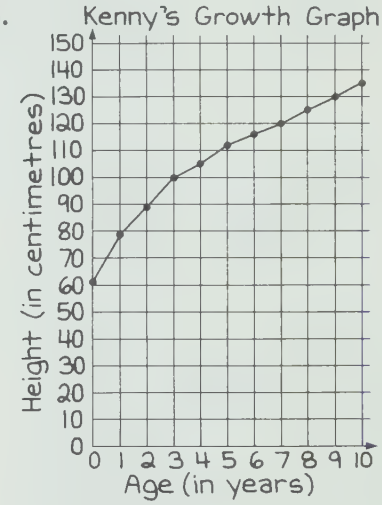
2.



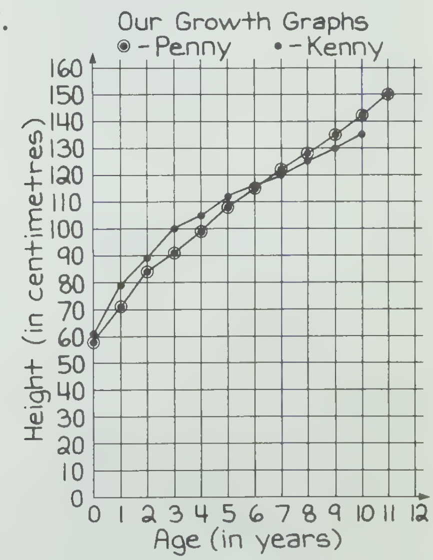
3.



6.

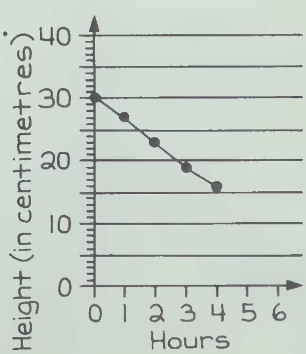


7.



Page T 81 (Assessment)

1.



Page 81 (Working Together)

1. $1 \times 2 = 2$
$2 \times 2 = 4$
$3 \times 2 = 6$
$4 \times 2 = 8$
$5 \times 2 = 10$
$6 \times 2 = 12$
$7 \times 2 = 14$
$8 \times 2 = 16$
$9 \times 2 = 18$

2. $1 \times 3 = 3$
$2 \times 3 = 6$
$3 \times 3 = 9$
$4 \times 3 = 12$
$5 \times 3 = 15$
$6 \times 3 = 18$
$7 \times 3 = 21$
$8 \times 3 = 24$
$9 \times 3 = 27$

3. $1 \times 4 = 4$
$2 \times 4 = 8$
$3 \times 4 = 12$
$4 \times 4 = 16$
$5 \times 4 = 20$
$6 \times 4 = 24$
$7 \times 4 = 28$
$8 \times 4 = 32$
$9 \times 4 = 36$

4. $1 \times 5 = 5$
$2 \times 5 = 10$
$3 \times 5 = 15$
$4 \times 5 = 20$
$5 \times 5 = 25$
$6 \times 5 = 30$
$7 \times 5 = 35$
$8 \times 5 = 40$
$9 \times 5 = 45$

5. $1 \times 6 = 6$
$2 \times 6 = 12$
$3 \times 6 = 18$
$4 \times 6 = 24$
$5 \times 6 = 30$
$6 \times 6 = 36$
$7 \times 6 = 42$
$8 \times 6 = 48$
$9 \times 6 = 54$

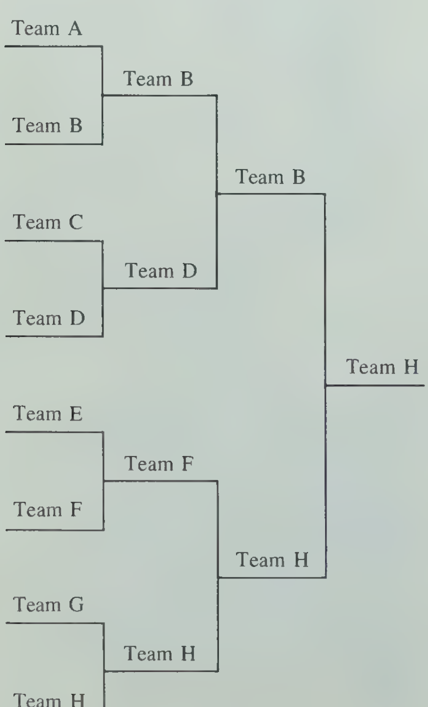
6. $1 \times 7 = 7$
$2 \times 7 = 14$
$3 \times 7 = 21$
$4 \times 7 = 28$
$5 \times 7 = 35$
$6 \times 7 = 42$
$7 \times 7 = 49$
$8 \times 7 = 56$
$9 \times 7 = 63$

7. $1 \times 8 = 8$
$2 \times 8 = 16$
$3 \times 8 = 24$
$4 \times 8 = 32$
$5 \times 8 = 40$
$6 \times 8 = 48$
$7 \times 8 = 56$
$8 \times 8 = 64$
$9 \times 8 = 72$

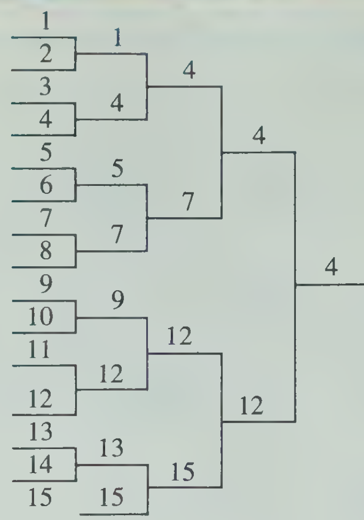
8. $1 \times 9 = 9$
$2 \times 9 = 18$
$3 \times 9 = 27$
$4 \times 9 = 36$
$5 \times 9 = 45$
$6 \times 9 = 54$
$7 \times 9 = 63$
$8 \times 9 = 72$
$9 \times 9 = 81$

Page 110 (Problem Solving)

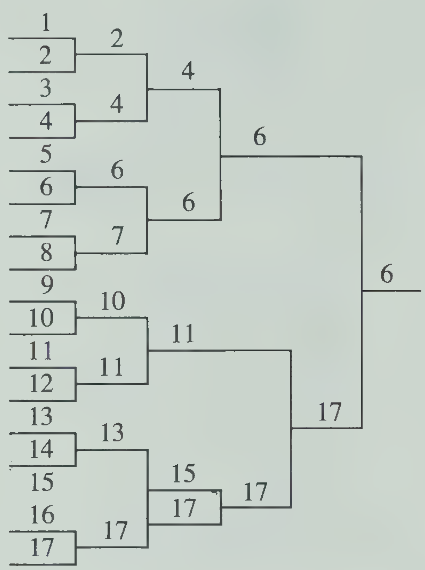
1.



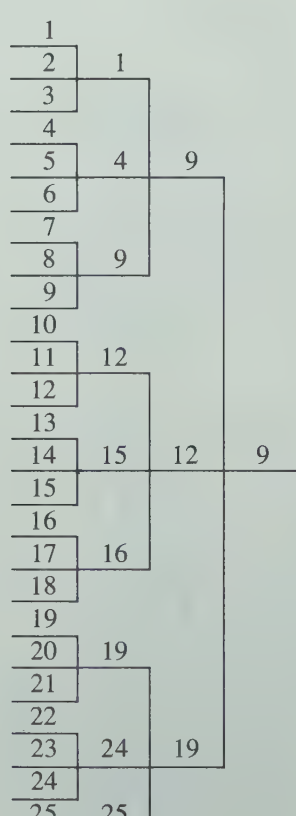
2.



3.



4.



Page 124 (Problem Solving)

5.

Place / Day	1	2	3
1	AB	CD	EF
2	DF	BE	AC
3	CE	AD	BF
4	AE	CF	BD
5	BC	AF	BE
6	CD	EF	AB
7	BE	AC	DF
8	AD	BF	CE
9	CF	BD	AE
10	AF	DE	BC

6.

Place / Day	1	2	3	4
1	CD	AB	EF	GH
2	EG	FH	AC	BD
3	AD	BC	EH	FG
4	CG	DH	BF	AE
5	BE	AF	DG	CH
6	DF	CE	BH	AG
7	AH	BG	CF	DE

Page 142 (Exercises)

	Perimeter (units)	Width (units)	Length (units)	Area (square units)
1.	8	1	3	3
		2	2	4
2.	12	1	5	5
		2	4	8
		3	3	9
3.	24	1	11	11
		2	10	20
		3	9	27
		4	8	32
		5	7	35
		6	6	36
4.	16	1	7	7
		2	6	12
		3	5	15
		4	4	16

Page 143 (Exercises)

	Area (square units)	Width (units)	Length (units)	Perimeter (units)
1.	4	1	4	10
		2	2	8
2.	16	1	16	34
		2	8	20
		4	4	16
3.	24	1	24	50
		2	12	28
		3	8	22
		4	6	20
4.	30	1	30	62
		2	15	34
		3	10	26
		5	6	22

Page 159 (Try This)

1.

1 kg (kilogram)	=	1000 g
1 hg (hectogram)	=	100 g
1 dag (decagram)	=	10 g
1 g (gram)	=	1 g
1 dg (decigram)	=	0.1 g
1 cg (centigram)	=	0.01 g
1 mg (milligram)	=	0.001 g

2.

1 kL (kilolitre)	=	1000 L
1 hL (hectolitre)	=	100 L
1 daL (decalitre)	=	10 L
1 L (litre)	=	1 L
1 dL (decilitre)	=	0.1 L
1 cL (centilitre)	=	0.01 L
1 mL (millilitre)	=	0.001 L

3.

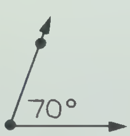
1 m	=	1000 mm
1 m	=	100 cm
1 m	=	10 dm
1 m	=	1 m
1 m	=	0.1 dam
1 m	=	0.01 hm
1 m	=	0.001 km

1 g	=	1000 mg
1 g	=	100 cg
1 g	=	10 dg
1 g	=	1 g
1 g	=	0.1 dag
1 g	=	0.01 hg
1 g	=	0.001 kg

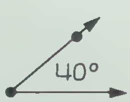
1 L	=	1000 mL
1 L	=	100 cL
1 L	=	10 dL
1 L	=	1 L
1 L	=	0.1 daL
1 L	=	0.01 hL
1 L	=	0.001 kL

Page 178 (Exercises)

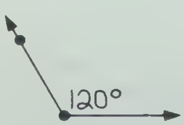
1.



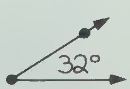
2.



3.



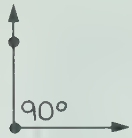
4.



5.

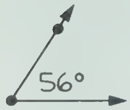
6.

7.



8.

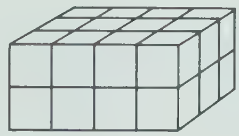
9.



10.

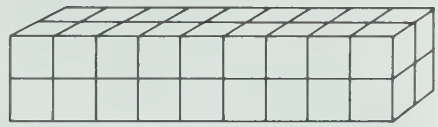
Page 225 (Problem Solving)

5.

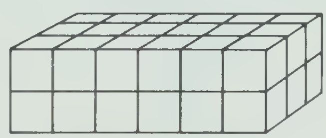


How many cubes have blue
on 3 faces only? 8
on 2 faces only? 12
on 1 face only? 4
on 0 faces? 0

6.

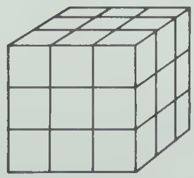


How many cubes have blue
on 3 faces only? 8
on 2 faces only? 28
on 1 face only? 0
on 0 faces? 0



How many cubes have blue
on 3 faces only? 8
on 2 faces only? 20
on 1 face only? 8
on 0 faces? 0

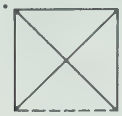
7.



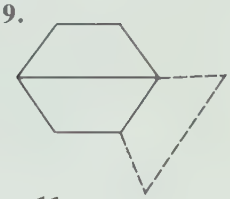
How many cubes have blue
on 3 faces only? 8
on 2 faces only? 12
on 1 face only? 6
on 0 faces? 1

Page 259 (Exercises)

8.

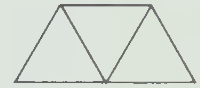


9.

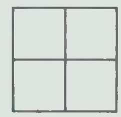


10.

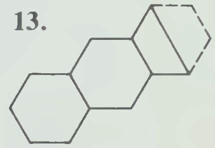
11.



12.

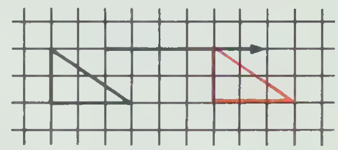


13.

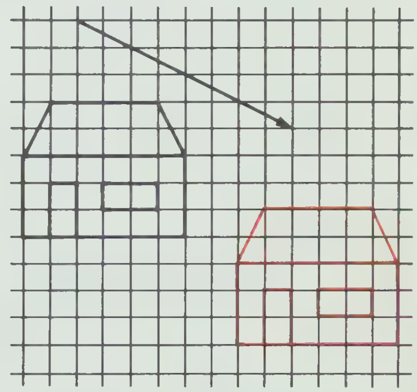


Page 291 (Exercises)

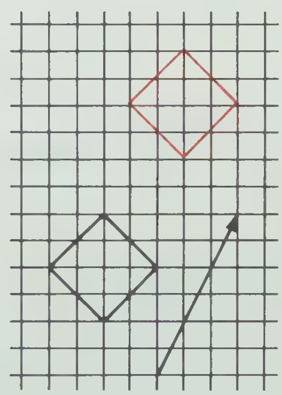
1.



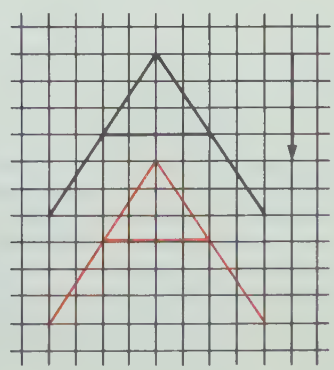
2.



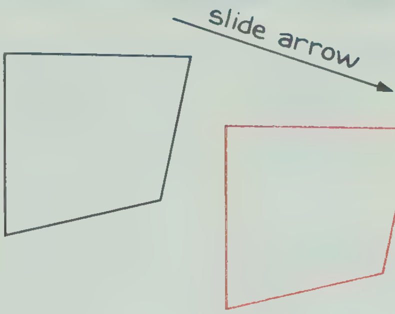
3.



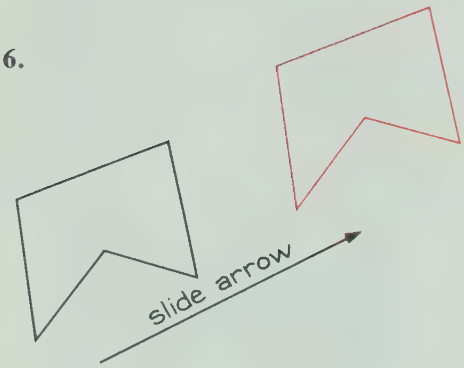
4.



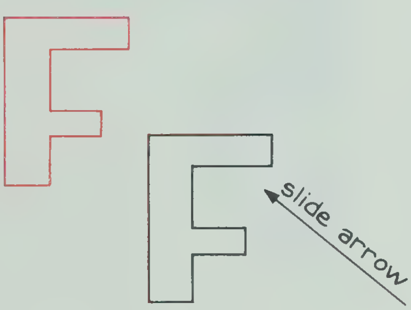
5.



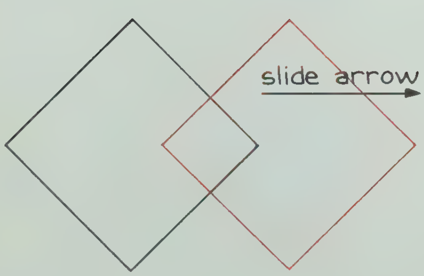
6.



7.

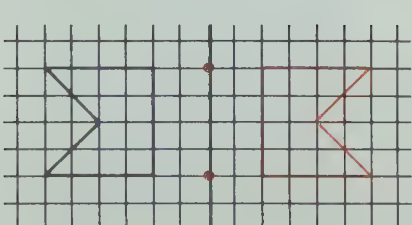


8.

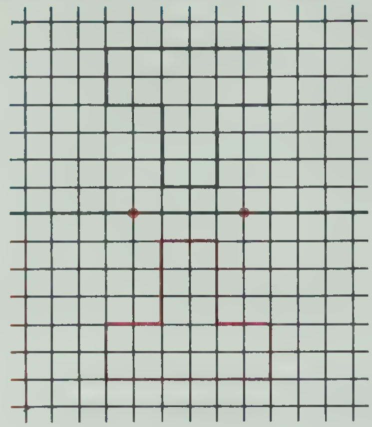


Page 295 (Exercises)

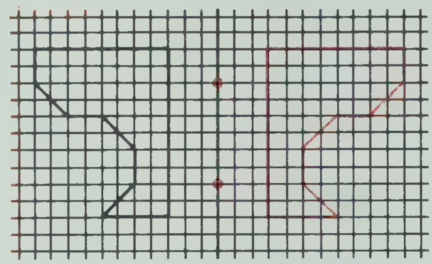
1.



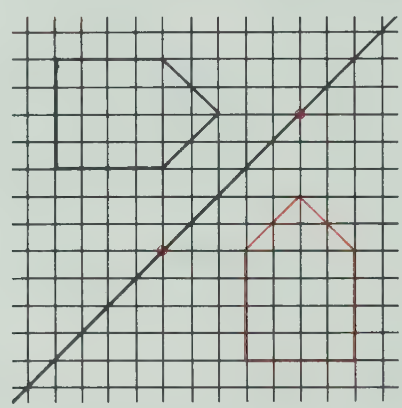
2.



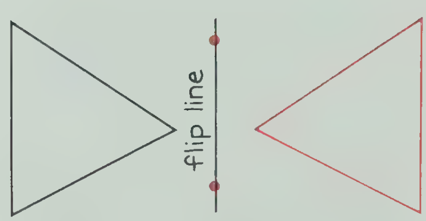
3.



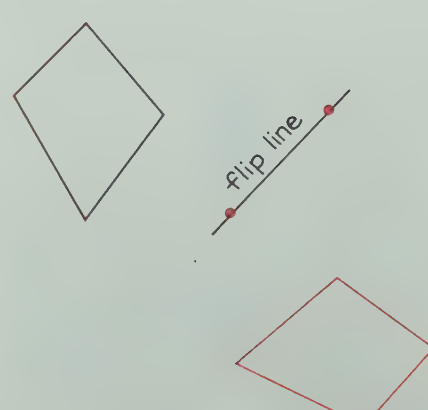
4.



5.

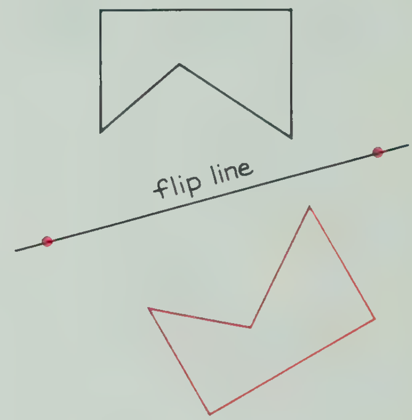


6.



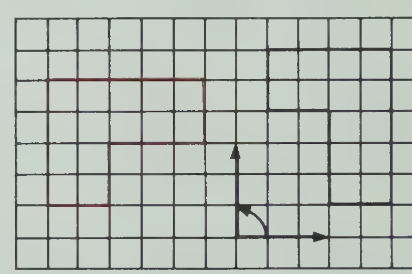
7.

8.

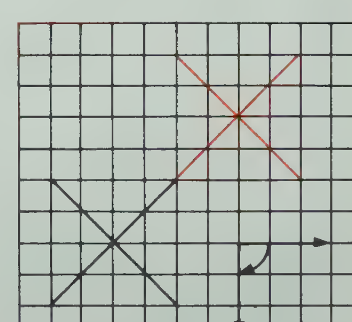


Page 299 (Exercises)

1.

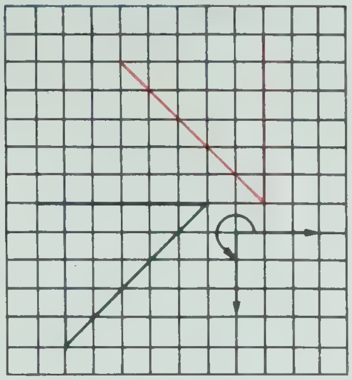


2.



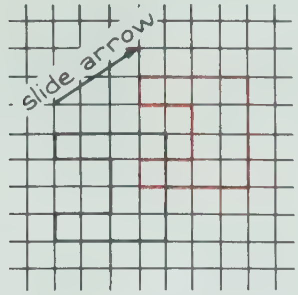
3.

4.

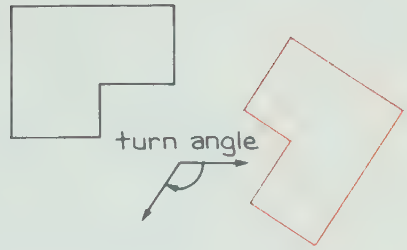


Pages 300-301 (Practice)

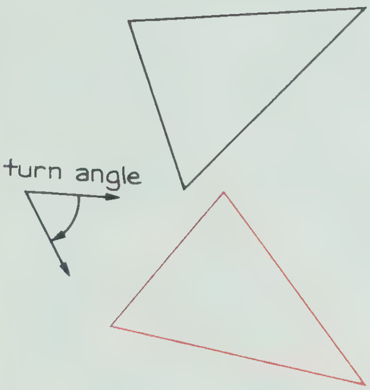
4.



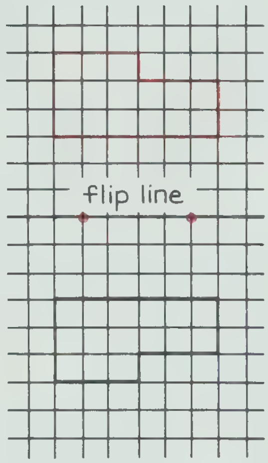
9.



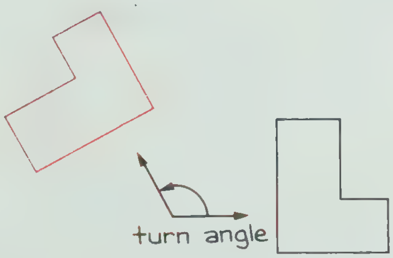
5.



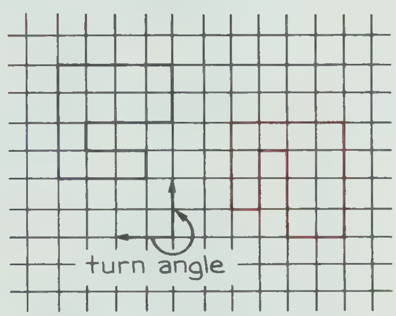
5.



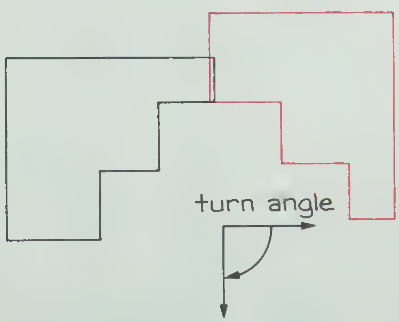
6.



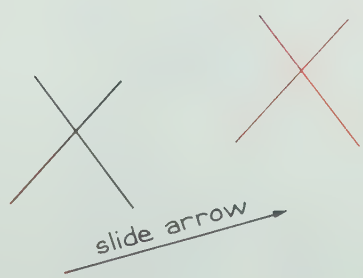
6.



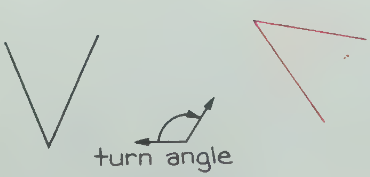
7.



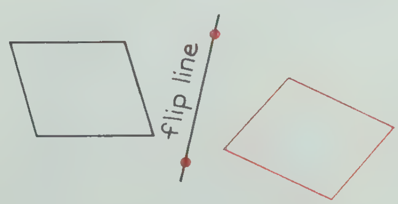
7.



8.

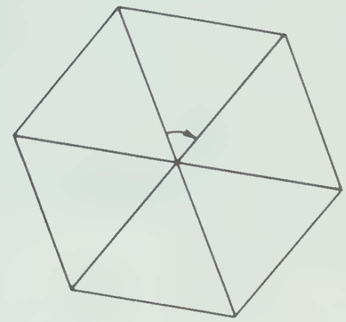


8.

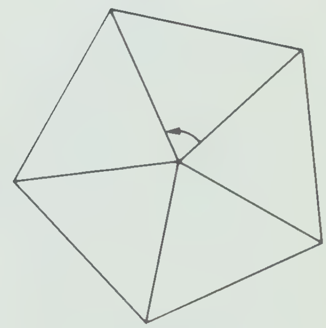


Page 303 (Exercises)

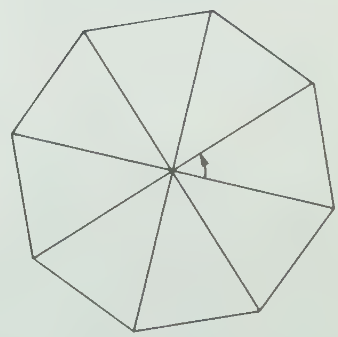
1.



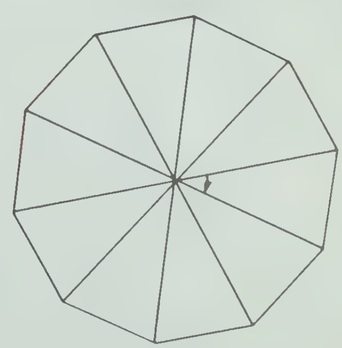
2.



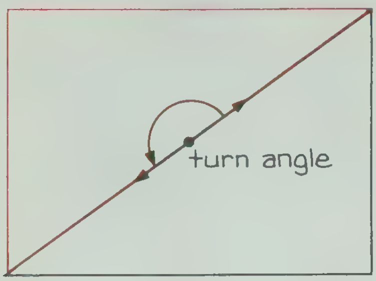
3.



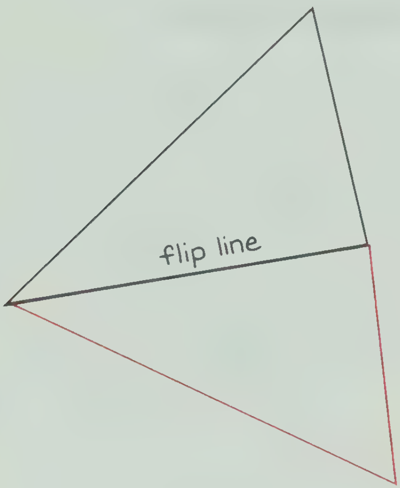
4.



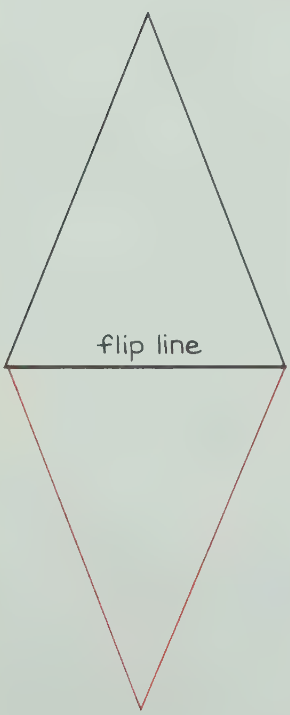
6.



7.

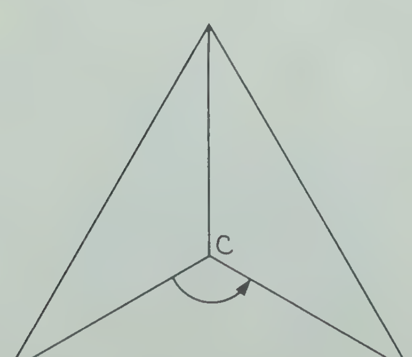


8.



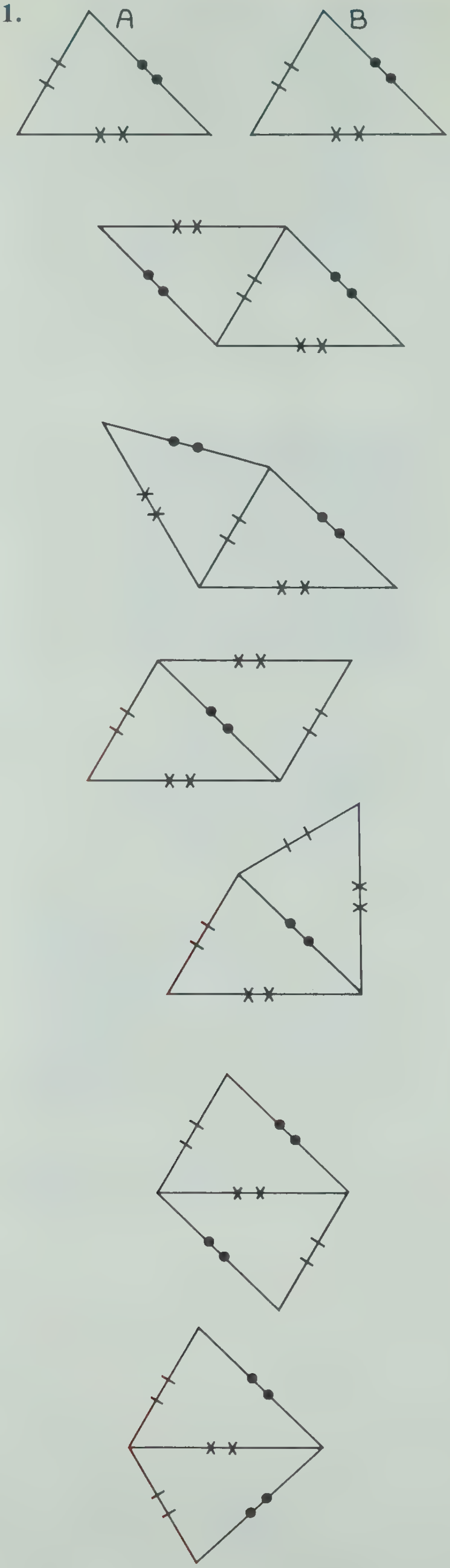
Page T 329 (Assessment)

2.

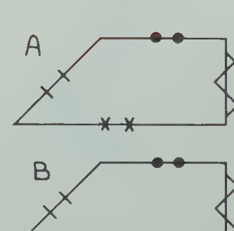


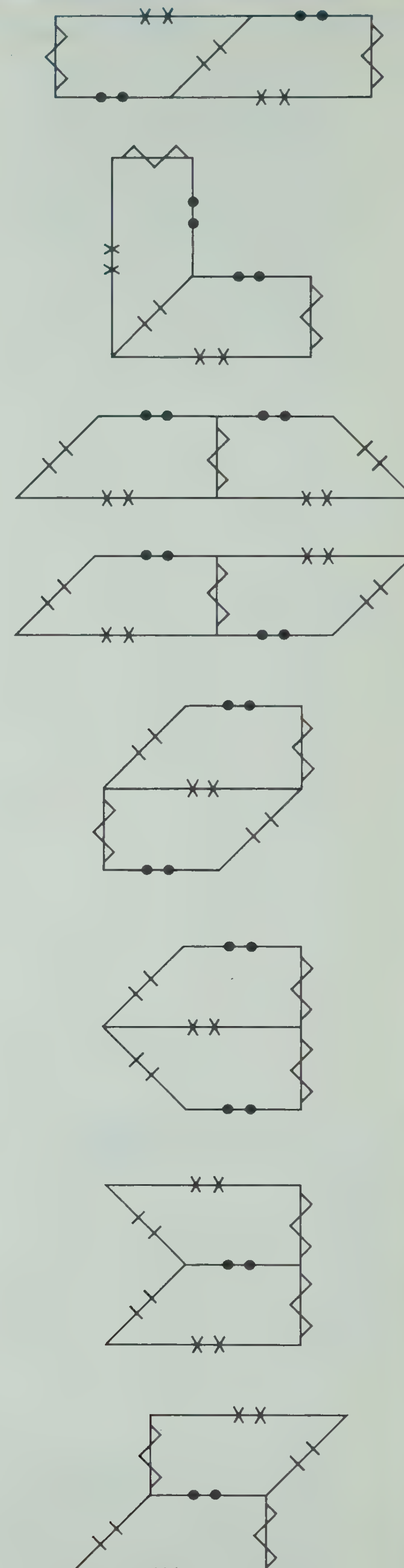
Page 305 (Exercises)

1.

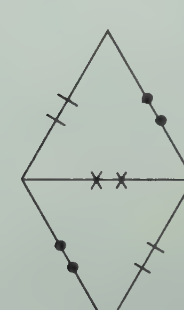


2.

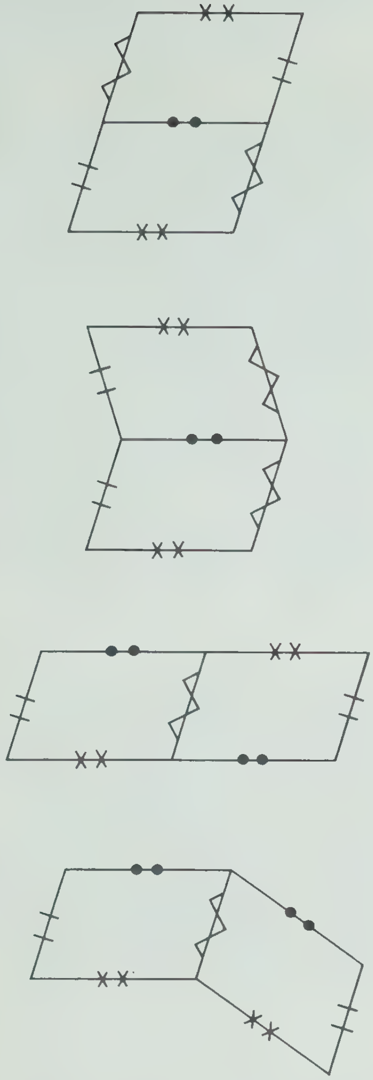


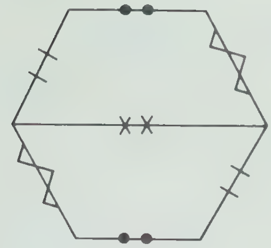


3.

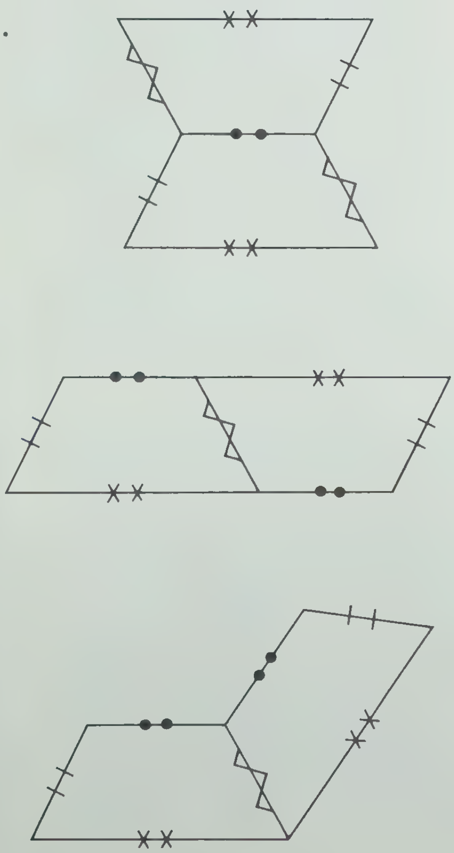


4.

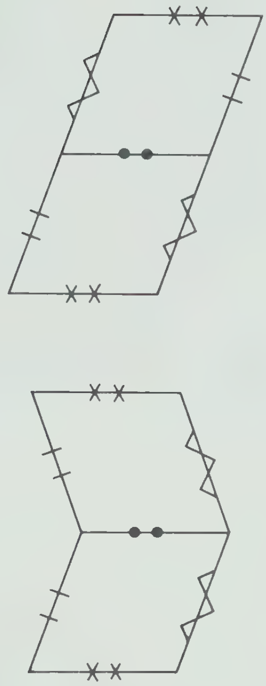




5.

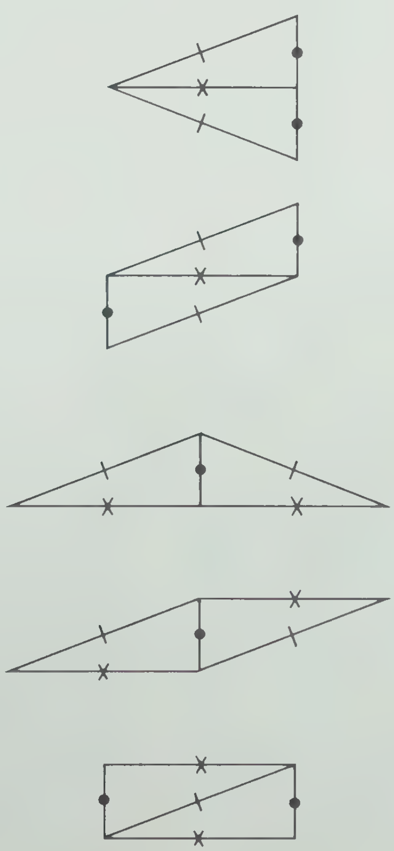


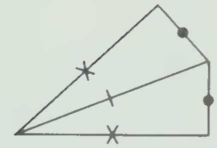
6.



Page T 331 (Assessment)

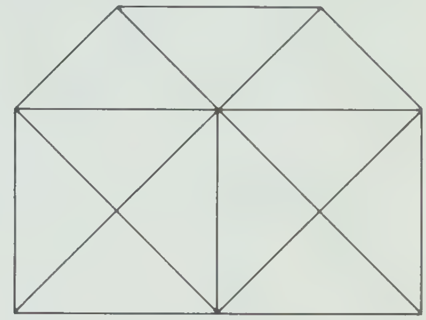
1.



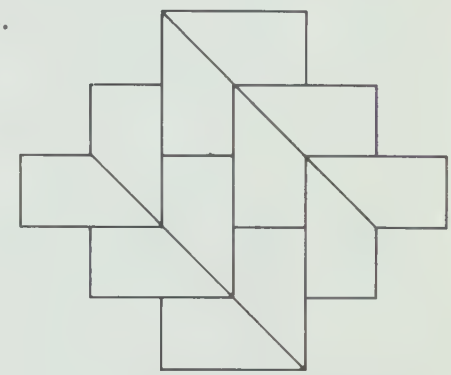


Page 307 (Exercises)

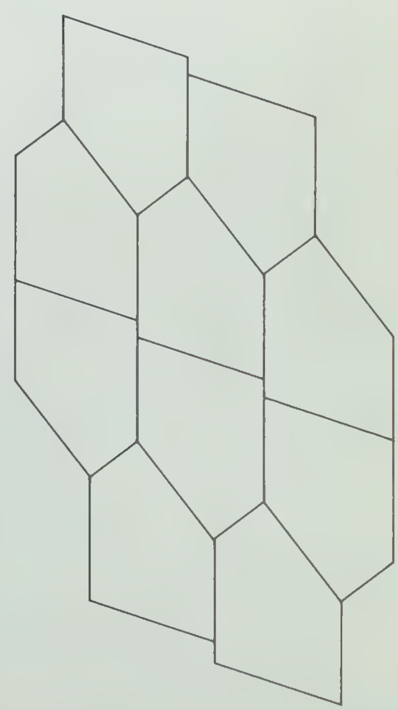
1.



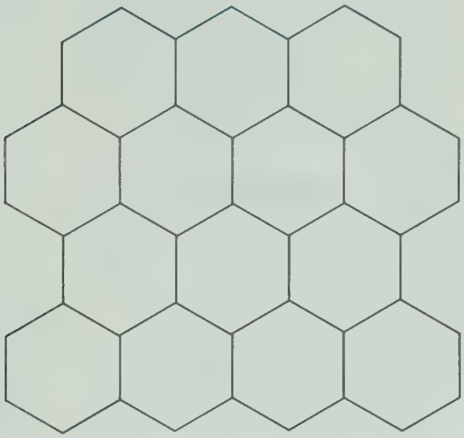
2.



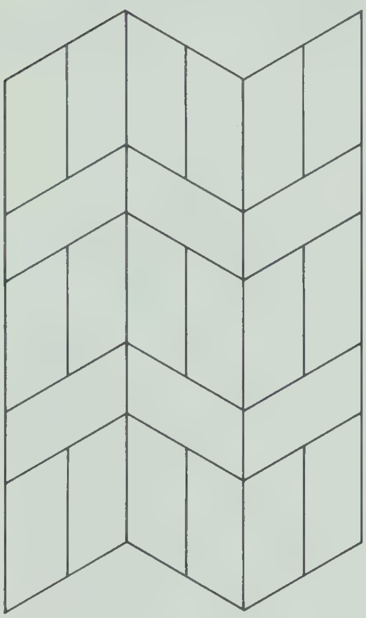
3.



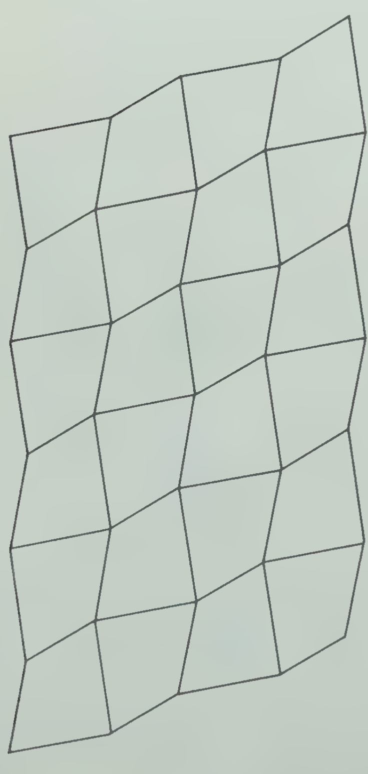
4.



5.

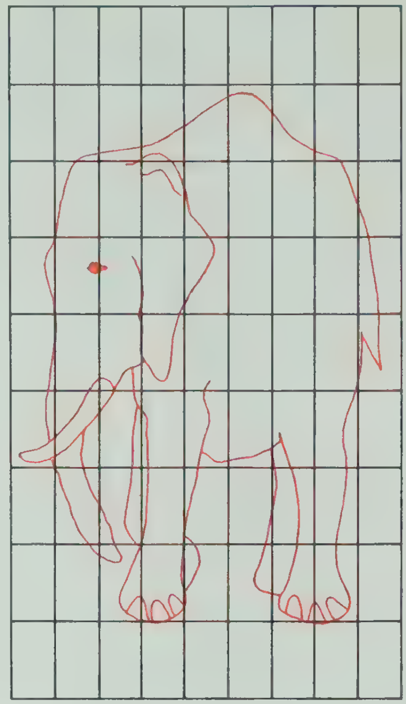


6.

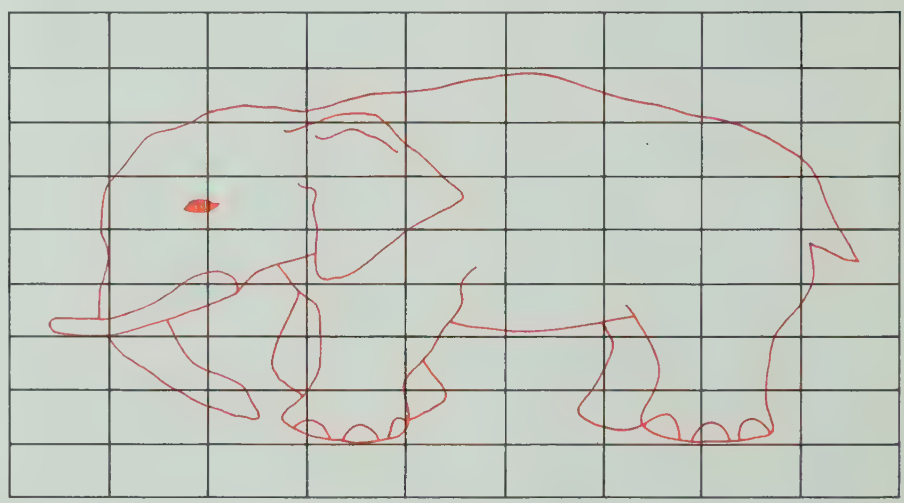


Page 309 (Exercises)

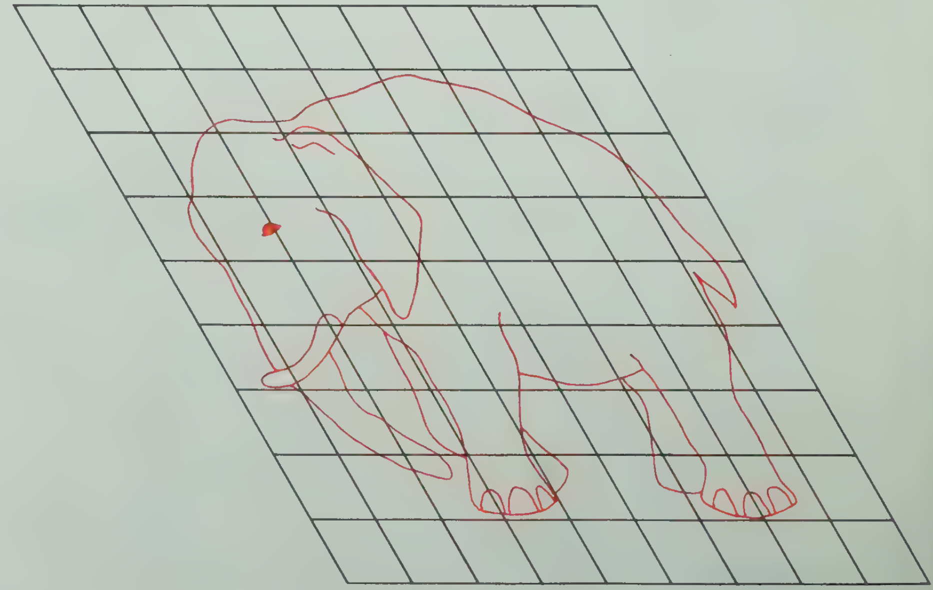
2.



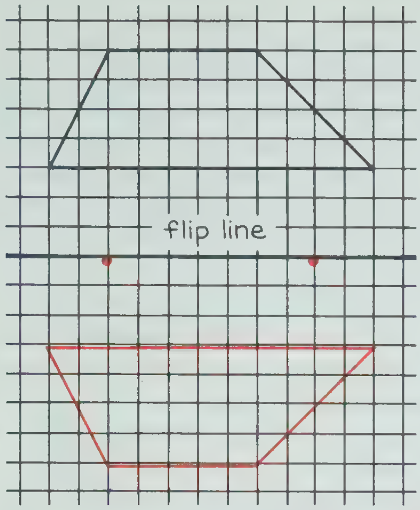
3.



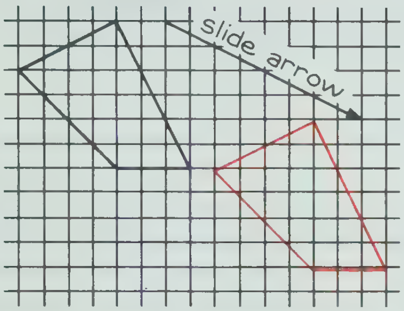
4.



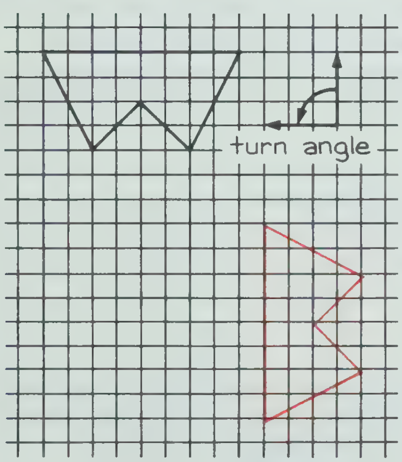
4.



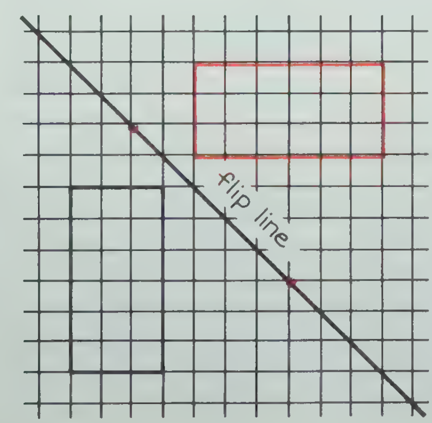
5.



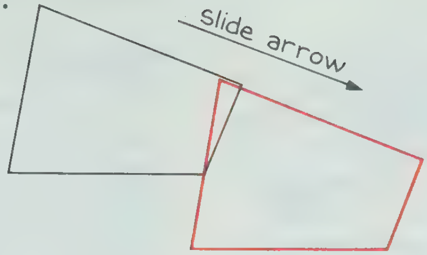
6.



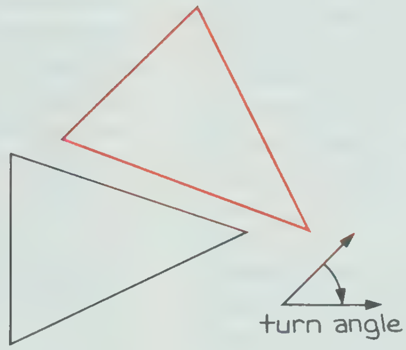
7.



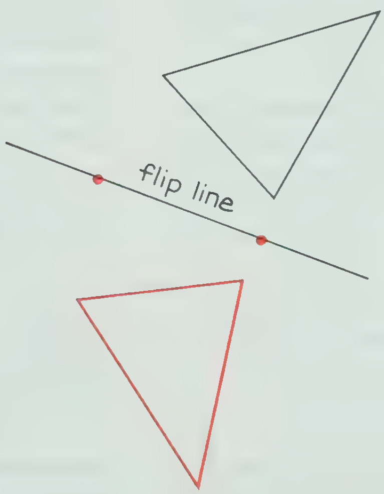
8.



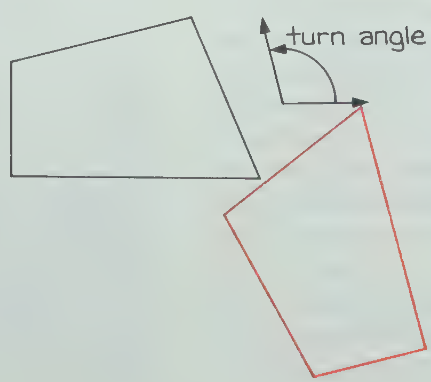
9.



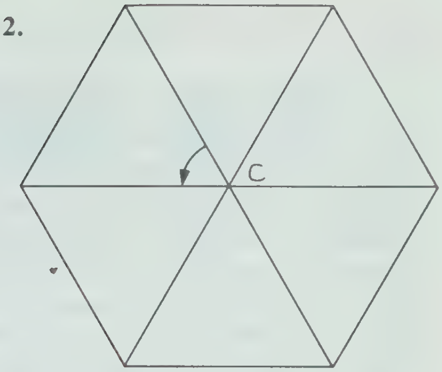
10.



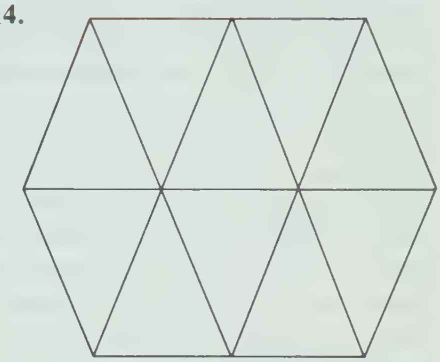
11.



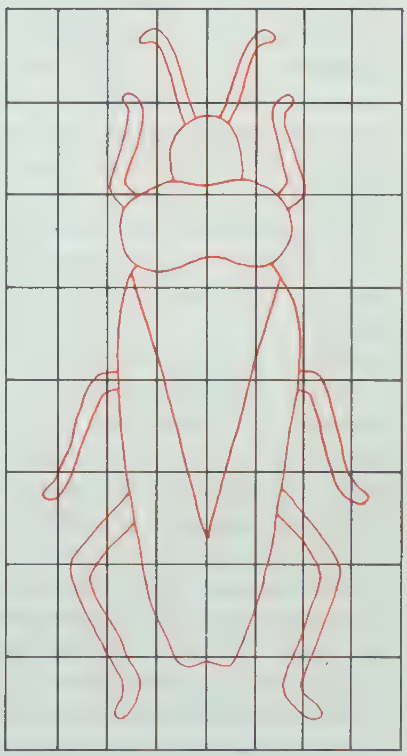
12.



14.



15.



Pages T 377 to T 400 include materials that have been referred to in the teaching suggestions for various lessons. Although suggestions for using many of these materials are given in the related lesson outlines, other suggestions for some of the materials are given below.

The addition table on page T 380 and the multiplication table on page T 381 may be used in mastering basic facts. As facts are memorized, students may color inside the appropriate squares on copies of these pages. By folding a copy of each table along the appropriate diagonal, students may observe that numbers on one side of the diagonal match numbers on the other side. Students may search for patterns in the tables, for example, even numbers, odd numbers, multiples of five, and square numbers. The tables are useful in demonstrating the commutative properties of addition and multiplication, and in relating inverse operations (addition and subtraction, multiplication and division).

The shapes on pages T 382 to T 385 may be used for making attribute blocks (see page xxiv) since the actual size is shown. The large circle is a suitable size for making individual number spinners. The triangles, rectangles, and squares also lend themselves for work with fractions and area since their dimensions are such that four of one small shape exactly cover the corresponding large shape.

The patterns for the three-dimensional shapes on pages T 386 to T 388 are marked with recommended dimensions. You may find it easier to construct some of these shapes if the pattern is outlined first on squared paper (page T 397), using the centimetre grid lines as a guide.

Because the number lines on page T 389 are marked into centimetres and half centimetres, copies of several of these may be pasted together to make "metre tapes" for the students to use in their measuring activities. Copies of the line which is marked in millimetres are useful in measuring small lengths and in relating millimetres and centimetres.

Copies of the 10-by-10 grid on page T 391 may be used for preparing individual game boards or work sheets for activities. The 11-by-11 grid may be used for preparing tables of basic addition or multiplication facts.

Copies of page T 392 will be suitable for preparing models of hundreds, tens, and ones for the students. These may be prepared by pasting the cutouts on cardboard, laminating the surface (optional), and then cutting the models apart as desired.

To make models for thousands, ten single models of hundreds may be stapled or taped together to represent one thousand.

Copies of page T 396 may be used in many different ways. Some suggestions are as follows: for activities similar to those in the *Problem Solving* feature on page 137 and Ex. 1-8 on pages 142 and 143; for reinforcement for pages 170 to 187 (dots are joined to form line segments, angles, or polygons, and then the students make appropriate measurements); for reinforcement for pages 182 and 183 (dots are joined to form a simple shape having one or more lines of symmetry); for providing arrays for which the students write the multiplication or division sentences; for students to show the size of the arrays for given multiplication and division sentences.

Copies of pages T 397 and T 398 may be used for activities involving slides, flips, and turns; line symmetry; bar graphs, line graphs, and coordinate geometry (naming points on a grid).

Teaching Aids, Activities, and Games

Peg Abacus

A simple peg abacus can be made from a Styrofoam tray, three wires of equal length cut from coat hangers, and plaster of Paris. Fill the tray with a mixture of plaster of Paris. Before it sets, insert the three wires and then allow the plaster to set. Use objects (colored wooden beads, washers, or empty spools) for representing numbers. The wires should be about 5 cm long unless empty spools are to be used, in which case they should be about 30 cm long.

A sturdier peg abacus may be made using wooden dowels on a wooden base. It is important that students view the abacus from the same side and not from opposite sides, which would result in a reversal of the place values.

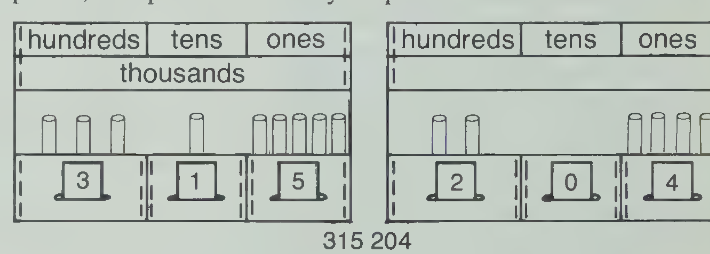
To show numerals with more than three digits, place one abacus to the left of another.

Place-value Pocket Chart

Pocket charts for demonstrating place value may be made in a variety of ways. One of the simplest forms is illustrated below.

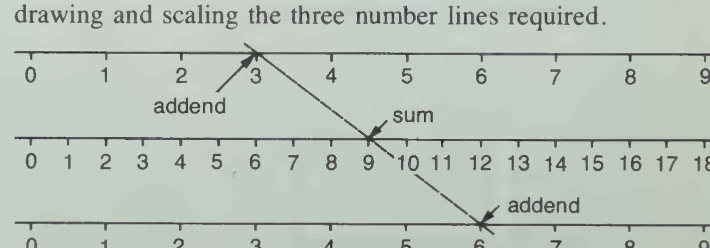
Bristol board or thick plastic is cut in one piece, folded, and stitched or stapled where indicated to make three pockets for working with hundreds, tens, and ones. Slits may be made in the pockets to accommodate numeral cards for showing the standard numeral for the number of hundreds, tens, and ones displayed.

This chart may be fastened to the display board or other convenient location. For showing numerals with more than three places, one pocket chart may be placed beside another.



Nomograph

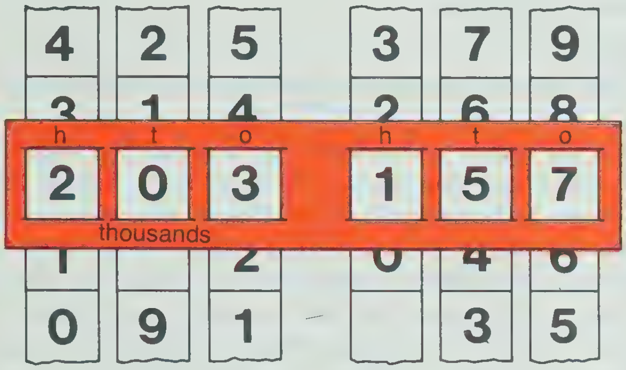
This device is useful for reviewing families of related addition and subtraction facts. Copies of page T 396 may be used for drawing and scaling the three number lines required.



Have the students place a straight edge in the position indicated by the broken line. The following sentences are derived for this position: $3 + 6 = 9$, $6 + 3 = 9$, $9 - 6 = 3$, and $9 - 3 = 6$. All positions of the straight edge through 9 on the middle number line show the basic facts having a sum of 9, and these may be interpreted for both addition and subtraction. Other basic facts are derived in the same way.

Digital Device

Prepare a digital device for six-digit numerals using six strips of paper, each showing eleven squares (copies of page T 391 or page T 397 may be used) and a card with six "windows". The numerals 0 to 9 are written on the squares of each strip and one square is left blank to allow for showing numerals with fewer than six digits. The two ends of each strip may be joined to form a loop. Students may use the device to display the standard form for a number named or for a number for which the expanded form is given. The devices may be constructed to make it possible to demonstrate nine-digit numerals.



Shopping Spree (Game for pages 34 and 35)

Materials: a game board prepared as suggested below;
a marker for each player;
one regular die;
a set of "limit" cards naming amounts of money such as $300.00, $250.00, and $174.95;
paper and pencil for each player.

Players: two to four

Rules:

1. The "limit" cards are shuffled and each player draws one card. The amount named on the card indicates the total amount of money a player may "spend" during the game.
2. Markers are placed on "START". Each player, in turn, tosses the die and advances her/his marker the number of spaces indicated by the die.
3. If the space occupied by a marker indicates an item for sale, the player may "purchase" the item. The price shown is copied onto a piece of paper.
4. The player who, at the completion of the game, comes closest to spending the amount indicated on her/his limit card, without exceeding that limit, wins the game.

Game Board

Prepare a game board similar to the one shown at the right. On some of the spaces, draw or paste pictures of items (or print the names of the items) and indicate their prices. If you prefer, number or letter the spaces in sequence and prepare an accompanying list to identify objects for sale in the spaces. To add interest to the game, mark some of the spaces with directives such as "Return to START". Other suggestions are provided at the right. Using a list of directives and prices enables you to vary the difficulty of the game and to change the "limit" cards accordingly.

Reference List

A	Hockey Stick	$ 3.89
B	Toaster	$29.95
C	Smoke Detector	$13.88
D	Hockey Puck	44¢
E	Return to START. Do not return your purchases.	
F	Bicycle, reconditioned	$55.00
G	Fishing Rod	$ 8.59
H	Stores closed today.	
I	Lunch	$ 4.36
J	Digital Alarm Clock	$24.98
K	Earn $3.00. Add to your limit.	
L	Hair Dryer	$16.33
M	Electric Can Opener	$11.88
N	Lose $3.00. Subtract from your limit.	
O	Battery	35¢
P	Tape Cassettes, 1 or 2	$ 1.75 each
Q	Everything you buy from now on is one dollar cheaper than the marked price.	
R	Key Case	$ 1.49
S	Record Album	$ 5.69
T	Pocket Calculator	$23.10
U	Pocket Portable Radio	$ 7.49
V	Return one of your purchases to the store and get a refund.	
W	Sun Glasses	$ 3.64
X	Watch	$15.99
Y	Take a chance! Exchange your limit card for a new one.	
Z	Toboggan	$11.49

Napier's Bones (Activity for page 61)

John Napier, a Scottish nobleman, who lived from 1550 to 1617, invented a set of ten rectangular rods with multiples of one of the numbers from 0 to 9 on each of the four long faces. These became known as Napier's Rods, or Napier's Bones, and were used to perform multiplication.

To make a set of Napier's Bones, complete a multiplication table on a copy of page T 391, as shown below. Note that each multiple is written as a two-digit numeral with the digits separated by a diagonal line.

Cut the table into ten vertical strips so that each strip displays multiples of the number shown at the top of the strip. The strip from the left of the table can be used to identify the rows on the ten strips.

X	0	1	2	3	4	5	6	7	8	9
1	0/0	0/1	0/2	0/3	0/4	0/5	0/6	0/7	0/8	0/9
2	0/0	0/2	0/4	0/6	0/8	1/0	1/2	1/4	1/6	1/8
3	0/0	0/3	0/6	0/9	1/2	1/5	1/8	2/1	2/4	2/7
4	0/0	0/4	0/8	1/2	1/6	2/0	2/4	2/8	3/2	3/6
5	0/0	0/5	1/0	1/5	2/0	2/5	3/0	3/5	4/0	4/5
6	0/0	0/6	1/2	1/8	2/4	3/0	3/6	4/2	4/8	5/4
7	0/0	0/7	1/4	2/1	2/8	3/5	4/2	4/9	5/6	6/3
8	0/0	0/8	1/6	2/4	3/2	4/0	4/8	5/6	6/4	7/2
9	0/0	0/9	1/8	2/7	3/6	4/5	5/4	6/3	7/2	8/1

To multiply 638 by 92, place the strips for 6, 3, and 8 side by side in that order. The partial products of multiplying 638 by 92 are then read from rows 2 and 9 by adding the digits from right to left in each parallelogram. These results are recorded as shown and then the partial products are added to give the final product. Extra strips will be needed if digits in a factor are repeated, for example, for multiplying 633 by 92.

X	6	3	8
1	0/6	0/3	0/8
2	1/2	0/6	1/6
3	1/8	0/9	2/4
4	2/4	1/2	3/2
5	3/0	1/5	4/0
6	3/6	1/8	4/8
7	4/2	2/1	5/6
8	4/8	2/4	6/4
9	5/4	2/7	7/2

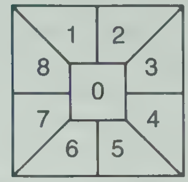
1 2 7 6

5 6 14 2
5 7 4 2

```
   638
 ×  92
 1 276
 57 42
 58 696
```

Lucky Zero (Game for page 99)

Materials: a game board as shown;
15 markers for each player;
a set of numeral cards for numbers to 99;
a number spinner for the numbers 2 to 9.



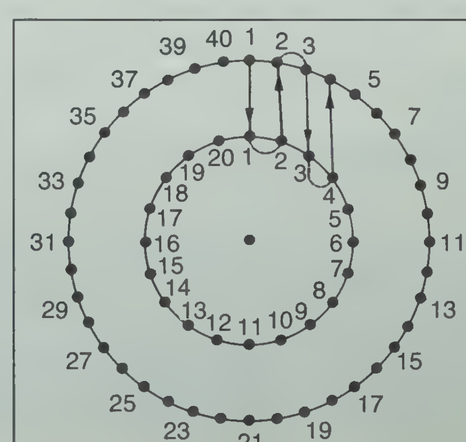
Players: two to four

Rules:

1. The numeral cards are shuffled and placed face down in a pile.
2. The first player turns up one card to obtain a number for a dividend and spins the spinner to obtain a number for a divisor. For example, if the card shows 46 and the spinner indicates 3, the division is $3\overline{)46}$. The player completes the division, states the remainder, 1 in this case, and places a marker on the space for 1 on the game board. If there is a marker on the space already, the player removes it and places it in her/his pile of markers, instead of placing a marker on the game board.
3. A player who obtains a division fact for which the remainder is 0 claims all the markers on the game board.
4. The game ends when one player has no more markers. The winner is the player with the most markers.

Curve Stitching (Activity for pages 174 and 175)

Prepare a large copy on Bristol board of the diagram shown below. The two concentric circles are marked with equally spaced dots. The outer circle requires twice as many dots as the inner circle. The dots are numbered in sequence, as shown. Stitch with colored thread, or yarn, to join the dots in the sequence 1⟶1⟶2⟶2⟶3⟶3⟶4 . . . . The dotted parts indicate stitches on the opposite side of the Bristol board. Half the dots on the outer circle will have been used when all the dots on the inner circle have been used. Continue stitching from 21⟶1⟶2⟶22⟶23⟶3⟶4 . . . until all the dots of the inner circle have been used again in sequence, to finish stitching the dots on the outer circle. The resulting pattern is called a *cardioid*.

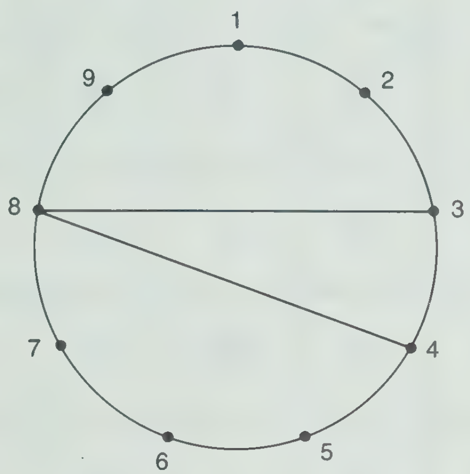


Multipatterns (Activity for pages 180 and 181)

Have students write the products for 4×1, 4×2, 4×3, …, 4×12 in the first row of a table similar to the following. In the second row, have them write the sum of the digits for each product. If a two-digit sum is obtained, the digits of the sum are added to obtain a one-digit number. For example, $4 \times 3 = 12$, $1 + 2 = 3$.

Multiple of 4	4	8	12	…	48
Sum of digits	4	8	3	…	3

Prepare large copies of the following diagram and give one to each student.



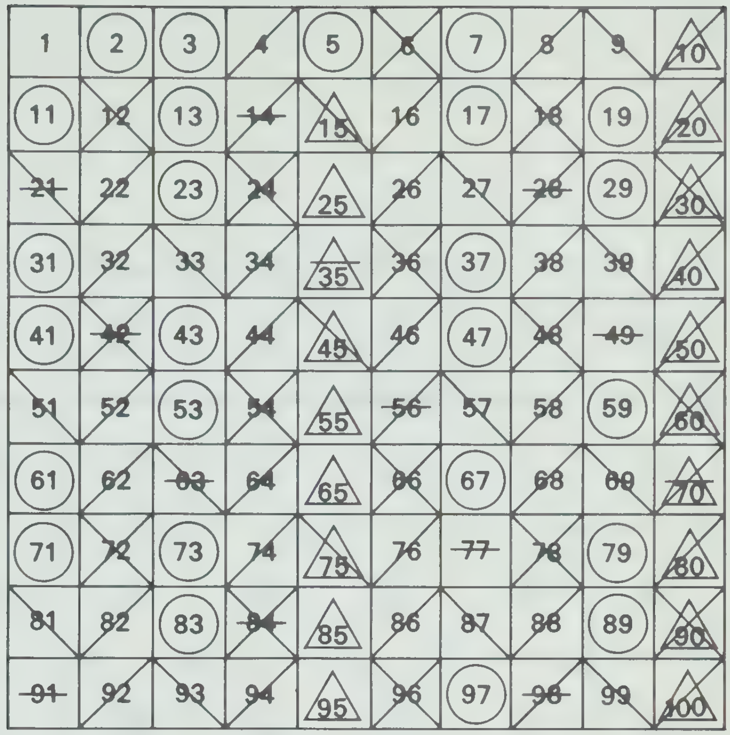
Have the students use a straight edge to join dots in sequence, following the sequence of numbers 4, 8, 3, …, 4 for the sums of the digits of the products in the table. Discuss the pattern obtained. Have students explore the symmetry in the pattern using a semitransparent plexiglass mirror. The design may be traced and tested for rotational symmetry. Students can color the completed patterns and use a protractor to measure some of the angles. This activity may be repeated for multiples of other numbers.

Investigating Prime Numbers (Activity for page 259)

Provide each student with a copy of page T 391 and have them outline the diagram to show a 10-by-10 grid. Have them write the numerals 1 to 100 in order in the appropriate squares. Then instruct them to mark squares as follows (using a different color for each symbol, if desired) and observe the patterns.

The steps describe a procedure known as the sieve of Eratosthenes.

1. Ring the numeral 2. Mark a diagonal (/) through the other multiples of 2 (4, 6, 8, …, 100).
2. Ring the numeral 3. Mark a diagonal (\) through the other multiples of 3 (6, 9, 12, …, 99).
3. Ring the numeral 5. Mark a triangle (△) around the other multiples of 5 (10, 15, 20, …, 100).
4. Ring the numeral 7. Mark a dash (—) through the other multiples of 7 (14, 21, 28, …, 98).
5. Ring all the remaining numerals with the exception of the numeral 1.

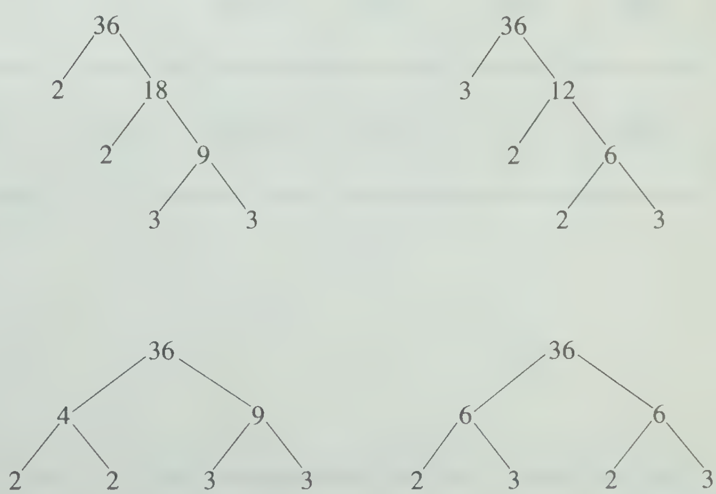


Recall that a prime number is one that has exactly two different factors. Thus, the number 1 is not a prime number. (You may wish to have the students color inside the square for the number 1.) All numerals marked with a ring represent prime numbers. The remaining numbers (with the exception of 1) are composite numbers. Have the students note the location of prime numbers. Have them discuss whether there is a pattern.

The students may write each prime number as a product.

$2 = 1 \times 2$
$3 = 1 \times 3$
$5 = 1 \times 5$
$7 = 1 \times 7$
$11 = 1 \times 11$
$13 = 1 \times 13$

You may wish to have the students select one or more composite numbers as shown on their 10-by-10 grid and show a factor tree to derive the prime factors. For some numbers, the factor tree may be drawn in more than one way.

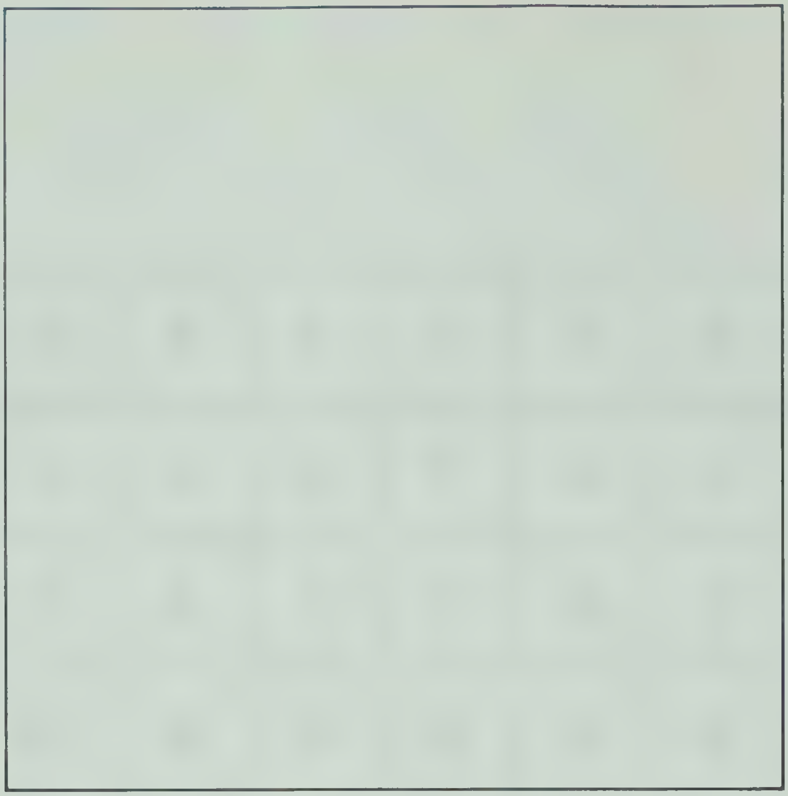


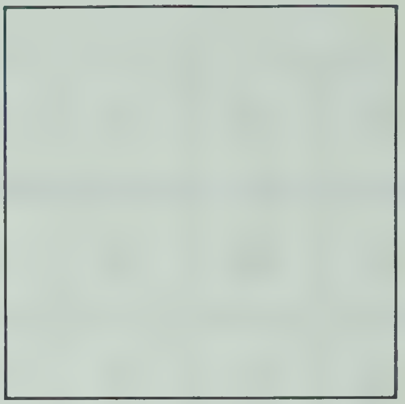
+	0	1	2	3	4	5	6	7	8	9
0	0	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9	10
2	2	3	4	5	6	7	8	9	10	11
3	3	4	5	6	7	8	9	10	11	12
4	4	5	6	7	8	9	10	11	12	13
5	5	6	7	8	9	10	11	12	13	14
6	6	7	8	9	10	11	12	13	14	15
7	7	8	9	10	11	12	13	14	15	16
8	8	9	10	11	12	13	14	15	16	17
9	9	10	11	12	13	14	15	16	17	18

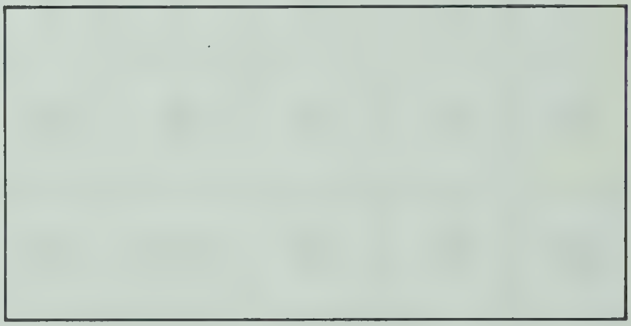
© Copyright, Canada, 1980, Ginn and Company. Permission to reproduce this page is granted to users of STARTING POINTS IN MATHEMATICS.

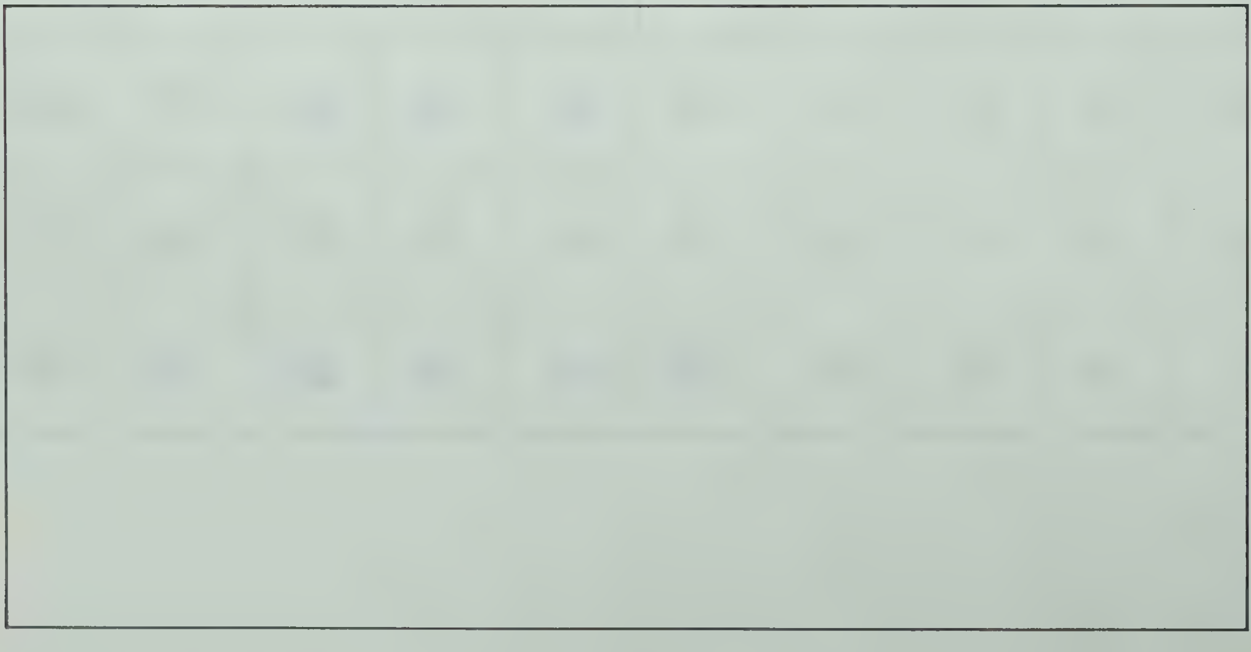
×	0	1	2	3	4	5	6	7	8	9
0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9
2	0	2	4	6	8	10	12	14	16	18
3	0	3	6	9	12	15	18	21	24	27
4	0	4	8	12	16	20	24	28	32	36
5	0	5	10	15	20	25	30	35	40	45
6	0	6	12	18	24	30	36	42	48	54
7	0	7	14	21	28	35	42	49	56	63
8	0	8	16	24	32	40	48	56	64	72
9	0	9	18	27	36	45	54	63	72	81

© Copyright, Canada, 1980, Ginn and Company. Permission to reproduce this page is granted to users of STARTING POINTS IN MATHEMATICS.

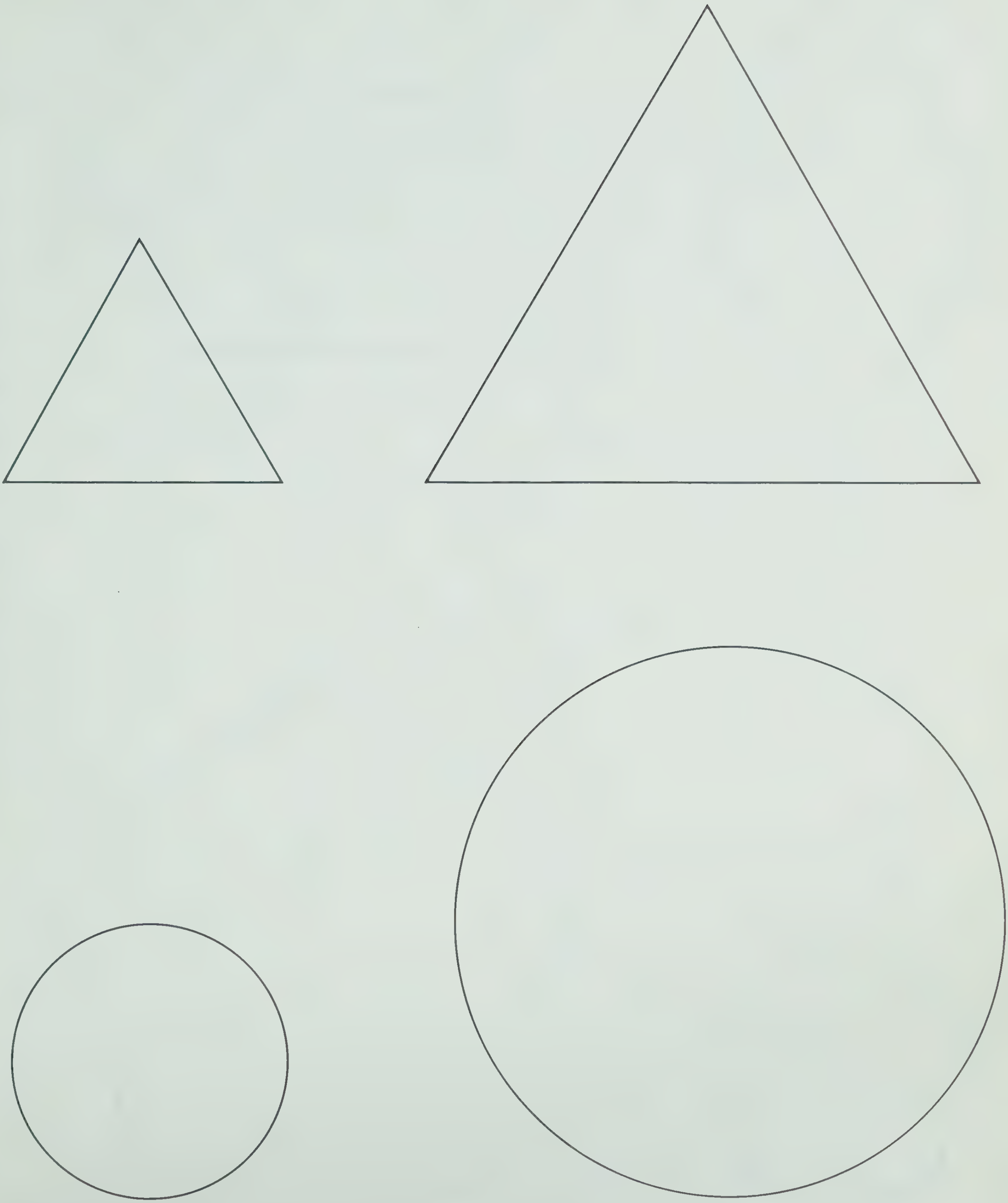






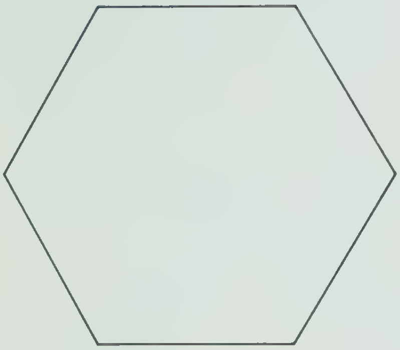


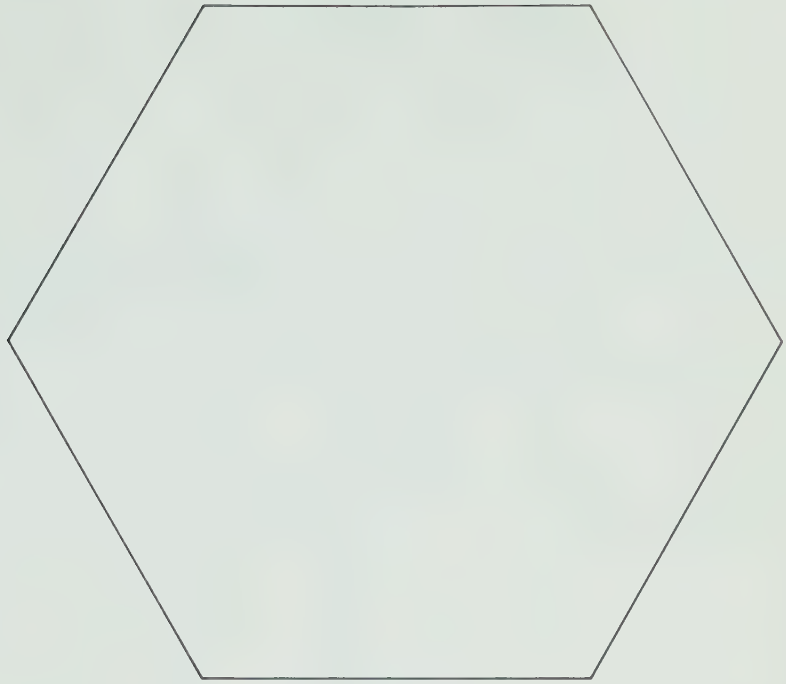
© Copyright, Canada, 1980, Ginn and Company. Permission to reproduce this page is granted to users of STARTING POINTS IN MATHEMATICS.

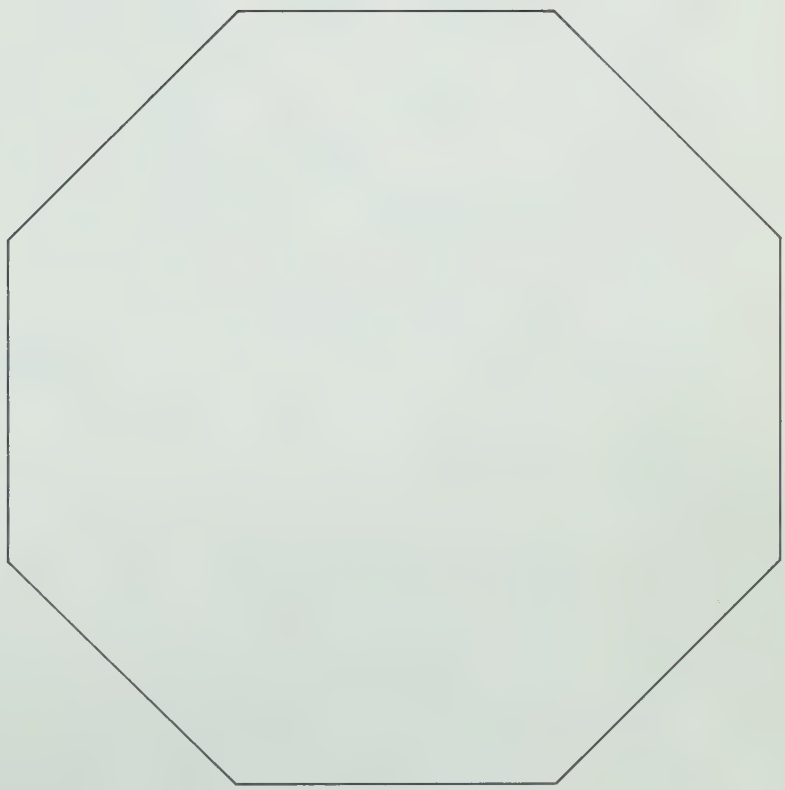


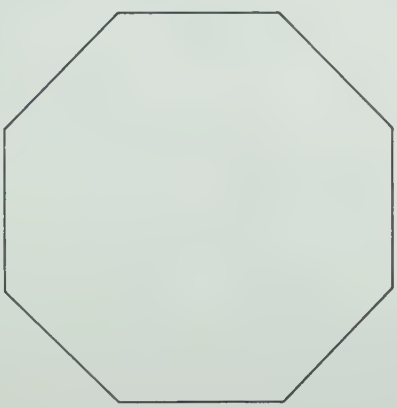
© Copyright, Canada, 1980, Ginn and Company. Permission to reproduce this page is granted to users of STARTING POINTS IN MATHEMATICS.

© Copyright, Canada, 1980, Ginn and Company. Permission to reproduce this page is granted to users of STARTING POINTS IN MATHEMATICS.



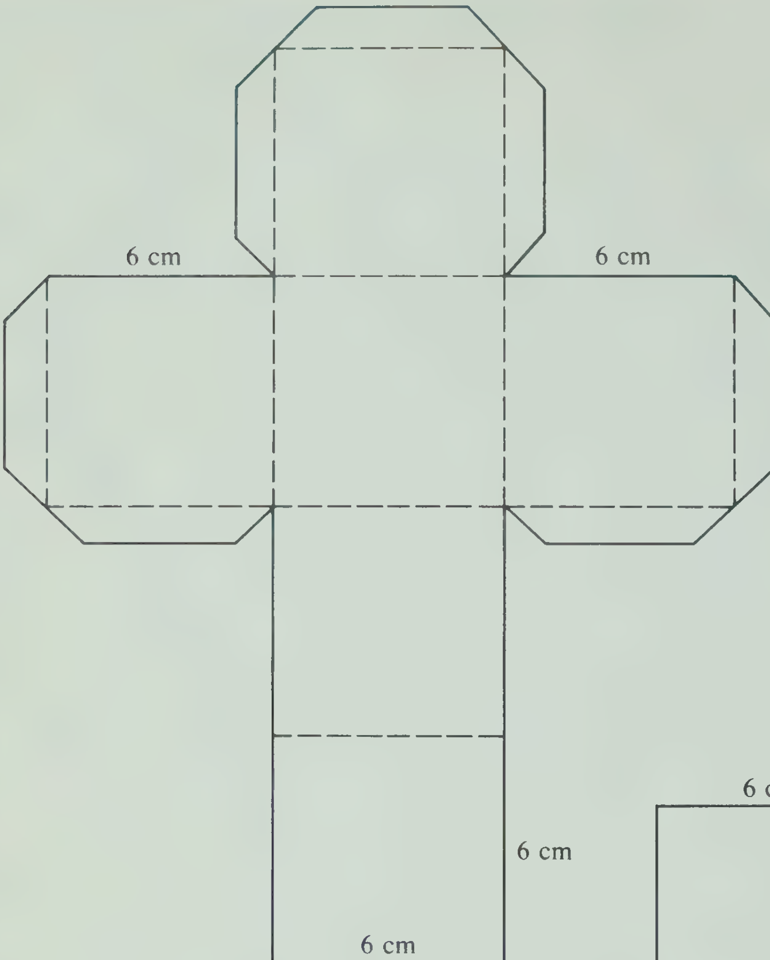




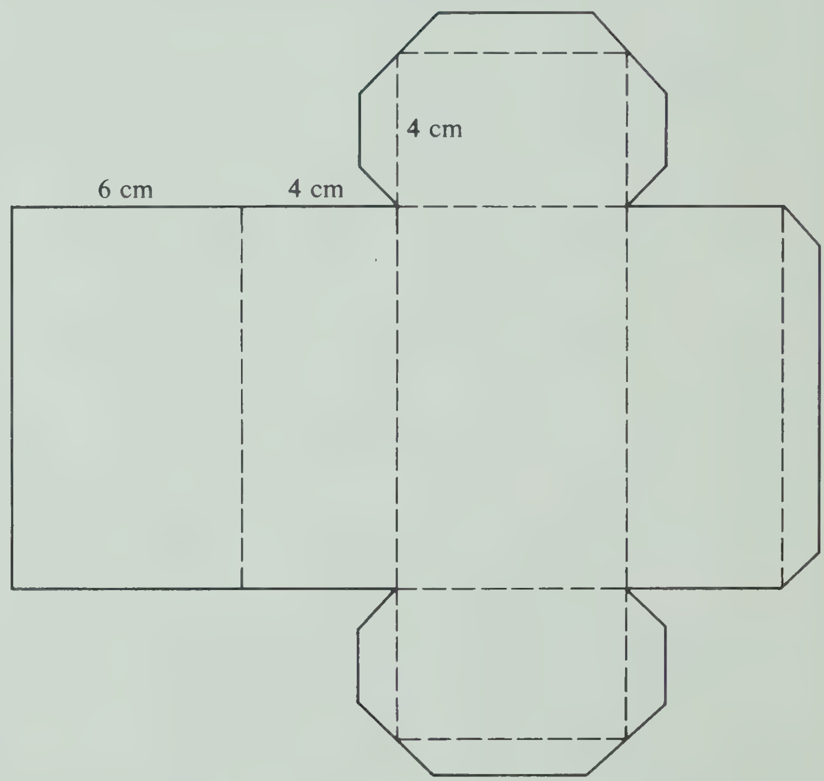


© Copyright, Canada, 1980, Ginn and Company. Permission to reproduce this page is granted to users of STARTING POINTS IN MATHEMATICS.

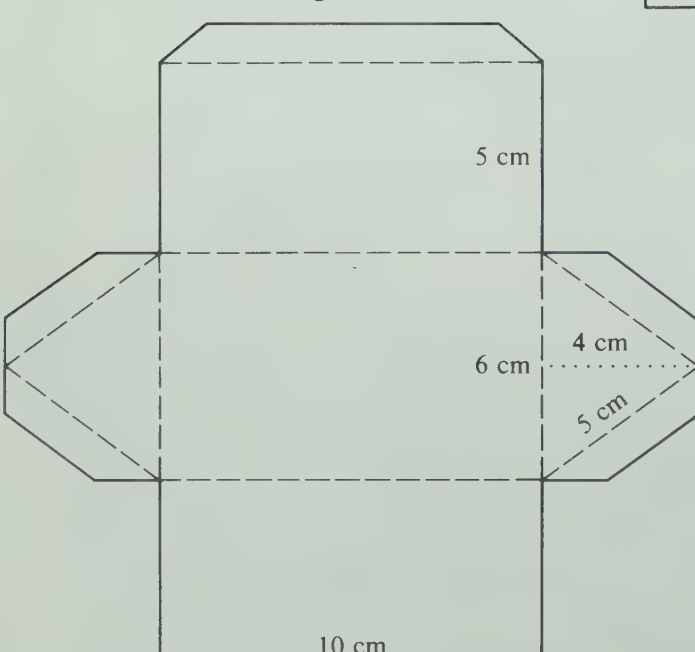
Cube



Rectangular Prism

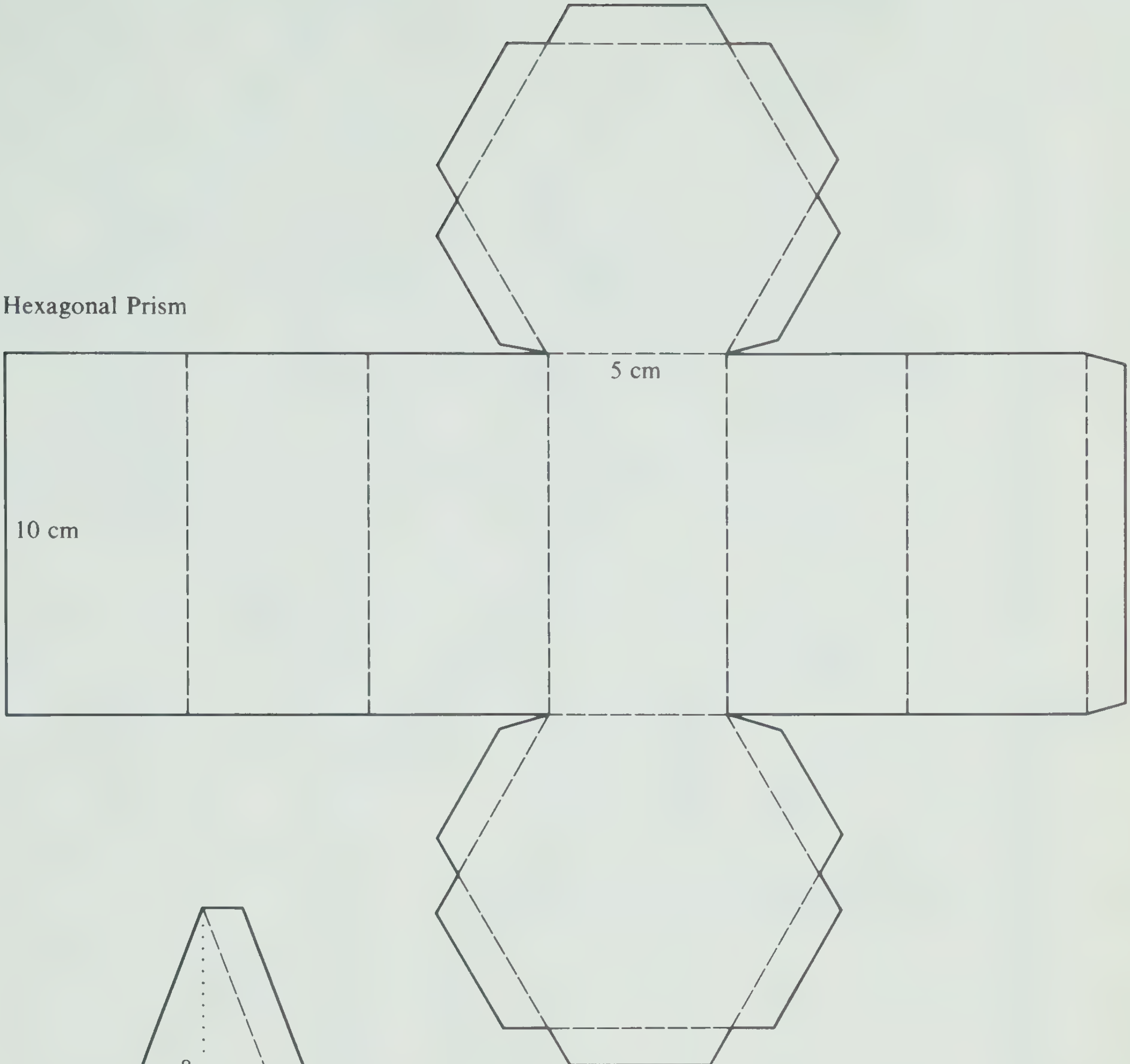


Triangular Prism

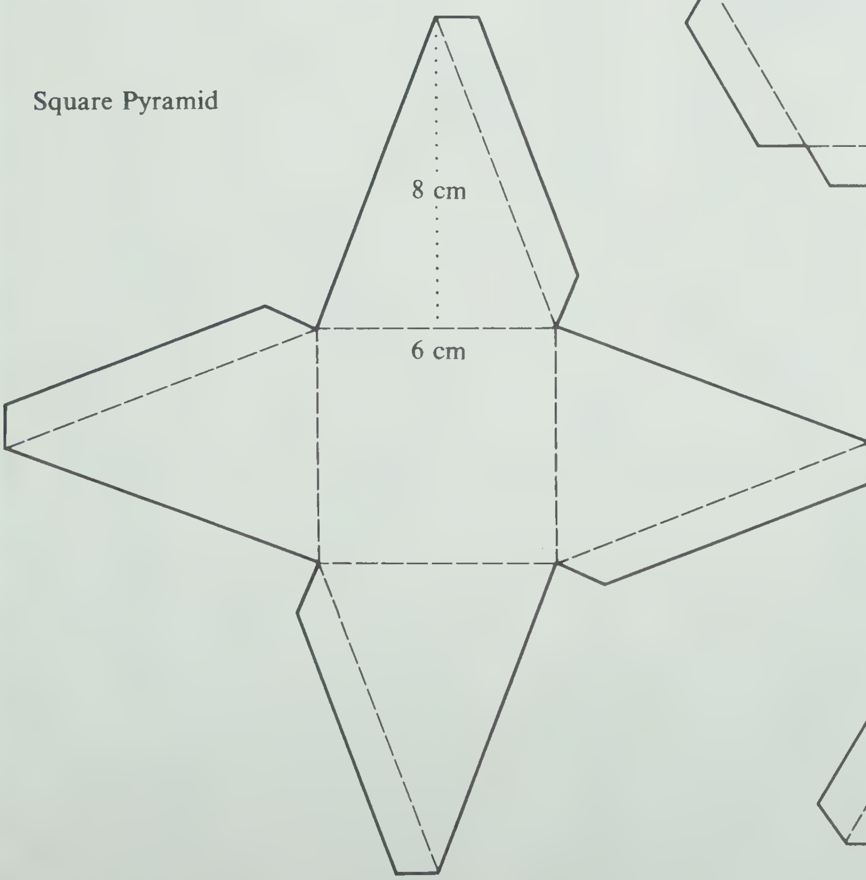


© Copyright, Canada, 1980, Ginn and Company. Permission to reproduce this page is granted to users of STARTING POINTS IN MATHEMATICS.

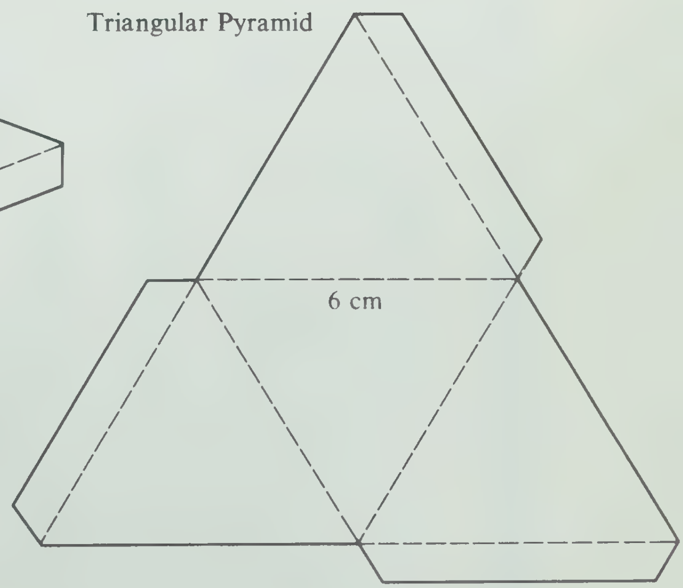
Hexagonal Prism



Square Pyramid

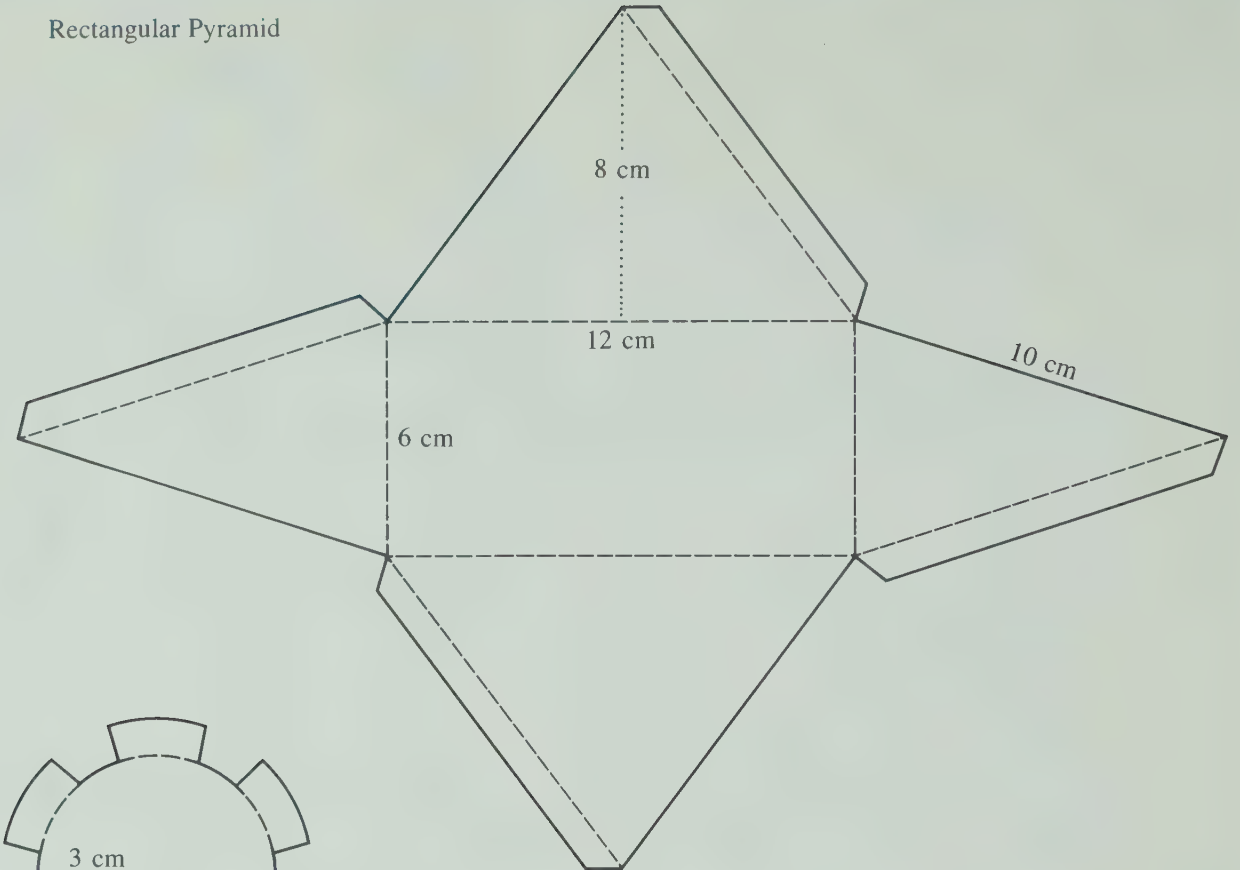


Triangular Pyramid

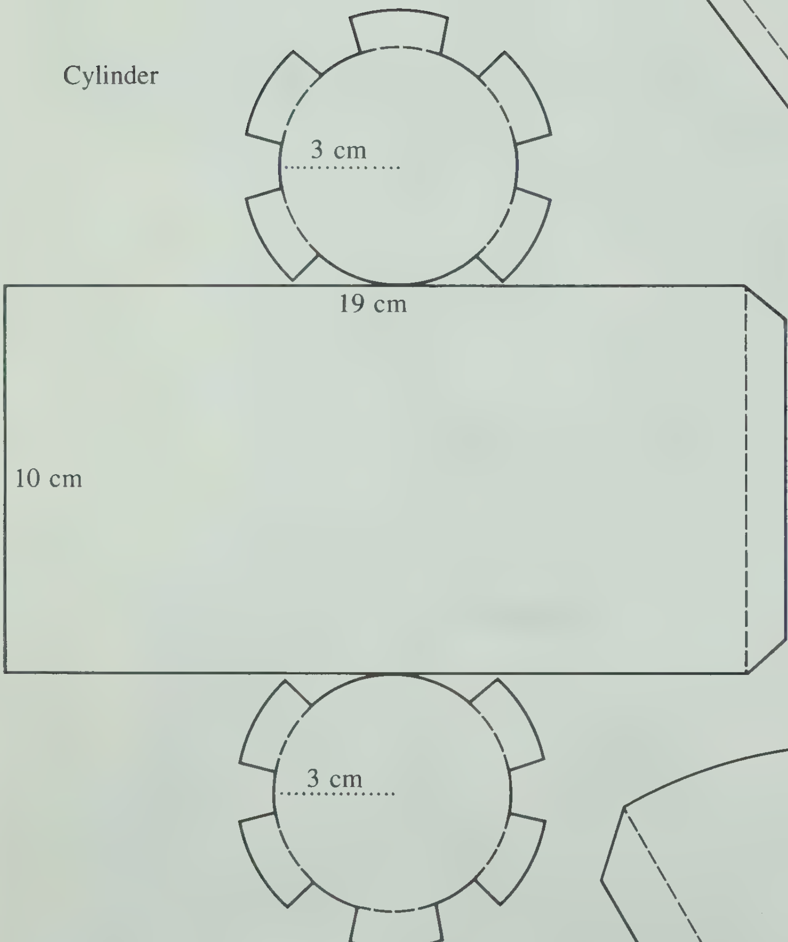


© Copyright, Canada, 1980, Ginn and Company. Permission to reproduce this page is granted to users of STARTING POINTS IN MATHEMATICS.

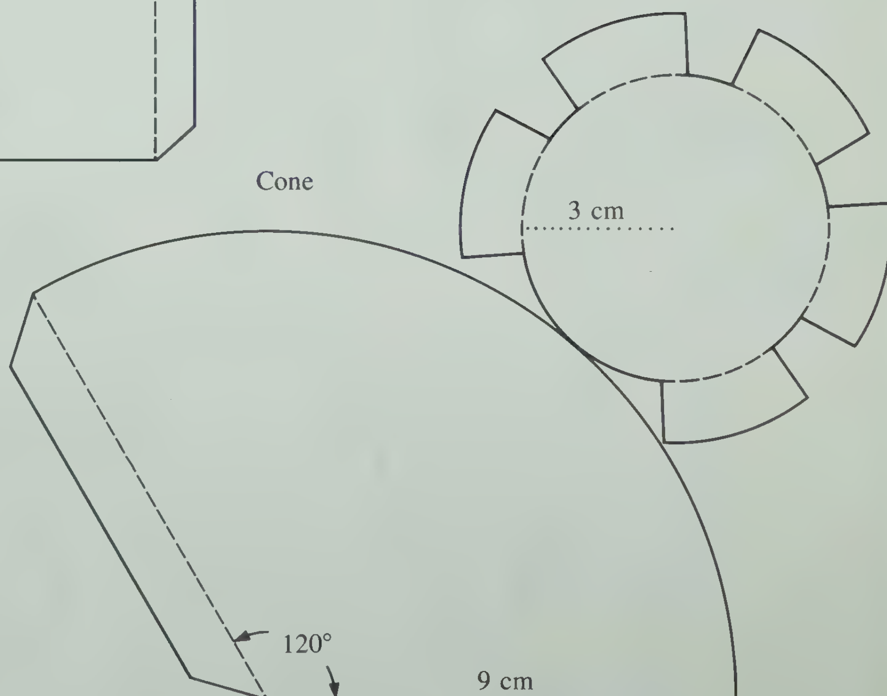
Rectangular Pyramid



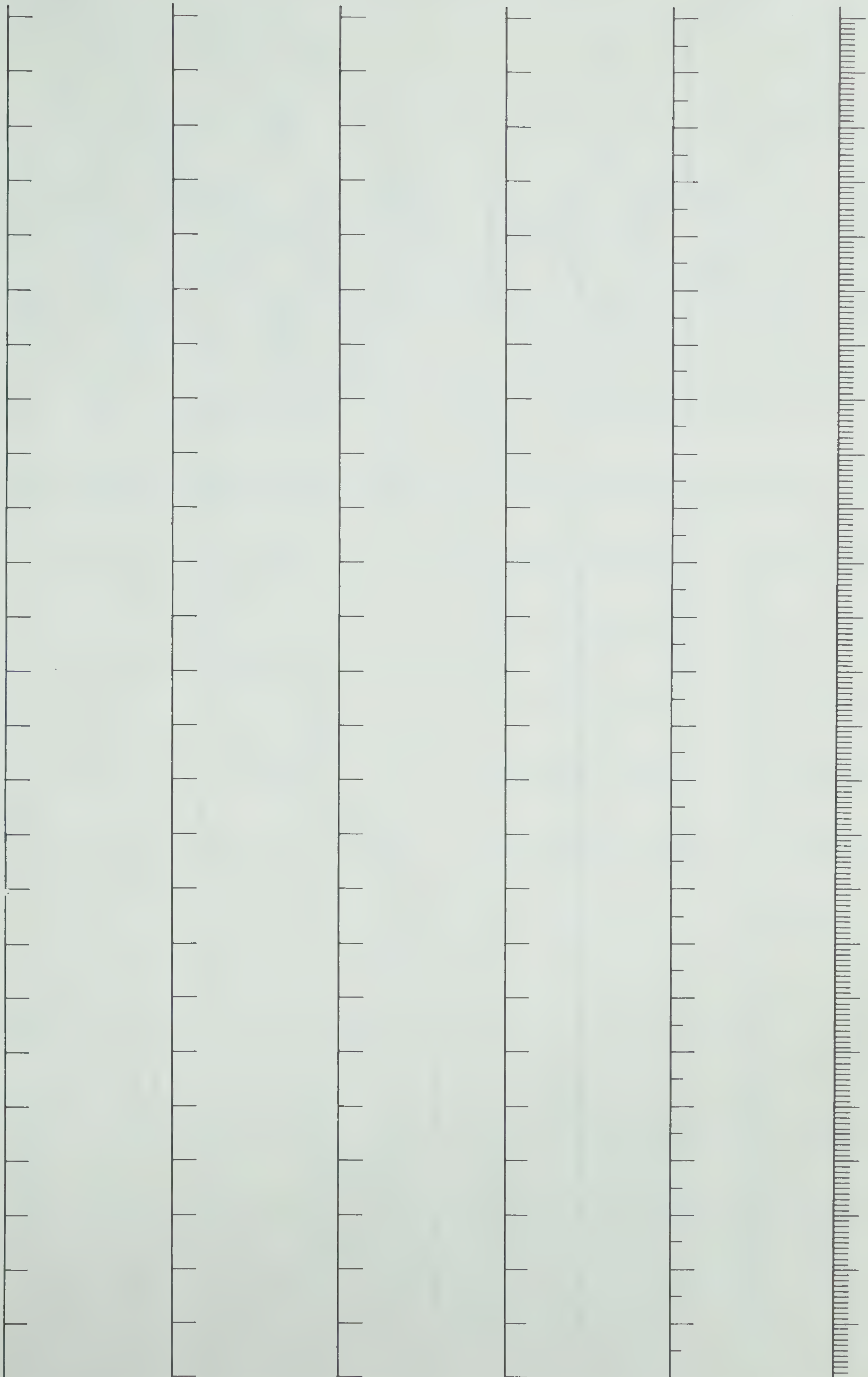
Cylinder



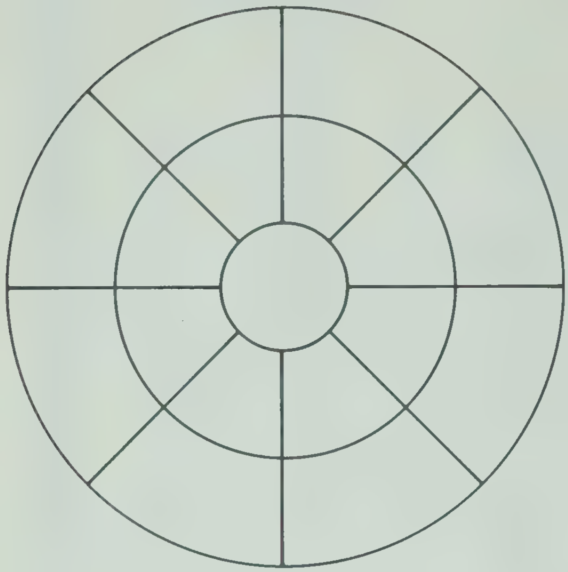
Cone

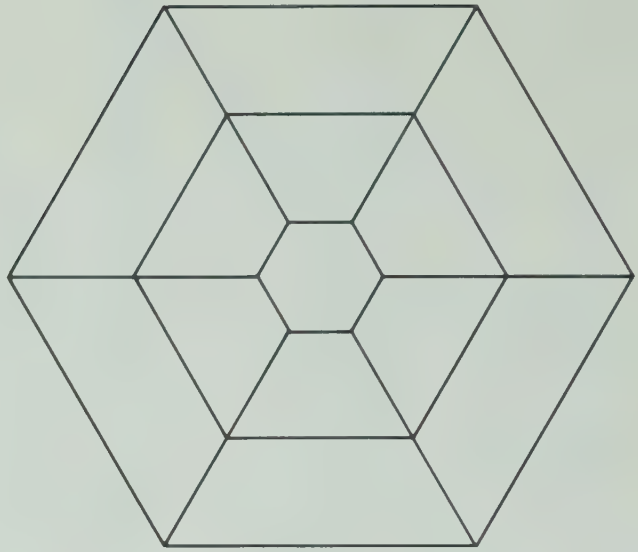


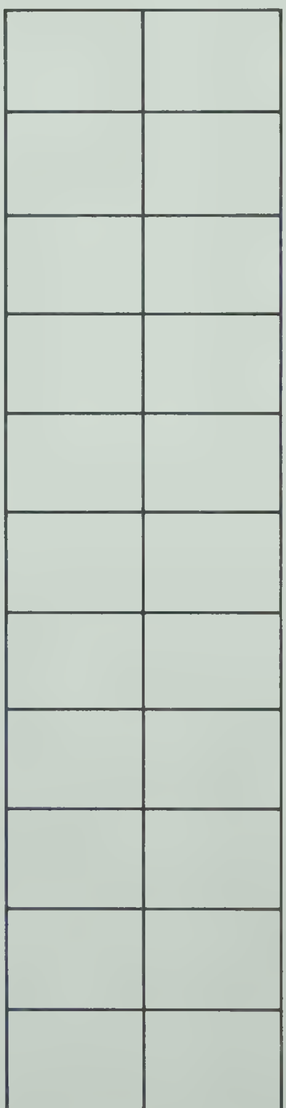
© Copyright, Canada, 1980, Ginn and Company. Permission to reproduce this page is granted to users of STARTING POINTS IN MATHEMATICS.

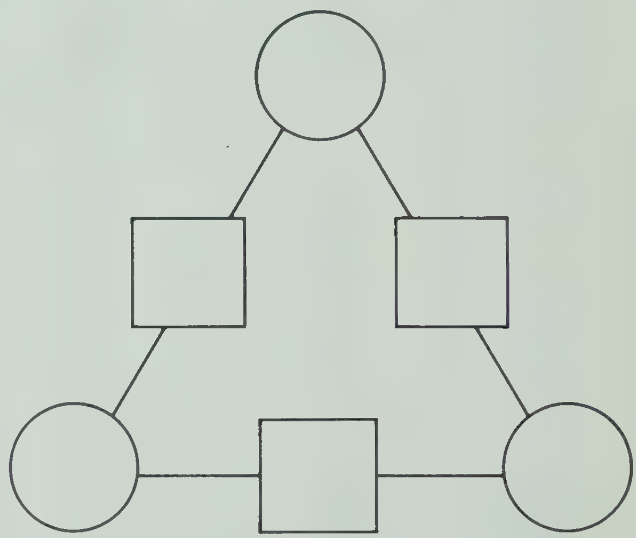


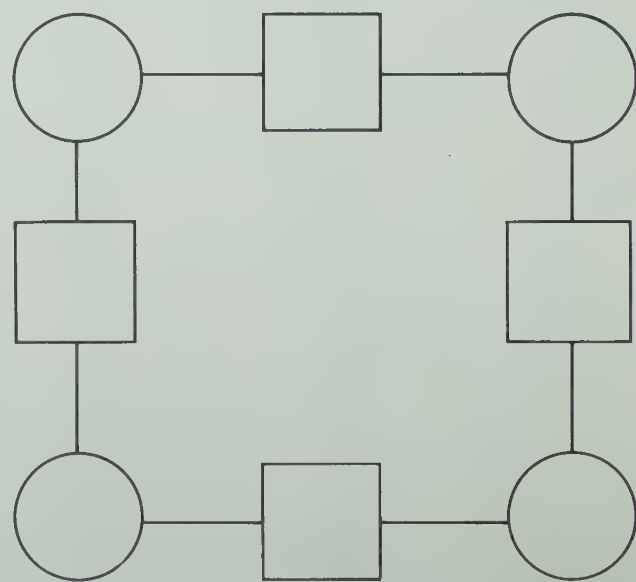
© Copyright, Canada, 1980, Ginn and Company. Permission to reproduce this page is granted to users of STARTING POINTS IN MATHEMATICS.



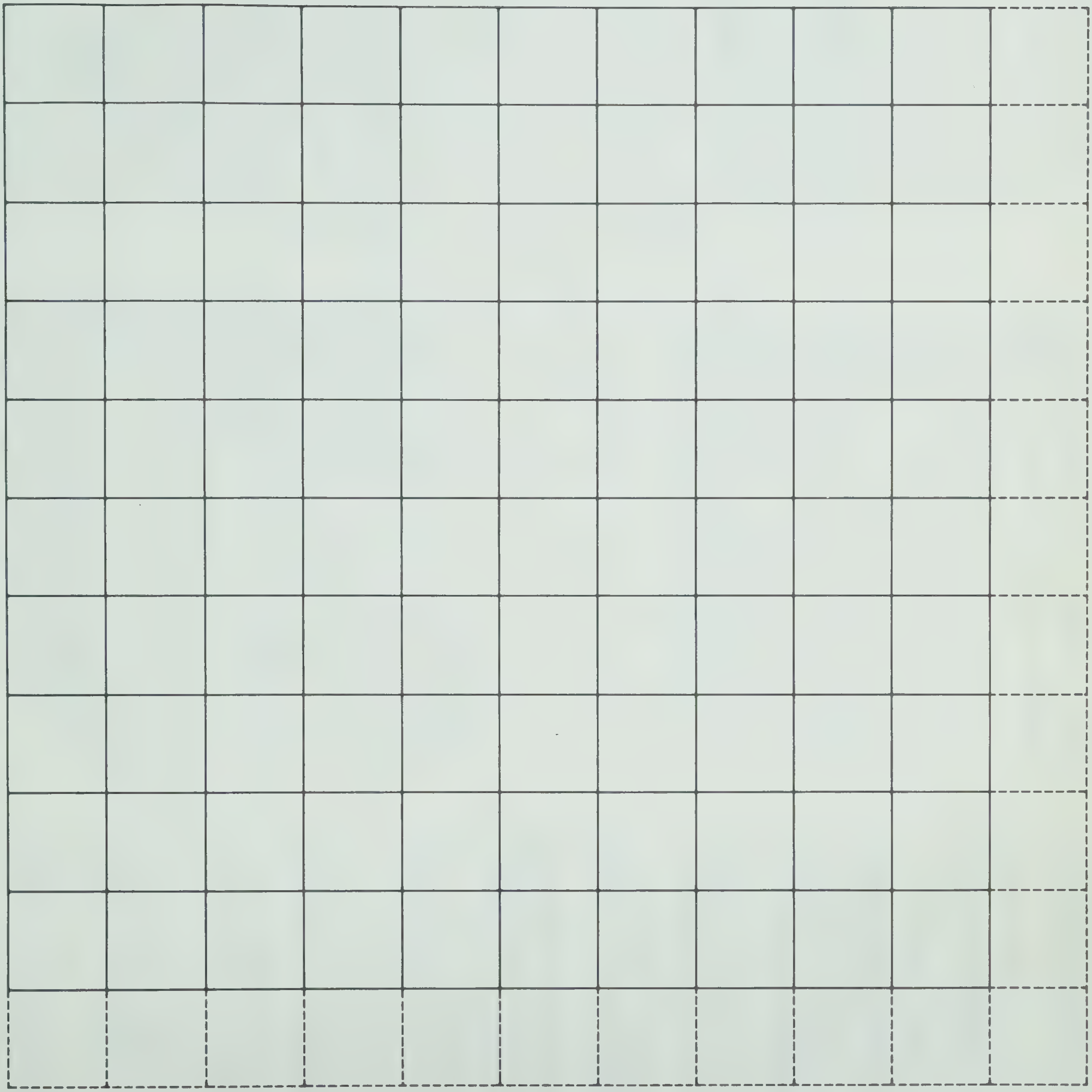




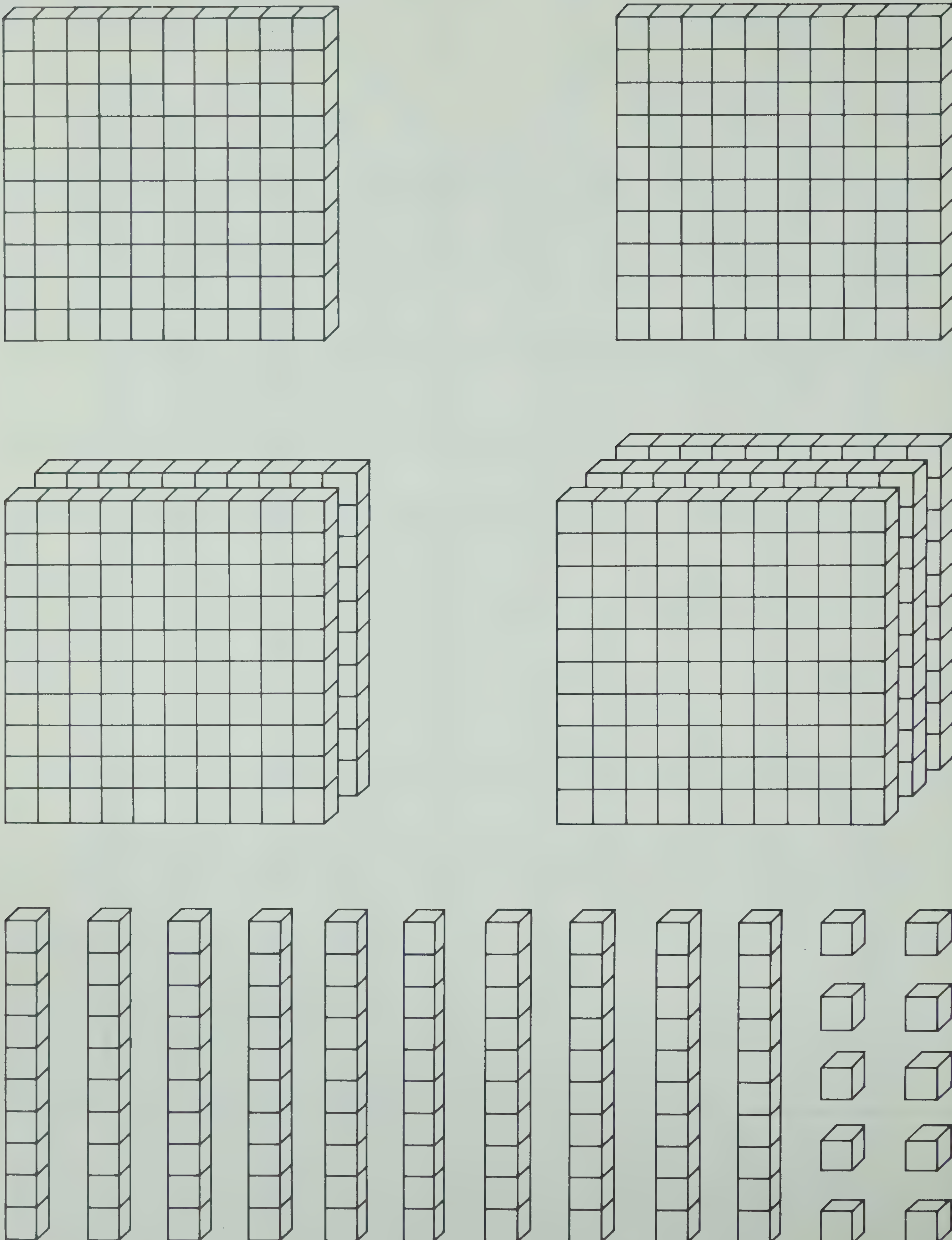




© Copyright, Canada, 1980, Ginn and Company. Permission to reproduce this page is granted to users of STARTING POINTS IN MATHEMATICS.

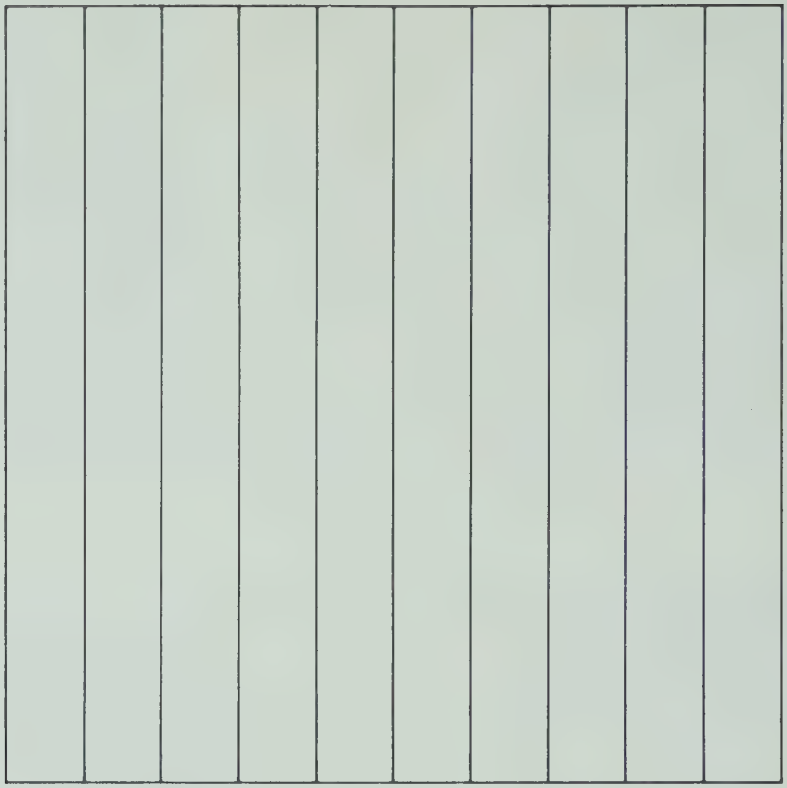


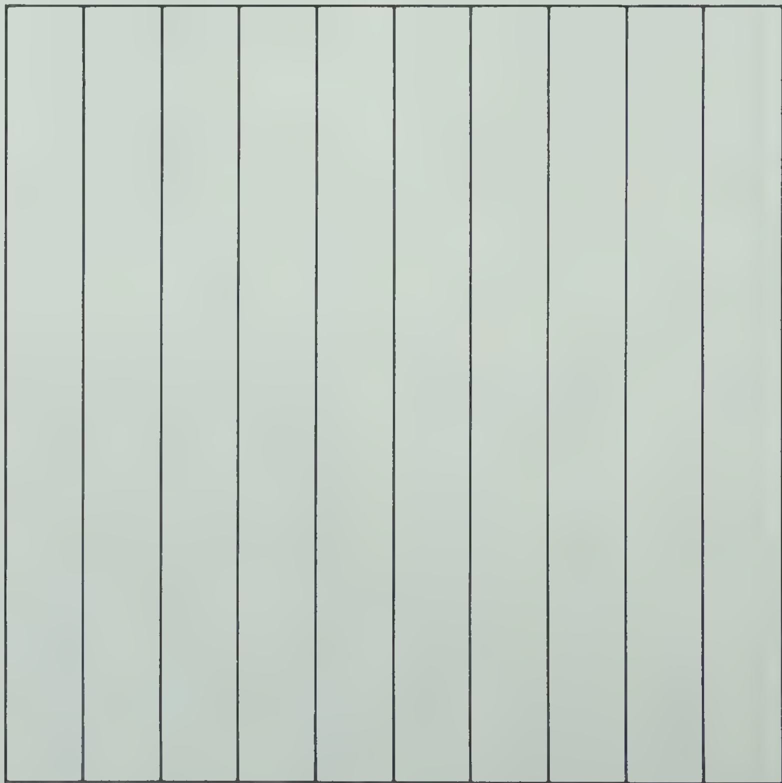
© Copyright, Canada, 1980, Ginn and Company. Permission to reproduce this page is granted to users of STARTING POINTS IN MATHEMATICS.



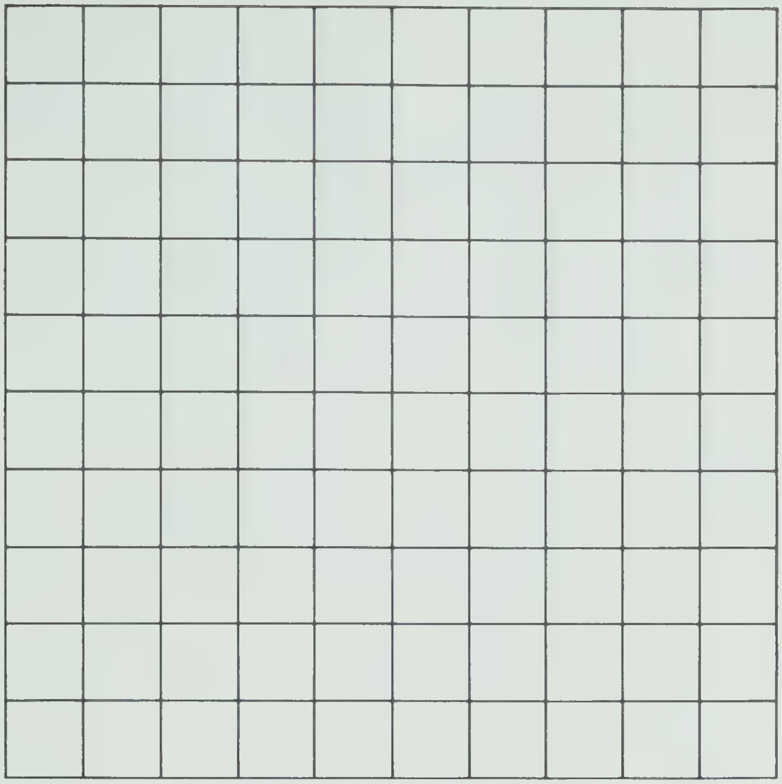
© Copyright, Canada, 1980, Ginn and Company. Permission to reproduce this page is granted to users of STARTING POINTS IN MATHEMATICS.

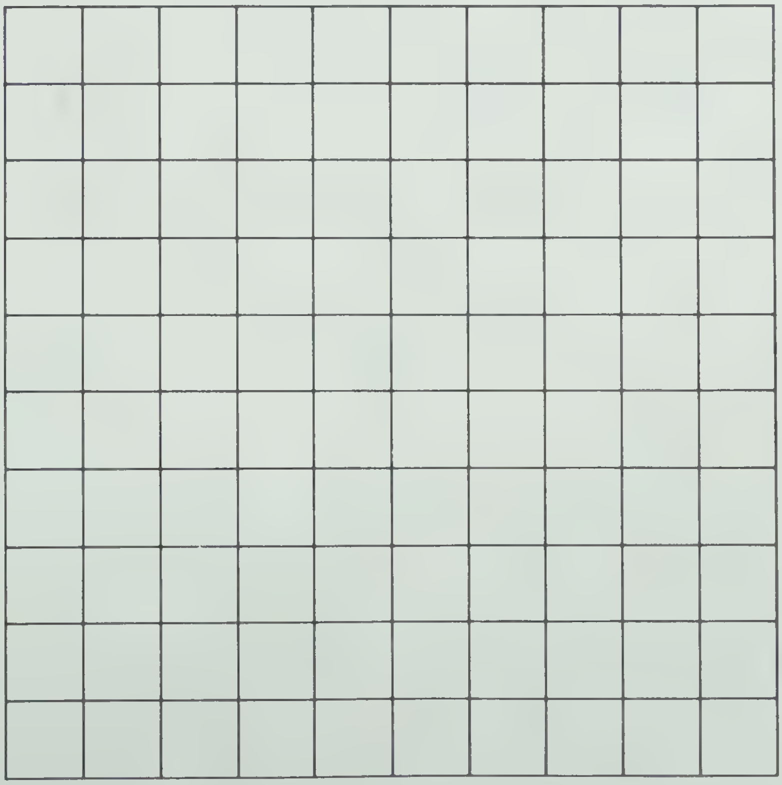
© Copyright, Canada, 1980, Ginn and Company. Permission to reproduce this page is granted to users of STARTING POINTS IN MATHEMATICS.





© Copyright, Canada, 1980, Ginn and Company. Permission to reproduce this page is granted to users of STARTING POINTS IN MATHEMATICS.





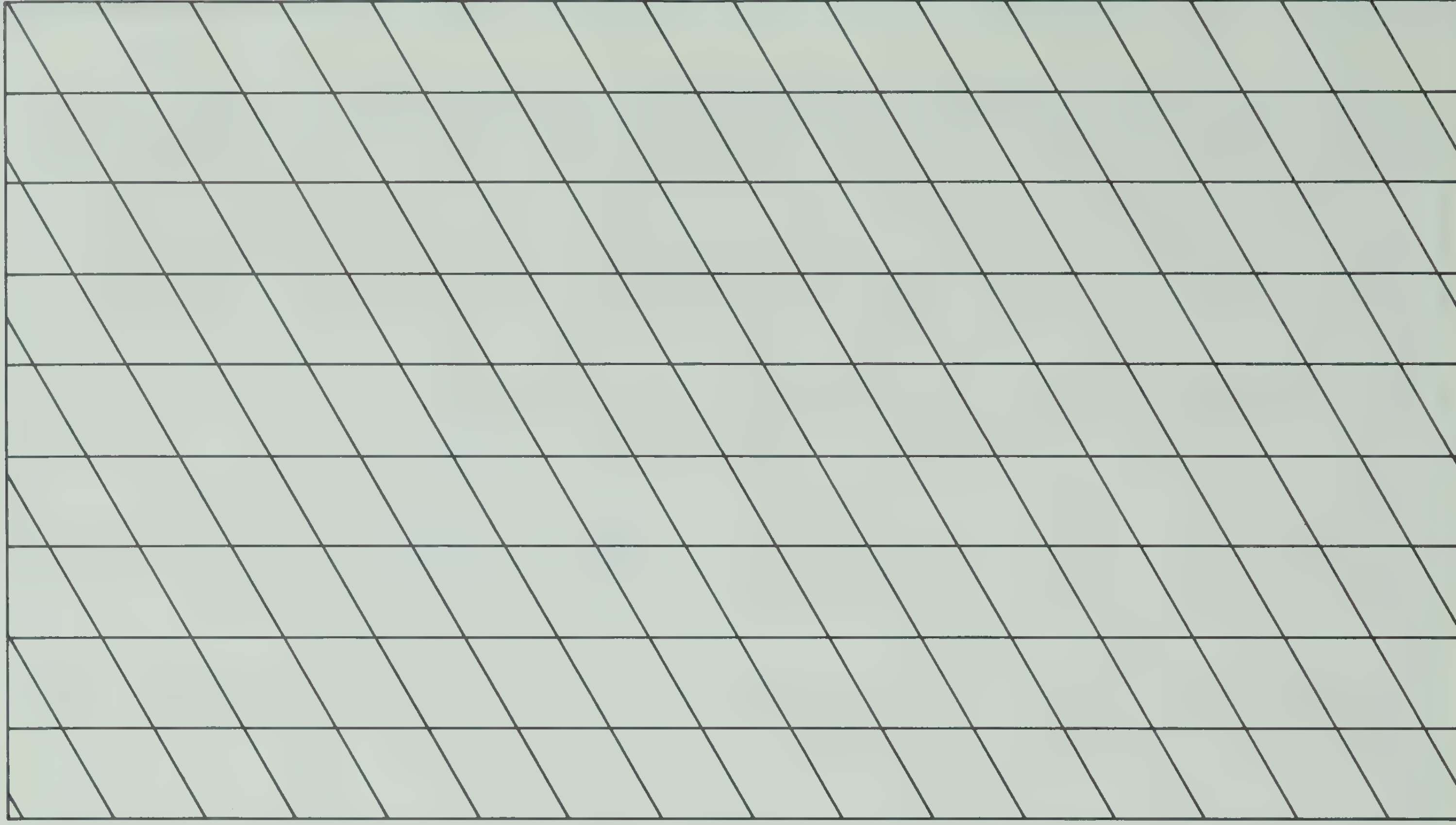
© Copyright, Canada, 1980, Ginn and Company. Permission to reproduce this page is granted to users of STARTING POINTS IN MATHEMATICS.

© Copyright, Canada, 1980, Ginn and Company. Permission to reproduce this page is granted to users of STARTING POINTS IN MATHEMATICS.

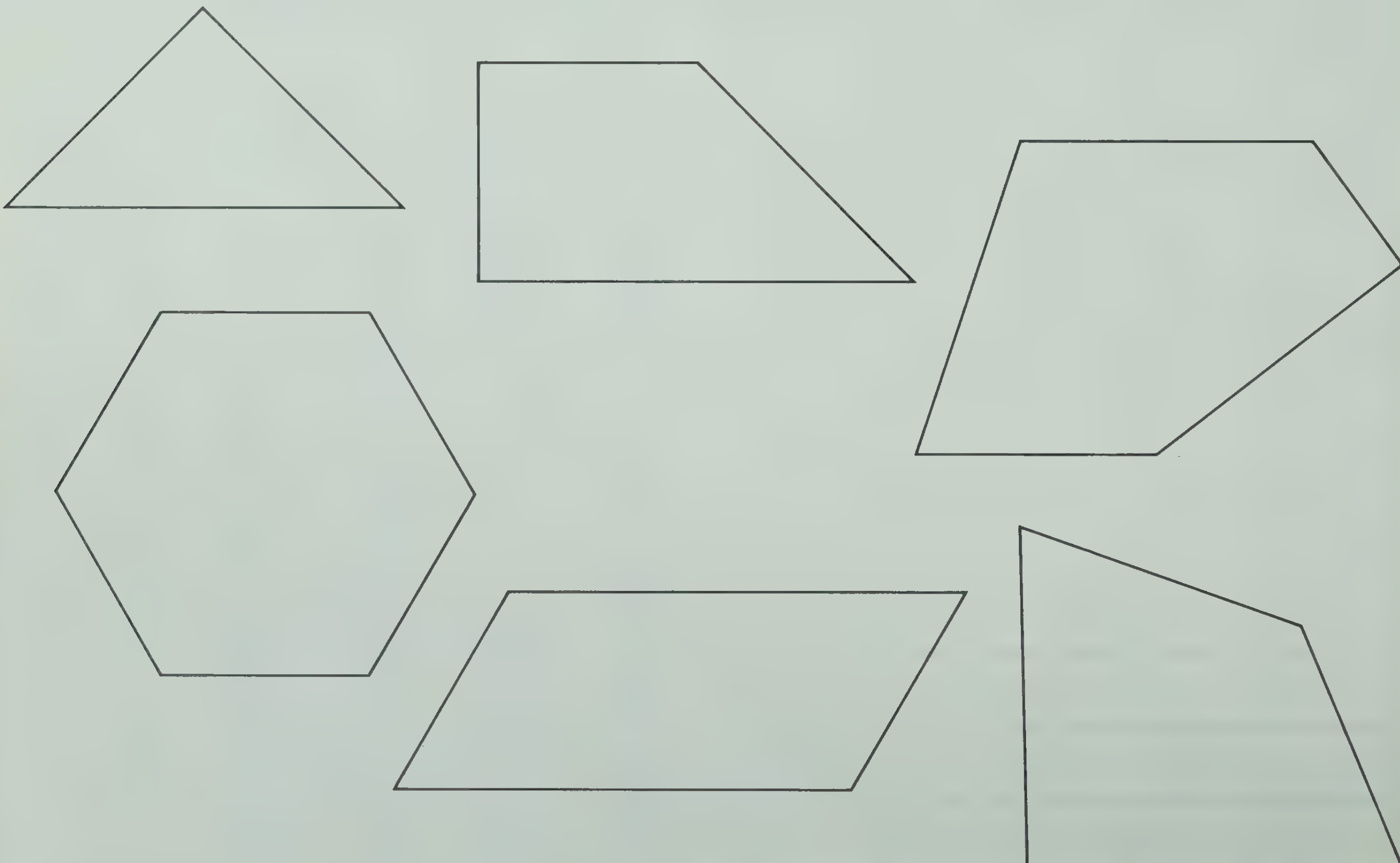
© Copyright, Canada, 1980, Ginn and Company. Permission to reproduce this page is granted to users of STARTING POINTS IN MATHEMATICS.

© Copyright, Canada, 1980, Ginn and Company. Permission to reproduce this page is granted to users of STARTING POINTS IN MATHEMATICS.

© Copyright, Canada, 1980, Ginn and Company. Permission to reproduce this page is granted to users of STARTING POINTS IN MATHEMATICS.



Shapes for page 307



© Copyright, Canada, 1980, Ginn and Company. Permission to reproduce this page is granted to users of STARTING POINTS IN MATHEMATICS.

Year-End Evaluation (See page xv.)

Name ______________________

Numeration

1. Reads/Writes standard numerals to 999 999 999 ☐
2. Interprets place value in numerals to 999 999 999 ☐
3. Understands/Writes expanded form for numbers to 999 999 999 ☐
4. Compares/Orders numbers to 999 999 ☐
5. Rounds numbers to the nearest thousand, ten thousand, or hundred thousand ☐
6. Knows meaning and use of the symbol:
   a. ¢ ☐ b. $ ☐ c. = ☐ d. = ☐
   e. > ☐ f. < ☐ g. + ☐ h. − ☐
   i. × ☐ j. ÷, $\overline{)\ }$ ☐ k. : ☐ l. % ☐
7. Understands/Uses the term:
   a. greater than ☐ b. less than ☐
   c. equals ☐ d. not equal to ☐
8. Reads/Writes Roman numerals for numbers to 2000 ☐
9. Expresses amounts of money using decimals ☐
10. Reads/Writes decimals for
    a. tenths ☐ b. hundredths ☐ c. thousandths ☐
11. Interprets place value for
    a. tenths ☐ b. hundredths ☐ c. thousandths ☐
12. Finds equivalent decimals ☐
13. Compares/Orders decimals to thousandths ☐
14. Rounds decimals, to thousandths ☐
15. Reads/Writes fractions for numbers less than one ☐
16. Identifies the numerator and denominator of a fraction ☐
17. Reads/Writes numerals for numbers in mixed form ☐
18. Identifies/Finds equivalent fractions ☐
19. Finds the missing term for equivalent fractions ☐
20. Finds like denominators for fractions ☐
21. Compares fractions using common denominators ☐
22. Expresses a whole number or a number in mixed form as an improper fraction ☐
23. Expresses an improper fraction as a whole number or as a number in mixed form ☐
24. Identifies/Finds
    a. prime numbers ☐ b. common factors ☐
25. Expresses fractions as decimals ☐
26. Reads/Writes ratios using colon and fraction notation ☐
27. Finds equivalent ratios, rates ☐
28. Finds the missing term in equivalent ratios, rates ☐
29. Reads/Writes percents ☐
30. Converts among decimals, fractions, ratios, percents ☐

Operations

Addition

1. Understands/Uses the term:
   a. add ☐ b. plus ☐ c. addend ☐ d. sum ☐
2. Knows the basic addition facts ☐
3. Understands the steps in the algorithm ☐
4. Understands addition properties:
   a. commutative (order) ☐ b. associative (grouping) ☐
5. Adds two numbers, up to six digits, no regrouping ☐
6. Adds two numbers, up to six digits, regrouping ☐
7. Adds more than two numbers, up to five digits each, regrouping ☐
8. Estimates sums by rounding ☐
9. Adds amounts of money ☐
10. Adds decimals, sums to thousandths ☐
11. Adds fractions with like denominators ☐
12. Solves problems using addition ☐

Subtraction

1. Understands/Uses the term:
   a. subtract ☐ b. minus ☐ c. difference ☐
2. Knows the basic subtraction facts ☐
3. Writes families of related facts ☐
4. Understands the steps in the algorithm ☐
5. Subtracts, up to six-digit numbers, no regrouping ☐
6. Subtracts, up to six-digit numbers, regrouping
   a. without zeros in the minuend ☐
   b. with zeros in the minuend ☐
7. Subtracts, uses addition to check ☐
8. Estimates differences by rounding ☐
9. Simplifies number expressions with parentheses ☐
10. Subtracts amounts of money ☐
11. Subtracts decimals, minuends to thousandths ☐
12. Subtracts fractions with like denominators ☐
13. Solves problems using subtraction ☐

Multiplication

1. Understands/Uses the term:
   a. multiply ☐ b. factor ☐ c. product ☐
2. Knows the basic multiplication facts ☐
3. Understands the steps in the algorithm ☐
4. Understands multiplication properties:
   a. commutative (order) ☐ b. associative (grouping) ☐
5. Multiplies by a one-digit number, multiplicands up to five digits ☐
6. Multiplies by multiples of 10, 100, and 1000 ☐
7. Multiplies by a two-digit number, multiplicands up to four digits ☐
8. Multiplies by a three-digit number, multiplicands up to four digits ☐
9. Multiplies amounts of money ☐
10. Simplifies number expressions with parentheses ☐
11. Multiplies a decimal to 0.9, 0.09, and 0.009 by a one-digit whole number ☐
12. Multiplies a decimal and a whole number ☐
13. Multiplies a whole number or a decimal
    a. by 10, 100, or 1000 ☐ b. by 0.1, 0.01, or 0.001 ☐
14. Multiplies a decimal by a decimal, products to thousandths when
    a. one factor is less than one ☐
    b. the factors are less than one ☐
    c. the factors are greater than one ☐
15. Estimates products by rounding when
    a. the factors are whole numbers ☐
    b. one factor is a whole number, the other, a decimal ☐
    c. the factors are decimals ☐
16. Uses multiplication and division to find a fractional part of a number ☐
17. Solve problems using multiplication ☐

Division

1. Understands/Uses the term:
   a. divide ☐ b. quotient ☐
   c. divisor ☐ d. remainder ☐
2. Knows the basic division facts ☐
3. Relates multiplication and division through families of related facts ☐
4. Understands the steps in the algorithm ☐
5. Divides a three-digit number by a one-digit number, no regrouping, remainder zero ☐

6. Divides a two-digit number by a one-digit number, regrouping, remainder greater than zero ☐
7. Divides a three-digit number by a one-digit number, regrouping, remainder greater than zero ☐
8. Divides by a one-digit number, dividends to five digits
   a. no zeros in the quotient ☐
   b. zeros in the quotient ☐
9. Divides by a multiple of 10, up to six-digit dividends
   a. remainder zero ☐ b. remainder greater than zero ☐
10. Divides by a two-digit number when
   a. the trial estimate for the quotient is correct ☐
   b. the trial estimate for the quotient is not correct ☐
11. Uses multiplication and addition to check results ☐
12. Estimates quotients by rounding ☐
13. Divides to find an average ☐
14. Divides amounts of money ☐
15. Divides a one-place decimal by a one-digit number
   a. no regrouping ☐ b. regrouping ☐
16. Divides a decimal to thousandths or a whole number
   a. by a one-digit number, quotients greater than one ☐
   b. by a one-digit number, quotients less than one ☐
   c. by a one-digit number, using more decimal places ☐
   d. by a two-digit number, using more decimal places ☐
17. Solves problems using division ☐

Measurement

Area

1. Recognizes/Uses the term and its symbol:
   a. square centimetre (cm^2) ☐ b. square metre (m^2) ☐
2. Finds area in square centimetres by counting ☐
3. Finds the area of a rectangle by multiplying
   a. the number of rows of centimetre squares and the number of centimetre squares in one row ☐
   b. the length and the width ☐
4. Draws rectangles having a given area ☐

Capacity

1. Recognizes/Uses the term and its symbol:
   a. millilitre (mL) ☐ b. litre (L) ☐
2. Chooses the best estimate for a capacity ☐
3. Relates millilitres and litres ☐
4. Relates cubic centimetres
   a. and millilitres ☐ b. and litres ☐

Length

1. Recognizes/Uses the term and its symbol:
   a. millimetre (mm) ☐ b. centimetre (cm) ☐
   c. decimetre (dm) ☐ d. metre (m) ☐
   e. hectometre (hm) ☐ f. kilometre (km) ☐
2. Estimates/Measures in
   a. millimetres ☐ b. centimetres ☐ c. metres ☐
3. Chooses the preferred unit for measuring a given length ☐
4. Relates millimetres, centimetres, metres, using decimals ☐
5. Uses addition to find the perimeter of a shape ☐
6. Uses multiplication and addition to find the perimeter of a square or a rectangle ☐
7. Draws rectangles having a given perimeter ☐

Mass

1. Recognizes/Uses the term and its symbol:
   a. gram (g) ☐ b. kilogram (kg) ☐ c. tonne (t) ☐
2. Chooses the best estimate for a mass ☐
3. Relates grams and kilograms ☐
4. Relates capacity, mass, and volume units for water ☐

Time

1. Reads/Records time to the nearest second ☐
2. Adds/Subtracts time, no regrouping ☐
3. Recognizes/Uses numeric dating (year, month, day) ☐

Volume

1. Recognizes/Uses the term and its symbol: cubic centimetre (cm^3) ☐
2. Finds volume by counting centimetre cubes ☐
3. Finds the volume of a rectangular prism by multiplying the number of centimetre cubes in one layer by the number of layers ☐

Graphing

1. Collects/Organizes information ☐
2. Draws and interprets
   a. pictographs ☐ b. bar graphs ☐
   c. line graphs ☐
3. Matches points on a grid with ordered pairs of numbers ☐

Problem Solving

1. Identifies missing information ☐
2. Recognizes that different situations affect answers ☐
3. Finds the number of possibilities of an event ☐
4. Collects, organizes, and displays information ☐
5. Gives the most reasonable answer ☐
6. Uses models to obtain solutions ☐
7. Writes/Solves an equation for a word problem ☐
8. Solves problems involving two or more steps ☐
9. Finds/Continues patterns ☐
10. Thinks logically ☐
11. Identifies relevant and irrelevant information ☐
12. Estimates with ratios ☐

Geometry

1. Identifies/Names/Draws lines, segments, rays ☐
2. Identifies/Draws intersecting, parallel, or perpendicular lines, line segments, rays ☐
3. Identifies/Names angles ☐
4. Identifies congruent angles, shapes, sides ☐
5. Measures/Draws angles ☐
6. Classifies angles as:
   a. acute ☐ b. obtuse ☐
   c. right ☐ d. straight ☐
7. Identifies/Names each of these polygons:
   a. hexagon ☐ b. octagon ☐ c. pentagon ☐
   d. quadrilateral ☐ e. rectangle ☐ f. square ☐
8. Identifies/Counts vertices, sides, angles of polygons ☐
9. Identifies/Counts edges, faces, vertices of solids ☐
10. Identifies/Draws lines of symmetry ☐
11. Classifies triangles as:
   a. equilateral ☐ b. isosceles ☐ c. scalene ☐
12. Classifies quadrilaterals as:
   a. parallelogram ☐ b. rectangle ☐ c. rhombus ☐
   d. square ☐ e. trapezoid ☐
13. Identifies the term as it relates to a circle:
   a. center ☐ b. radius ☐
   c. diameter ☐ d. circumference ☐
14. Classifies prisms and pyramids ☐
15. Identifies cones, cylinders, and spheres ☐
16. Identifies/Draws images of shapes for
   a. flips ☐ b. slides ☐ c. turns ☐
17. Fits polygons together to make a tiling pattern ☐
18. Copies pictures using grids, including distortions ☐

Index for Student's Book

QA 135-5 S79 1982 GR-5 TCH-ED-
C-2
STARTING POINTS IN MATHEMATICS/
/REV --
M2 39584808 CURR

000011546462

DATE DUE SLIP

F255

B40010